*INT, Seattle 23.05.2022*

# Thinking outside the box:

# Neutron star mergers with the *Lagrangian* Numerical Relativity code `SPHINCS_BSSN`

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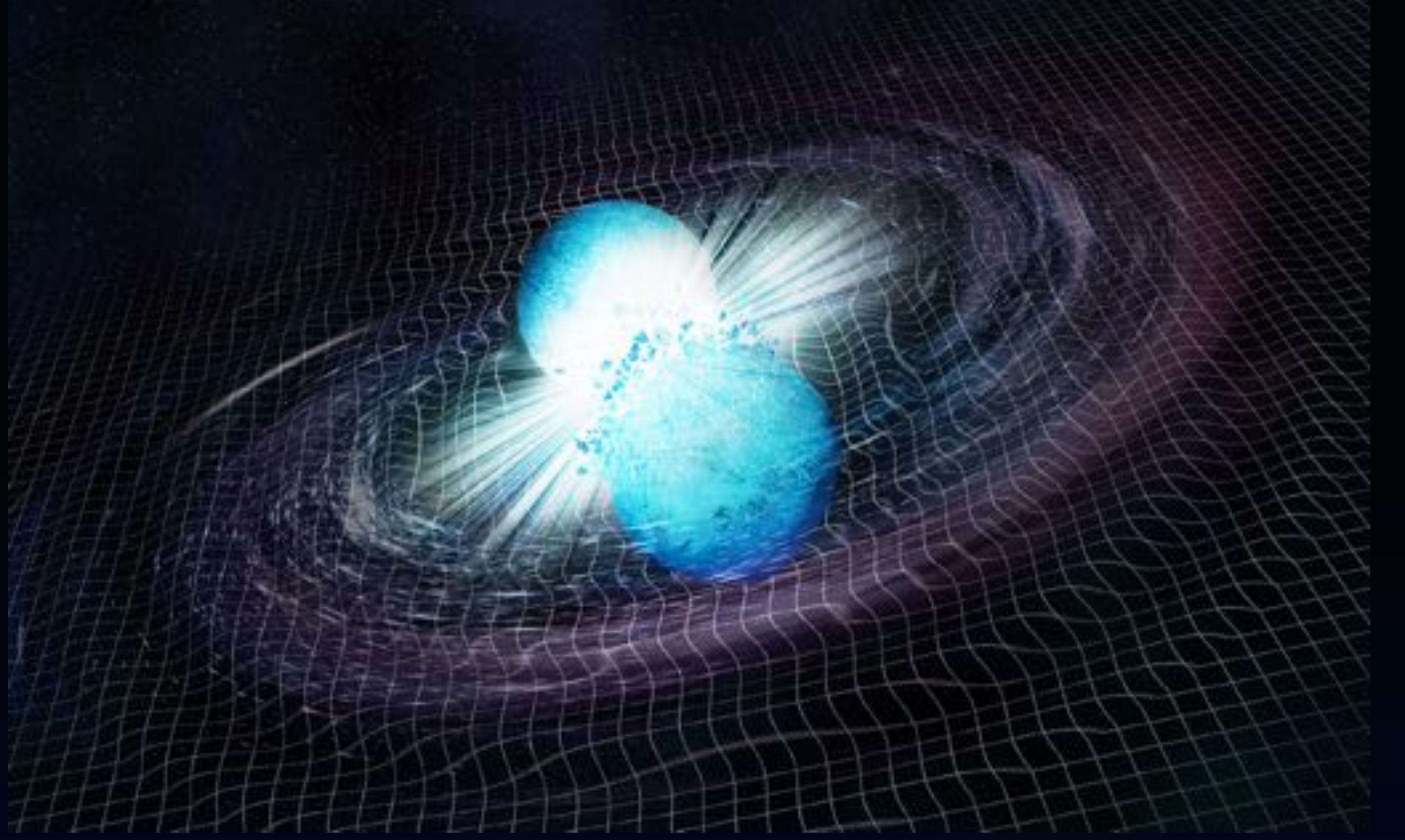
# First neutron star merger detection GW170817

- Gravitational Waves
- Electromagnetic emission all across the spectrum
- Hubble parameter
- Nuclear matter
- Propagation speed of gravity (= $c_{light}$ to within $1:10^{15}$ !)
- Heavy elements!

- combined gravitational & electromagnetic information crucial

- electromagnetic waves ⟹ from only 1% of binary mass!

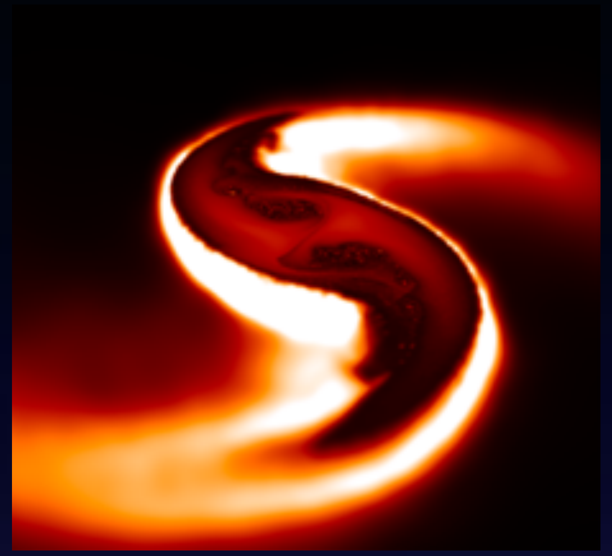
**⟹ understanding ejecta is key to multi-messenger astrophysics!**

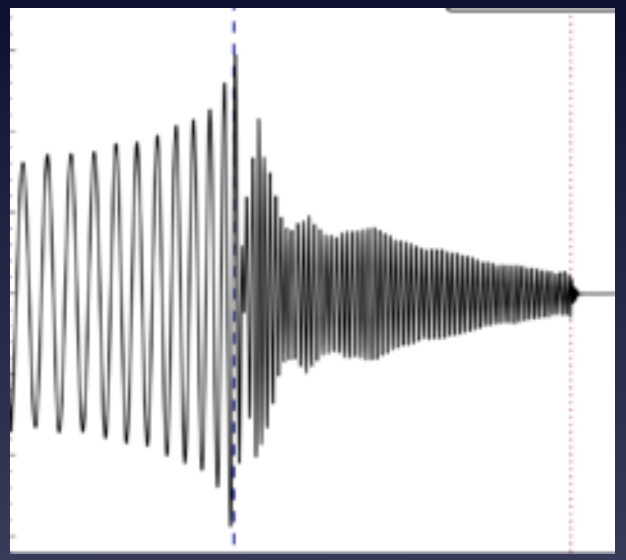


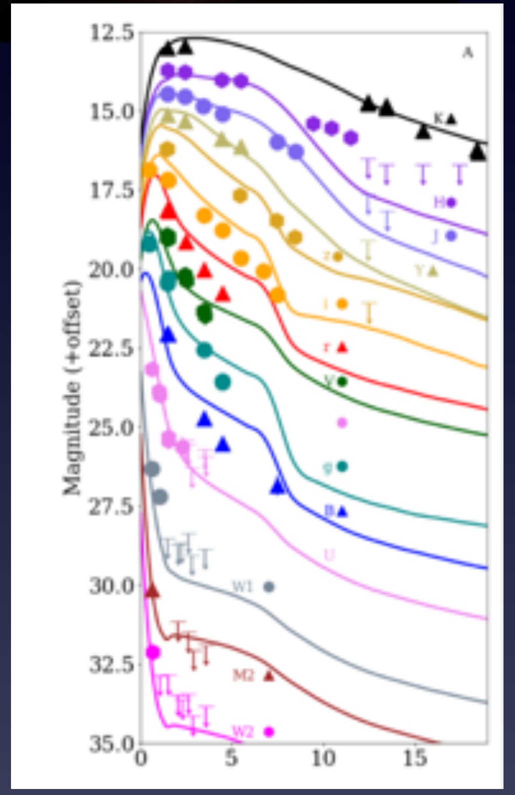
# The challenge

gravitational waves (GWs)

electromagnetic waves (EM)







both signals crucial for
multi-messenger astrophysics !

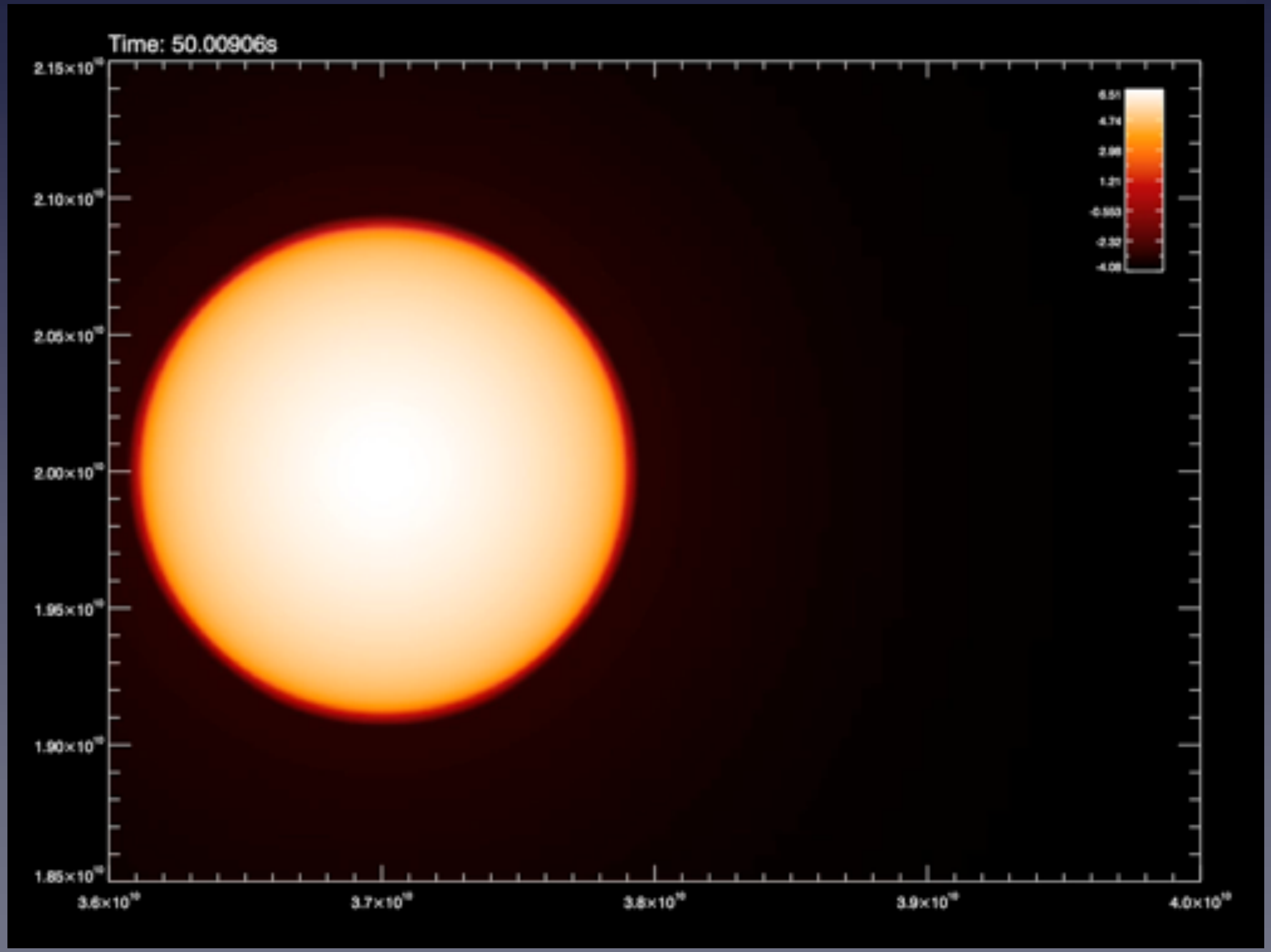
| | | |
|---|---|---|
| mass: | produced by bulk motion $\sim 3M_\odot$ | $\sim 0.01\ M_\odot$ |
| lengths: | $\sim 10$ km | $\sim 10^{10}$ km |
| times: | $\sim 10$ ms | ~ days to weeks |
| composition: | “irrelevant” | crucial |

# Why *Lagrangian* hydrodynamics?

- Advection is exact

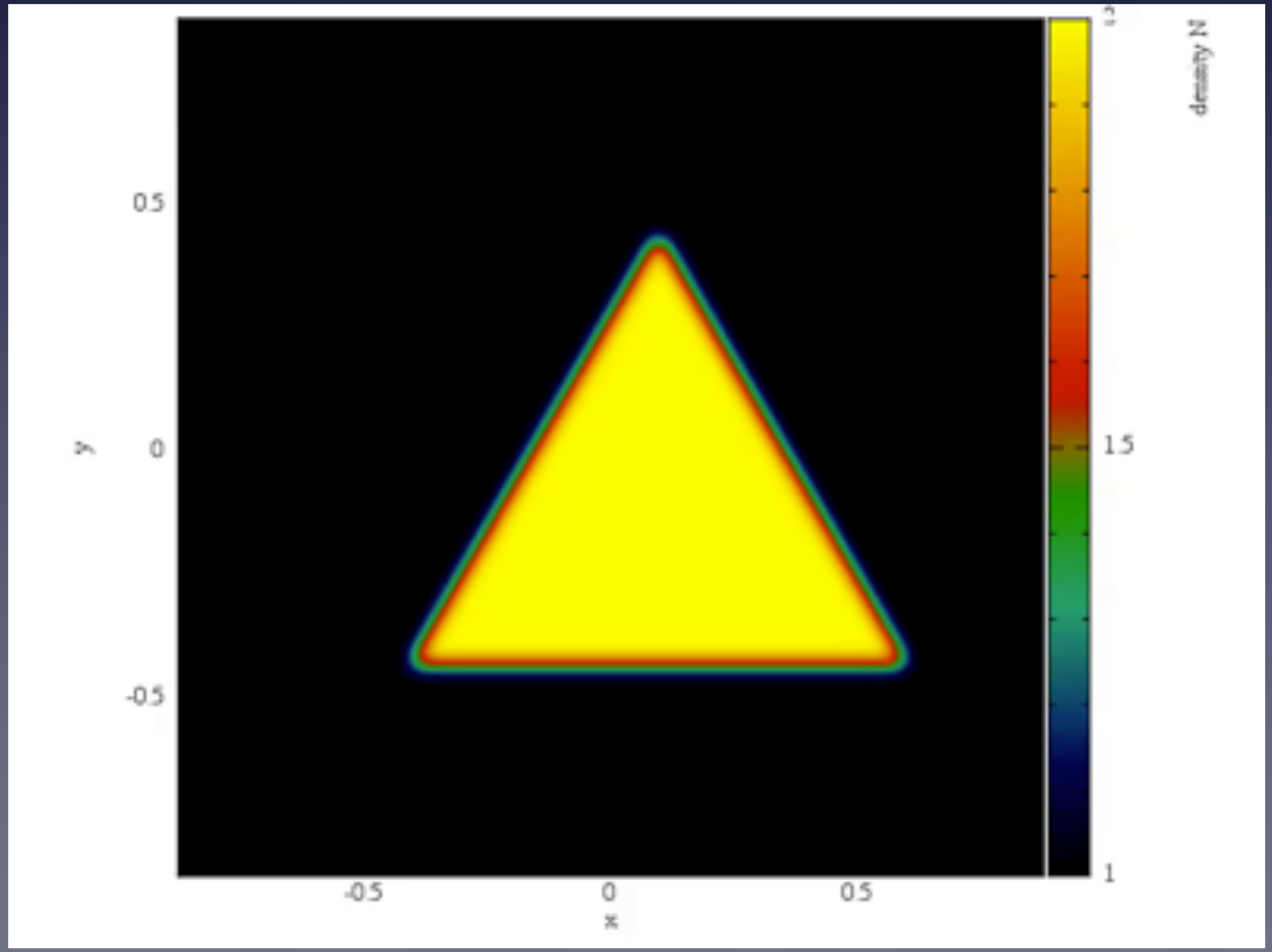
## Eulerian example:

star advected through vacuum
(AMR code FLASH; Fryxell+ 2000)

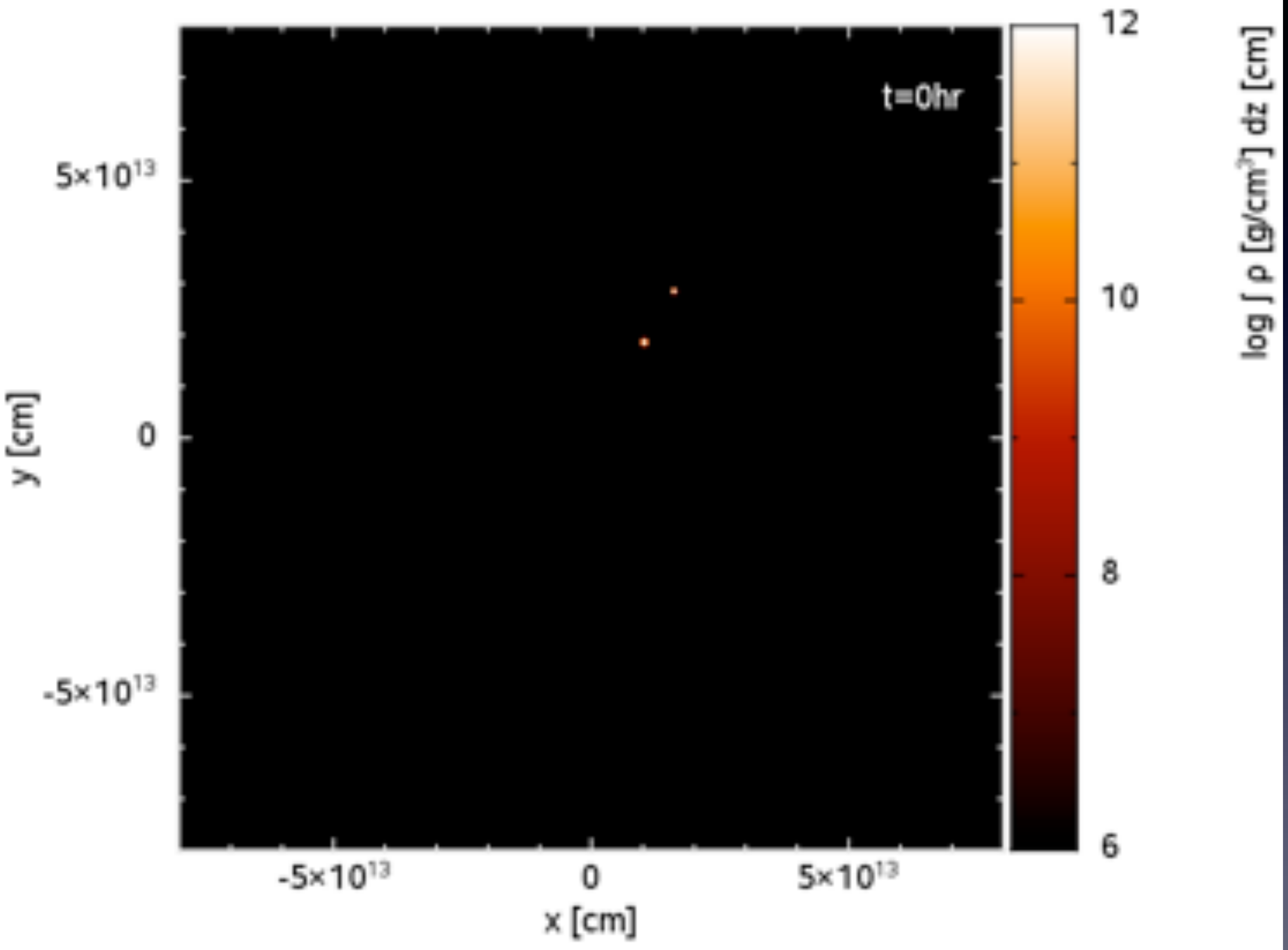


## Lagrangian example:

high-density triangle advected
through periodic box, Lorentz factor $\Gamma = 70.7$
(SPH code SPHINCS_SR; Rosswog 2015)



# The power of particles



simulation code: MAGMA2 (Rosswog 2020)

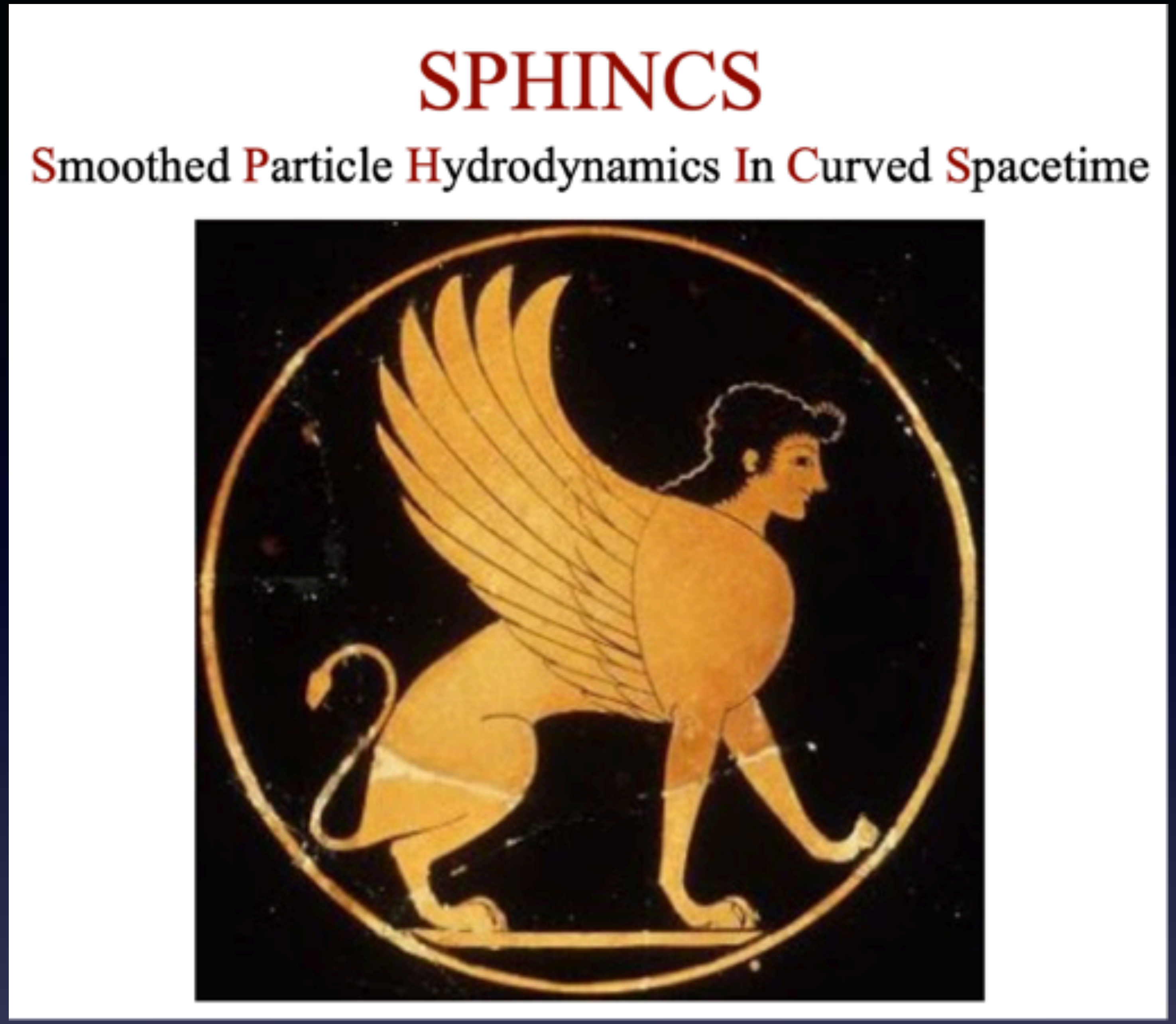
Tidal disruption of a stellar binary system

- stars: 36.8 M⊙ + 67.0 M⊙
- black hole: $10^6$ M⊙
- initial stars cover ~$10^{-9}$ of finally shown volume

Major advantages

- geometric flexibility
- exact advection
- not bound to “computational volume”
- “vacuum is vacuum”

# SPHINCS_BSSN: Lagrangian hydrodynamics in full General Relativity



- detailed code papers:

(i) “SPHINCS BSSN: A general relativistic Smooth Particle Hydrodynamics code for dynamical spacetimes”
S. Rosswog & P. Diener; Class. Quant. Gravity 38, 11, 115002 (2021)

(ii) “Simulating neutron star mergers with the Lagrangian Numerical Relativity code SPHINCS_BSSN”
P. Diener, S. Rosswog, F. Torsello, European Physical Journal A, 58, 74 (2022), arXiv:2203.06478 (2022)

(iii) “Thinking outside the box: Numerical Relativity with particles”,
S. Rosswog, P. Diener, F. Torsello, submitted; arXiv:2205.08130v1

# Our strategy:

1. Spacetime evolution:

“Baumgarte-Shapiro-Shibata-Nakamura-Oohara-Kojima”

(BSSN-OK) with fixed mesh refinement

2. Matter evolution:

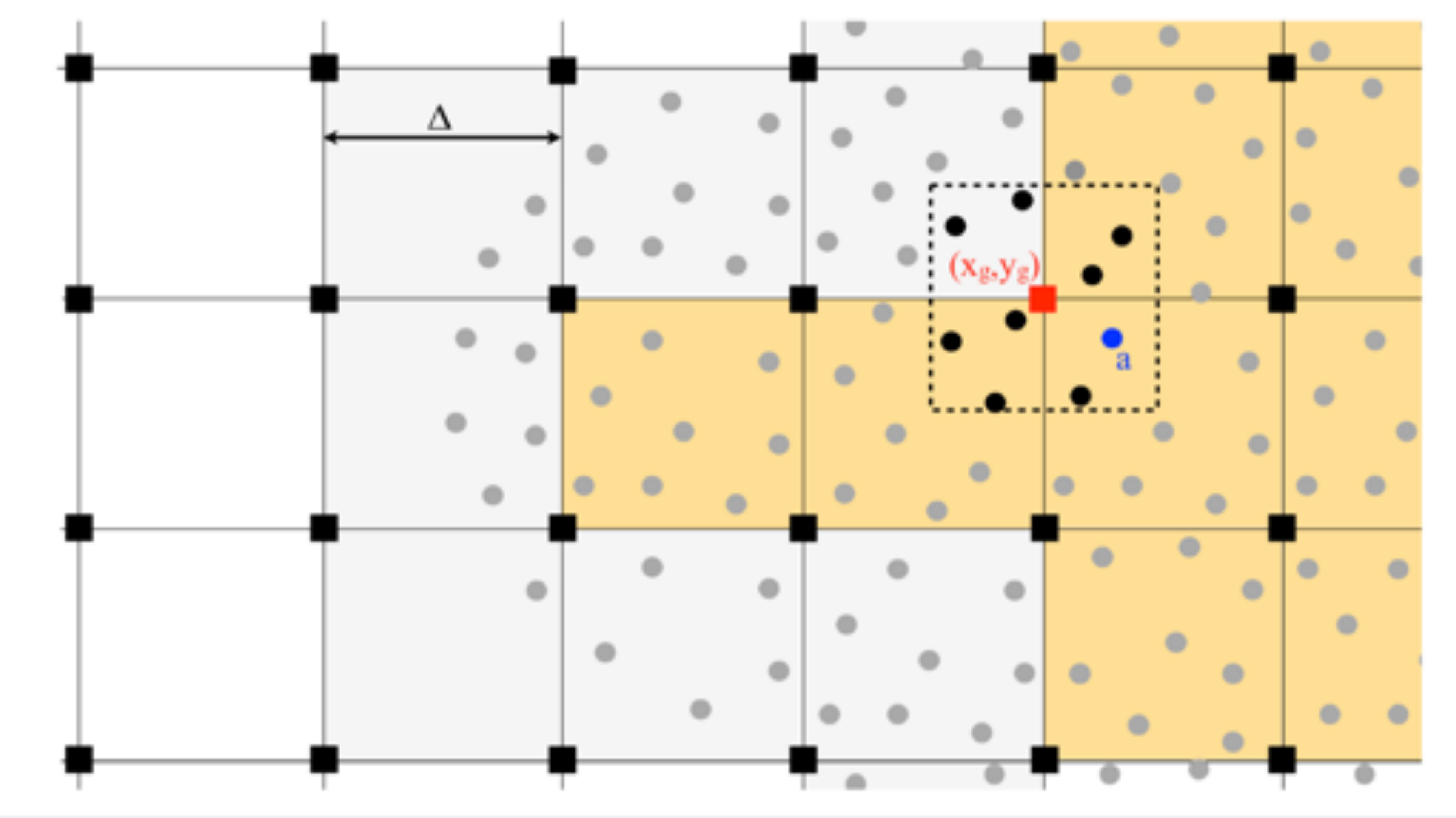
freely moving SPH-particles

3. Coupling between the particles and the mesh

…code written from scratch…

S.R. Peter Diener, LSU

@ LIGO Livingston



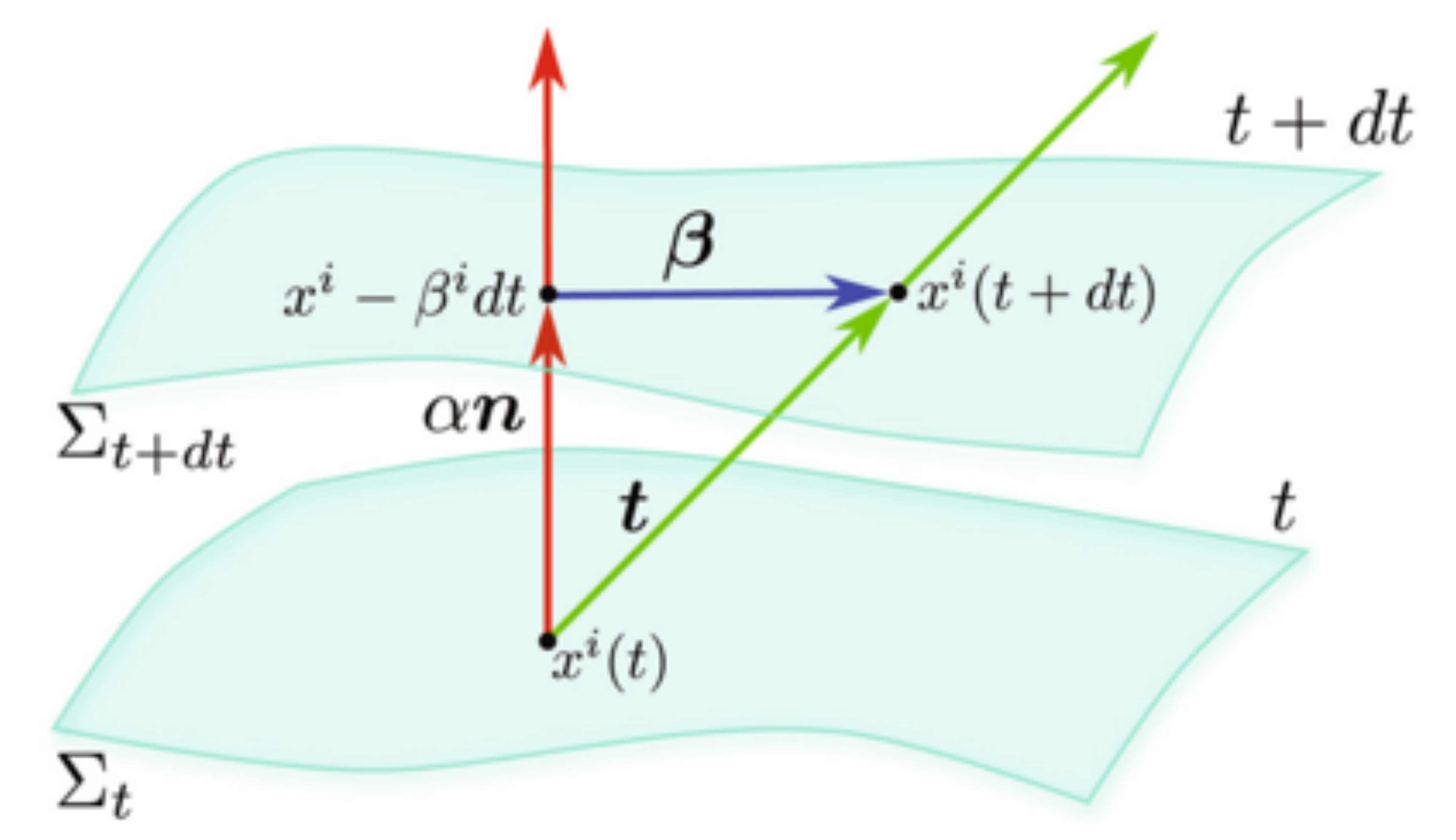
# 1. Spacetime evolution

- essentially same approach that is taken in Eulerian Numerical Relativity

- “3 + 1 - split”: evolve spacelike hypersurfaces forward in time

- spacetime line element

$$\boxed{ds^2 = -\underset{\uparrow\,\text{“lapse”}}{\alpha^2} dt^2 + \underset{\uparrow\,\text{“spatial metric”}}{\gamma_{ij}}(dx^i + \beta^i dt)(dx^j + \underset{\uparrow\,\text{“shift”}}{\beta^j} dt)}$$



- evolution via BSSN-OK equations on a structured mesh

# 2. General-relativistic SPH

- similarly to Newtonian SPH: can be derived from Lagrangian

$$L_{\rm GR} = -\int T^{\mu\nu} U_\mu U_\nu \sqrt{-g}\, dV$$

energy-momentum tensor

4-velocities

determinant of metric tensor

- use as numerical variables:

canonical momentum per baryon $(S_i)_a \equiv \dfrac{1}{\nu_a}\dfrac{\partial L}{\partial v_a^i} = (\Theta \mathcal{E} v_i)_a,$

canonical energy: $E = \sum_a (\partial L / \partial \vec{v}_a) \cdot \vec{v}_a - L \implies e_a = \left( S_i v^i + \dfrac{1+u}{\Theta} \right)_a$ canonical energy per baryon

- (after a lot of algebra, see Rosswog 2009 for details) one finds:

momentum

- "look and feel" very similar to Newtonian SPH

- BUT: we are not evolving the physical variables we are interested in

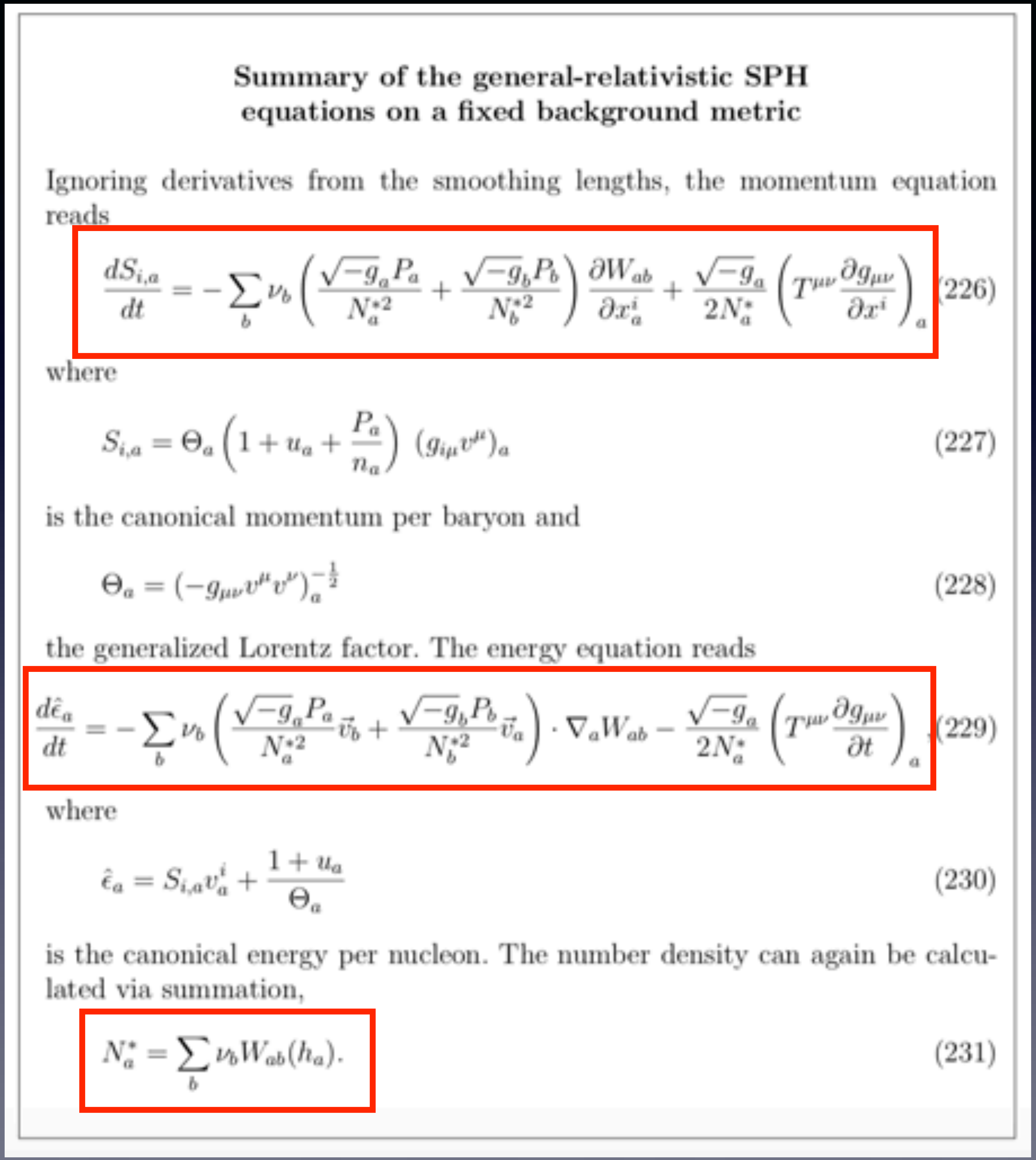
energy

$\Longrightarrow$ need to recover "physical variables" $(n, v^i, u)$

from "numerical variables" $(N^*, S_i, \hat{e})$

$\Longrightarrow$ use techniques very similar to Eulerian Numerical Relativity

baryon number



### Summary of the general-relativistic SPH equations on a fixed background metric

Ignoring derivatives from the smoothing lengths, the momentum equation reads

$$\boxed{\frac{dS_{i,a}}{dt} = -\sum_b \nu_b \left( \frac{\sqrt{-g}_a P_a}{N_a^{*2}} + \frac{\sqrt{-g}_b P_b}{N_b^{*2}} \right) \frac{\partial W_{ab}}{\partial x_a^i} + \frac{\sqrt{-g}_a}{2N_a^*} \left( T^{\mu\nu} \frac{\partial g_{\mu\nu}}{\partial x^i} \right)_a} \quad (226)$$

where

$$S_{i,a} = \Theta_a \left( 1 + u_a + \frac{P_a}{n_a} \right) (g_{i\mu} v^\mu)_a \quad (227)$$

is the canonical momentum per baryon and

$$\Theta_a = (-g_{\mu\nu} v^\mu v^\nu)_a^{-\frac{1}{2}} \quad (228)$$

the generalized Lorentz factor. The energy equation reads

$$\boxed{\frac{d\hat{\epsilon}_a}{dt} = -\sum_b \nu_b \left( \frac{\sqrt{-g}_a P_a}{N_a^{*2}} \vec{v}_b + \frac{\sqrt{-g}_b P_b}{N_b^{*2}} \vec{v}_a \right) \cdot \nabla_a W_{ab} - \frac{\sqrt{-g}_a}{2N_a^*} \left( T^{\mu\nu} \frac{\partial g_{\mu\nu}}{\partial t} \right)_a} \quad (229)$$

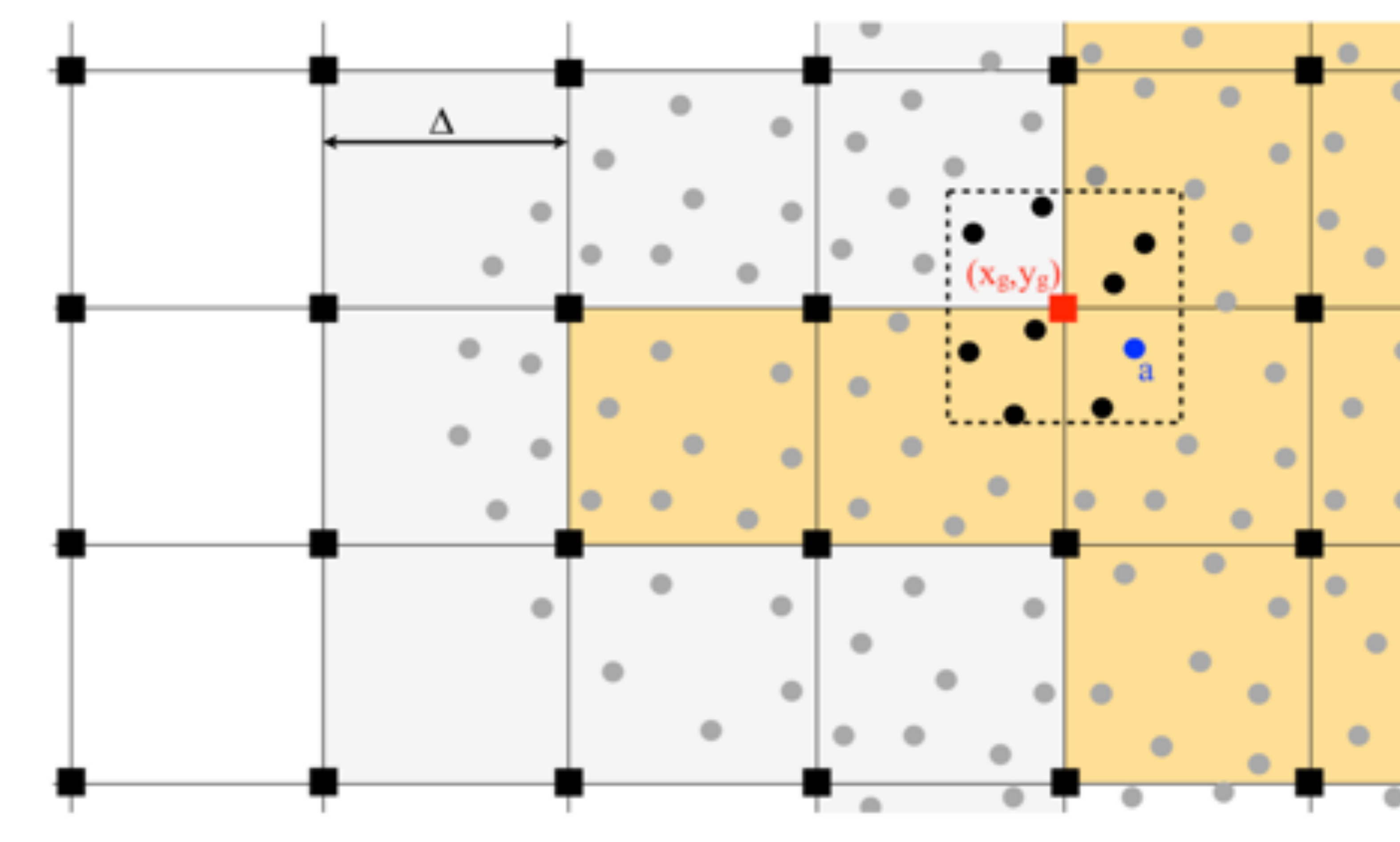
where

$$\hat{\epsilon}_a = S_{i,a} v_a^i + \frac{1 + u_a}{\Theta_a} \quad (230)$$

is the canonical energy per nucleon. The number density can again be calculated via summation,

$$\boxed{N_a^* = \sum_b \nu_b W_{ab}(h_a).} \quad (231)$$

# 3. Coupling between matter and spacetime



- “mesh needs”: energy-momentum tensor $T_{\mu\nu}$ (known at particles)
- “particles need”: derivatives of metric/gravitational acceleration terms
- Our Particle-Mesh method

  (A) particle ⟶ mesh step

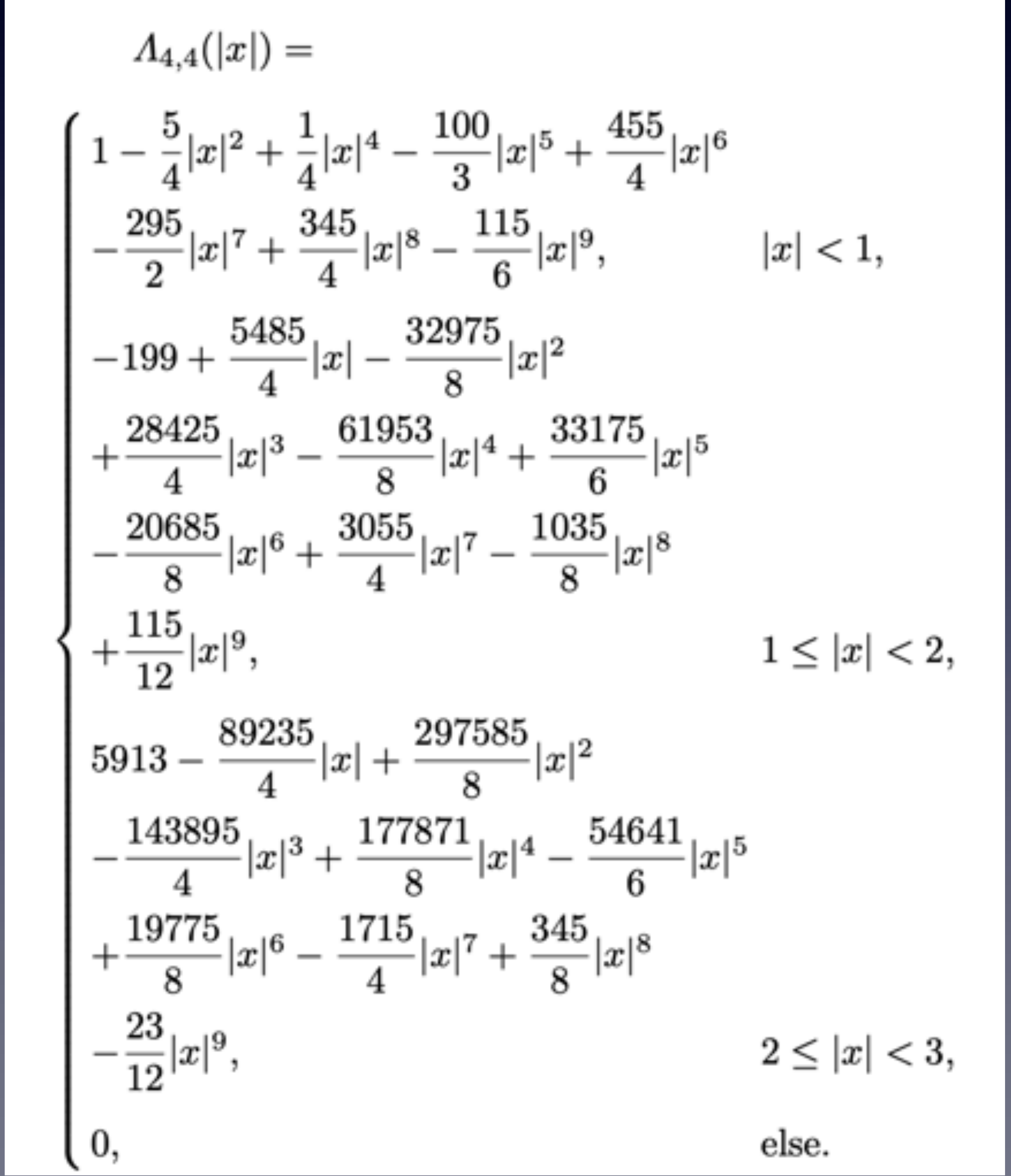
  $\Longrightarrow$ use hierarchy of sophisticated kernel functions borrowed from “vortex methods”, e.g. $\Lambda_{4,4}$

  $\Longrightarrow$ MOOD (“multi-dimensional optimal order detection”)

  - hierarchy of kernels
  - choose the kernel that best reproduces the particle-$T_{\mu\nu}$ on the grid

  (B) mesh ⟶ particle step

  $\Longrightarrow$ 5th order Hermite interpolation



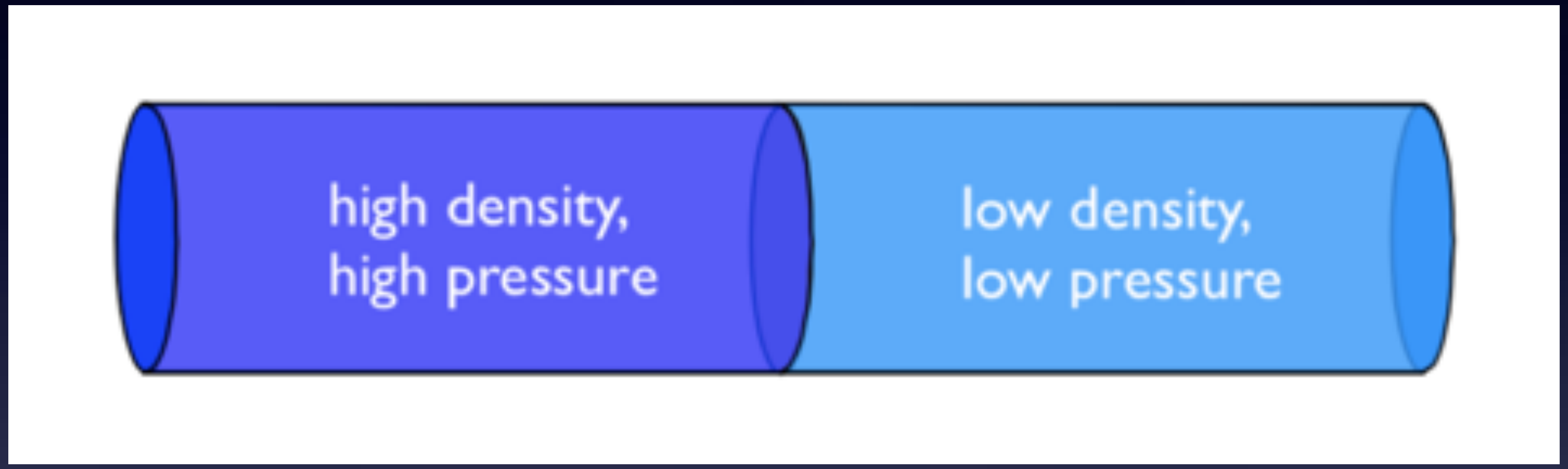
$$\Lambda_{4,4}(|x|) =$$

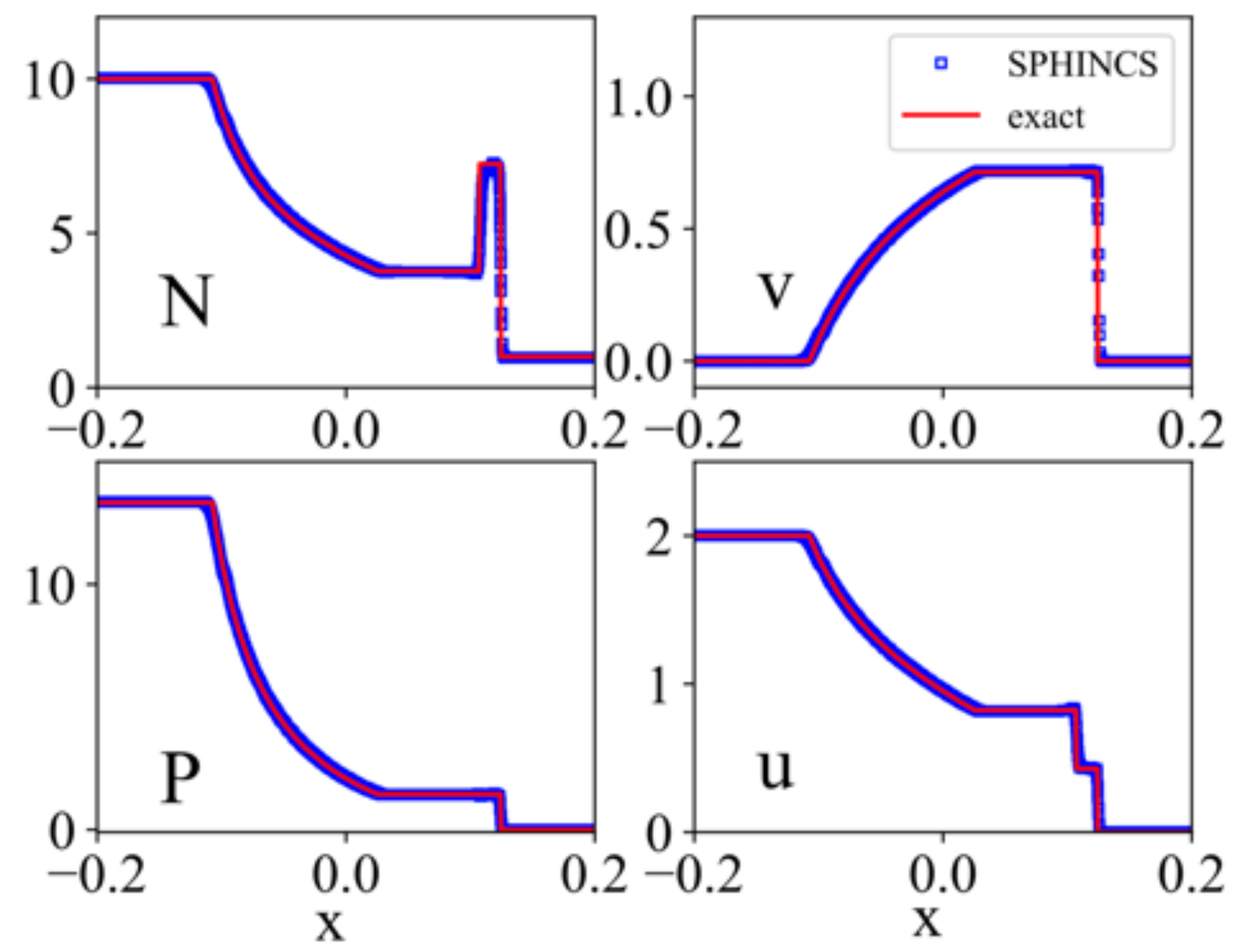
$$\begin{cases} 1-\frac{5}{4}|x|^2+\frac{1}{4}|x|^4-\frac{100}{3}|x|^5+\frac{455}{4}|x|^6 \\ -\frac{295}{2}|x|^7+\frac{345}{4}|x|^8-\frac{115}{6}|x|^9, & |x|<1, \\ -199+\frac{5485}{4}|x|-\frac{32975}{8}|x|^2 \\ +\frac{28425}{4}|x|^3-\frac{61953}{8}|x|^4+\frac{33175}{6}|x|^5 \\ -\frac{20685}{8}|x|^6+\frac{3055}{4}|x|^7-\frac{1035}{8}|x|^8 \\ +\frac{115}{12}|x|^9, & 1\leq|x|<2, \\ 5913-\frac{89235}{4}|x|+\frac{297585}{8}|x|^2 \\ -\frac{143895}{4}|x|^3+\frac{177871}{8}|x|^4-\frac{54641}{6}|x|^5 \\ +\frac{19775}{8}|x|^6-\frac{1715}{4}|x|^7+\frac{345}{8}|x|^8 \\ -\frac{23}{12}|x|^9, & 2\leq|x|<3, \\ 0, & \text{else.} \end{cases}$$

# 4. Tests

"Does special-relativistic hydrodynamics work?"

$\Longrightarrow$ special-relativistic "shock-tube" test in 3D:



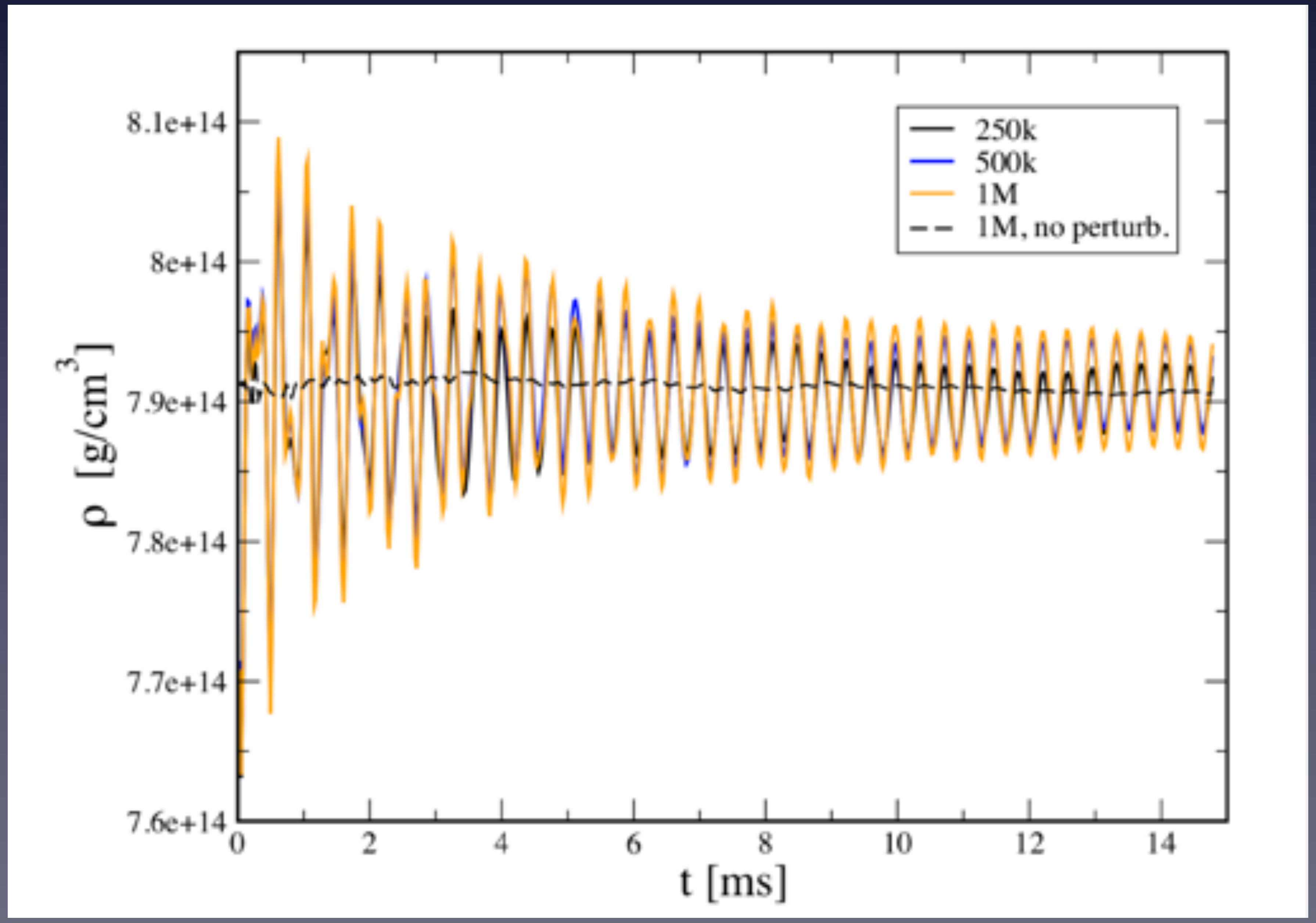


# "Does general-relativistic hydrodynamics work?"

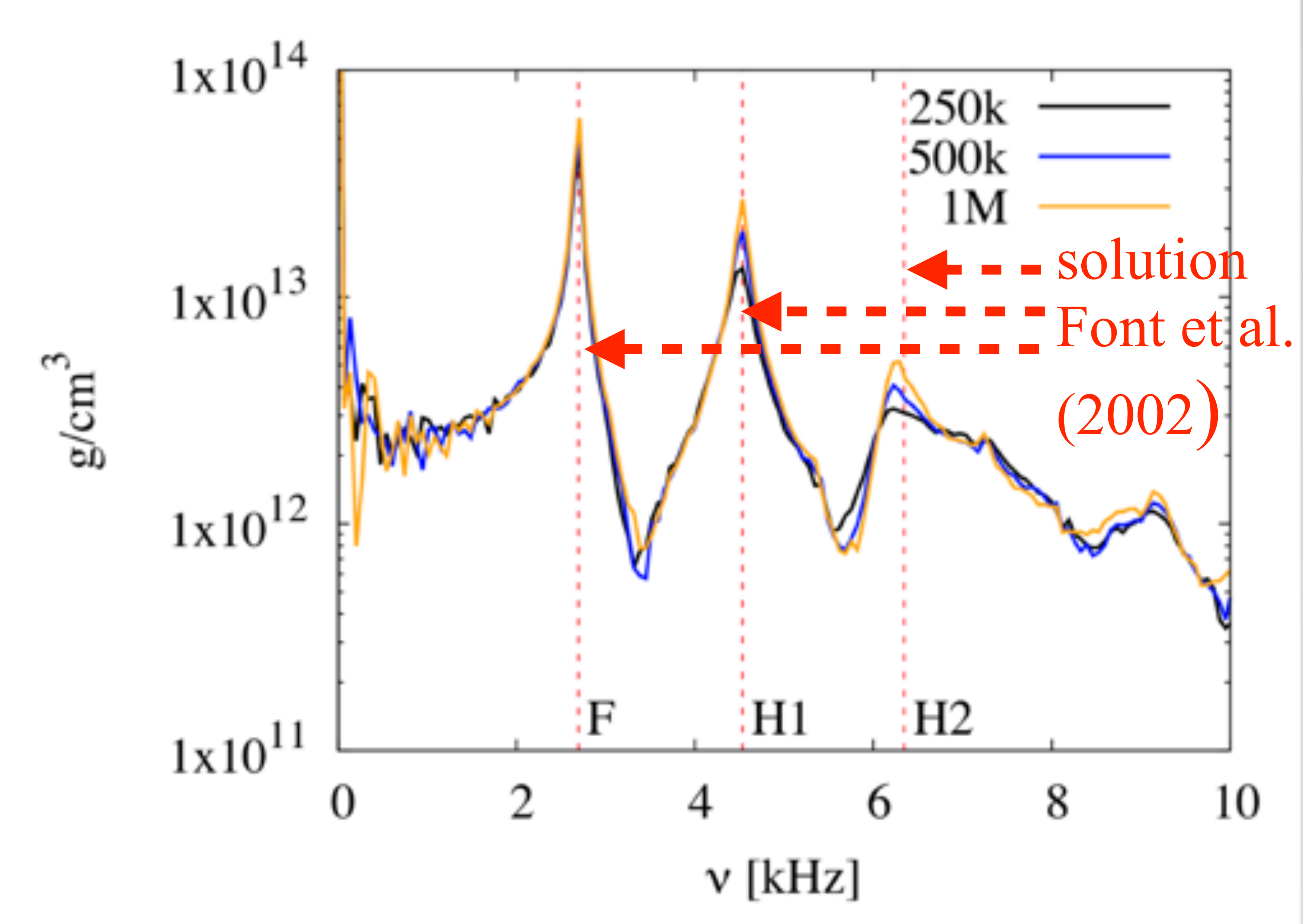
⟹ oscillating neutron star in a "frozen spacetime" ("Cowling approximation")

- construct neutron star (polytropic EOS, Γ= 2.0), solve Tolman-Oppenheimer-Volkoff equations
- perturb star → oscillation frequencies
- let matter evolve, keep spacetime fix

with correct frequencies?



star oscillates around initial stellar profile!



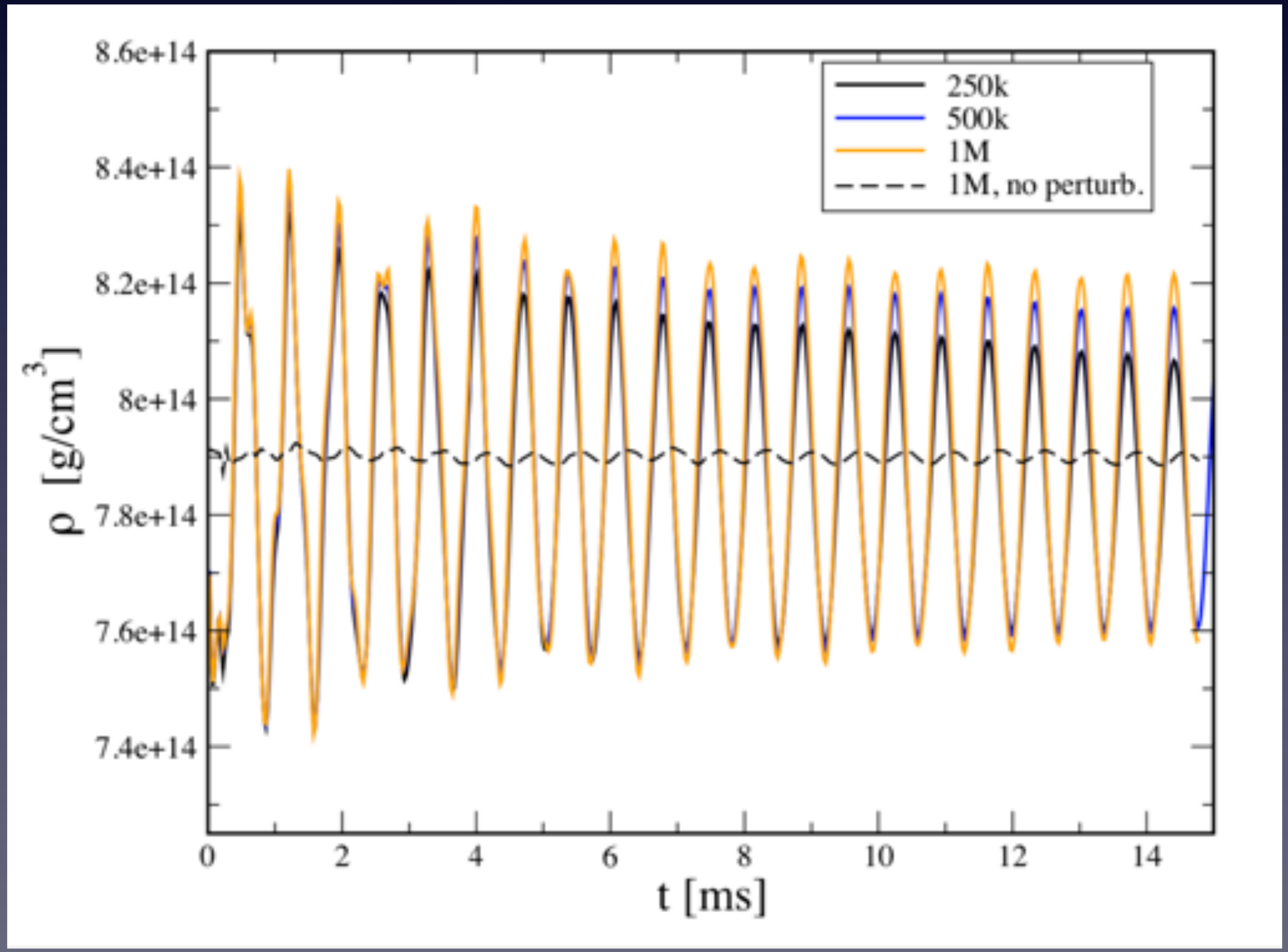
central density evolution

Fourier spectrum central density evolution

# “Does coupling between spacetime and hydrodynamics work?”

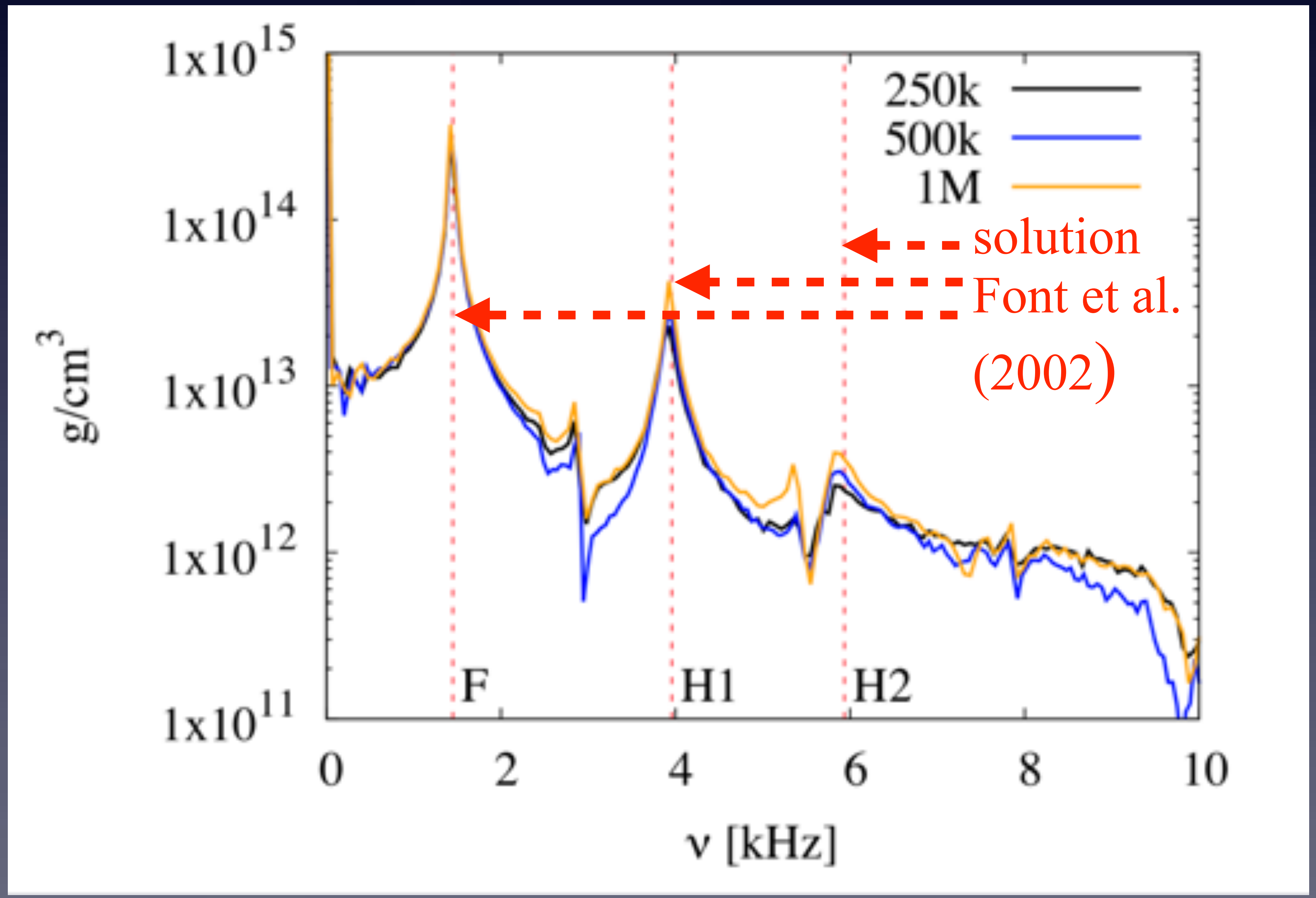
$\Longrightarrow$ oscillating neutron star, but now full hydrodynamics + spacetime evolution

central density evolution



star oscillates around initial stellar profile!

Fourier spectrum central density evolution

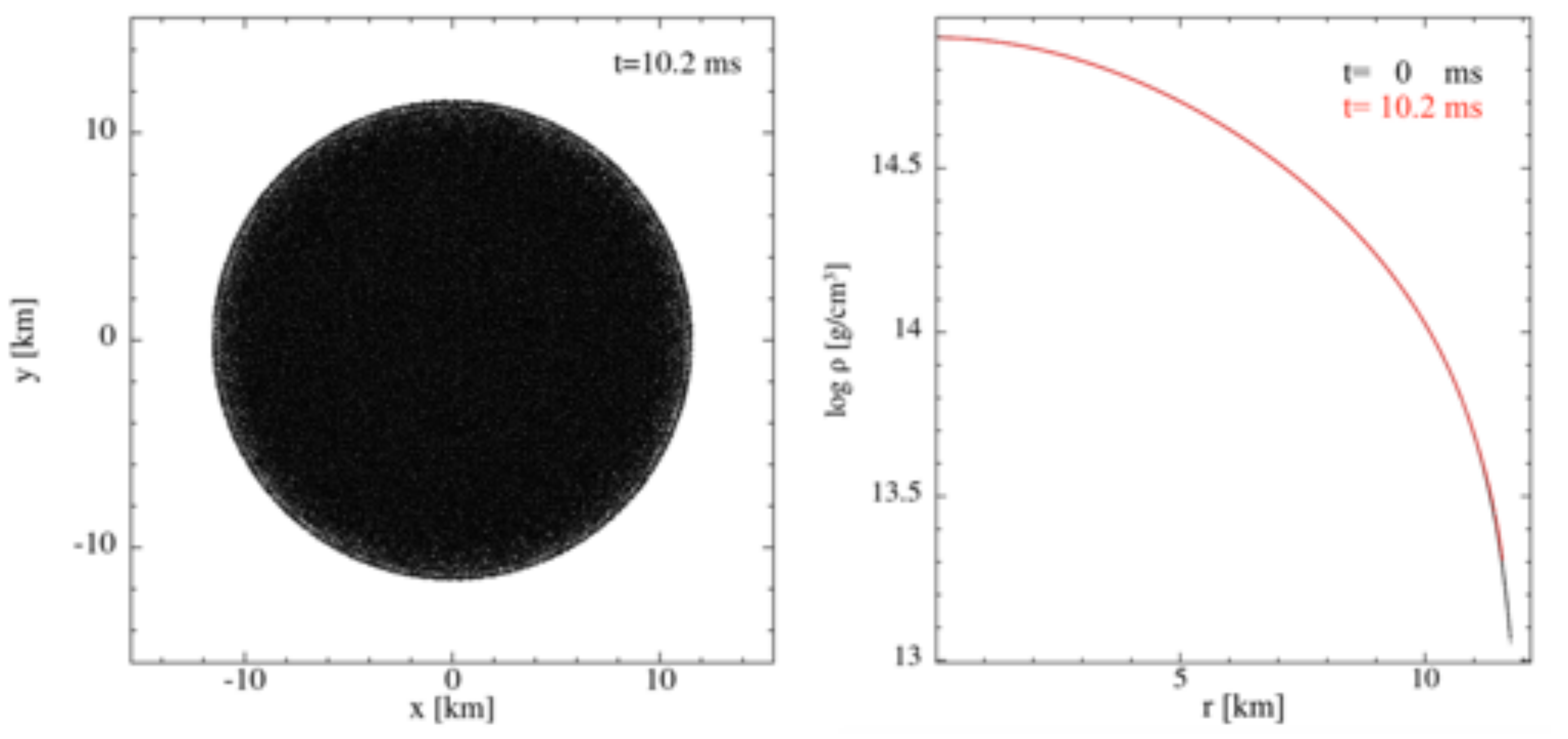


solution Font et al. (2002)

$\Longrightarrow$ oscillates at same frequencies as reference solution

# “How close does the star stay to the TOV-solution?”

neutron star after 10 ms full evolution (~14 oscillation periods)



- surface remains “perfectly” well behaved (no “special treatment” necessary)
- star remains very close to initial solution

# “Evolution of an unstable neutron star”

- prepare neutron star on unstable branch
- extremely relativistic: central density $5 \times 10^{15}$ g/cm$^3 \approx 20 \times \rho_{nuc}$
- literature (e.g. Baiotti et al. 2005; Bernuzzi & Hilditch 2010):

  “evolution sensitively depends on initial state”

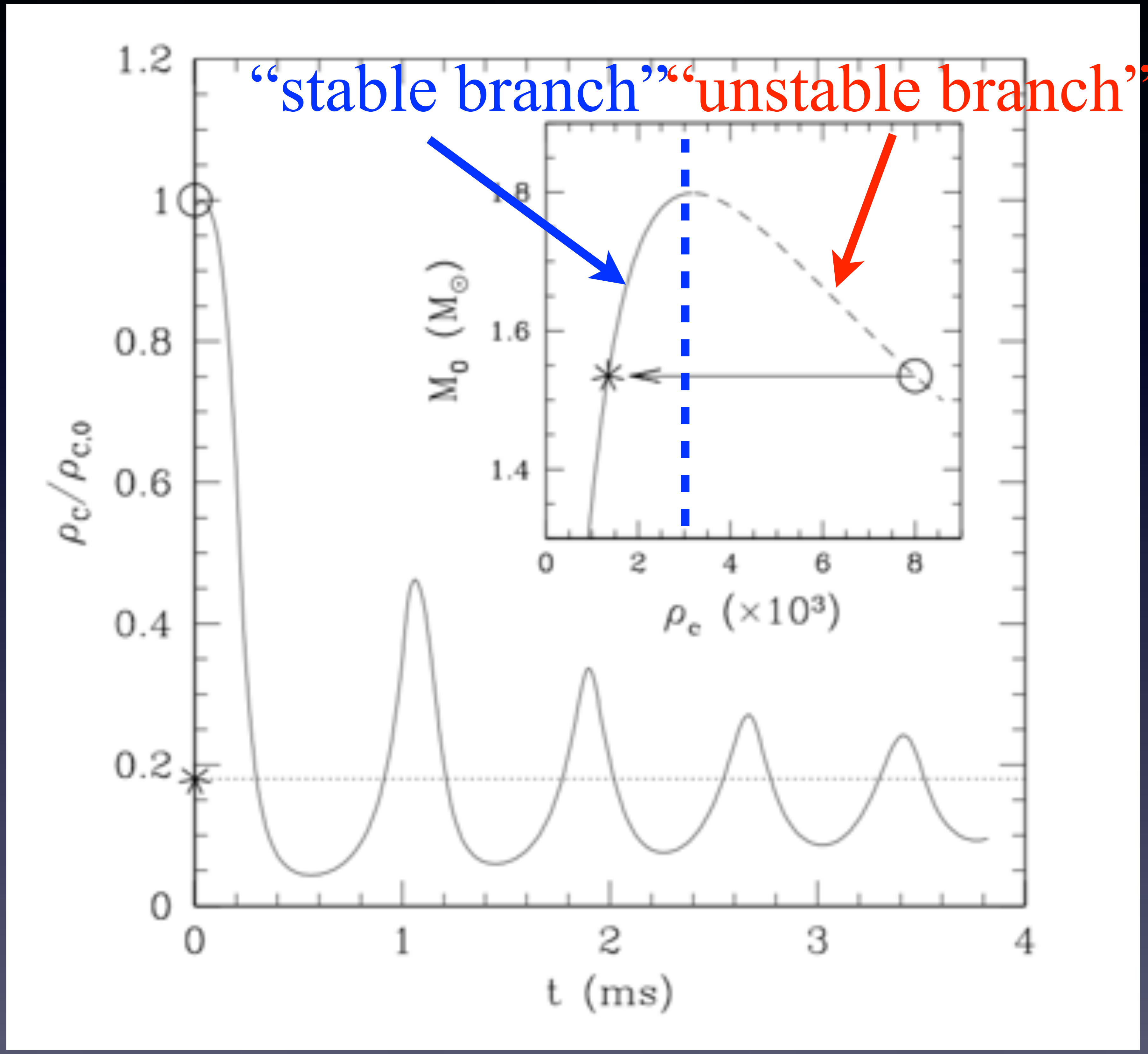
  (a) IF just evolved:

  truncation error $\Longrightarrow$ violent (!) oscillations (v~0.5 c)

  (b) with small perturbation: $v_r = -0.005c$

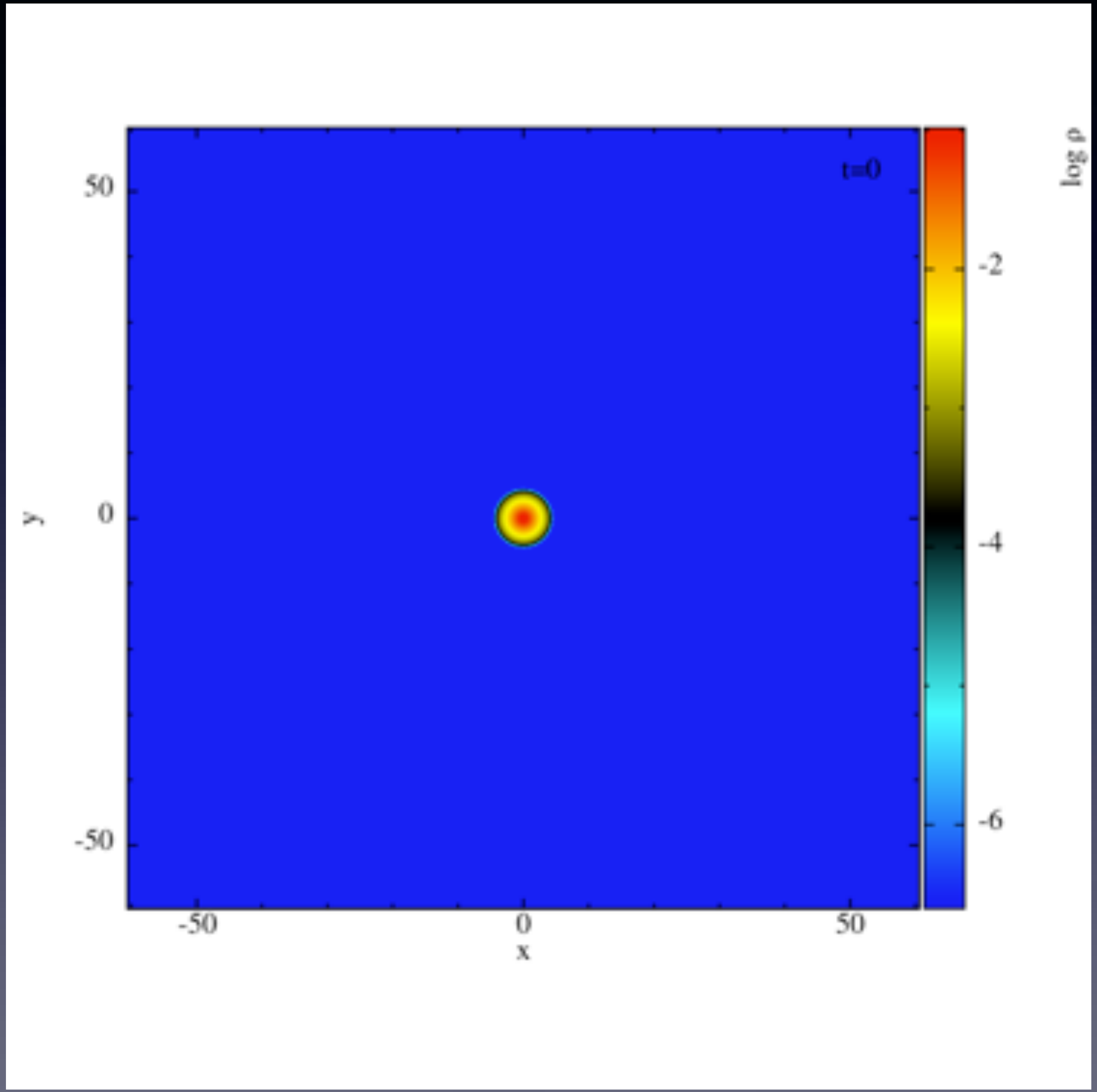
  $\Longrightarrow$ collapse to black hole

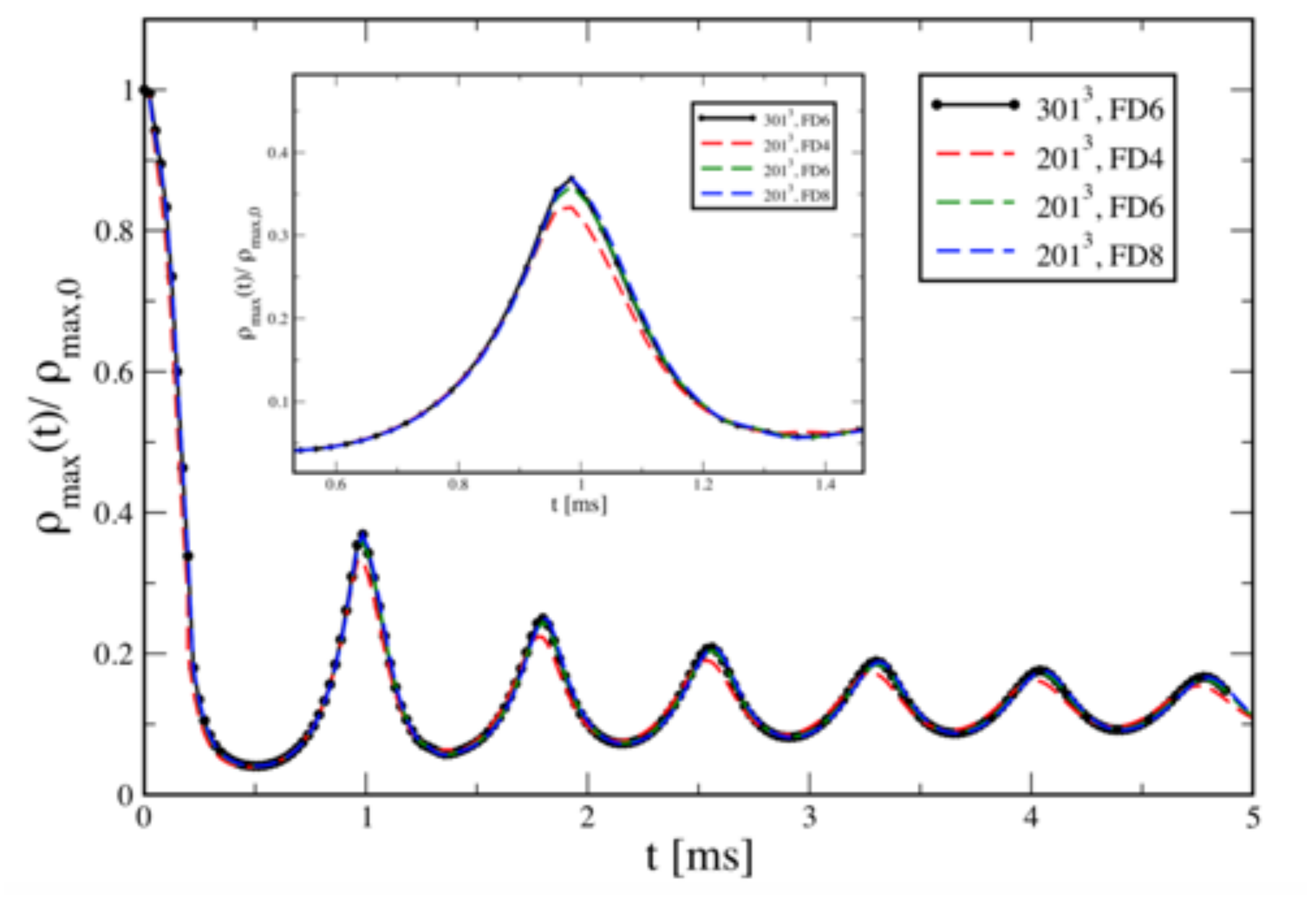
  $\Longrightarrow$ can we confirm this?



(from Baiotti et al. 2005)

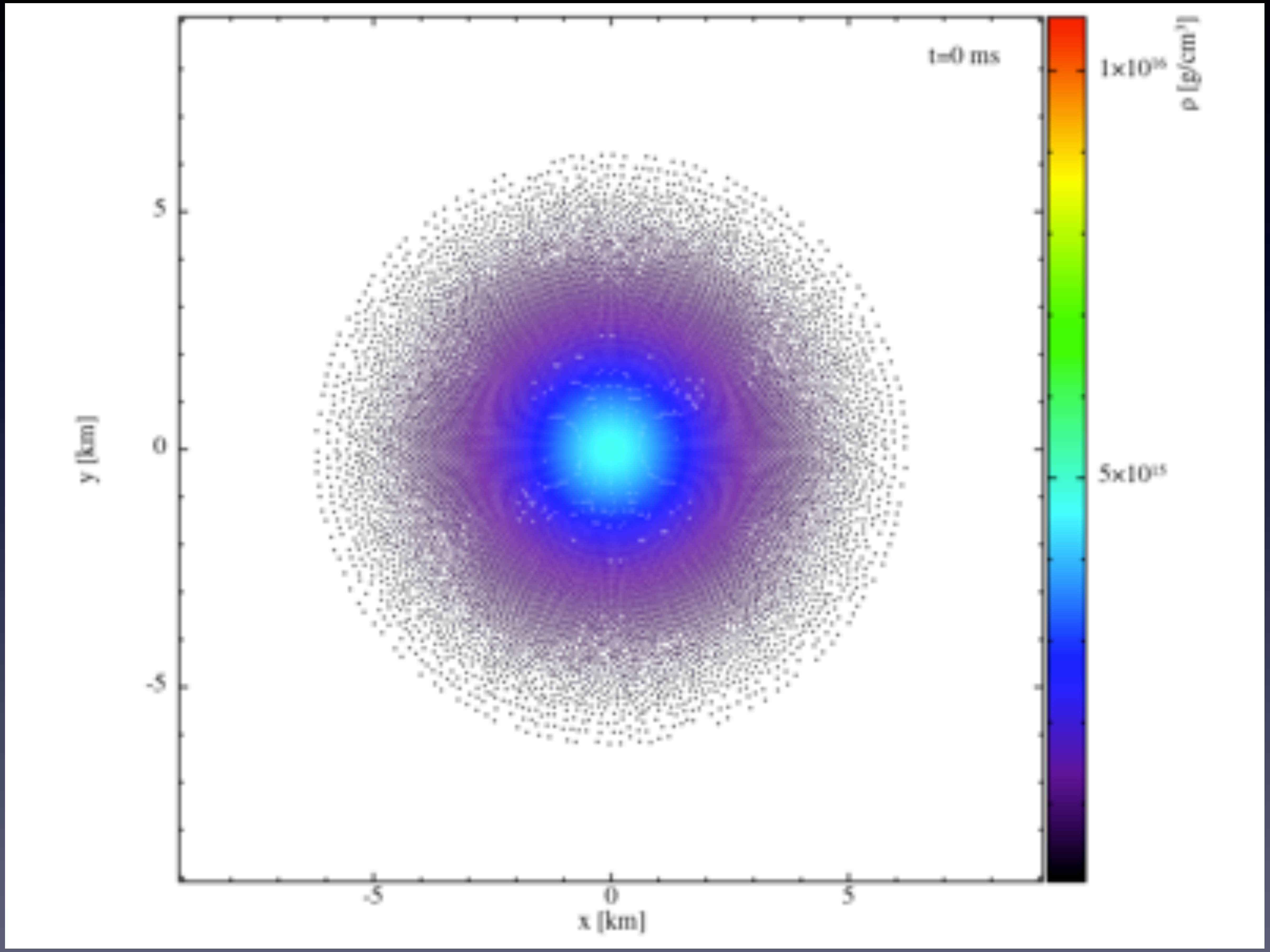
## “just evolve” (i.e. initial perturbation from truncation error)





$\Longrightarrow$ very similar to results of Eulerian Numerical Relativity!

# “small radial perturbation” ($v_r$= -0.005c)



- particles in thin slice ($z < 0.1$) shown
- black hole formation ⟹ “collapse of lapse”
- below a critical value (lapse $\alpha < 0.03$) particles are removed

⟹ again: very similar to results of Eulerian Numerical Relativity!

# Full GR Lagrangian neutron star mergers

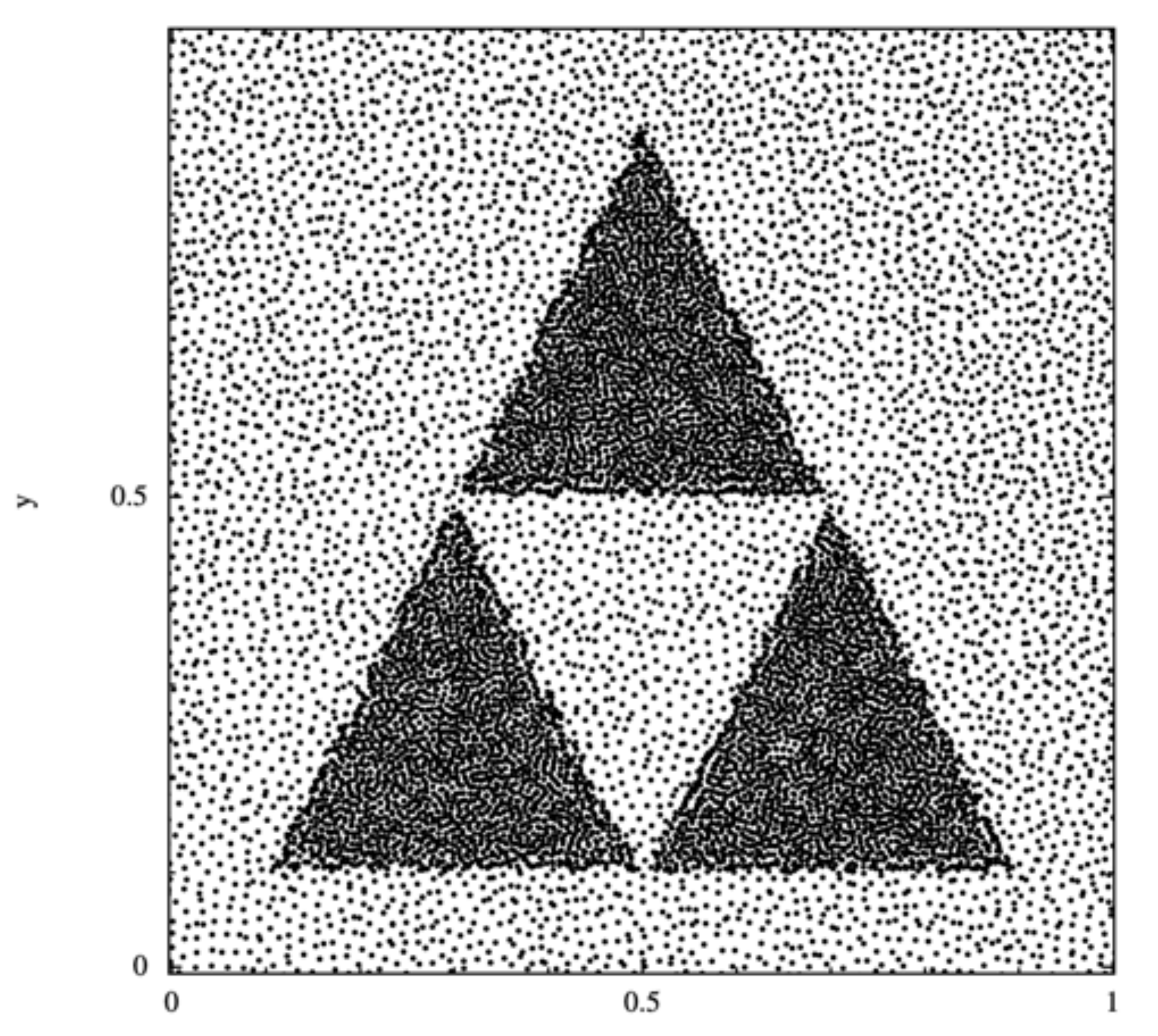
- initial conditions from LORENE library

- needs to be “translated to particles”!

- done via “Artificial Pressure Method” (APM; Rosswog 2020)

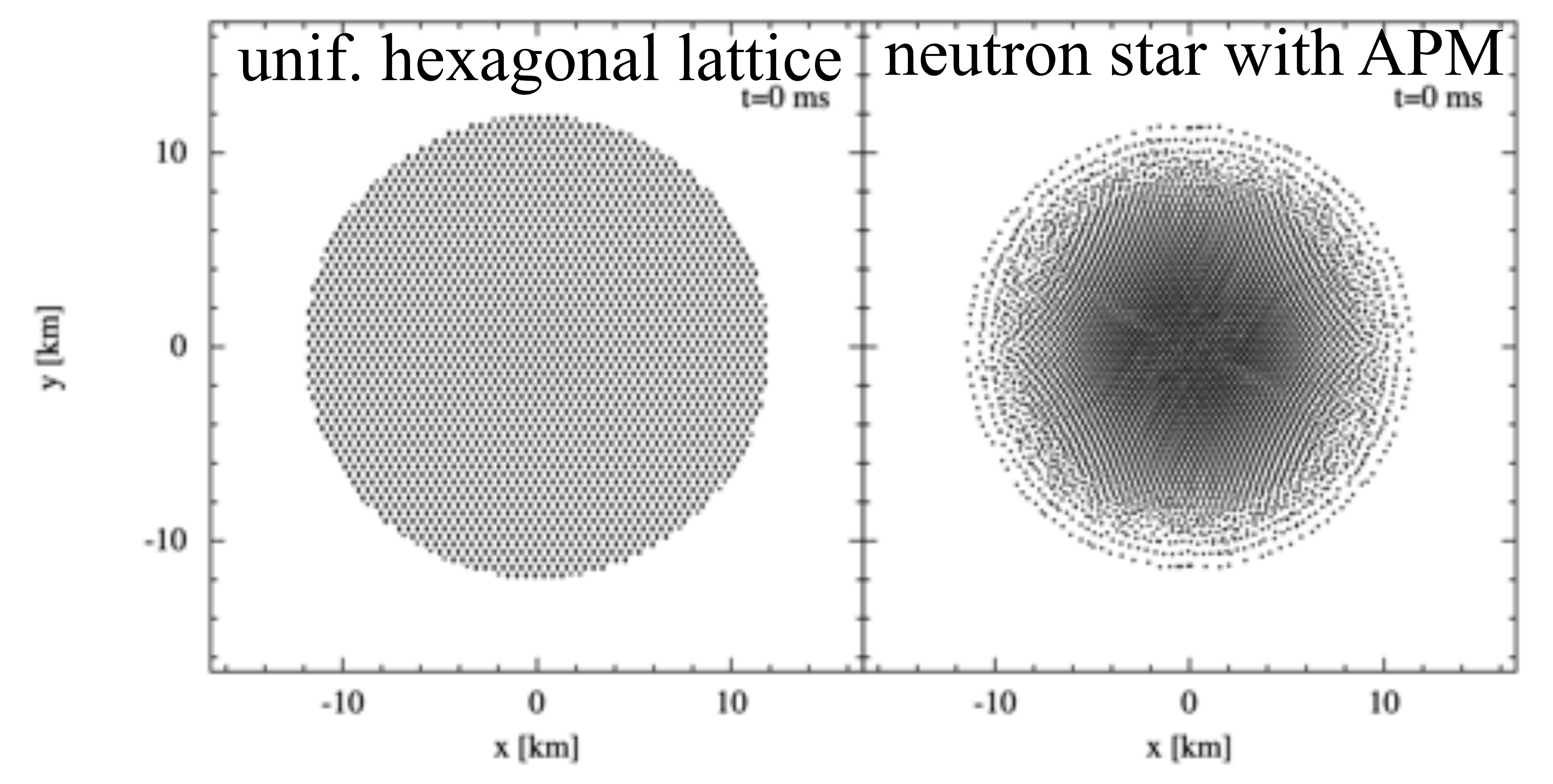
  “particles move according to hydrodynamic momentum equation to minimize their density error with respect to given profile”

set of high-density triangles



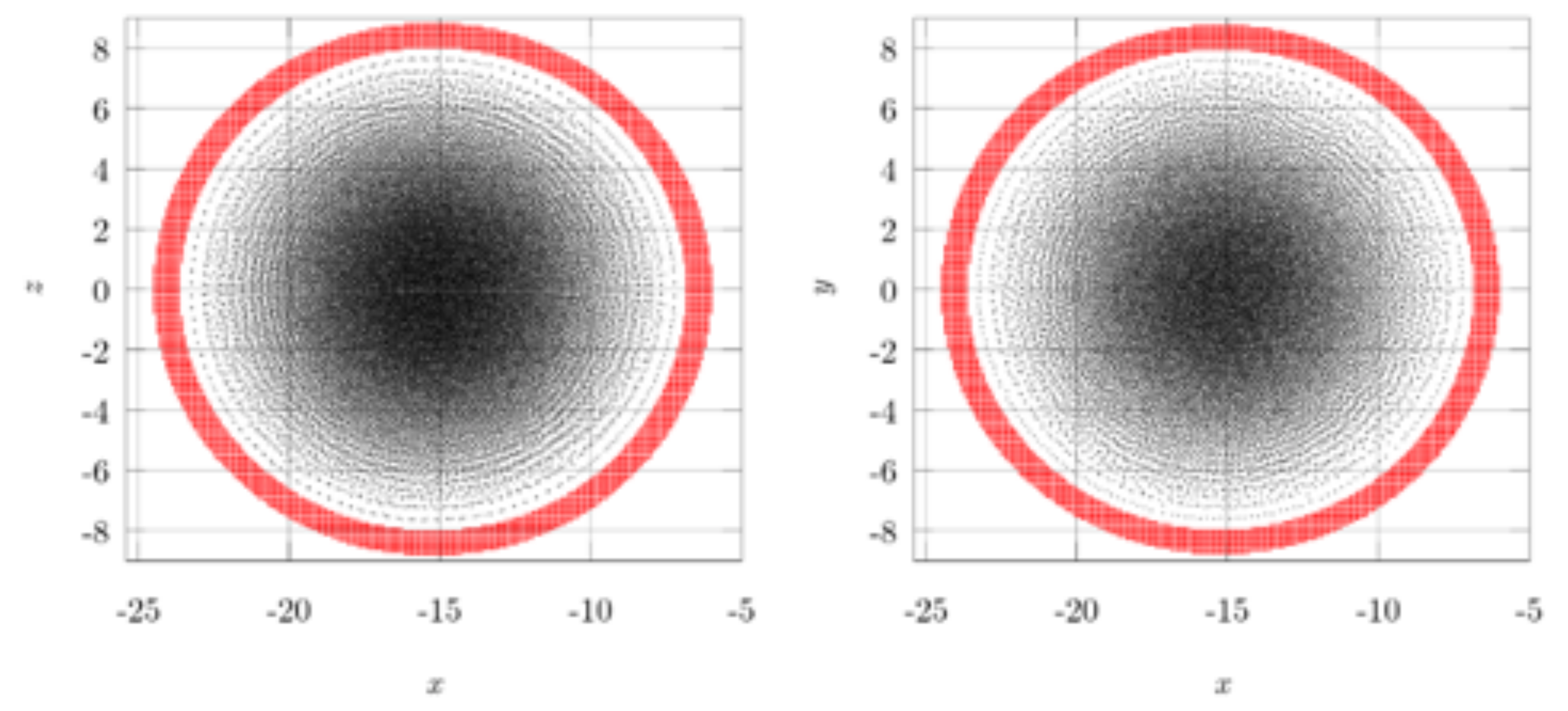
(Rosswog 2020)

relativistic, single neutron star



(Rosswog & Diener 2021)

# APM for relativistic binary system (based on LORENE)



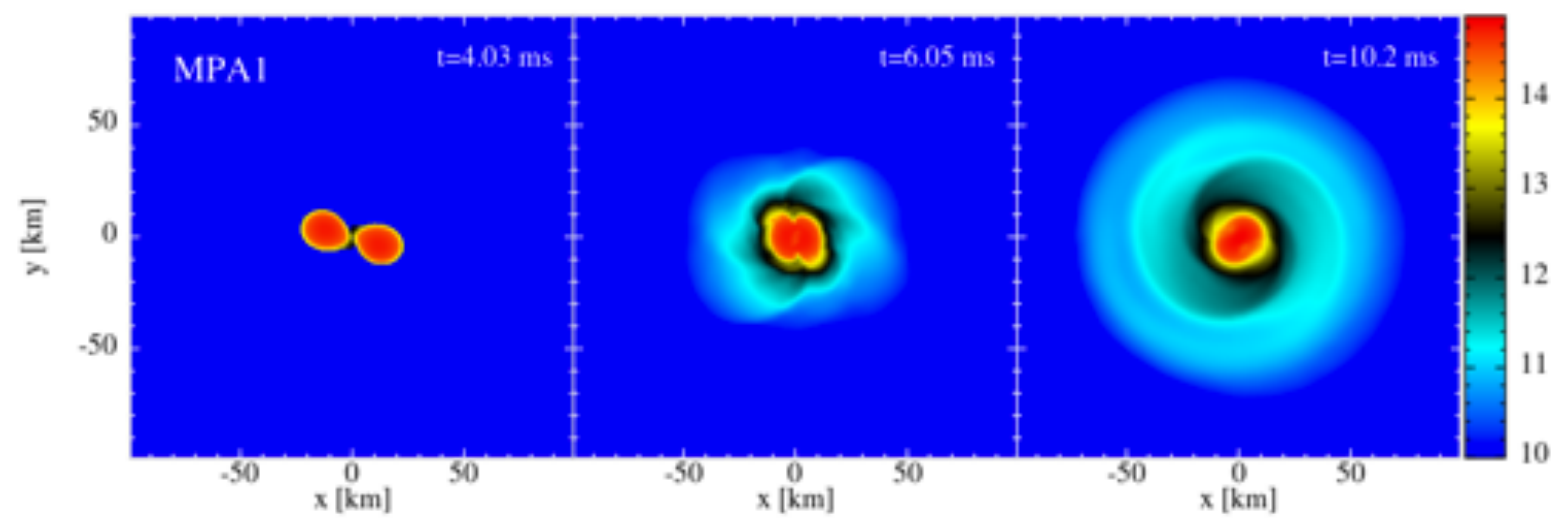
(Diener, Rosswog & Torsello, European Physical Journal A, 58, 74 (2022)

Francesco Torsello

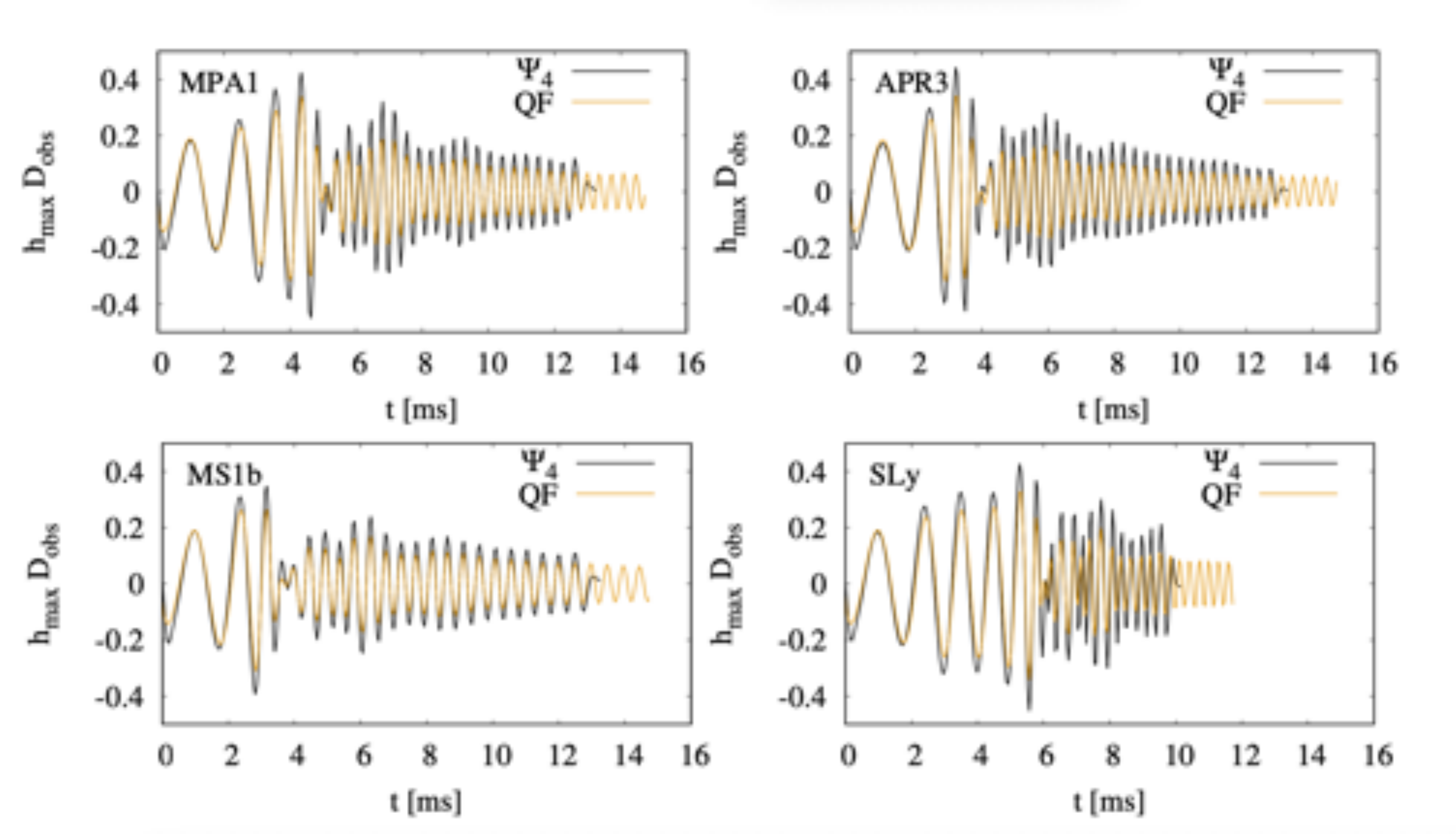
code `SPHINCS_ID`

# First Lagrangian mergers in full GR

- binary: 2 x 1.3 M☉, irrotational
- spacetime: 7 (fixed) refinement levels
- fluid: up to 5 million SPH particles
- EOS: piecewise polytropic + thermal component (MPA1, APR3, SLy, MS1b)



# Extracting gravitational waves



(Rosswog, Diener, Torsello (2022); arXiv:2205.08130v1)

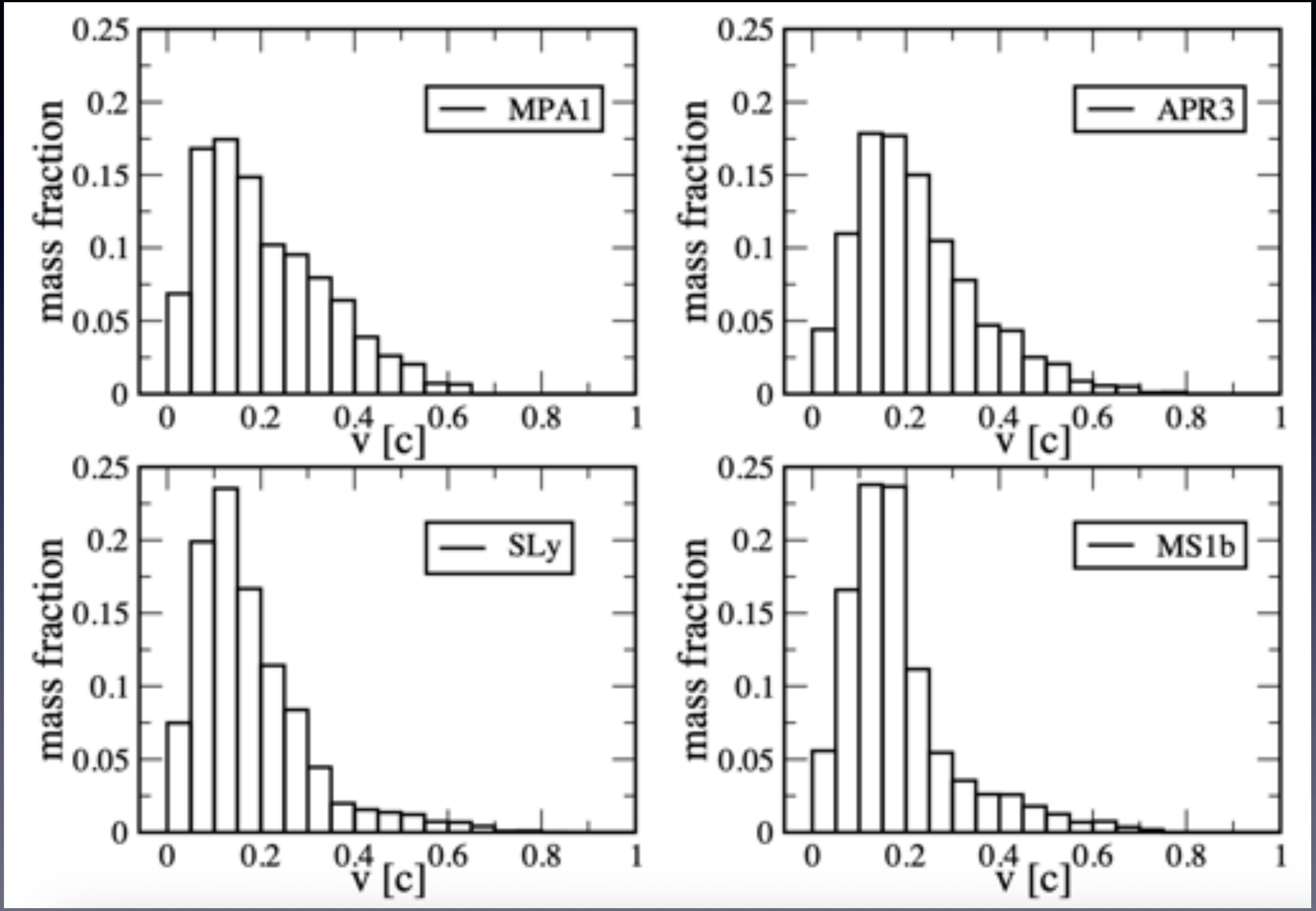
a) directly from particles, **quadrupole formalism**

b) from spacetime, **$\psi_4$ formalism**

⇒ quadrupole approximation reasonably good

# Dynamic ejecta

asymptotic velocities



(Rosswog, Diener, Torsello (2022); arXiv:2205.08130v1)

- ejecta mass: few x $10^{-3}$ $M_\odot$
- soft EOSs eject more (shocks!)
- high velocity component: $\sim 10^{-4}$ $M_\odot$ are above 0.5 c

$\Rightarrow$ a) early “blue” kilonova precursor

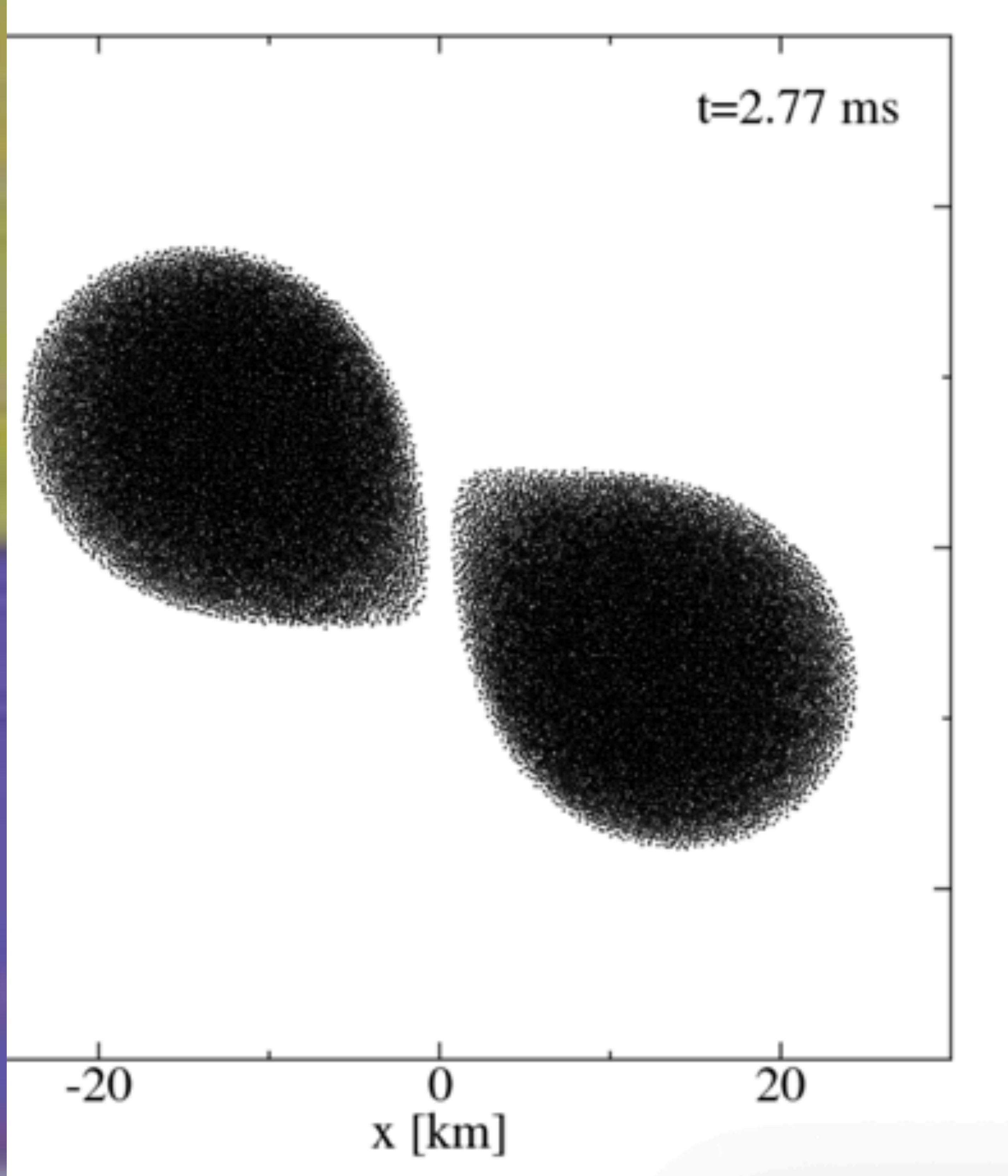
(Metzger et al. 2015)

b) “kilonova afterglow”

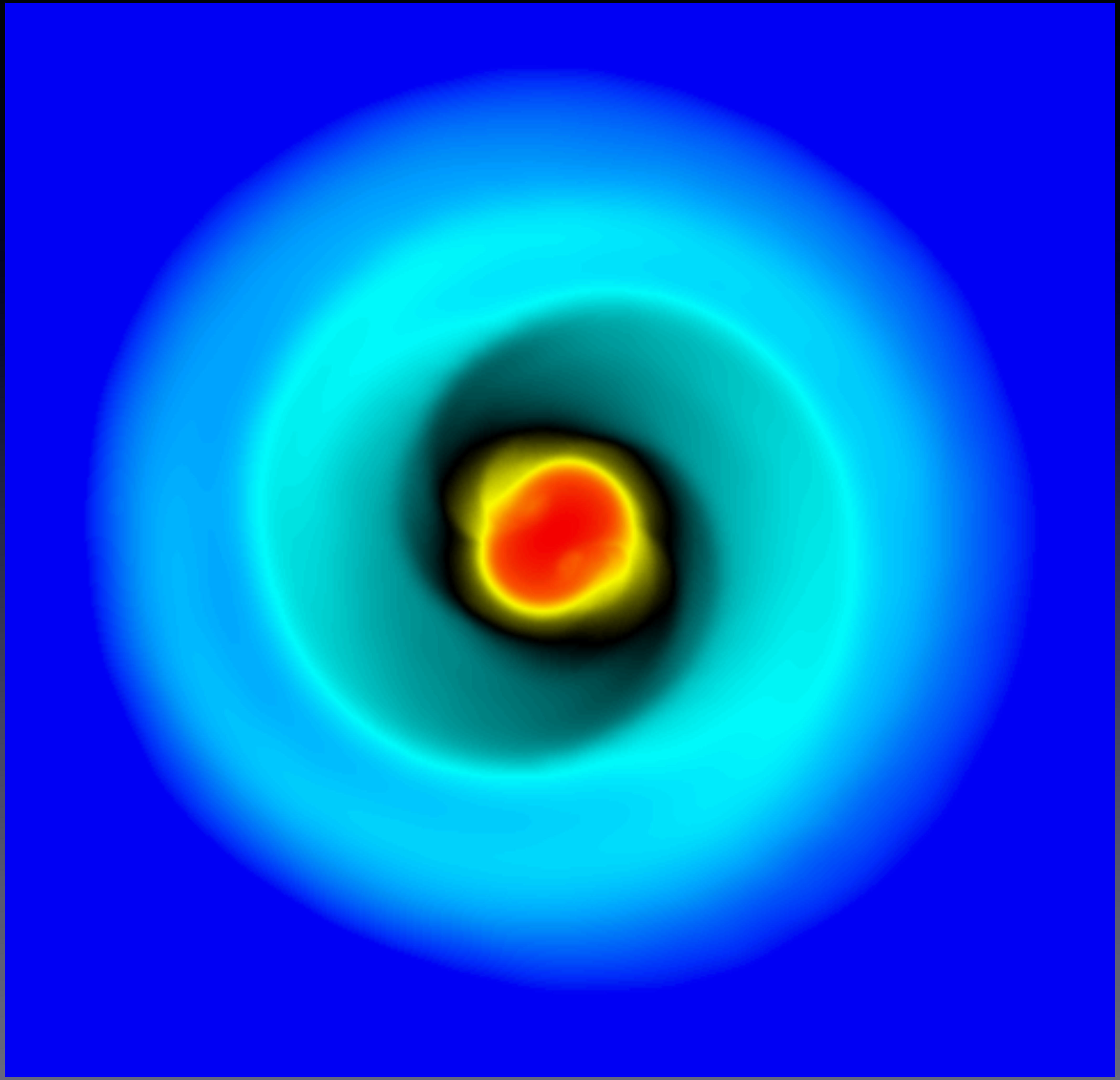
(Hajela et al. 2022)

# Summary

- spacetime evolution: “s
- matter evolution: freely
- successfully passed ma
- major advantages:
  - neutron star surface n
  - “vacuum is vacuum”
  - ejecta evolution
- future work:
  - code performance
  - microphysics



# 7 Postdoc positions at University Hamburg, Germany
(starting from autumn 2022)



Excellence Cluster "Quantum Universe"
University Hamburg

If you are interested, get in contact with me: stephan.rosswog@astro.su.se