Factorization of power corrections in DIS in the $x \to 1$ limit

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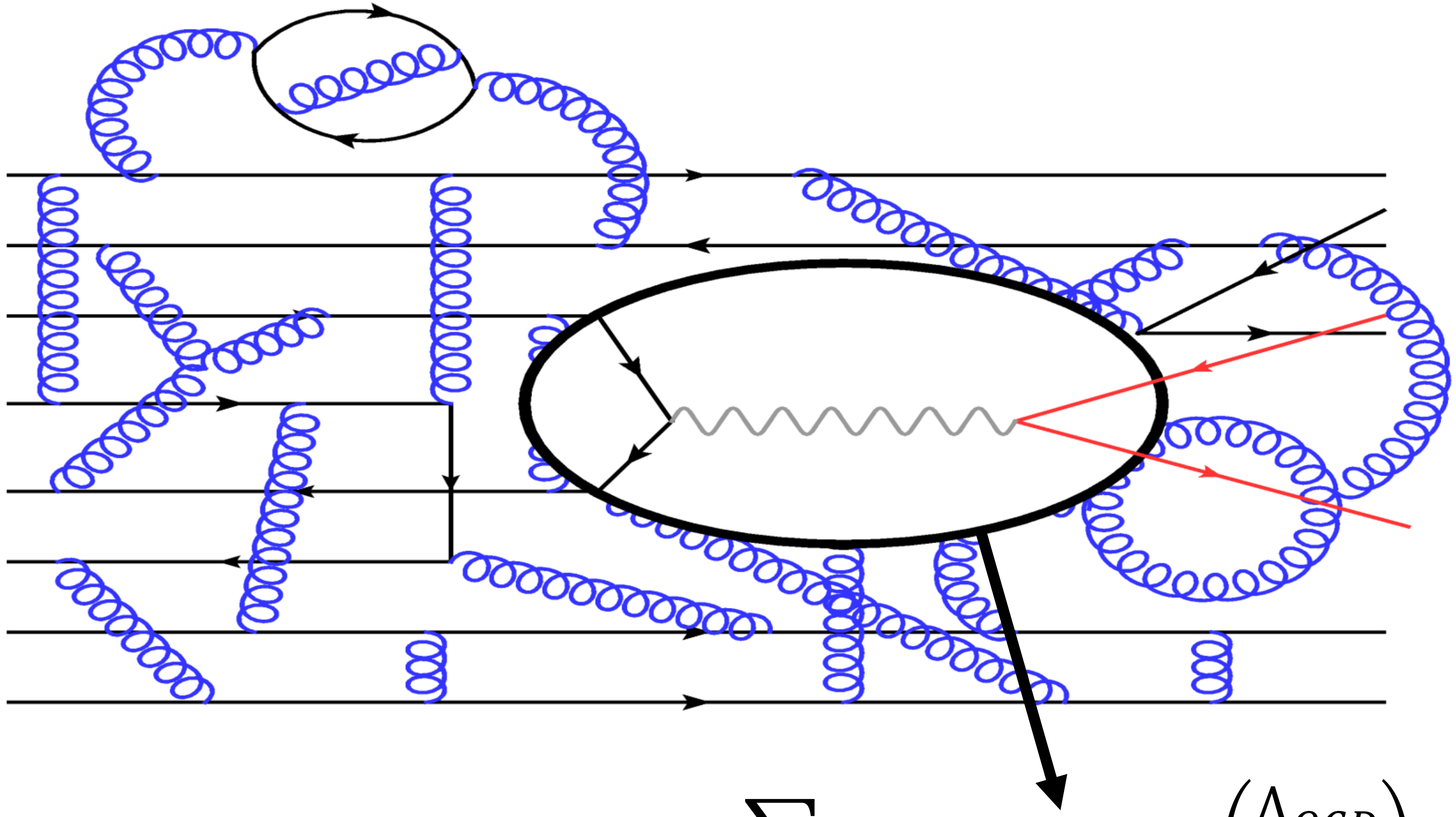
Outline

- Factorization and SCET
- Power corrections and endpoint divergence
- Factorization without modes
- DIS (and DY) at NLP
- Conclusion



$$\sigma(P + P \to l^+ l^- + X) = \sum_{ij} f_i \otimes f_j \otimes H_{ij} + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)$$

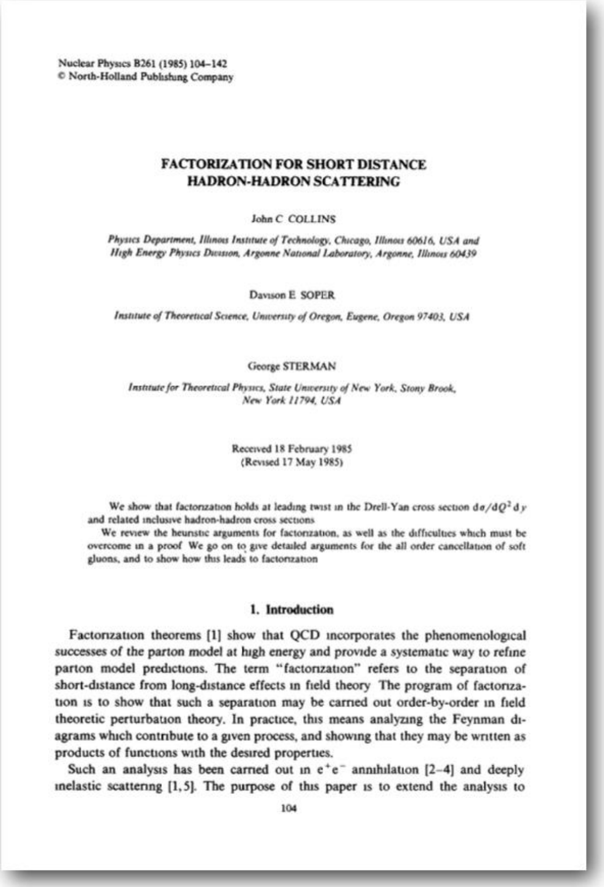
f_i: Parton distribution function

H_{ij}: Hard scattering cross-section

“ Factorization ”

- Most observables do not factorize in a simple manner.

- Proofs of factorization are long and complicated (based on analysis of Feynman diagrams).



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FACTORIZATION FOR SHORT DISTANCE HADRON-HADRON SCATTERING

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We show that factorization holds at leading twist in the Drell-Yan cross section $d\sigma/dQ^2\,dy$ and related inclusive hadron-hadron cross sections.
We review the heuristic arguments for factorization, as well as the difficulties which must be overcome in a proof. We go on to give detailed arguments for the all order cancellation of soft gluons, and to show how this leads to factorization.

1. Introduction

Factorization theorems [1] show that QCD incorporates the phenomenological successes of the parton model at high energy and provide a systematic way to refine parton model predictions. The term "factorization" refers to the separation of short-distance from long-distance effects in field theory. The program of factorization is to show that such a separation may be carried out order-by-order in field theoretic perturbation theory. In practice, this means analyzing the Feynman diagrams which contribute to a given process, and showing that they may be written as products of functions with the desired properties.
Such an analysis has been carried out in e^+e^- annihilation [2–4] and deeply inelastic scattering [1,5]. The purpose of this paper is to extend the analysis to

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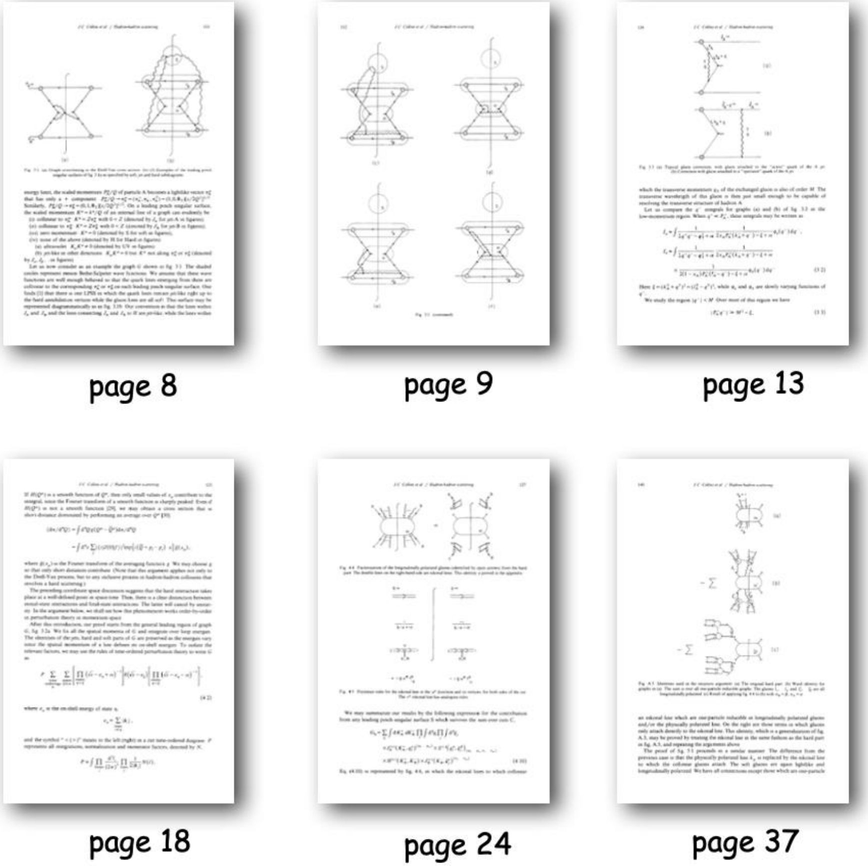
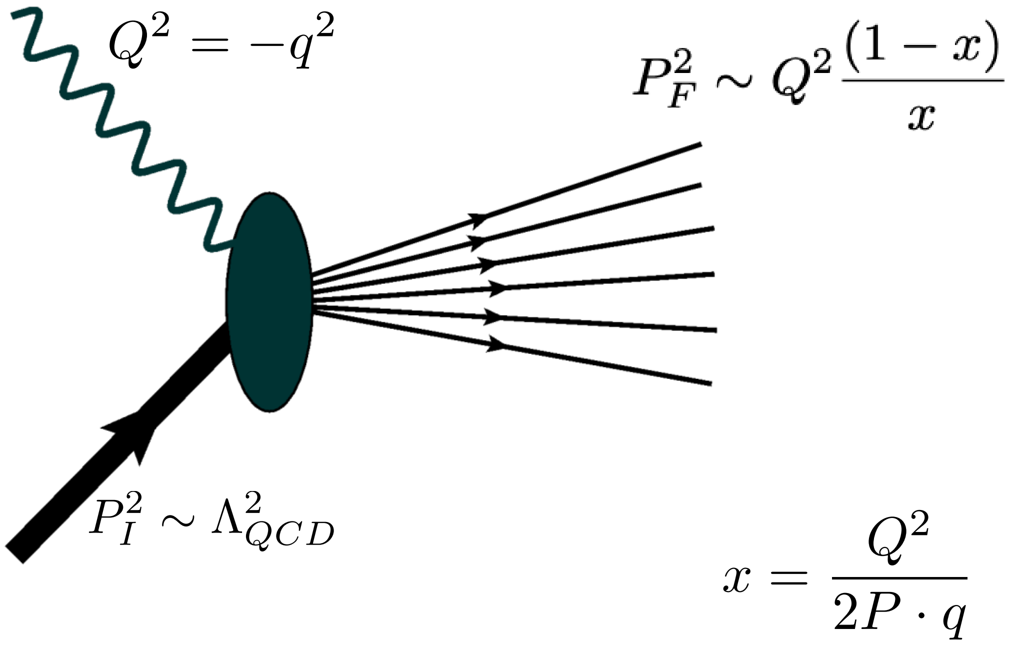


(Collins, Soper, Sterman, 1980's)

Slightly more complicated factorization

- Inclusive Deep Inelastic Scattering (DIS) in $x \to 1$
  - $P + \gamma^* \to X$
  - Hard scale: Invariant mass of the off-shell photon, $-Q^2$
  - Invariant mass of the outgoing final state, $Q^2 \frac{(1-x)}{x}$



Cross-section: $$\frac{d\sigma}{dx} = \int d\xi \sum_a f_a(\xi, \mu) \cdot \frac{d\hat{\sigma}}{dx}(a(\xi P) + \gamma \to X), \mu)$$

- Except the partonic cross-section is singular for $x \to 1$
- The integrated partonic rate, for $\mu \sim Q$

$$\int_{1-\Delta}^{1} \frac{d\hat{\sigma}}{dx} dx \sim$$

$$\begin{aligned}
&1 + \alpha_s \left(a_0 + a_1 \log\Delta + a_2 \log^2\Delta\right) \\
&\quad + \alpha_s^2 \left(b_0 + b_1 \log\Delta + b_2 \log^2\Delta + b_3 \log^3\Delta + b_4 \log^4\Delta\right) \\
&\quad + \alpha_s^3 \left(c_0 + c_1 \log\Delta + c_2 \log^2\Delta + c_3 \log^3\Delta + c_4 \log^4\Delta + c_5 \log^5\Delta + c_6 \log^6\Delta\right) \\
&\quad + O(\alpha_s^4) \\
&+\Delta \left[\alpha_s \left(d_0 + d_1 \log\Delta + d_2 \log^2\Delta\right)\right. \\
&\quad \left. + \alpha_s^2 \left(e_0 + e_1 \log\Delta + e_2 \log^2\Delta + e_3 \log^3\Delta + e_4 \log^4\Delta\right) + O(\alpha_s^3)\right] \\
&+O(\Delta)^2
\end{aligned}$$

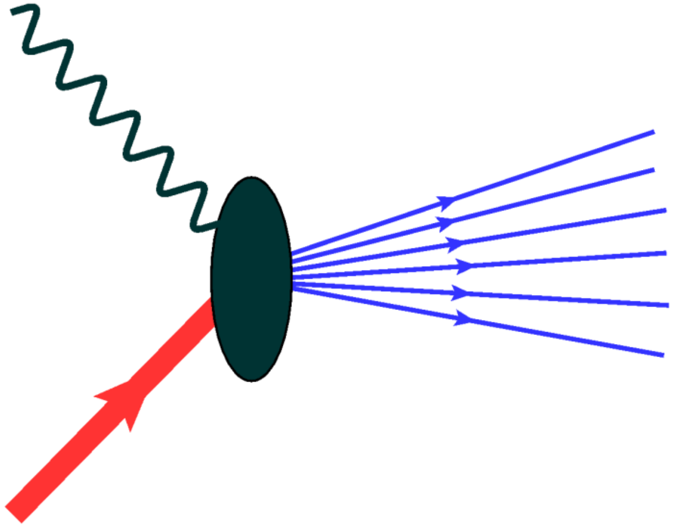
- So even if α_s is small, the large double logarithms (Sudakov logs) spoil the convergence of perturbation theory.

$$\frac{d\hat{\sigma}}{dx} = H(Q^2, \mu) \cdot J(Q^2(1-x), \mu)$$

"hard function" "jet function"

Factorization in SCET

[C. Bauer, S. Fleming and M. Luke (2000); C. Bauer, S. Fleming, D. Pirjol and I. Stewart (2001); M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann (2002)]



Relevant degrees of freedom (target rest frame)

$$p_c^\mu = (p^+, p^-, \bar{p}_\perp) \sim (\lambda^2 Q, Q, \lambda Q)$$

$$p_{us}^\mu = (p^+, p^-, \bar{p}_\perp) \sim (\lambda^2 Q, \lambda^2 Q, \lambda^2 Q)$$

1. Hard interaction:

$$J^{\mathrm{QCD}} = \bar{\psi}\gamma^\mu\psi \to J^{\mathrm{SCET}} = J^{(0)} + J^{(1)} + \ldots$$

$$J^{(0)} = C^{(0)}\bar{\xi}_n W_n \Gamma q$$

Collinear quark

Ultrasoft quark

Factorization in SCET

2. Manifest decoupling of ultrasoft and collinear d.o.f in the Lagrangian [Bauer et. al (2002)]

LP SCET Lagrangian: $$\mathcal{L}^{(0)}_{\xi\xi} = \sum_{\tilde{p},\tilde{p}'} \bar{\xi}_{n,\tilde{p}'} \left[in\cdot D + i\not{D}^{\perp}_{n} W_n \frac{1}{\bar{n}\cdot\mathcal{P}} W^{\dagger}_{n} i\not{D}^{\perp}_{n} \right] \frac{\not{\bar{n}}}{2} \xi_{n,\tilde{p}}$$

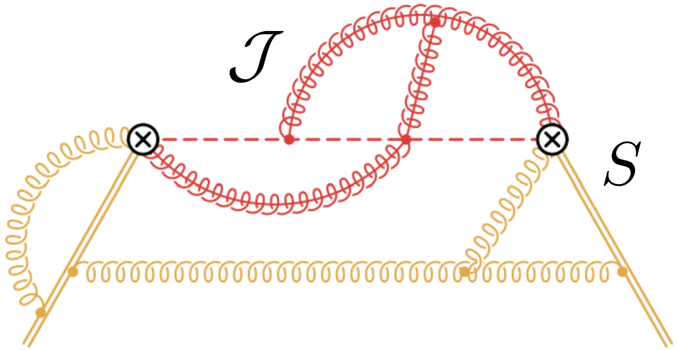
$$\xi_{n,p}(x) = Y_n(x)\xi^{(0)}_{n,p}(x)$$

$$A^{\mu}_{n,p}(x) = Y_n(x)\, A^{(0)\mu}_{n,p}(x) Y^{\dagger}_{n}(x)$$

$\Rightarrow$

$$J^{(0)} = C^{(0)} \left(\bar{\xi}^{(0)}_{n} W_n \right) \Gamma(Y^{\dagger}_{n} q)$$

3. Cross-sections (T-product of currents) fierzed into factorized form:



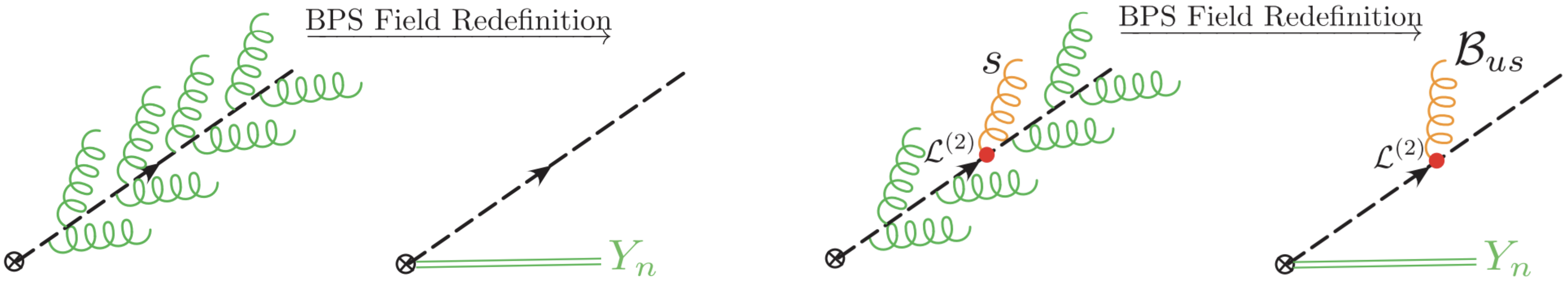
$$d\sigma = H \otimes \mathcal{J} \otimes S$$

[A. Manohar (2003); T. Becher et. al. (2006)]

What about power corrections?

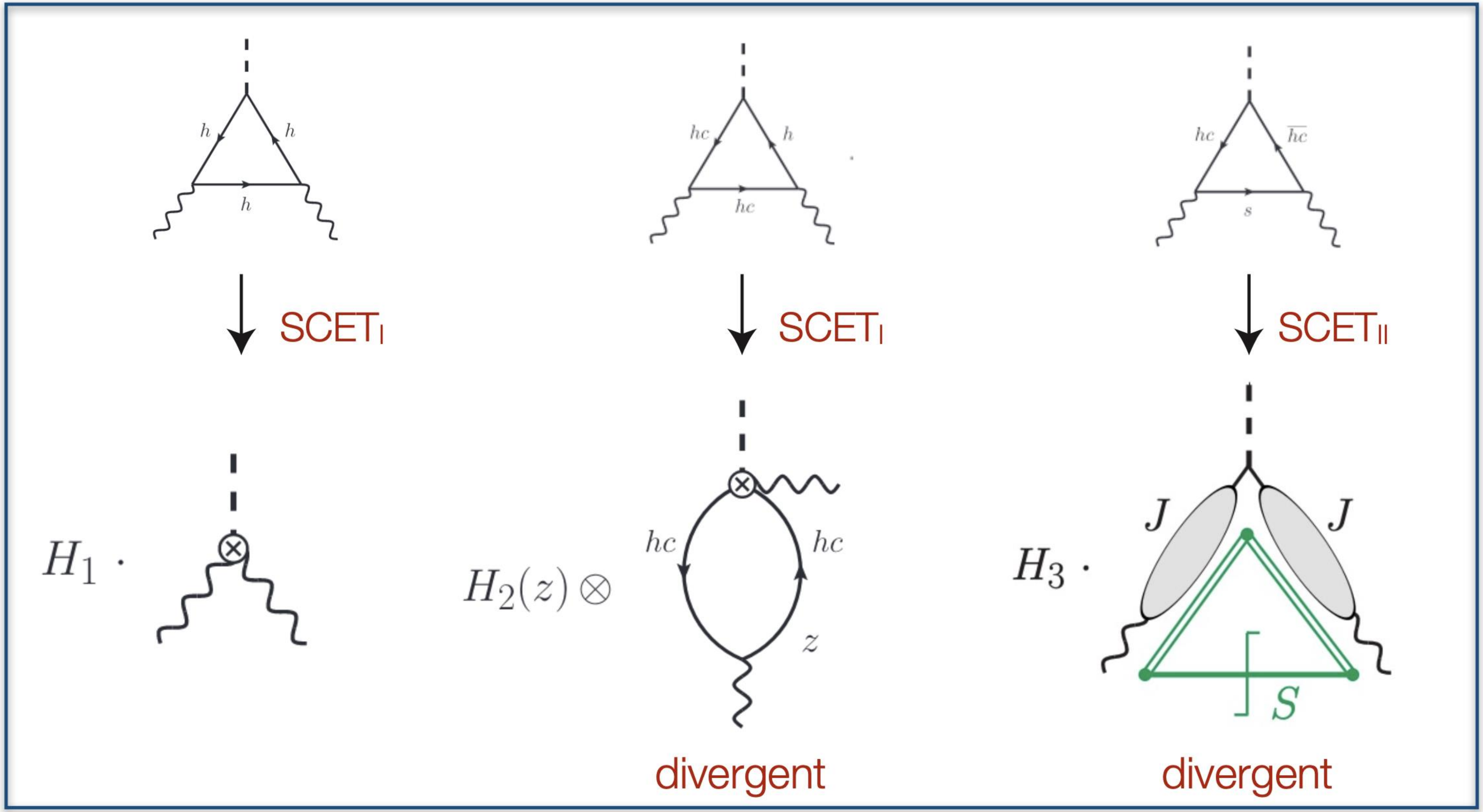
Should be easy. SCET has been around for >20 years but power corrections have been studied only in past few years...why?

- Decoupling of soft/ultrasoft from collinear in the Lagrangian fails at subleading power (can be extended using *radiative functions*) [I. Moult et. al. (2019)]



- Naïve factorization formulas break down for radiative corrections due to appearance of spurious divergences. [Z. L. Liu et. al. (2020, 2021), M. Beneke et. al. (2020, 2022)]

Ex: $h \to \gamma\gamma$ (via b quark loop)



[Z. L. Liu and M. Neubert (2020); Z. L. Liu et. al. (2021)]

$$
\begin{aligned}
\mathcal{M}_b = & \left(H_1^{(0)} + \Delta H_1^{(0)}\right) \langle\gamma\gamma|\, O_1^{(0)}|h\rangle \\
& + 2 \lim_{\delta\to 0} \int_\delta^{1-\delta} dz \left[H_2^{(0)}(z) \, \langle\gamma\gamma|\, O_2^{(0)}(z)\,|h\rangle - \frac{[\![\bar{H}_2^{(0)}(z)]\!]}{z} [\![\langle\gamma\gamma|\, O_2^{(0)}(z)\,|h\rangle]\!] \right. \\
& \qquad \left. - \frac{[\![\bar{H}_2^{(0)}(1-z)]\!]}{1-z} [\![\langle\gamma\gamma|\, O_2^{(0)}(1-z)\,|h\rangle]\!] \right] \\
& + g_\perp^{\mu\nu} \lim_{\sigma\to -1} H_3^{(0)} \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{d\ell_+}{\ell_+} \, J^{(0)}(M_h \ell_-)\, J^{(0)}(-M_h \ell_+)\, S^{(0)}(\ell_+ \ell_-)
\end{aligned}
$$

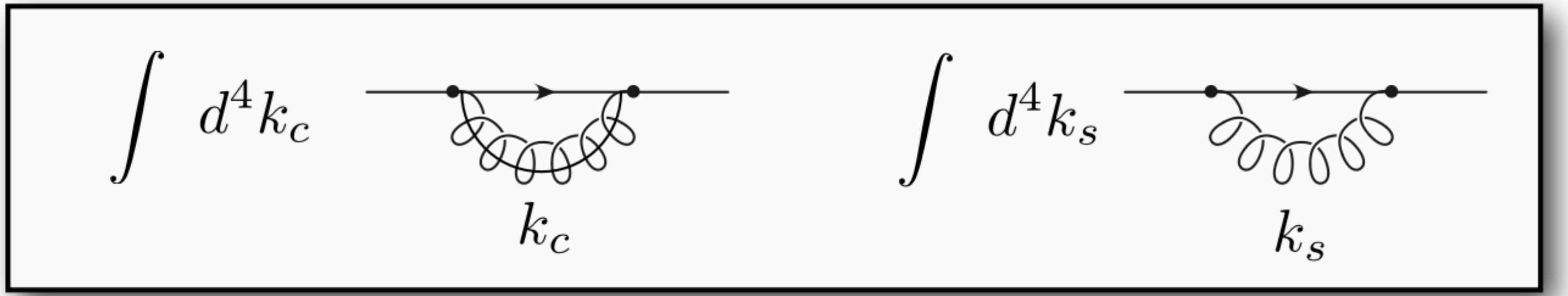
$$
[[\langle\gamma\gamma|O_2(z)|h\rangle]] \equiv \langle\gamma\gamma|O_2(z)|h\rangle|_{z\to 0}
$$

- SCET amplitude is finite, and terms can be rearranged to make individual contributions finite (“refactorization”).

- More complex refactorization conditions for other processes [G. Bell et. al. (2022)]

- No universal construction for *rearrangement*

SCET has different modes which decouple at LP but loops complicate stuff.



- double counting of degrees of freedom
- spurious divergences when loop integrals contain regions where the mode expansion fails (rapidity divergences, endpoint divergences).

Alternative framework

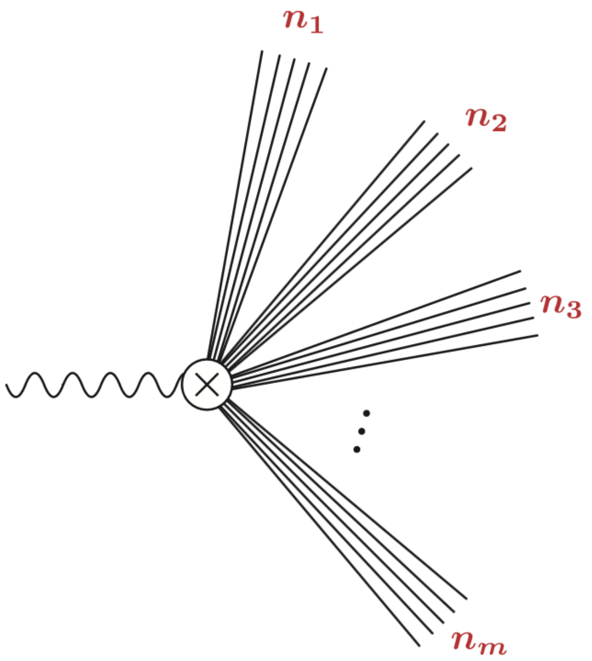
Drop the mode expansion.

Might provide another perspective if thought in terms of "traditional" EFT (4-Fermi, HQET).

- **What you don't get:** factorization into modes (e.g. $H \otimes J \otimes J \otimes S$)
- **What you get:** resummed cross-section factorized into matching coefficients and RG (virtuality and rapidity) evolution factors.
- **Simplifies power correction**

The EFT

[R. Goerke and M. Luke (2018)]



$$p_{n_i} \cdot p'_{n_i} \ll Q^2$$

$$p_{n_i} \cdot p'_{n_j} \sim Q^2$$

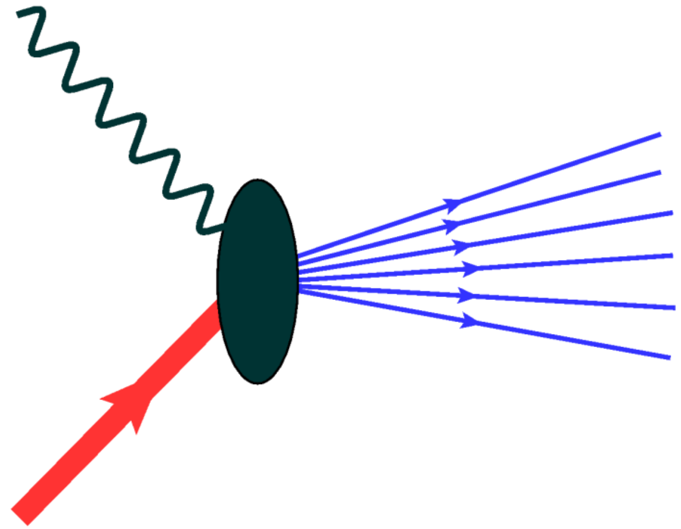
- Integrating out physics above $\mu^2 \geq Q^2$ requires us to define "sectors": states in the same sector have small invariant mass; invariant mass between different sectors is large. Sectors contain all degree of freedom below the cutoff.

- Dynamics within each sector is described by QCD while interaction between different sectors is given by effective operators suppressed by the hard scale.

DIS (2-sector theory)

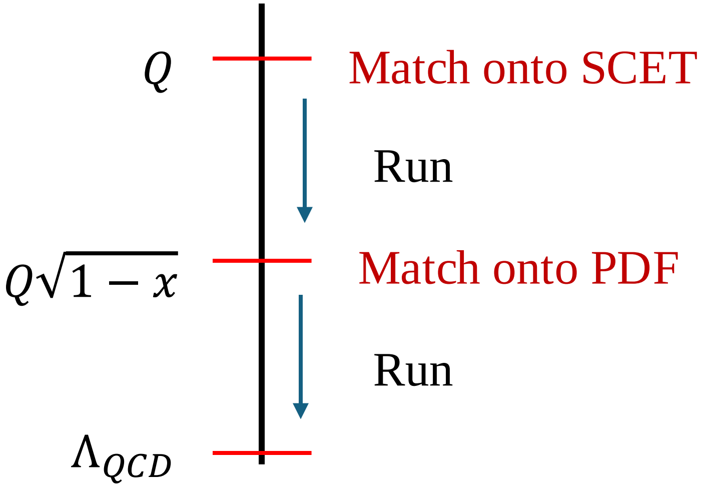
$$\mathcal{L} = \mathcal{L}^{n}_{QCD} + \mathcal{L}^{\bar{n}}_{QCD} + \mathcal{J}^{\mu} A_{\mu}$$

$$\mathcal{J}^{\mu}(x) = \sum_i \frac{1}{Q^{[i]}} C_2^{(i)}(\mu)\, O_2^{(i)\mu}(x,\mu)$$

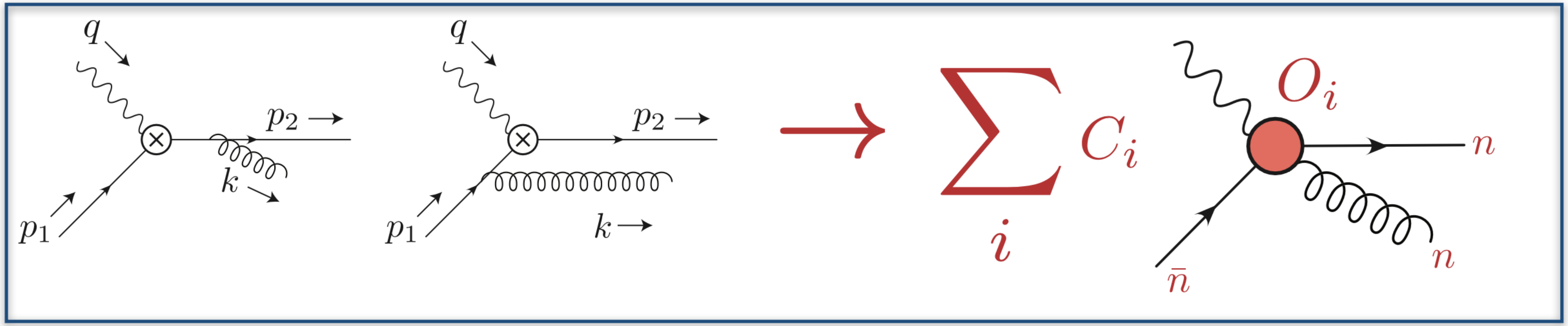


$$p_n^2 \sim Q^2(1-x) << Q^2$$
$$p_{\bar{n}}^2 \sim \Lambda^2_{QCD} << Q^2$$
$$p_n \cdot p_{\bar{n}} \sim Q^2$$

- Power corrections only arise from expansion of the current



$\mu = Q$: Match QCD to SCET



Expand QCD amplitude in powers of $\frac{p_1 \cdot \bar{n}}{Q}, \frac{p_2 \cdot n}{Q}, \frac{k \cdot n}{Q}, \frac{p_{i\perp}}{Q}, \frac{k_\perp}{Q}$

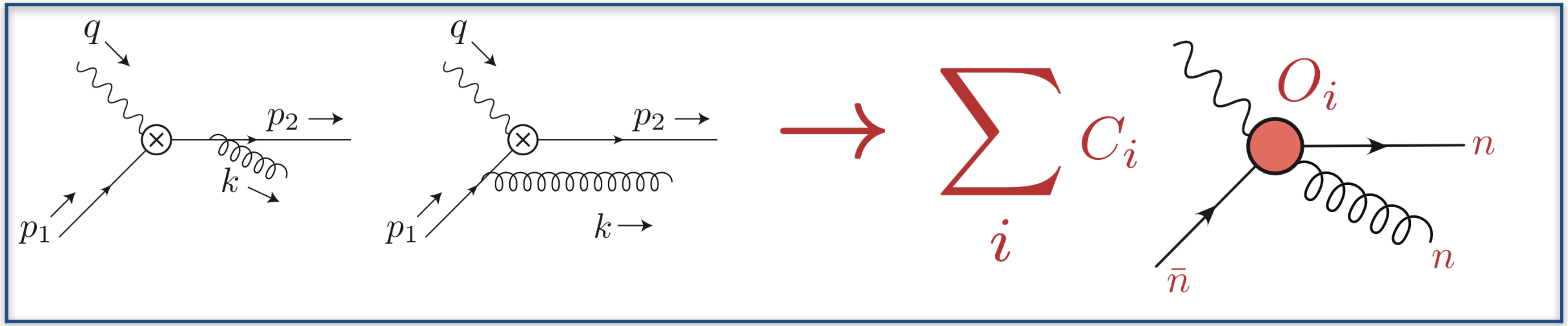
Gauge invariant operator blocks:

$$\bar{\chi}_{\bar{n}}(x) = \bar{\psi}_{\bar{n}}(x) \overline{W}_{\bar{n}}(x) P_n$$

$$\chi_n(x) = \overline{W}_n^\dagger(x) P_n \psi_n(x)$$

$$\mathcal{B}_{\bar{n}}^{\mu_1 \cdots \mu_N}(x) = \overline{W}_{\bar{n}}^\dagger(x) i D_{\bar{n}}^{\mu_1}(x) \cdots i D_{\bar{n}}^{\mu_N}(x) \overline{W}_{\bar{n}}(x)$$

$\mu = Q$: Match QCD to SCET



$$i\mathcal{A}^\mu = -\, g_s T^a \bar{u}(p_2)\Bigg[\frac{2p_2^\alpha + \gamma^\alpha \not{k}}{2p_2\cdot k} P_{\bar{n}}\gamma^\mu P_{\bar{n}} - P_{\bar{n}}\gamma^\mu P_{\bar{n}}\frac{\bar{n}^\alpha}{\bar{n}\cdot k}$$

$$O_2^{(0)\mu}(x) = \bar{\chi}_n(x)\gamma^\mu \chi_{\bar{n}}(x)$$

$$+\frac{1}{Q}\left(\bar{\Delta}^{\alpha\beta}(k)\gamma_\beta^\perp \frac{\not{\bar{n}}}{2}\gamma^\mu P_{\bar{n}} + P_{\bar{n}}\gamma^\mu \frac{\not{n}}{2}\gamma_\beta^\perp \bar{\Delta}^{\alpha\beta}(k)\right)$$

$$O_2^{(1A)\mu}(x,\hat{t}) = -\,\bar{\chi}_n(x)\mathcal{B}_n^\alpha(x+\bar{n}t) \times \left(\gamma_\alpha^\perp \frac{\not{\bar{n}}}{2}\gamma^\mu + \gamma^\mu \frac{\not{n}}{2}\gamma_\alpha^\perp\right)\chi_{\bar{n}}(x)$$

$$+\frac{1}{Q^2}P_{\bar{n}}\gamma^\mu P_{\bar{n}}\gamma_\beta^\perp\gamma_\gamma^\perp k^\beta \bar{\Delta}^{\alpha\gamma}(k)\left(1+\frac{\bar{n}\cdot p_2}{\bar{n}\cdot k}\right)$$

$$-\frac{1}{Q^2}\gamma_\beta^\perp \frac{\not{\bar{n}}}{2}\gamma^\mu \frac{\not{n}}{2}\gamma_\gamma^\perp k^\beta \bar{\Delta}^{\alpha\gamma}(k)\left(1+\frac{\bar{n}\cdot k}{\bar{n}\cdot p_2}\right)\Bigg]$$

$$O_2^{(2A_1)\mu}(x,\hat{t}) = 2\pi i\theta(\hat{t}) \otimes \bar{\chi}_n(x)\mathcal{B}_n^{\alpha\beta}(x+\bar{n}t)\gamma^\mu\gamma_\alpha^\perp\gamma_\beta^\perp\chi_{\bar{n}}(x)$$

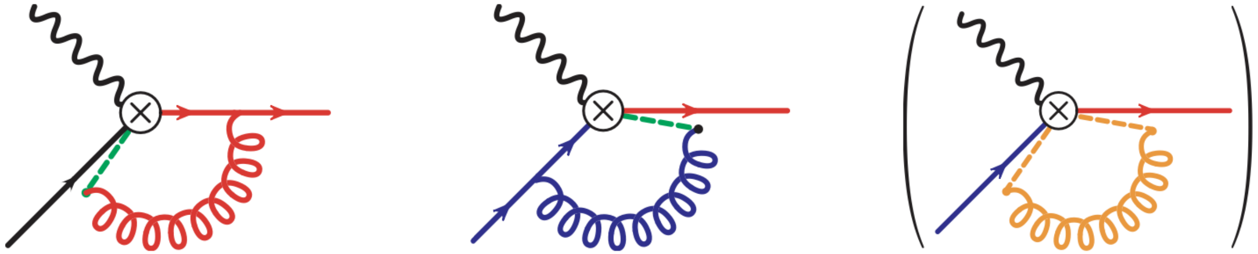
$$O_2^{(2A_2)\mu}(x,\hat{t}) = -\,2\pi i\theta(\hat{t}) \otimes \bar{\chi}_n(x+\bar{n}t)\mathcal{B}_n^{\alpha\beta}(x)\gamma_\alpha^\perp\frac{\not{\bar{n}}}{2}\gamma^\mu\frac{\not{n}}{2}\gamma_\beta^\perp\chi_{\bar{n}}(x)$$

$$\times\, u(p_1)\varepsilon_\alpha^*(k) + \ldots$$

The complication: Double counting

[A. Manohar and I. Stewart (2007); A. Idilbi and T. Mehen (2007); C. Lee and G. Sterman (2007)]

- some degree of freedom have momenta that fall below the cutoff of more than one sector - these get double counted
- matrix elements in SCET are well-defined if double counting between modes (0-bin)/ sectors (overlap subtraction) has been removed
- acute in this framework: without subtraction, loop graphs have IR-dependent counterterms

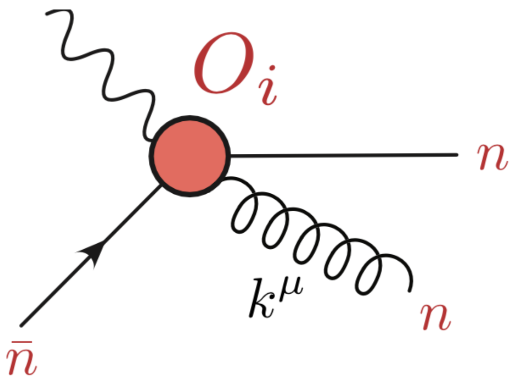


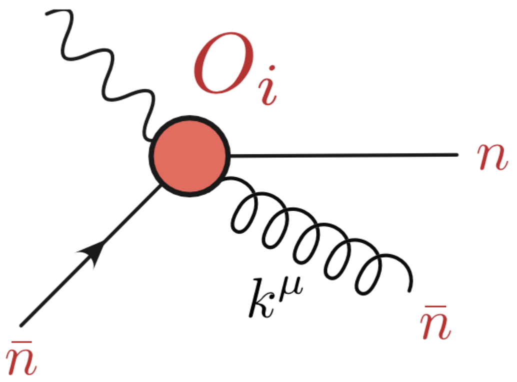
$$I \sim -i\frac{\mu^{2\epsilon}e^{\gamma_E \epsilon}}{(4\pi)^{2-2\epsilon}}\Gamma(\epsilon)\left(\int_0^1 m_g^{-2\epsilon}(1-z)^{1-\epsilon}\frac{dz}{z} - \int_0^\infty \left(m_g^{-2\epsilon} - (Q^2 z + m_g^2)^{-\epsilon}\right)\frac{dz}{z} - \int_0^\infty m_g^{-2\epsilon}\frac{dz}{z}\right)$$

(+ wave function)

$$= \frac{C_F \alpha_s}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{3 - 2\log\frac{Q^2}{\mu^2}}{\epsilon} - \log^2\frac{m_g^2}{\mu^2} + 2\log\frac{Q^2}{\mu^2}\log\frac{m_g^2}{\mu^2} - 3\log\frac{m_g^2}{\mu^2} - \frac{5\pi^2}{6} + \frac{9}{2}\right]$$

Overlap subtraction prescription

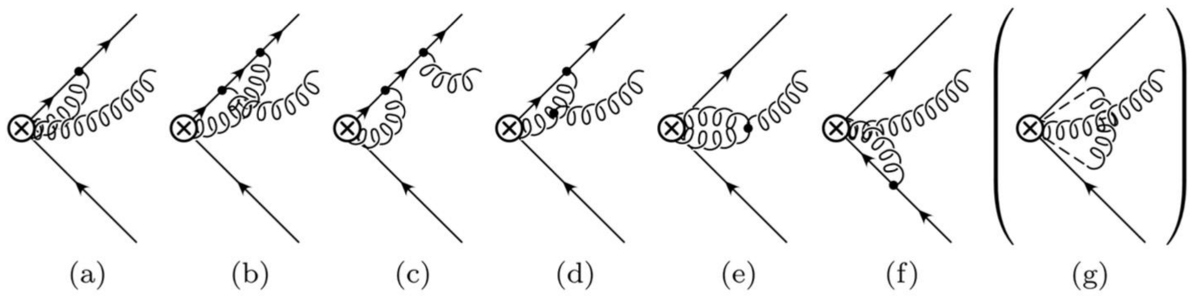




- in the regime $k \cdot n, k \cdot \bar{n} \ll Q$, the same gluon is double counted in the EFT
- as in 0-bin, expand "wrong sector" in $\bar{n} \to n$ limit and subtract
- this will be crucial in the NLP calculation for cancelling of endpoint divergences.

[M. Inglis-Whalen and R. Goerke (2018)]

$Q > \mu > Q\sqrt{1-x}$: RG evolution



$$\frac{d}{d\log\mu} O_2^{(j)}(x,u) = -\int dv\, \gamma_2^{(j)}(u,v) O_2^{(j)}(x,v)$$

$$\begin{aligned}
\gamma_{(1a)}(u,v) = {} & \frac{\alpha_s \delta(u-v)}{\pi} \left[C_F \left(\log\frac{-Q^2}{\mu^2} - \frac{3}{2} + \log\bar{v} \right) + \frac{C_A}{2} \left(1 + \log\frac{v}{\bar{v}} \right) \right] \\
& + \frac{\alpha_s}{\pi} \left(C_F - \frac{C_A}{2} \right) \bar{u} \left(\frac{uv}{\bar{u}\bar{v}} \theta(1-u-v) + \frac{uv+u+v-1}{uv} \theta(u+v-1) \right) \\
& + \frac{\alpha_s}{\pi} \frac{C_A}{2} \bar{u} \left(\frac{\bar{v}-uv}{u\bar{v}} \theta(u-v) + \frac{\bar{u}-uv}{v\bar{u}} \theta(v-u) \right. \\
& \left. - \frac{1}{\bar{u}\bar{v}} \left[\bar{u}\frac{\theta(u-v)}{u-v} + \bar{v}\frac{\theta(v-u)}{v-u} \right]_+ \right)
\end{aligned}$$

$$\begin{aligned}
\gamma_{(1c)}(u,v) = {} & \frac{\alpha_s \delta(u-v)}{\pi} \left[\frac{1}{2} C_F + C_A \left(\log\frac{-Q^2}{\mu^2} - 1 + \frac{1}{2}\log v\bar{v} \right) \right] \\
& - \frac{\alpha_s}{\pi} \left(C_F - \frac{C_A}{2} \right) \frac{1}{v\bar{v}} \left(v\bar{u}\theta(u-v) + u\bar{v}\theta(v-u) \right. \\
& \left. + \left[\bar{u}v\frac{\theta(u-v)}{u-v} + \bar{v}u\frac{\theta(v-u)}{v-u} \right]_+ \right).
\end{aligned}$$

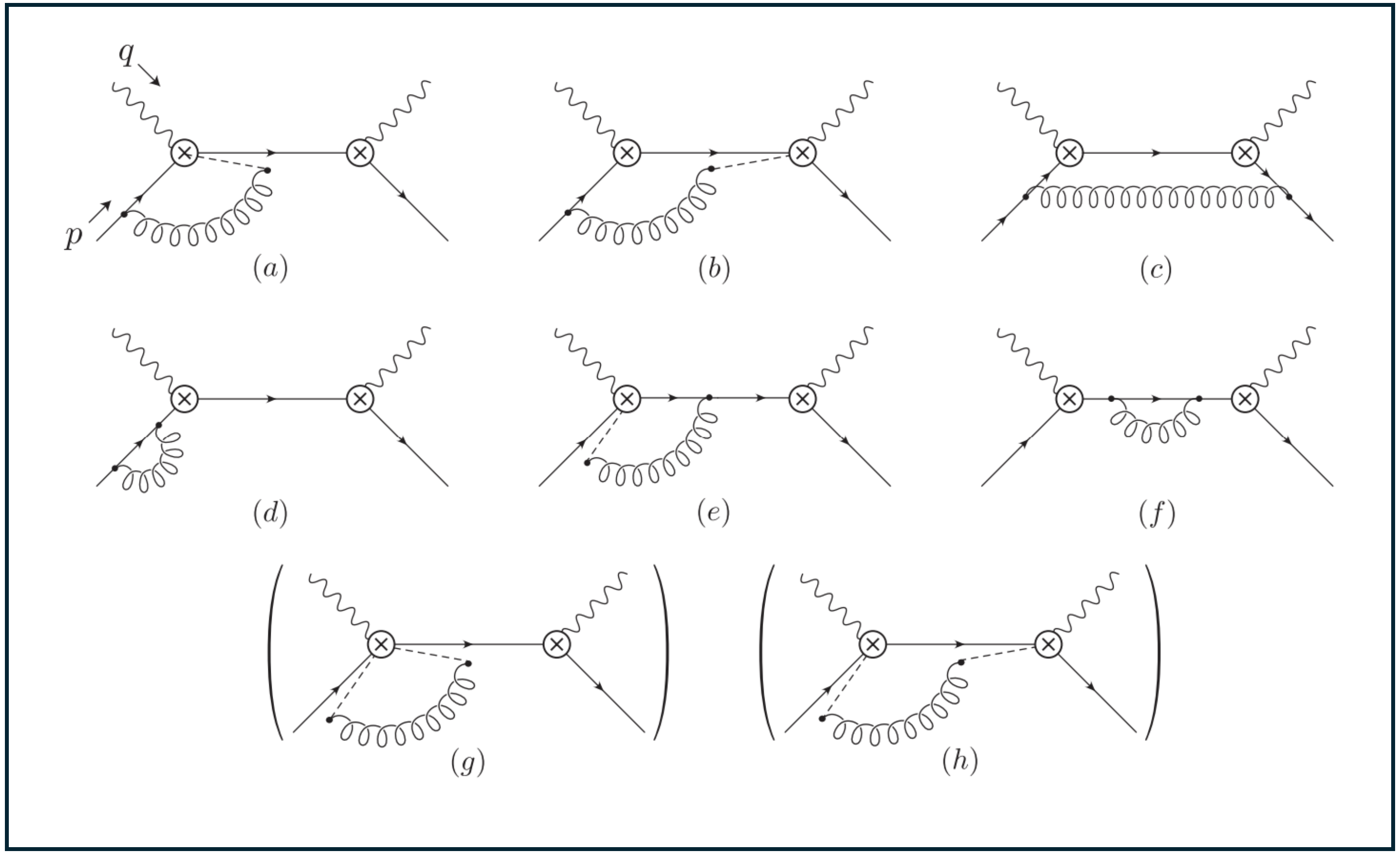
$$\begin{aligned}
\gamma_2^{(2a_1)}(u,v) = {} & \frac{\alpha_s}{\pi} \delta(u-v) \left[C_F \left(\log\frac{-Q^2}{\mu^2} + \log(\bar{v}) - \frac{3}{2} \right) + \frac{C_A}{2} \left(\log\frac{v}{\bar{v}} + \frac{5}{2} \right) \right] \\
& + \frac{\alpha_s}{\pi} \left(C_F - \frac{C_A}{2} \right) \frac{1}{v\bar{v}^2} \left(\bar{u}^2\bar{v}^2\, \theta(u+v-1) + uv(\bar{u}\bar{v} + \bar{u} + \bar{v} - 1)\theta(1-u-v) \right) \\
& - \frac{\alpha_s}{\pi} \frac{C_A}{2} \frac{1}{v\bar{v}^2} \left(v\bar{u}^2(1+\bar{v})\theta(u-v) + u\bar{v}^2(1+\bar{u})\theta(v-u) \right. \\
& \left. + \left[v\bar{u}^2\frac{\theta(u-v)}{u-v} + u\bar{v}^2\frac{\theta(v-u)}{v-u} \right]_+ \right),
\end{aligned}$$

$$\begin{aligned}
\gamma_2^{(2a_2)}(u,v) = {} & \frac{\alpha_s}{\pi} \delta(u-v) \left[C_F \left(\log\frac{-Q^2}{\mu^2} + \log(v) - \frac{3}{2} \right) + \frac{C_A}{2} \left(\log\frac{\bar{v}}{v} + \frac{5}{2} \right) \right] \\
& + \frac{\alpha_s}{\pi} \left(C_F - \frac{C_A}{2} \right) \frac{1}{\bar{v}v^2} \left(\frac{uv}{\bar{u}\bar{v}} (\bar{u}-v)(\bar{v}-u)\theta(1-u-v) \right) \\
& - \frac{\alpha_s}{\pi} \frac{C_A}{2} \frac{1}{\bar{v}v^2} \left(\frac{v\bar{u}(\bar{v}-u)}{\bar{v}}\theta(u-v) + \frac{u\bar{v}(\bar{u}-v)}{\bar{u}}\theta(v-u) \right. \\
& \left. + \left[\bar{u}v^2\frac{\theta(u-v)}{u-v} + \bar{v}u^2\frac{\theta(v-u)}{v-u} \right]_+ \right).
\end{aligned}$$

$\mu = Q\sqrt{1-x}$: Match onto PDF

$$T^{\mu\nu} = \text{Disc}\,\frac{1}{2\pi}\int d^dx\; e^{-iq\cdot x}T\left[\mathcal{J}^{\mu\dagger}(x)\mathcal{J}^{\nu}(0)\right] \quad \to \int \frac{dw}{w}C^{\mu\nu}(w)\phi(-q^+/w) + \ldots$$

$$\phi(r^+) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dt\, e^{-ir^+t}\,\bar{\psi}(nt)W(nt,0)\not{n}\psi(0)$$



Overlap graphs

$$I = \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{2}{\epsilon^2} + \frac{2\log\frac{\mu^2}{Q^2} + 3}{\epsilon}\right)\delta(1-y) - \frac{1}{\epsilon}\left(\frac{1+y^2}{(1+y)_+} + \frac{3}{2}\delta(1-y)\right) + \text{finite terms}\right]$$

Counterterm of $O_2^{(0)}$ Altarelli-Parisi splitting kernel

LP factorization:

$$T^{\mu\nu} = \left|C_2^{(0)}(\mu)\right|^2 C_J^{(0,T)}(w,\mu) g_\perp^{\mu\nu} \otimes \phi(w) + \ldots$$

$$C_J^{(0,T)}(w) = -\,\delta(1-w) - \frac{\alpha_s C_F}{2\pi}\left\{\left(\log^2\frac{Q^2}{\mu^2} - \frac{3}{2}\log\frac{Q^2}{\mu^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)\delta(1-w)\right.$$
$$\left. + \left((1+w^2)\log\frac{Q^2}{\mu^2 w} - \frac{3}{2}\right)\frac{1}{[1-w]_+} + (1+w^2)\left[\frac{\log(1-w)}{1-w}\right]_+ + \frac{1}{2}(3+w)\theta(1-w)\right\}$$

(same result everyone gets)

What's different?

Alternative formalism

- Similar to traditional EFT's: Hierarchy between d.o.f. below the cutoff not distinguished.

- Inclusive rates: perform OPE and match onto low energy theory.

- Factorization: automatically into Wilson coefficient (short distance) and matrix elements (long distance) at matching scale.

"Jet function" - matching coefficient onto the soft theory.

Standard SCET

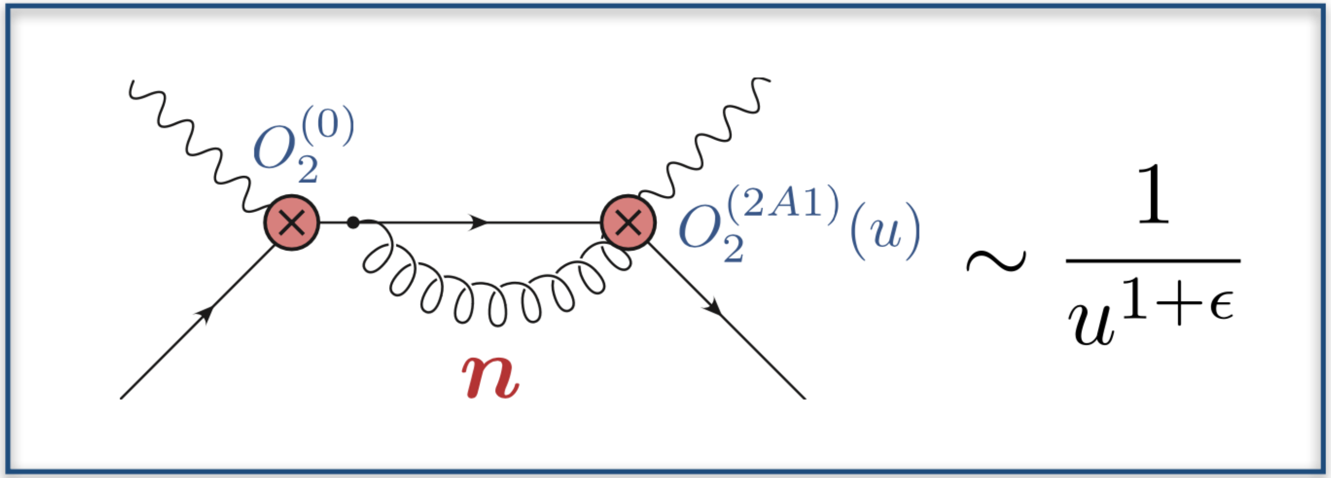
- d.o.f. below cutoff separated into modes.

- Inclusive rates: fierz into factorizable form, renormalize each factor at appropriate scale.

- Factorization: Soft and collinear separation in the Lagrangian. Hard physics separates as Wilson coefficient in the current.

Endpoint divergence at NLP

$$F_{T,n}^{(0,2A_1)} = T[O_2^{(2A_1)}(u)O_2^{(0)}]$$



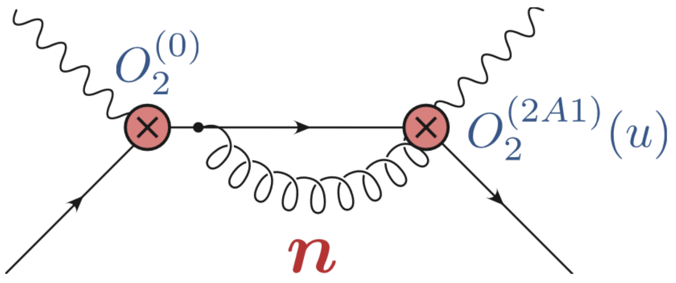
$$\int du\, F_{T,n}^{(0,2A_1)} = \frac{\alpha_s C_F}{2\pi}\frac{\theta(1-y)}{y}\left(-\frac{1}{\epsilon} + \log\frac{Q^2(1-y)}{\mu^2 y} - \frac{3}{2}\right)$$

$$u = k\cdot\bar{n}$$

- spurious divergence - neither UV (no counterterm) or IR (not in matrix element of distribution function)

- arises from region of phase space integration where both sectors contribute – should be fixed by overlap subtraction

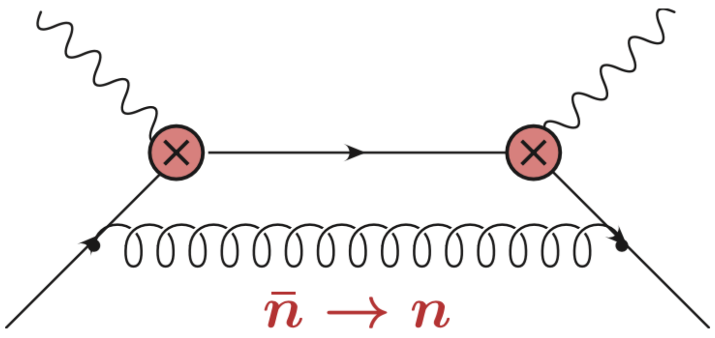
Endpoint divergence at NLP



$$\sim \frac{1}{u^{1+\epsilon}}$$

$$\int du\, F_{T,n}^{(0,2A_1)} = \frac{\alpha_s C_F}{2\pi}\frac{\theta(1-y)}{y}\left(-\frac{1}{\epsilon} + \log\frac{Q^2(1-y)}{\mu^2 y} - \frac{3}{2}\right)$$

- consistently expand the overlap to NLP order



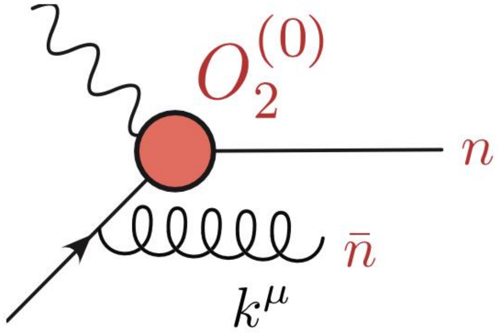
$O(1)$: LP overlap

$O(1/Q^2)$: NLP overlap

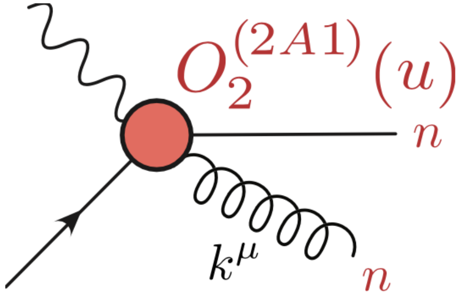
$$F_{T,\bar{n}\to n}^{(0,0),\mathrm{NLP}} = \frac{\alpha_s C_F}{\pi}\frac{\theta(1-y)}{y}\left(-\frac{1}{\epsilon} + \log\frac{Q^2(1-y)}{\mu^2 y} - 1\right)$$

Consistency

Cancellation happen between terms which run differently → nontrivial constraints on anomalous dimension



$$\gamma_2^{(0)} = \frac{\alpha_s C_F}{\pi}\left(\log\frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$



$$\begin{aligned}
\gamma_2^{(2A_1)}(u,v) = \frac{\alpha_s}{\pi}\Bigg(&\delta(u-v)\left\{C_F\left(\log\frac{Q^2}{\mu^2} - \frac{3}{2} + \log\bar{v}\right)\right.\\
&\left.+\frac{C_A}{2}\left(\frac{5}{2} + \log\frac{v}{\bar{v}}\right)\right\}\\
&+\left(C_F - \frac{C_A}{2}\right)\left\{\frac{\bar{u}^2}{u}\theta(u+v-1)\right.\\
&\left.+\frac{v}{\bar{v}^2}(\bar{u}\bar{v}+\bar{u}+\bar{v}-1)\theta(1-u-v)\right\}\\
&-\frac{C_A}{2u\bar{v}^2}\left\{v\bar{u}^2(1+\bar{v})\theta(u-v) + u\bar{v}^2(1+\bar{u})\theta(v-u)\right.\\
&\left.+\left[v\bar{u}^2\frac{\theta(u-v)}{u-v} + u\bar{v}^2\frac{\theta(v-u)}{v-u}\right]_+\right\}\Bigg),
\end{aligned}$$

Consistency

- QCD current at $\mathcal{O}(\alpha_s)$ matches onto the integrated operator

$$\overline{O}_2^{(2A_1)}(\mu) \equiv \int_0^1 du\ O_2^{(2A_1)}(u,\mu)$$

- Since

$$\int_0^1 du\ \gamma_2^{(2A_1)}(u,v) = \gamma_2^{(0)} = \frac{\alpha_s C_F}{\pi} \log\left(\frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$

$$\mu \frac{d}{d\mu} \overline{O}_2^{(2A_1)}(\mu) = -\gamma_2^{(0)} \overline{O}_2^{(2A_1)}(\mu)$$

- Subleading integrated operator runs the same as leading operator.

[M. Luke, JR and A. Spourdalakis (2023)]

- Factorization at NLP

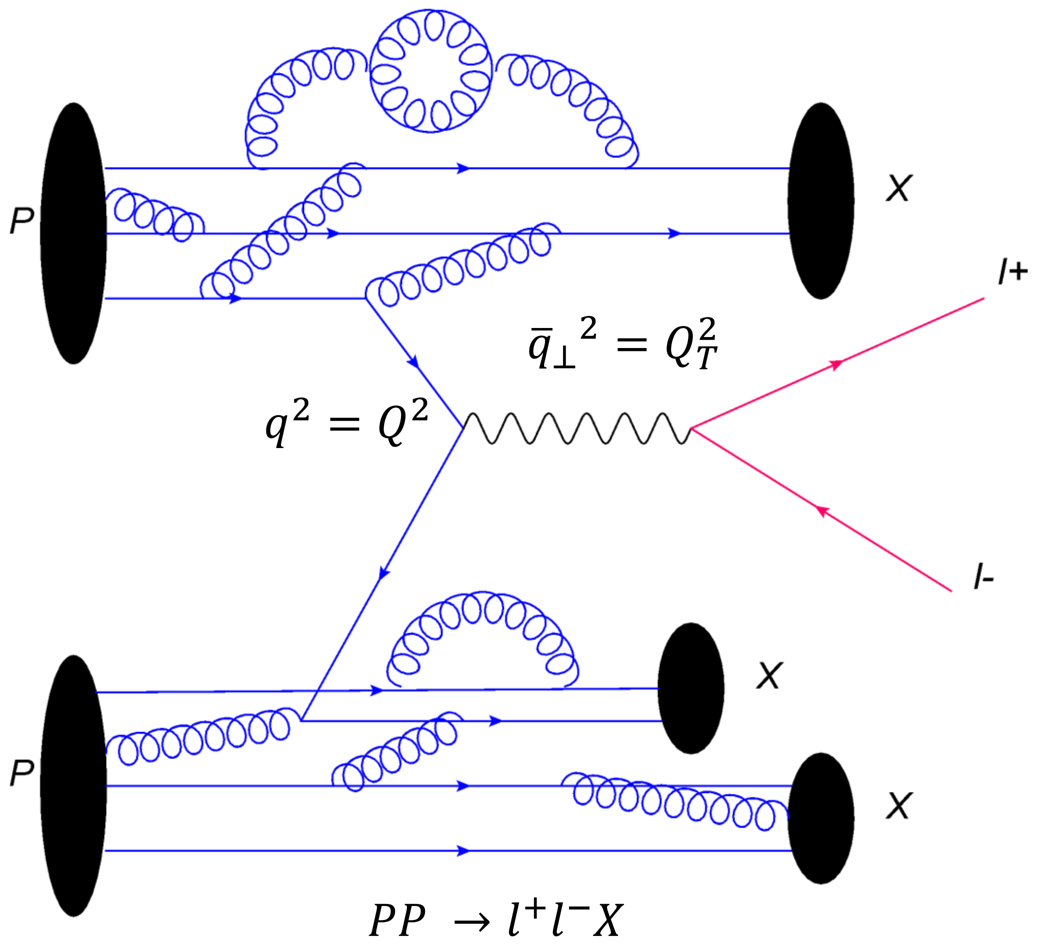
Matching onto PDF

$$C^{\mu\nu}(w) = \left|C_2^{(0)}(\mu)\right|^2 \left[C_J^{(0,T)}(w) + C_J^{(2,T)}(w)\right] g_\perp^{\mu\nu}$$
$$+ \int du\, dv\, C_2^{(1A)\dagger}(u,\mu) C_2^{(1A)}(v,\mu) C_J^{(2,L)}(u,v,w) L^{\mu\nu} + \ldots$$

Matching onto SCET

$$C_J^{(0,T)}(w) = -\,\delta(1-w) - \frac{\alpha_s C_F}{2\pi}\left\{\left(\log^2\frac{Q^2}{\mu^2} - \frac{3}{2}\log\frac{Q^2}{\mu^2} - \frac{\pi^2}{2} + \frac{7}{2}\right)\delta(1-w)\right.$$
$$\left. + \left((1+w^2)\log\frac{Q^2}{\mu^2 w} - \frac{3}{2}\right)\frac{1}{[1-w]_+} + (1+w^2)\left[\frac{\log(1-w)}{1-w}\right]_+ + \frac{1}{2}(3+w)\theta(1-w)\right\}$$
$$C_J^{(2,T)}(w) = -\,\frac{\alpha_s C_F}{2\pi}\frac{\theta(1-w)}{w}$$
$$C_J^{(2,L)}(u,v,w) = \frac{2\alpha_s C_F}{\pi}\frac{\theta(1-w)}{w}(1-u)\theta(u)\theta(1-u)\delta(u-v)$$

Drell-Yan at small Q_T



- SCET$_{II}$ process in the mode picture.
- Scales :

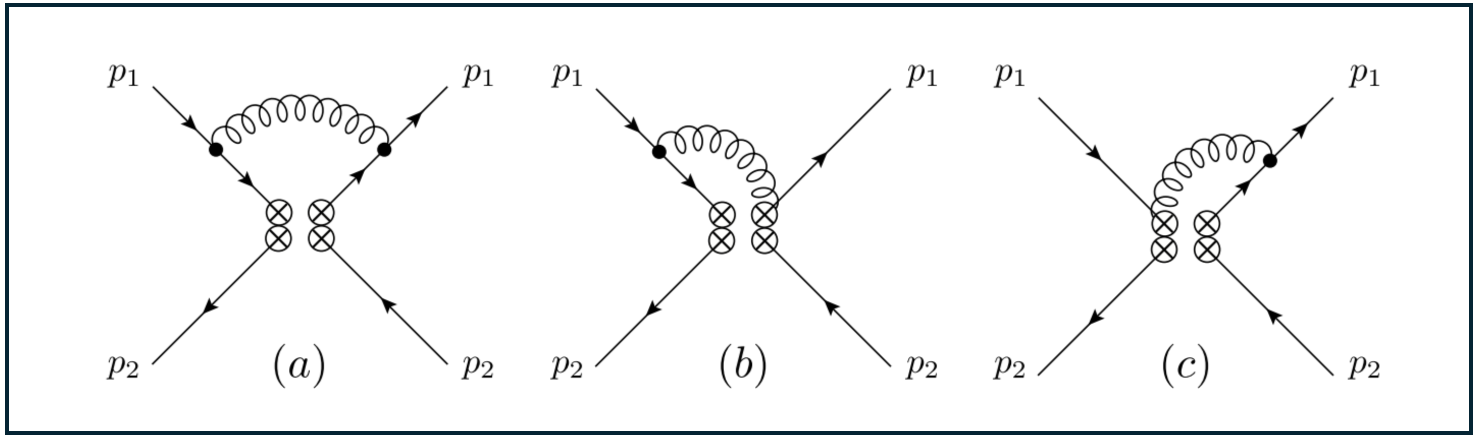
$$Q^2 \gg Q_T^2 \gg \Lambda_{QCD}^2$$

- At $\mu = Q$, same story

QCD ⟶ SCET

- Run down to Q_T and match the product of current onto PDFs.

- Inclusive rates given by matrix elements of operator products.



- can Fierz operator products into convolutions of subleading TMD's

$$T_{(k,\ell)}(q,\{u\})=\int\frac{d^dx}{2(2\pi)^d}e^{-iq\cdot x}\Phi_n^{(k)}(x_n,\{u\})\Phi_{\bar{n}}^{(\ell)}(x_{\bar{n}},\{u\})$$

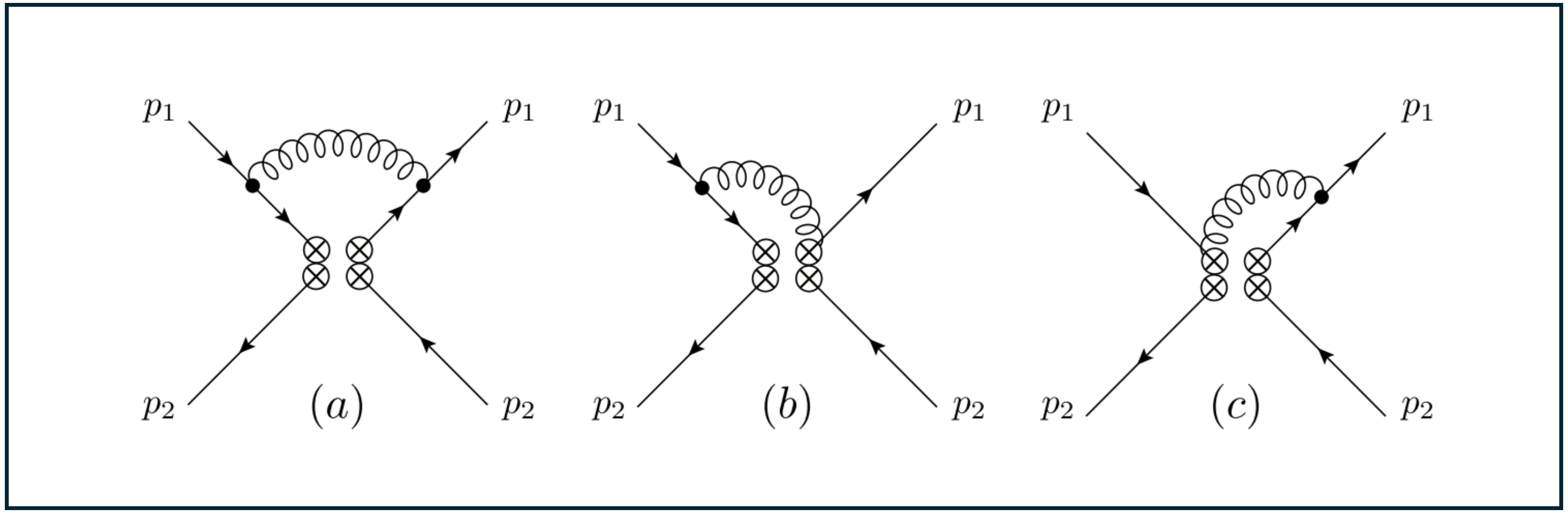
$$\Phi_n^{(0)}(x_n)\equiv\bar{\chi}_n(x_n)\frac{\bar{\not{n}}}{2}\chi_n(0)$$

$$\Phi_n^{(2_1)}(x_n,\hat{t})\equiv(i\partial^\mu\bar{\chi}_n(x_n))\frac{\bar{\not{n}}}{2}\gamma_\mu^\perp\gamma_\nu^\perp\mathcal{B}_n^{\dagger\nu}(-\bar{n}t)\chi_n(0)$$

$$\Phi_n^{(2_2)}(x_n,\hat{t}_1,\hat{t}_2)\equiv-\bar{\chi}_n(x_n)\mathcal{B}_n^\mu(x_n-\bar{n}t_1)\frac{\bar{\not{n}}}{2}\gamma_\mu^\perp\gamma_\nu^\perp$$

$$\Phi_n^{(2_3)}(x_n,\hat{t})\equiv 2\pi i\theta(\hat{t})\otimes\bar{\chi}_n(x_n)\mathcal{B}_n^{\mu\nu}(x_n-\bar{n}t)\frac{\bar{\not{n}}}{2}\gamma_\nu^\perp\gamma_\mu^\perp\chi_n(0)$$

$$\Phi_n^{(2_4)}(x_n)\equiv q^+q^-\frac{x^-}{2}(n\cdot\partial\,\bar{\chi}_n(x_n))\frac{\bar{\not{n}}}{2}\chi_n(0)$$



- n-sector, $\bar{n}$-sector and overlap graphs are individually divergent and unlike DIS cannot be regulated by dimensional regularization.

- Same picture as DIS: Overlap subtraction cancels the divergence.

- Except matrix elements now contains large logarithms of $\frac{Q_T^2}{Q^2}$.

Rapidity divergence and logarithms

- These logarithms are not generated by individual sector. They are rather rapidity logarithms of the form $\log\left(\frac{Q_T}{p_1^-}\right)$ and $\log\left(\frac{Q_T}{p_2^+}\right)$, where $p_1^- \sim p_2^+ \sim Q$.

- These needs to be resummed and can be done by introducing rapidity regulators.

E.g.: Variant of pure rapidity regulator [M. Ebert et. al. (2019)]

$$d^d k_n \to w_n^2 \left(\frac{q_L^2}{\nu_n^2}\right)^{\eta_n/2} \left(\frac{q^-}{q^+}\frac{k_n^+}{k_n^-}\right)^{\eta_n/2} d^d k_n$$

$$d^d k_{\bar{n}} \to w_{\bar{n}}^2 \left(\frac{q_L^2}{\nu_{\bar{n}}^2}\right)^{\eta_{\bar{n}}/2} \left(\frac{q^+}{q^-}\frac{k_{\bar{n}}^-}{k_{\bar{n}}^+}\right)^{\eta_{\bar{n}}/2} d^d k_{\bar{n}}$$

- $T_{(i,j)}$ now contains $\log\left(\frac{Q_T}{\nu_n}\frac{Q_T}{\nu_{\bar{n}}}\right)$ and $\frac{1}{\eta_n}$ and $\frac{1}{\eta_{\bar{n}}}$ poles corresponding to the rapidity divergence.

- Reproduces QCD when both sectors are regulated the same and $\nu_n = \nu_{\bar{n}} = Q$.

- Regulating the sectors differently introduces a factorization scale which can be used to obtain rapidity renormalization group (RRG) equations

$$\frac{d}{d\log\nu_{n,\bar{n}}}T_{(i,j)}\left(q^-, q^+, \mathbf{q}_T; \nu_{n,\bar{n}}\right) = \sum_{k,\ell}\left(\gamma^{n,\bar{n}}_{(i,j),(k,\ell)} * T_{(k,\ell)}\right)\left(q^-, q^+, \mathbf{q}_T; \nu_{n,\bar{n}}\right)$$

Features of NLP corrections in DY

[M. Inglis-Whalen, M. Luke, JR and A. Spourdalakis (2021)]

- At leading power, the only operator is $T_{(0,0)}$ which gets multiplicatively renormalized in the rapidity space. Reproduces known results [T. Becher and M. Neubert (2011)]

- At NLP multiple operators contribute to the inclusive rate which are individually rapidity divergent. The cancellation of the rapidity divergence takes places between different subleading operators and overlaps ⟹ operator mixing in rapidity space

- **Final factorization:** $\frac{1}{\sigma_0}\frac{d\sigma}{dq^2dydq_T^2} = \int \frac{dz_1}{z_1}\frac{dz_2}{z_2} C_{f\bar{f}}(z_1,z_2,q^2,q_T^2)\ f_{q/N_1}\left(\frac{\xi_1}{z_1}\right) f_{\bar{q}/N_2}\left(\frac{\xi_2}{z_2}\right) + O\left(\frac{\Lambda^2_{\rm QCD}}{q^2}\right)$

$$
\begin{aligned}
&C_{f\bar{f}}(z_1,z_2,q_L^2,q_T^2) = \\
&\int \frac{d\Omega_T}{2}\int\frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}\sum_{ijkk'\ell\ell'}\frac{H_{(i,j)}(\mu_S)\,K^{(i,j)}_{(k,\ell)}}{q_L^{[i]+[j]}} && \text{Hard Matching}\\
&\times d^2\mathbf{p}_T\ V_{(k,\ell),(k',\ell')}\left(\omega_1,\omega_2,\mathbf{p}_T;\mu_S;\nu^H_{n,\bar{n}},\nu^S_{n,\bar{n}}\right) && \text{Rapidity Running}\\
&\times C_{S,(k',\ell')}\left(\frac{z_1}{\omega_1},\frac{z_2}{\omega_2},\mathbf{q}_T-\mathbf{p}_T;\mu_S,\nu^S_{n,\bar{n}}\right). && \text{Soft Matching}
\end{aligned}
$$

Conclusions:

- useful to write SCET as a theory of decoupled QCD sectors interacting via Wilson lines. Factorization arises through matching/running procedure.

- using this we presented the first calculation of power corrections in Endpoint DIS and small-Q_T Drell-Yan.

- cancellation of endpoint and rapidity divergence happens naturally due to consistent application of overlap subtraction fixing the IR of the theory.

- at NLP, complicated pattern of divergence cancellation between different operators including NLP expansion of overlap.

- lots of future work: application to more processes, consistency conditions, Glaubers..