Superfluidity in an EFT expansion

Silas Beane

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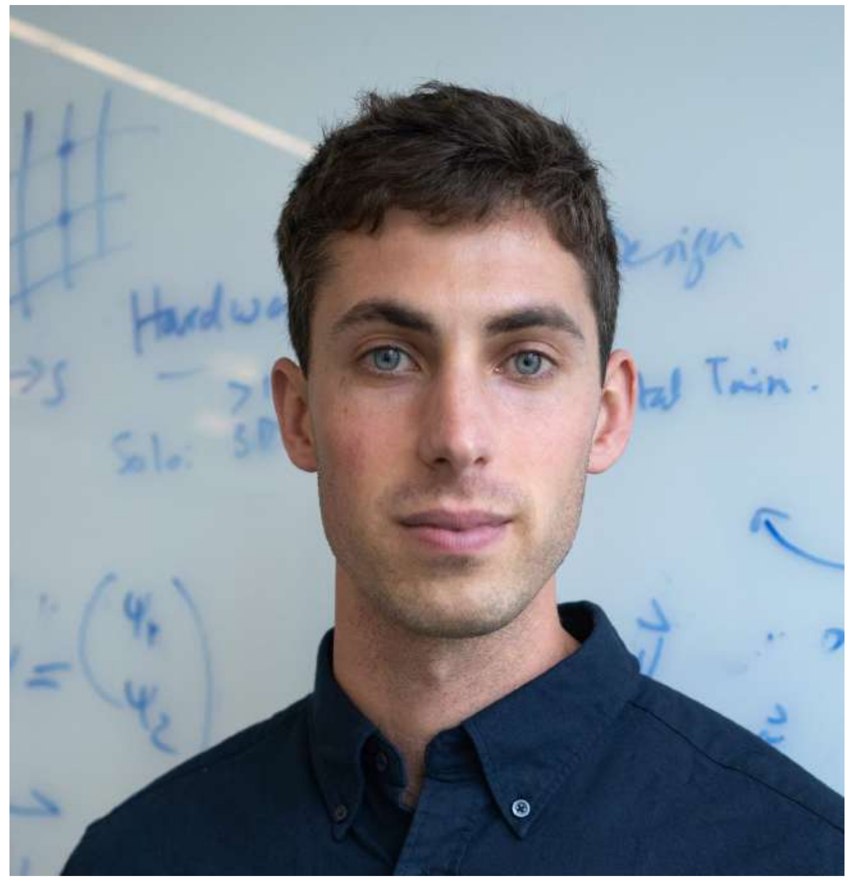
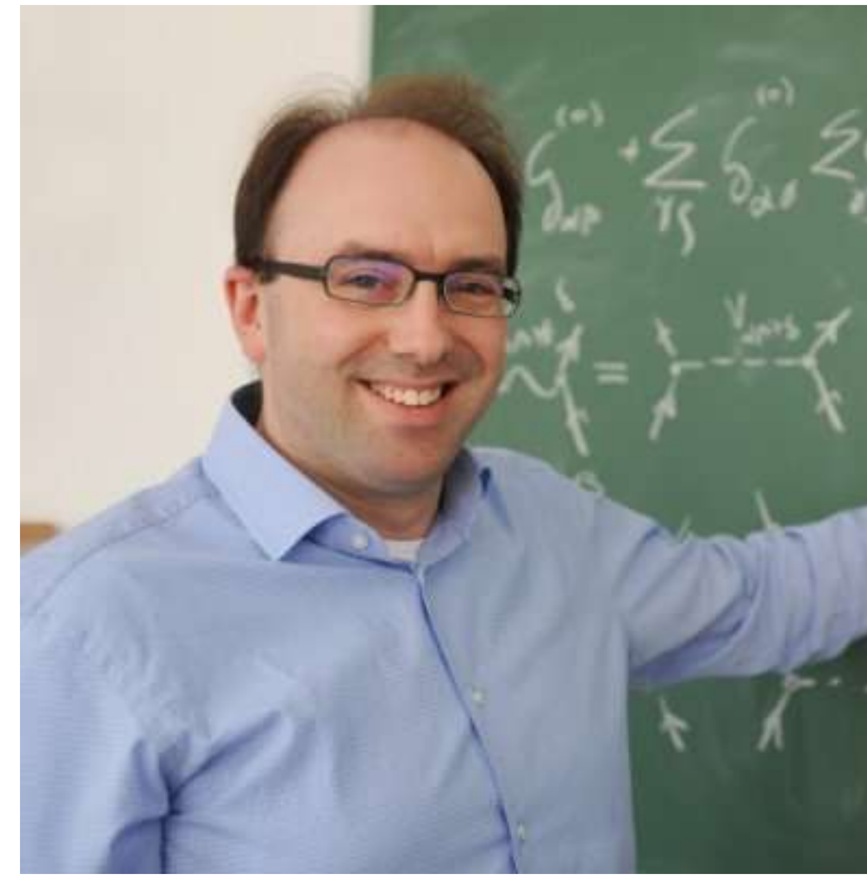
U.S. DEPARTMENT OF ENERGY Office of Science

Swiss National Science Foundation

I will discuss work done in collaboration with

<u>Zeno Cappatti</u> (Bern), <u>Roland Farrell</u> (Caltech), <u>Achim Schwenk</u> (Darmdstadt)



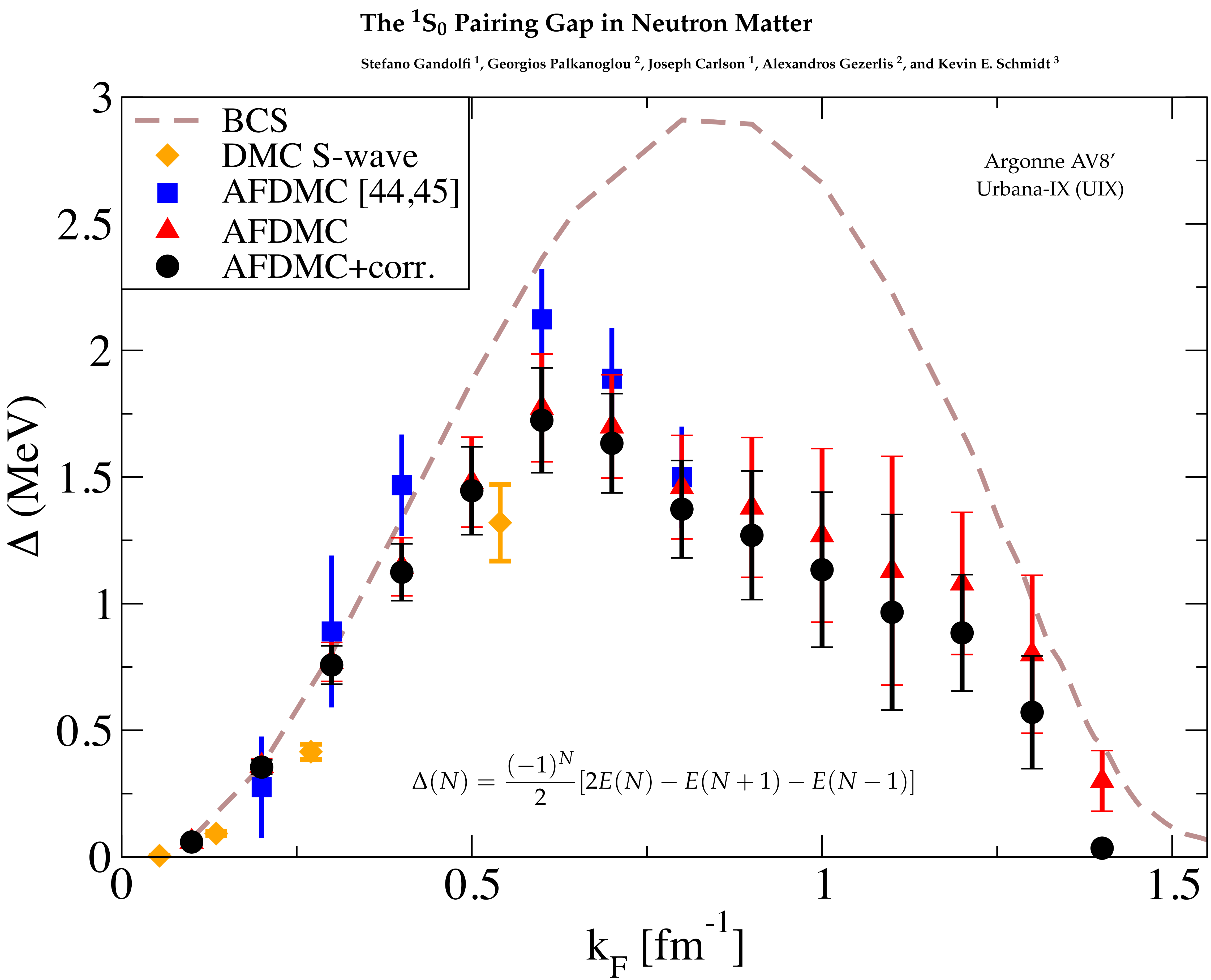


arXiv:2407.20168 [nucl-th]

+ in progress

- Motivation and set up
- Power counting the BCS singularity
- NLO (recovering Gor'kov,Melik-Barkhudarov effect)
- NNLO
- Pairing from realistic potentials: $\mathrm{EFT}(\not\pi)\ldots$
- Summary

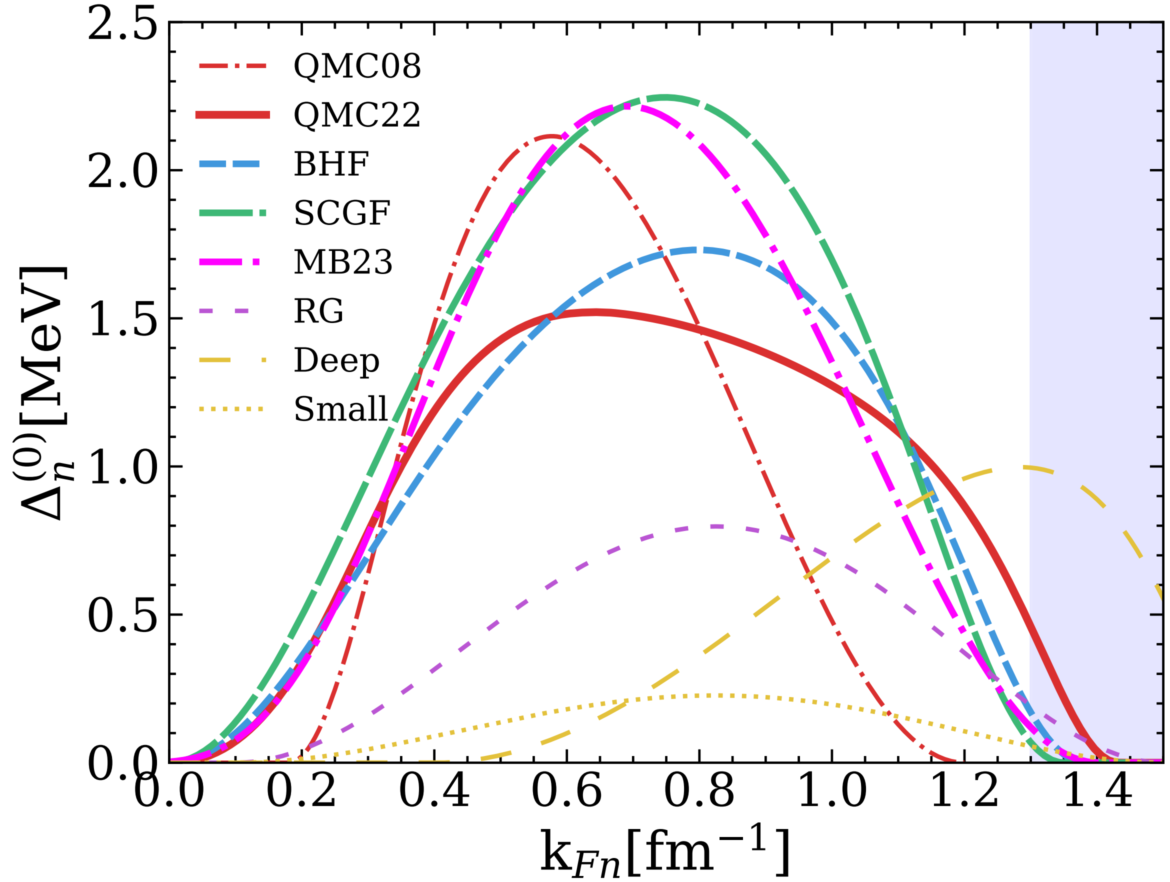
Motivation



The $^1S_0$ Pairing Gap in Neutron Matter
Stefano Gandolfi [1], Georgios Palkanoglou [2], Joseph Carlson [1], Alexandros Gezerlis [2], and Kevin E. Schmidt [3]
BCS
DMC S-wave
AFDMC [44,45]
AFDMC
AFDMC+corr.
Argonne AV8′
Urbana-IX (UIX)
$\Delta(N) = \frac{(-1)^N}{2}[2E(N) - E(N+1) - E(N-1)]$
Δ (MeV)
k_F [fm^{-1}]

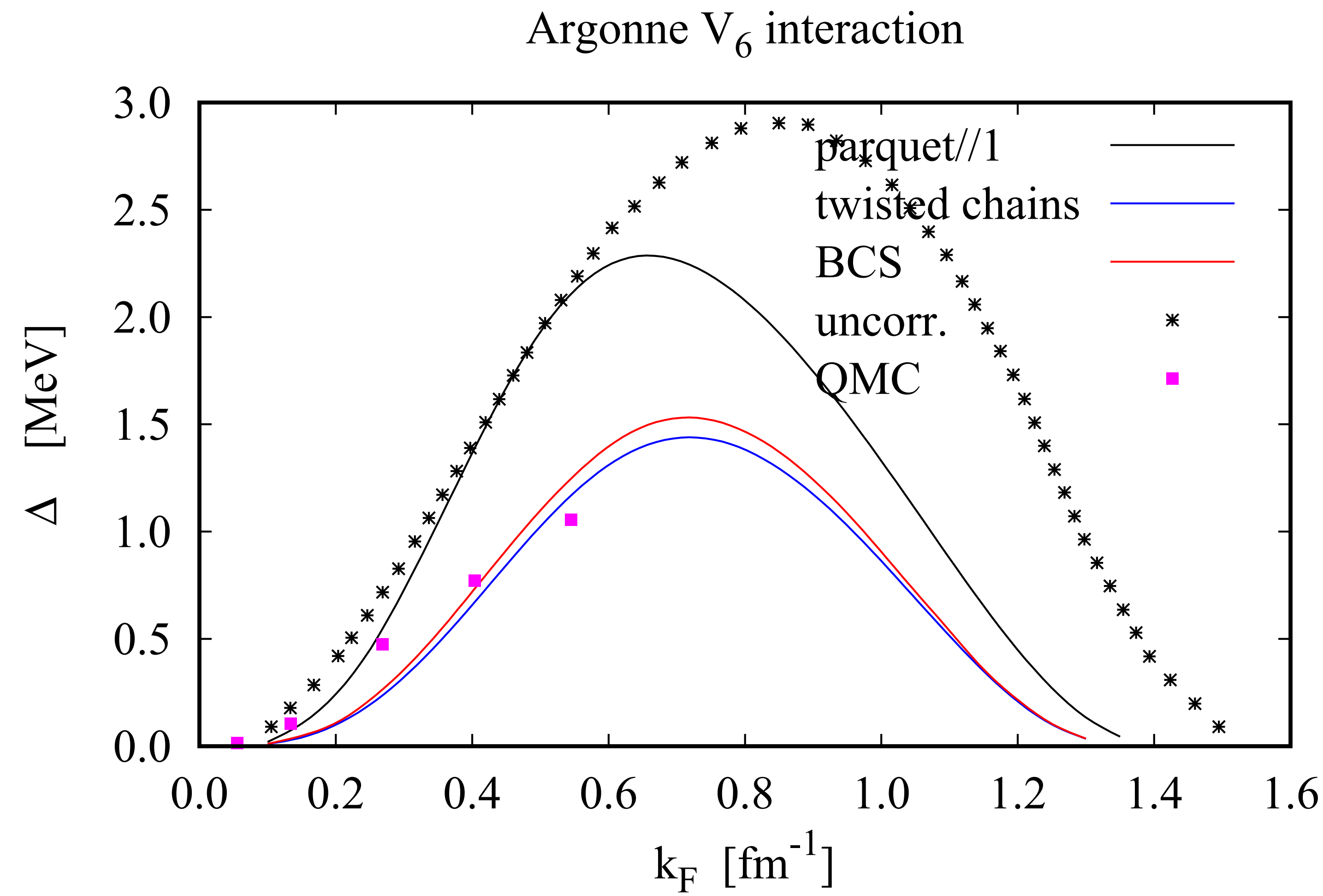
Gapless neutron superfluidity in the crust of the accreting neutron stars KS 1731−260 and MXB 1659−29

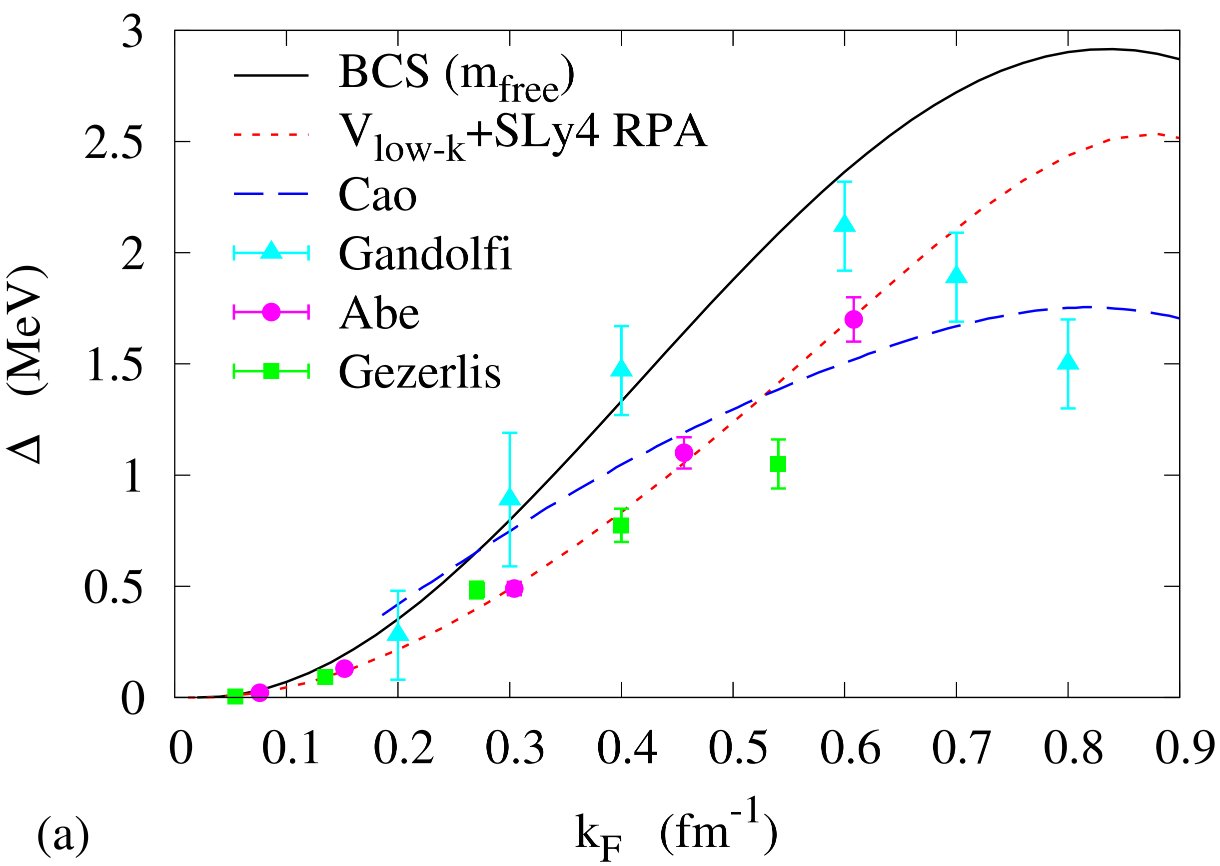
Valentin Allard[a], Nicolas Chamel[b]
Institute of Astronomy and Astrophysics, Université Libre de Bruxelles, Boulevard du Triomphe, Brussels 1050, Belgium



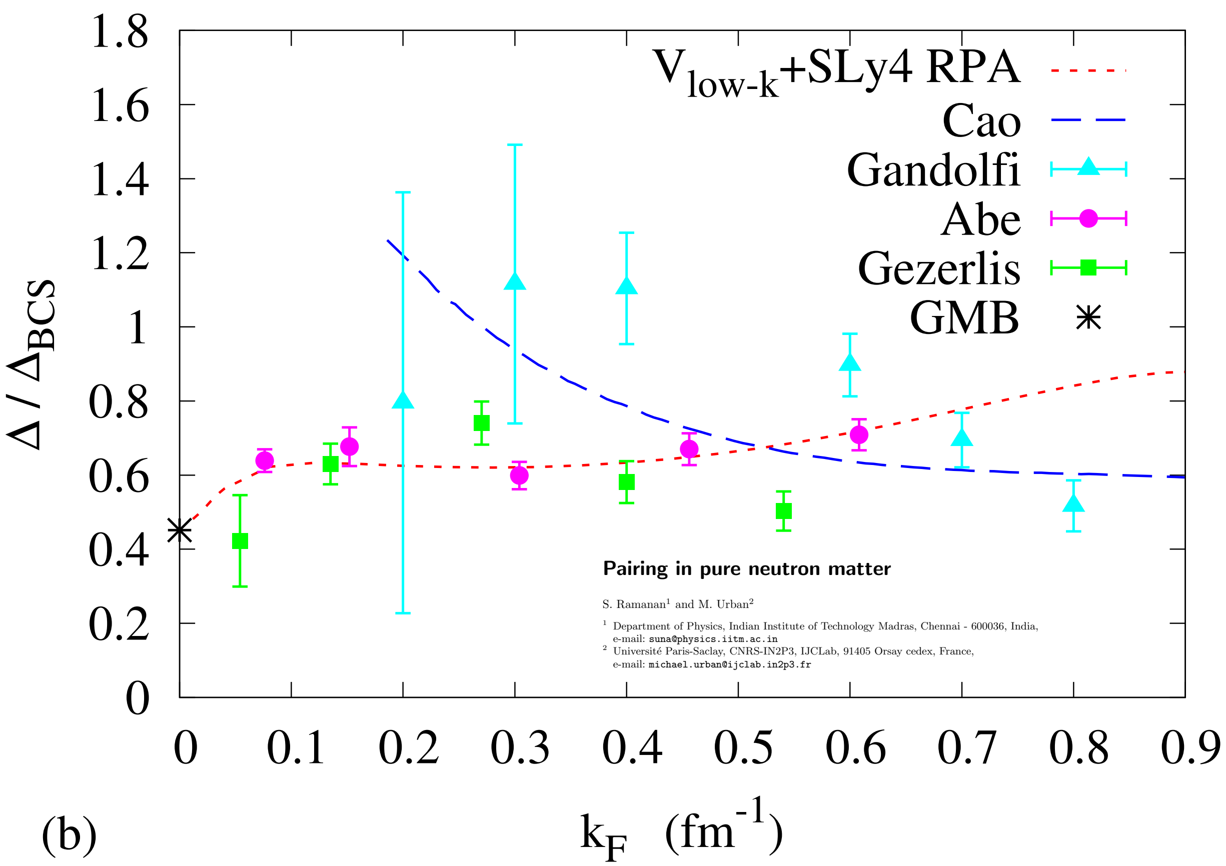
Variational and parquet-diagram calculations for neutron matter. III. *S*-wave pairing

E. Krotscheck and J. Wang
Department of Physics, University at Buffalo, SUNY Buffalo, Buffalo, New York 14260 and Institut für Theoretische Physik, Johannes Kepler Universität, A 4040 Linz, Austria





(a)



(b)

Set up

EFT of contact operators

$$\mathcal{L} = \sum_{\sigma=\uparrow,\downarrow} \left[\psi_\sigma^\dagger \left(i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} + \mu_F \right) \psi_\sigma - \frac{1}{2} g (\psi_\sigma^\dagger \psi_\sigma)^2 \right]$$

$$g \overset{\overline{MS}}{=} \frac{4\pi a}{M} < 0$$

Set up

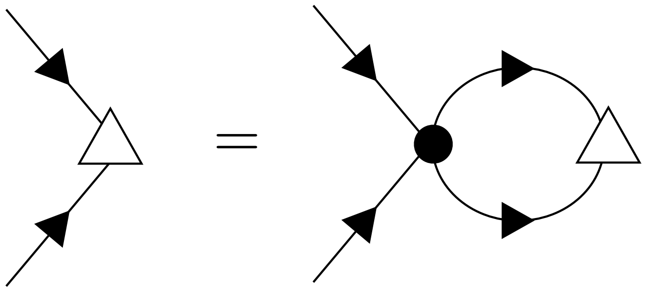
EFT of contact operators

$$\mathcal{L} = \sum_{\sigma=\uparrow,\downarrow} \left[\psi_\sigma^\dagger \left(i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} + \mu_F \right) \psi_\sigma - \frac{1}{2} g (\psi_\sigma^\dagger \psi_\sigma)^2 \right] \qquad g \overset{\overline{MS}}{=} \frac{4\pi a}{M} < 0$$

BCS equations

$$\Delta(\mathbf{k}) = -\int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{V(\mathbf{k}, \mathbf{q})\,\Delta(\mathbf{q})}{2\sqrt{(\omega_q - \mu_F)^2 + |\Delta(\mathbf{q})|^2}}$$

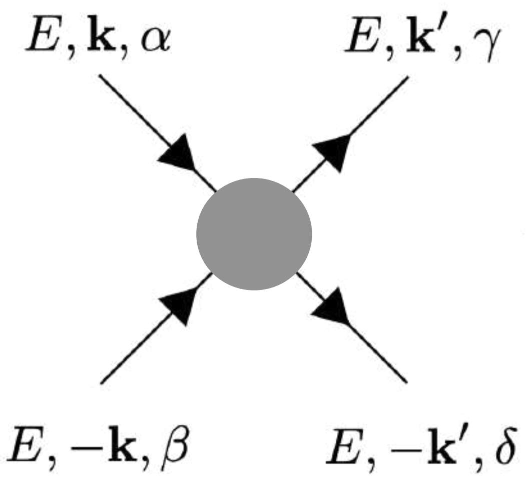
$$\rho = \frac{k_F^3}{3\pi^2} = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[1 - \frac{(\omega_q - \mu_F)}{\sqrt{(\omega_q - \mu_F)^2 + |\Delta(\mathbf{q})|^2}} \right]$$



$$\Delta_{\mathrm{BCS}} = \frac{8}{e^2} \omega_{k_F} \exp\left(\frac{\pi}{2 k_F a} \right) + \ldots$$

Here consider EFT of contact operators at T=0 and consider the in-medium 4-point function:

$$i\Gamma_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}';2E) \quad =$$



$$iG_0(k_0,\mathbf{k})\delta_{\alpha\gamma} = i\delta_{\alpha\gamma}\left(\frac{\theta(k-k_F)}{k_0-\omega_k+i\epsilon}+\frac{\theta(k_F-k)}{k_0-\omega_k-i\epsilon}\right)$$

$$\left[\Gamma(\mathbf{k},\mathbf{k}';\mathrm{Re}[2E]+i\Delta)\right]^{-1} = 0$$

Must get same answer as with BCS equations: <u>Kohn-Luttinger-Ward theorem</u>

Power counting the BCS singularity

Expansion in logarithm of the gap

$$\log\left(\frac{\Delta}{\omega_{k_F}}\right) = \frac{c_{-1}}{\lambda} + c_0 + c_1\lambda + \ldots$$

"gas parameter" $\lambda \equiv \frac{2}{\pi} k_F a$

$$c_{-1} = 1$$ BCS

$$c_0 = \frac{7}{3}(\log 2 - 1) = -0.71599\ldots$$ Gor'kov,Melik (1960)

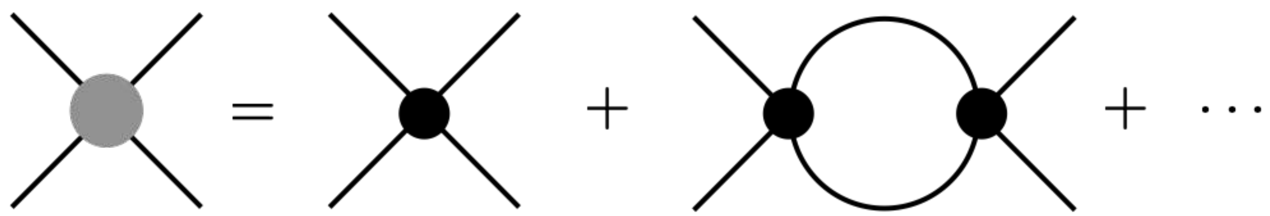
$$c_1 = 0.47619(20)$$ BCF

Consider free-space 4-point function

(van Kolck)

(Kaplan,Savage,Wise)

$\mathrm{EFT}(\not{\pi})$



$g \qquad g^2\,\mathbb{I}(k)$

$$\mathbb{I}(k) \sim Q$$

strong coupling: infrared enhancement

$$g \sim \frac{4\pi}{M\aleph} \sim Q^{-1}$$

$$T(k) = g + g^2\,\mathbb{I}(k) + g^3\,\mathbb{I}(k)^2 + \ldots = \left(\frac{1}{g} - \mathbb{I}(k)\right)^{-1}$$

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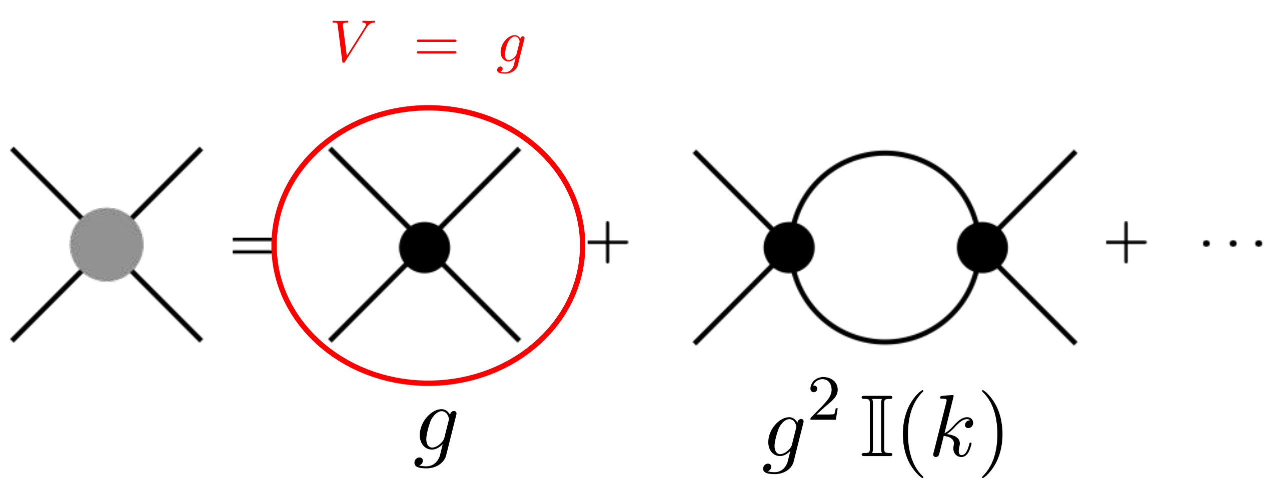

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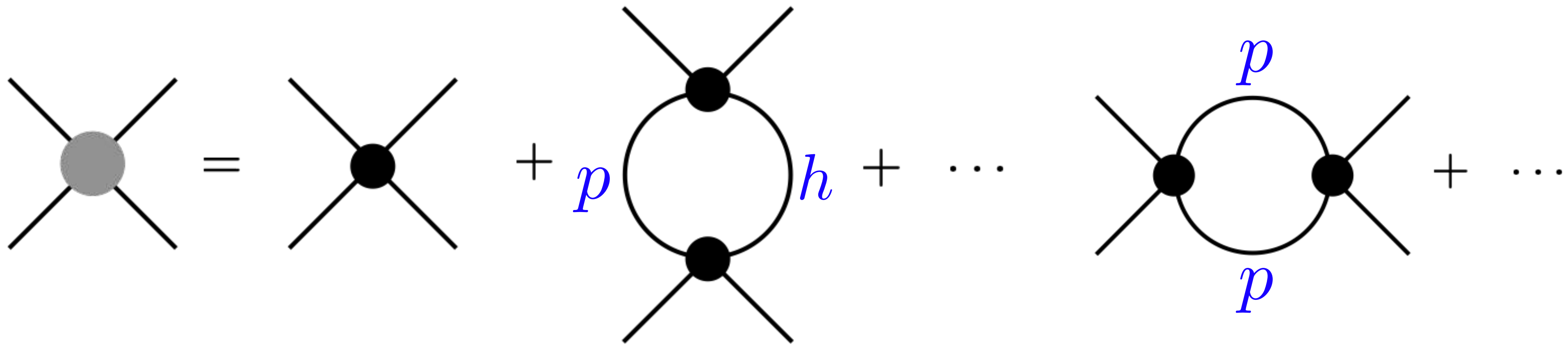
Consider in-medium 4-point function



g $\qquad g^2\,\Pi_{ph}$ $\qquad g^2\,\Pi_{pp}$

Have control only when g is "weak": $\quad k_F a \ll 1$

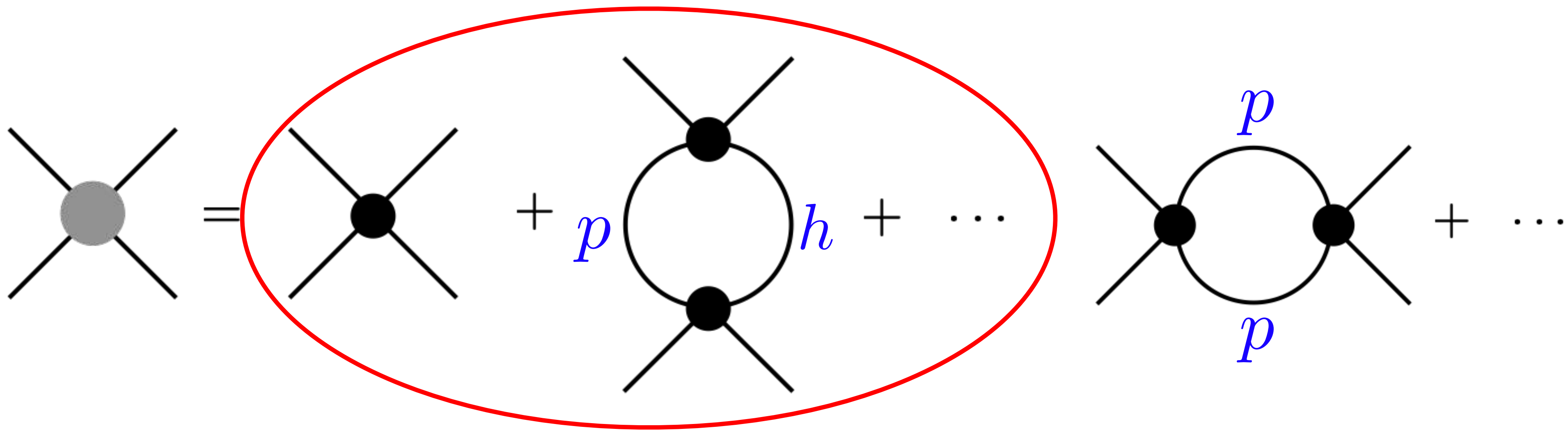
BCS kinematical singularity: infrared enhancement

$$\Pi_{pp} \;\sim\; g^{-1}$$

$$\Gamma_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}';2E) \;=\; -\frac{g}{1+g\Pi_{pp}(E)}(\delta_{\alpha\gamma}\delta_{\beta\delta} \;-\; \delta_{\alpha\delta}\delta_{\beta\gamma}) \;+\; \mathcal{O}(g^2)$$

Consider in-medium 4-point function

$$V = g + V^{(g^2)} + \dots$$



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Have control only when g is “weak”: $k_F a \ll 1$

BCS kinematical singularity: infrared enhancement

$$\Pi_{pp} \sim g^{-1}$$

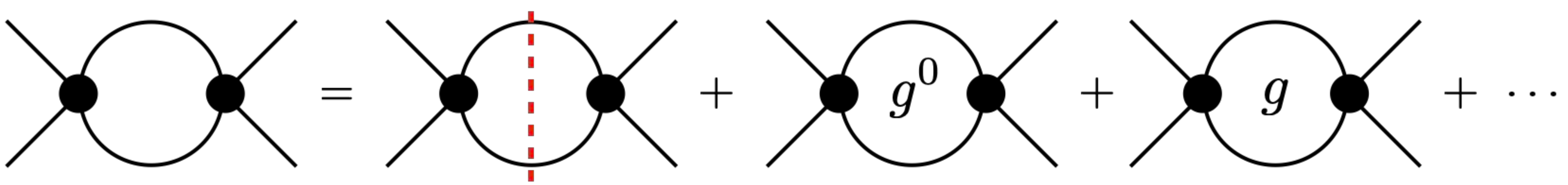
$$\Gamma_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}';2E) = -\frac{g}{1+g\Pi_{pp}(E)}(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}) + \mathcal{O}(g^2)$$

$$\Pi_{pp}(E) \;=\; -\mathbb{I}(E) + \frac{M}{2\pi^2}\left[\int_0^{k_F} dl\,\log\left(2ME - l^2\right) \;-\; \underline{k_F \log\left(2ME - k_F^2\right)}\right]$$

$$E \;=\; Re[E] + i\frac{\Delta}{2}$$

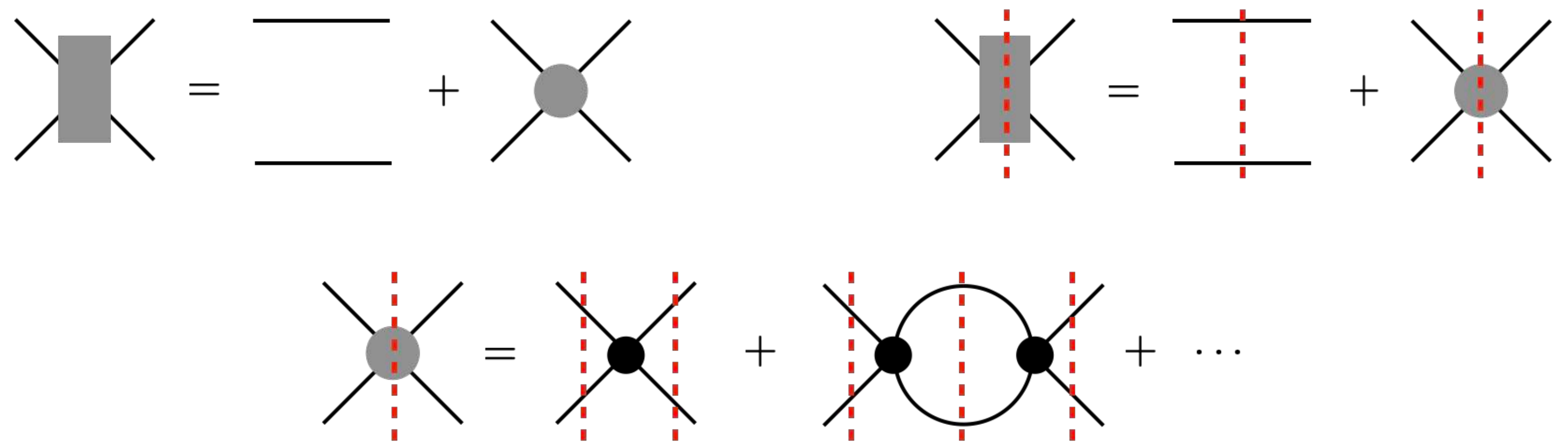
BCS singularity @ the Fermi surface: regulated by the gap

Evaluated @ the Fermi surface

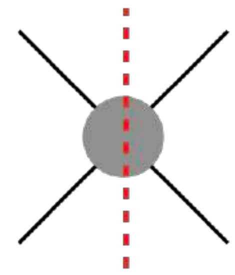


$$\Pi_{pp}(E) \;=\; \Pi_{pp}^{(g^{-1})} \;+\; \Pi_{pp}^{(g^{0})} \;+\; \Pi_{pp}^{(g)} \;+\ldots$$

Basic building blocks



LO :

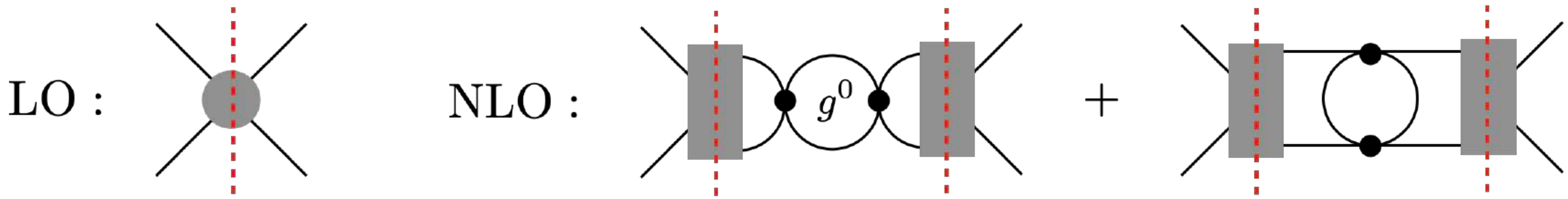


$$0 = 1 - g\frac{Mk_F}{2\pi^2}\log\Delta + \mathcal{O}(g)$$

$$\Delta_{\mathrm{LO}} \sim \exp\left(\frac{\pi}{2k_F a}\right)$$

prefactor undetermined at this order!

NLO: recovering Gor'kov,Melik-Barkhudarov effect

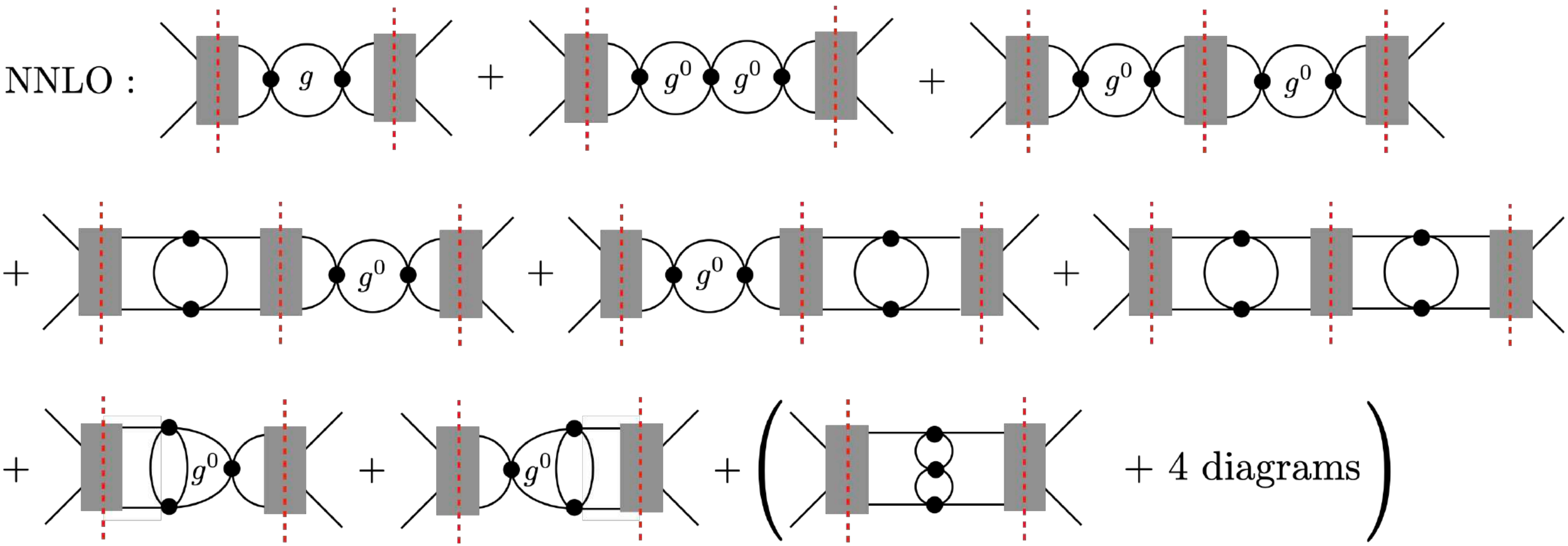


$$\Gamma_{\rm NLO} \sim -\frac{g}{1+g\Pi_{pp}^{(g^{-1})}}\left(1-\frac{g\Pi_{pp}^{(g^0)}}{1+g\Pi_{pp}^{(g^{-1})}}\right) \qquad \frac{-V^{(g^2)}}{\left(1+g\Pi_{pp}^{(g^{-1})}\right)^2}$$

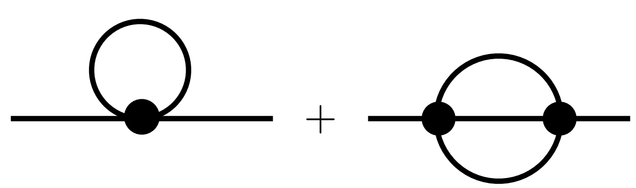
$$0 = \Gamma_{\rm NLO}^{-1} = 1 + g\left(\Pi_{pp}^{(g^{-1})} + \Pi_{pp}^{(g^0)}\right) - \frac{V^{(g^2)}}{g} + \mathcal{O}(g^2)$$

$$\Delta_{\rm NLO} = \Delta_{\rm GM} = \frac{8\omega_{k_F}}{e^2(4e)^{1/3}}\exp\left(\frac{\pi}{2k_F a}\right)$$

NNLO

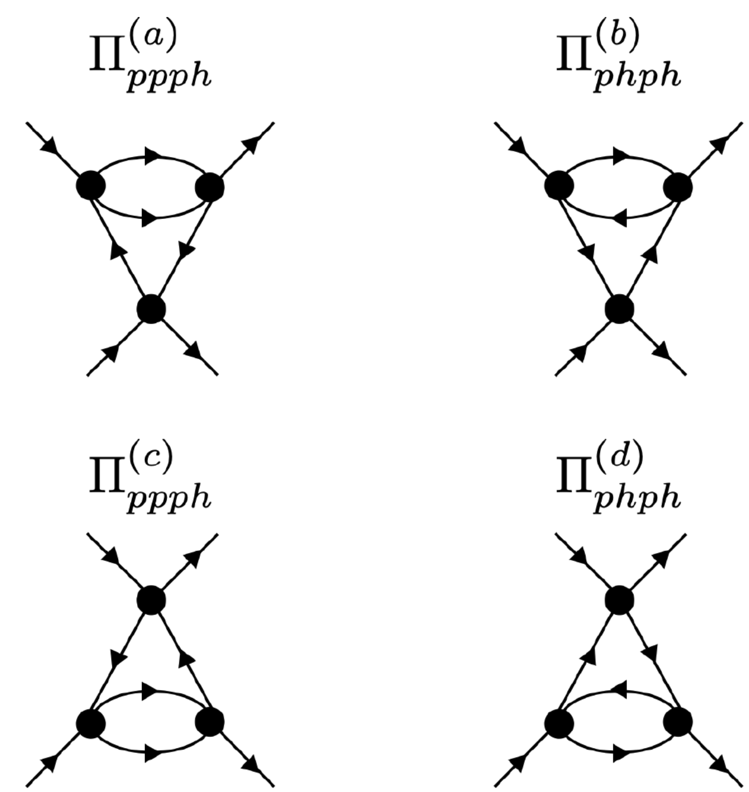


Self-energy corrections

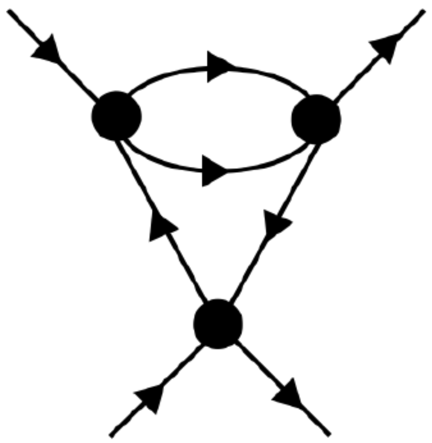


$$\frac{M^\star}{M} = 1 + \lambda^2 \frac{2}{15}(7\log 2 - 1) \; + \; \mathcal{O}(\lambda^3)$$

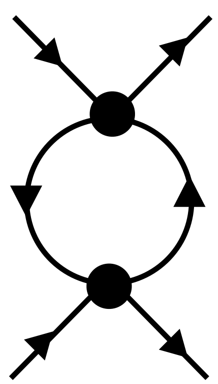
$$Z = 1 - \lambda^2 \log 2 \; + \; \mathcal{O}(\lambda^3)$$



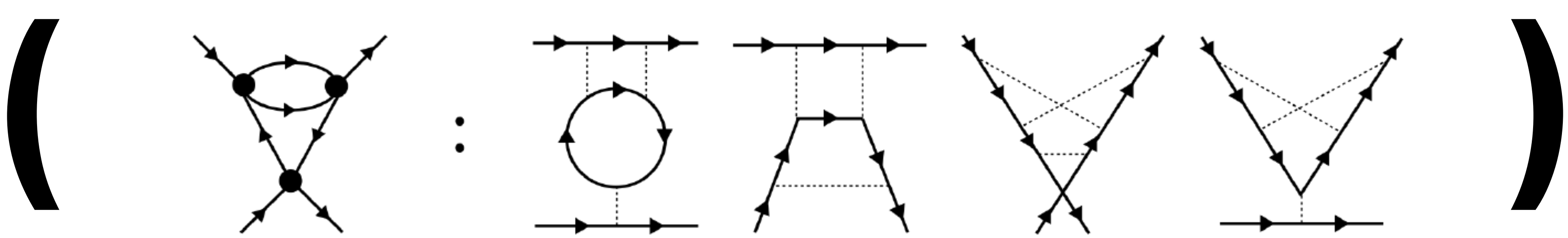
Advantages of the PDS scheme



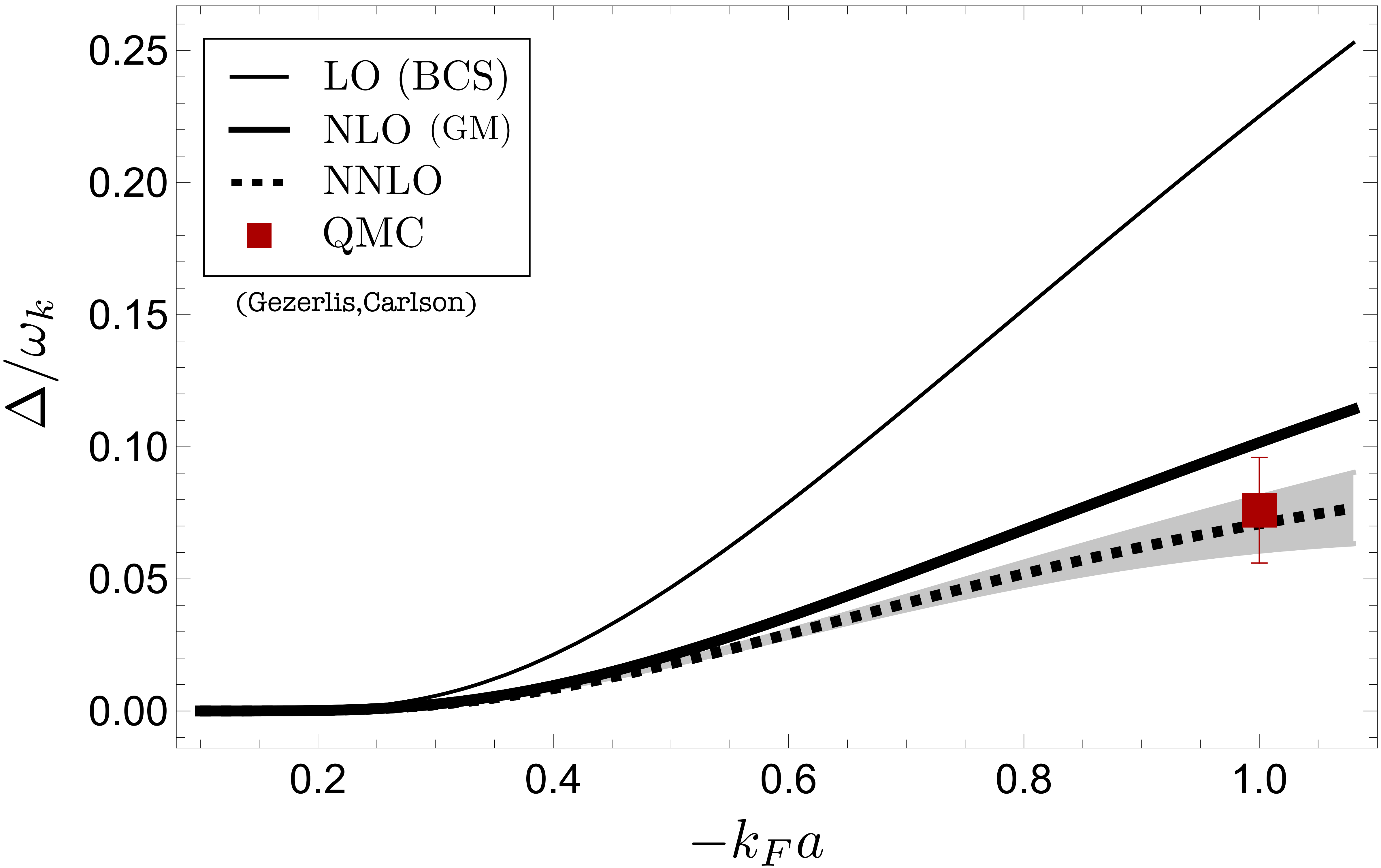
$$V^{(g^3)}_{\alpha,\beta,\gamma\delta}(\mathbf{k},\mathbf{k}';\omega)[\mu] \;=\; -\,2g(\mu)^3\frac{M\mu}{4\pi}\;\cdots\;\;=\;-\;\left(\frac{4\pi a}{M}\right)^2\left(2a\mu\;+\;\mathcal{O}[(a\mu)^2]\right)$$



$$V^{(g^2)}_{\alpha\beta,\gamma\delta}(\mathbf{k},\mathbf{k}';\omega)[\mu] \;=\; g(\mu)^2\;\cdots\;\;=\;\left(\frac{4\pi a}{M}\right)^2\left(1\;+\;2a\mu\;+\;\mathcal{O}([a\mu]^2)\right)\;\cdots$$



$$\Delta_{\mathrm{NNLO}} = \Delta_{\mathrm{NLO}} \left(1 + \frac{0.95238(40)}{\pi} k_F a\right)$$



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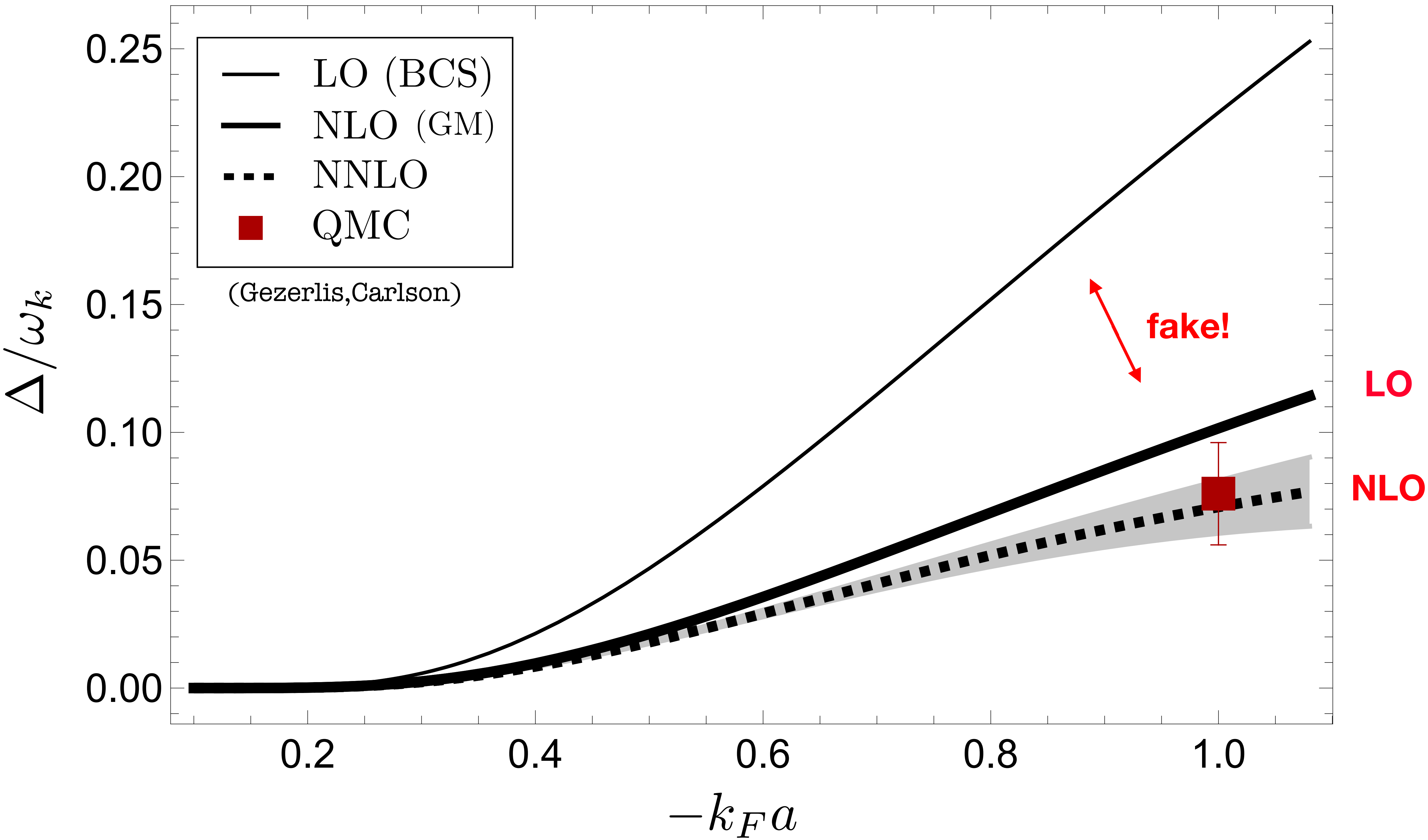
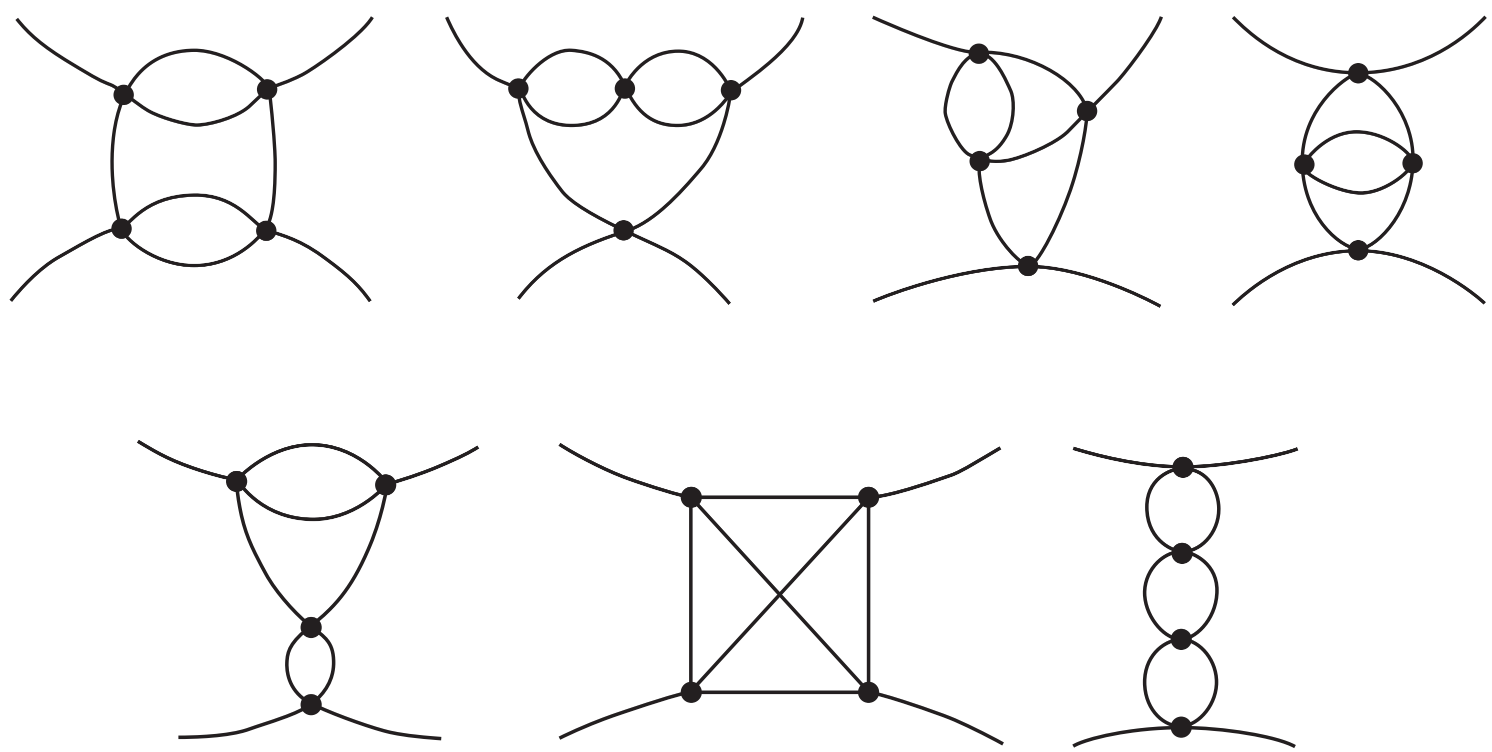
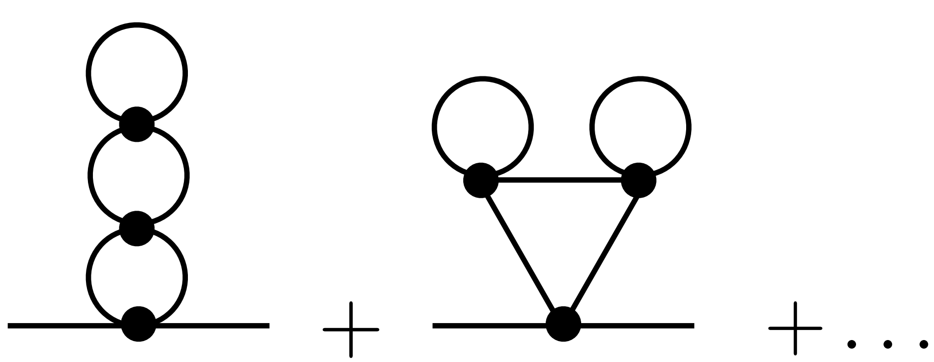


N3LO? Not crazy..



(Efremov,Mar'enko,Baranov,Kagan)

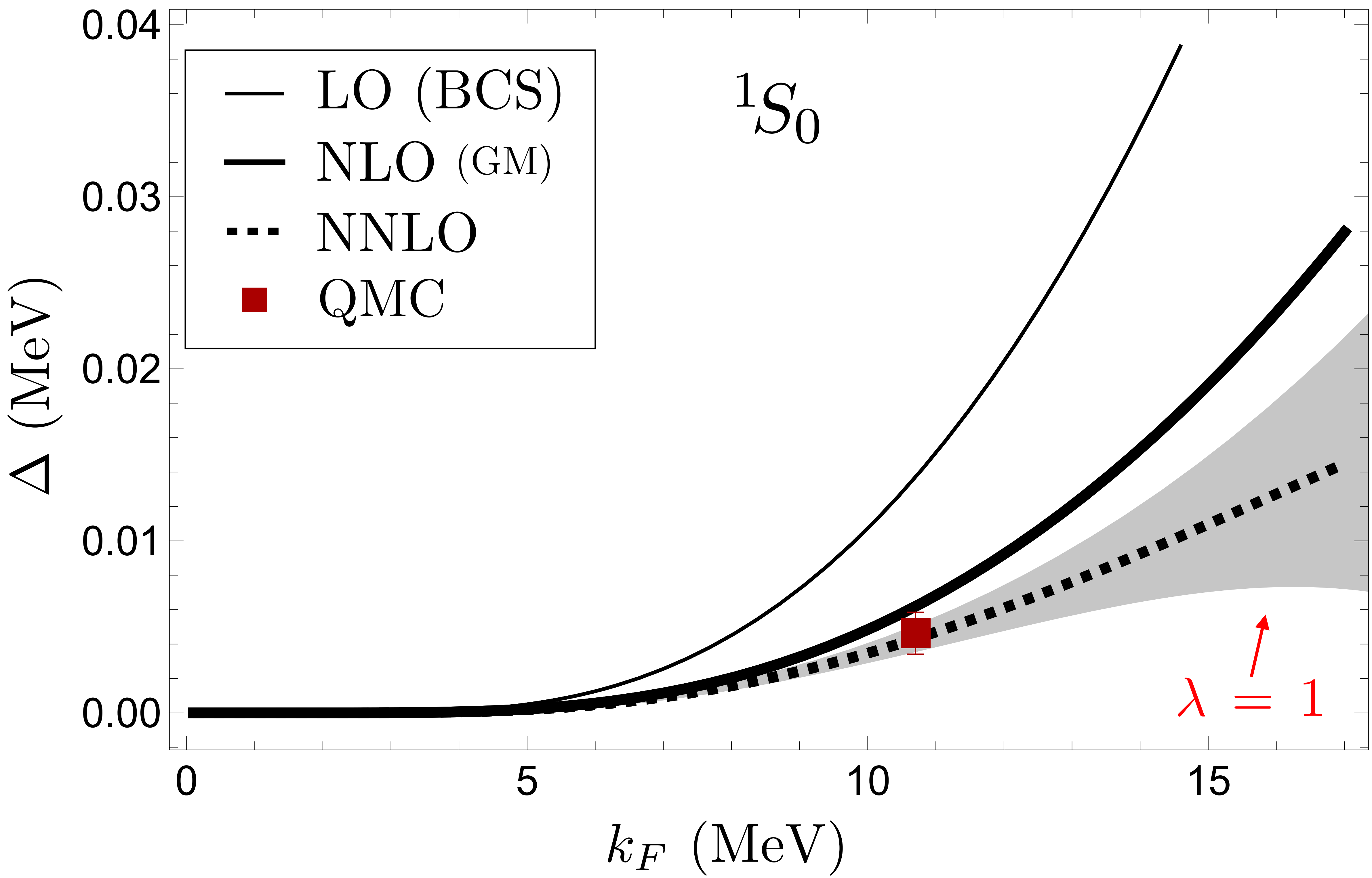
Self-energy corrections



(Hammer,Platter, Meißner)

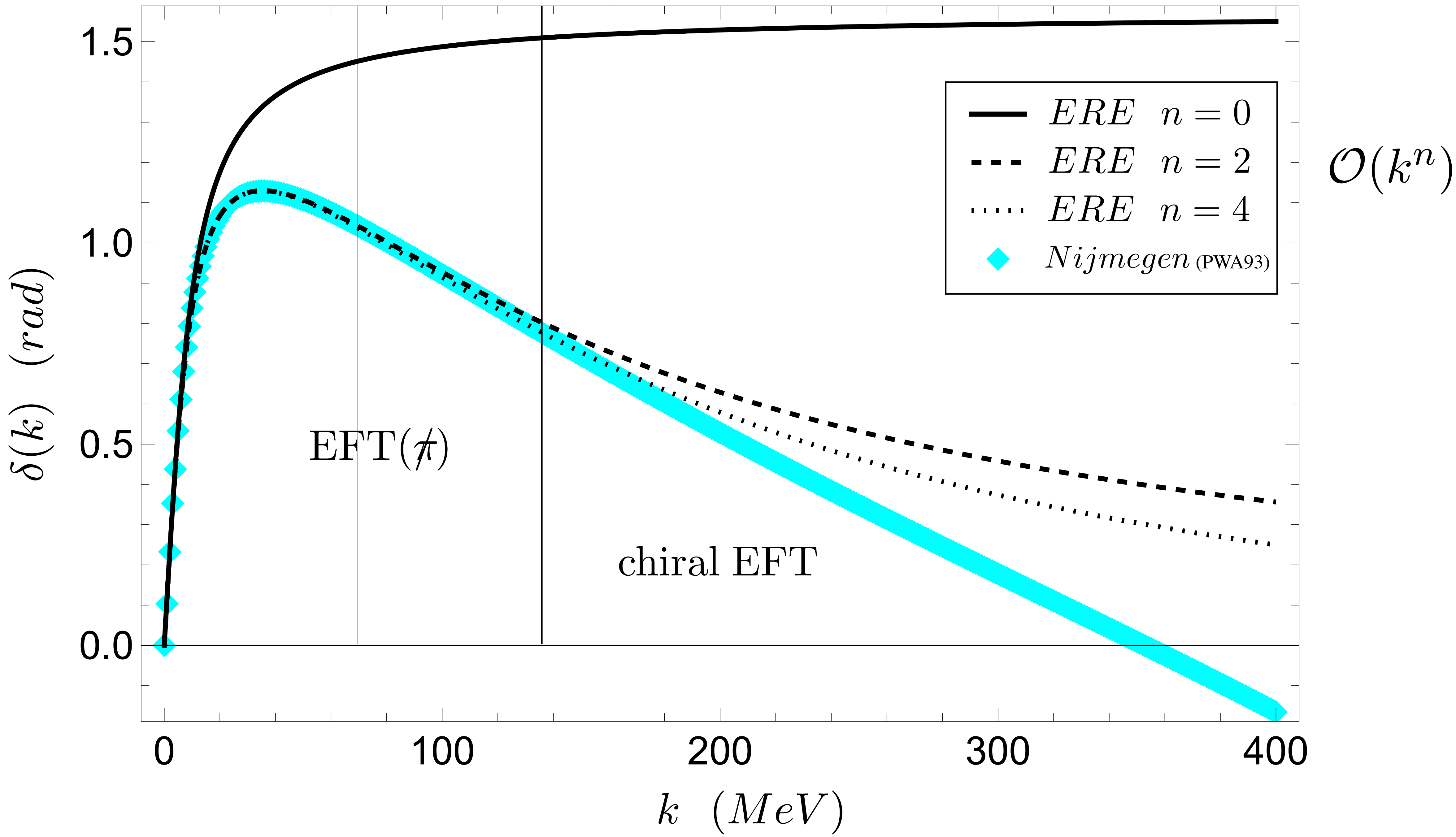
Relevance for superfluid neutrons?

$a = -18.5$ fm



Pairing from realistic potentials

Neutron-proton singlet s-wave phase shift



$$k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + \mathcal{O}(k^8)$$

$\mathrm{EFT}(\not\pi) \equiv$ Effective range theory = rational S-matrix

$$S = e^{i\phi} \prod_{a=1}^{n} \frac{k + i\gamma_a}{k - i\gamma_a}$$

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$$S_n : \qquad \mathbf{e}_k^{(n)} \equiv \mathbf{e}_k(\gamma_1, \ldots, \gamma_n) = \sum_{1 \le j_1 < j_2 < \cdots < j_k \le n} \gamma_{j_1} \cdots \gamma_{j_k}$$

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$$S \;=\; e^{i\phi}\prod_{a=1}^{n}\frac{k+i\gamma_a}{k-i\gamma_a}$$

S_n :

$$\mathbf{e}_k^{(n)} \equiv \mathbf{e}_k(\gamma_1,\ldots,\gamma_n) \;=\; \sum_{1\le j_1<j_2<\cdots<j_k\le n}\gamma_{j_1}\cdots\gamma_{j_k}$$

$\mathcal{O}(k^n)$:

$$V\left(k,k'\right) \;=\; -\frac{4\pi}{M}\left|\mathbf{e}_{n-1}^{(n)}\right|\prod_{a=1}^{n/2}\frac{1}{\sqrt{\gamma_a^2+k^2}\sqrt{\gamma_a^2+k'^2}}$$

$\mathrm{EFT}(\not{\pi})$ UV/IR

Treat range corrections to all orders

$$V\left(k,k'\right) = -\frac{4\pi}{M}\left|\gamma_1+\gamma_2\right|\frac{1}{\sqrt{\gamma_1^2+k^2}\sqrt{\gamma_1^2+k'^2}}$$

$$\gamma_1 = 157.93 \text{ MeV} \quad , \quad \gamma_2 = -7.93 \text{ MeV}$$

(Farrell,SB)

(Peng,Lyu,König,Long)

(Timóteo,van Kolck)

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(Peng,Lyu,König,Long)
(Timóteo,van Kolck)

Expansion about point of exact symmetry

$$k \;\to\; -\frac{\gamma_1\gamma_2}{k} \quad : \quad S \;\to\; S$$

$\mathrm{EFT}(\not\pi)$ UV/IR

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Expansion about point of exact symmetry

$$k \to -\frac{\gamma_1\gamma_2}{k} \quad : \quad S \to S$$

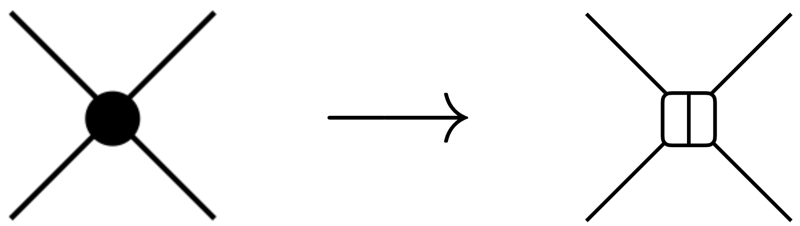
Corrections? shape parameter

$$V\left(k,k'\right) = -\frac{4\pi}{M}\left|\gamma_1\gamma_2\gamma_3+\gamma_1\gamma_2\gamma_4+\gamma_1\gamma_3\gamma_4+\gamma_2\gamma_3\gamma_4\right|\frac{1}{\sqrt{(\gamma_1^2+k^2)(\gamma_2^2+k^2)}\sqrt{(\gamma_1^2+k'^2)(\gamma_2^2+k'^2)}}$$

$$\gamma_1 = 172.04\ \mathrm{MeV} \quad , \quad \gamma_2 = 486.66\ \mathrm{MeV} \quad , \quad \gamma_3 = -650.77\ \mathrm{MeV} \quad , \quad \gamma_4 = -7.93\ \mathrm{MeV}$$

Many-body physics with momentum-dependent interaction

$$g \longrightarrow V(\mathbf{k}, \mathbf{k}')$$



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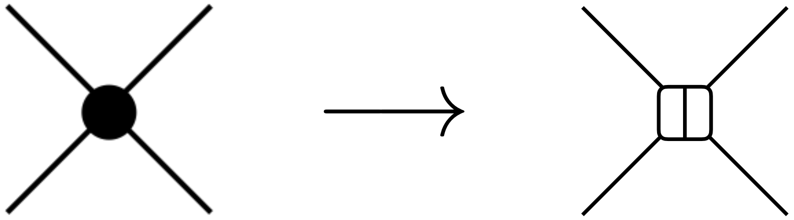
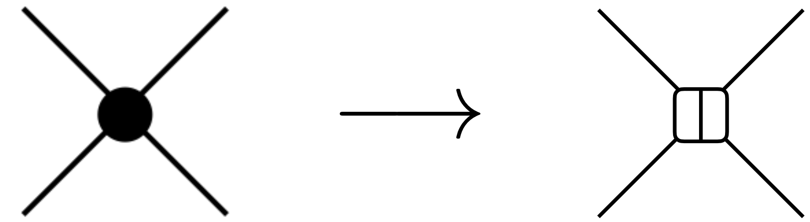


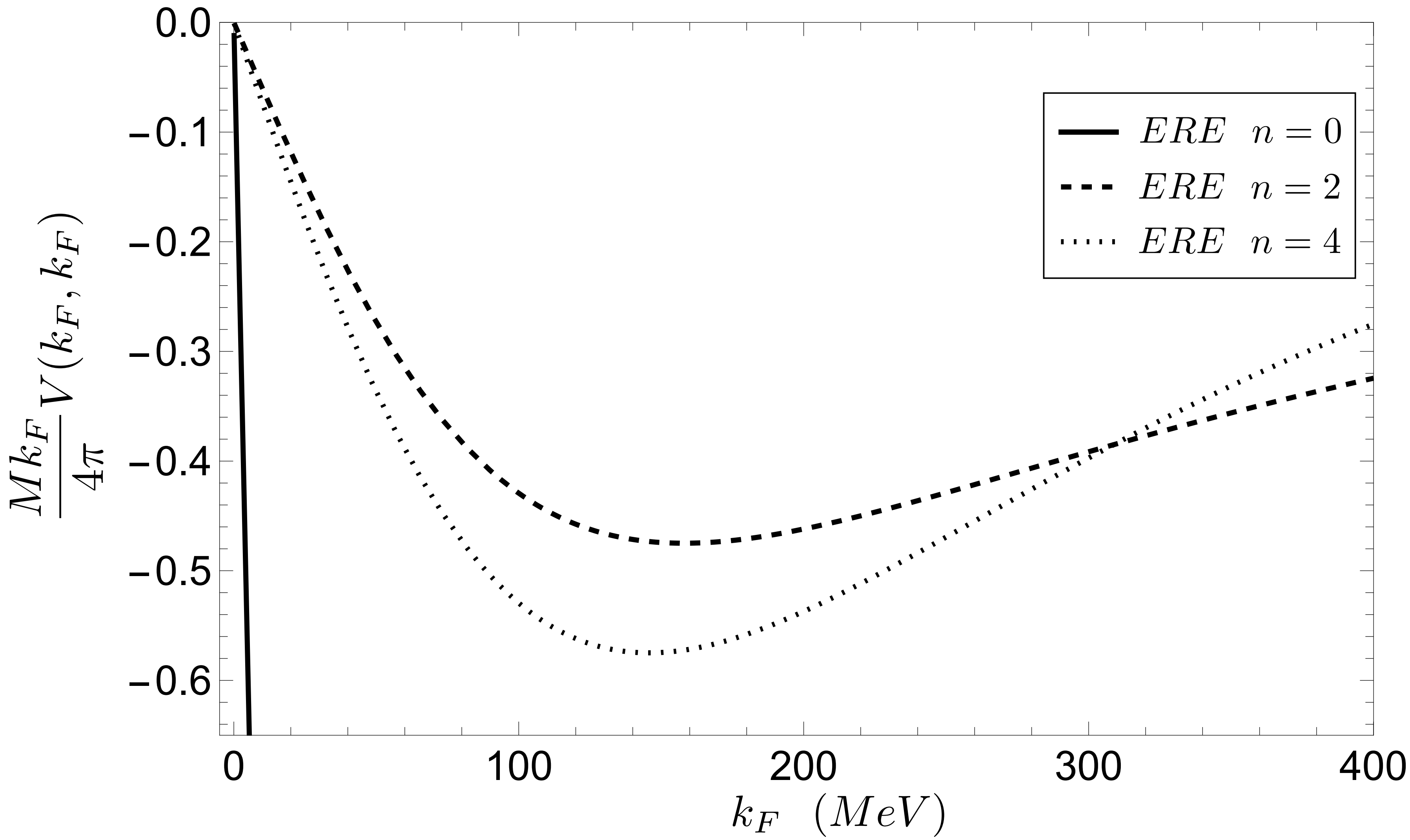
$$k_F a = \frac{M k_F}{4\pi} \cdot \frac{4\pi a}{M} \longrightarrow \frac{M k_F}{4\pi} V(k_F, k_F)$$

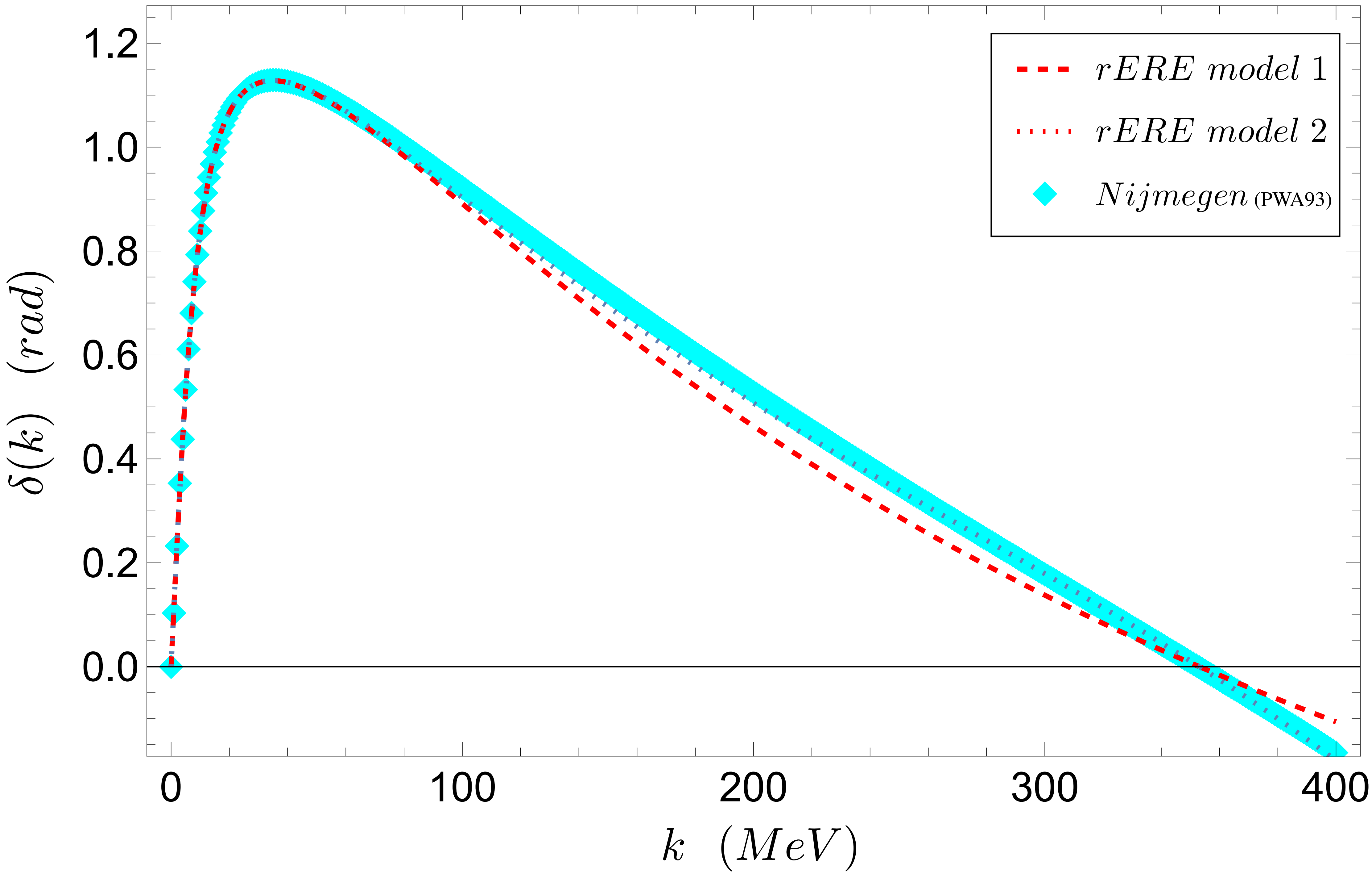
Many-body physics with momentum-dependent interaction

$$g \longrightarrow V(\mathbf{k}, \mathbf{k}')$$



$$k_F a = \frac{M k_F}{4\pi} \cdot \frac{4\pi a}{M} \longrightarrow \boxed{\frac{M k_F}{4\pi} V(k_F, k_F) \ll 1}$$





rERE Model 1 $$k\cot\delta(k) = -\frac{1}{a} + \frac{1}{2}rk^2\frac{k_*^2}{k_*^2-k^2}$$ (Sánchez Sánchez et al)

rERE Model 2 $$k\cot\delta(k) = -\frac{1}{a} + \left[\frac{1}{2}rk^2 + \left(v_2 + \frac{1}{2}rk_*^{-2}\right)k^4\right]\frac{k_*^2}{k_*^2-k^2}$$

$$V\left(k,k'\right) \;=\; c_0\sqrt{\frac{k_*^2-k^2}{\gamma_1^2+k^2}}\sqrt{\frac{k_*^2-k'^2}{\gamma_1^2+k'^2}}$$

$$c_0 \xrightarrow{\overline{\mathrm{MS}}} -\frac{4\pi}{M}\left(\mathbf{e}_1^{(3)}\right)^{-1}$$

$$\gamma_1 \;=\; 212.19\ \mathrm{MeV} \quad , \quad \gamma_2 \;=\; 618.74\ \mathrm{MeV} \quad , \quad \gamma_3 \;=\; -7.93\ \mathrm{MeV}$$

$$V\left(k,k'\right) = c_0\sqrt{\frac{k_*^2-k^2}{\gamma_1^2+k^2}}\sqrt{\frac{k_*^2-k'^2}{\gamma_1^2+k'^2}}$$

$$c_0 \xrightarrow{\overline{\text{MS}}} -\frac{4\pi}{M}\left(\mathbf{e}_1^{(3)}\right)^{-1}$$

S_2

$$\gamma_1 = 212.19\ \text{MeV} \quad , \quad \gamma_2 = 618.74\ \text{MeV} \quad , \quad \gamma_3 = -7.93\ \text{MeV}$$

$$\overline{V}\left(k,k'\right) = c_0\sqrt{\frac{k_*^2-k^2}{\gamma_2^2+k^2}}\sqrt{\frac{k_*^2-k'^2}{\gamma_2^2+k'^2}}$$

phase equivalence!

$$V\left(k,k'\right) = c_0 \sqrt{\frac{k_*^2 - k^2}{\gamma_1^2 + k^2}} \sqrt{\frac{k_*^2 - k'^2}{\gamma_1^2 + k'^2}}$$

$$c_0 \xrightarrow{\overline{\mathrm{MS}}} -\frac{4\pi}{M}\left(\mathbf{e}_1^{(3)}\right)^{-1}$$

S_2

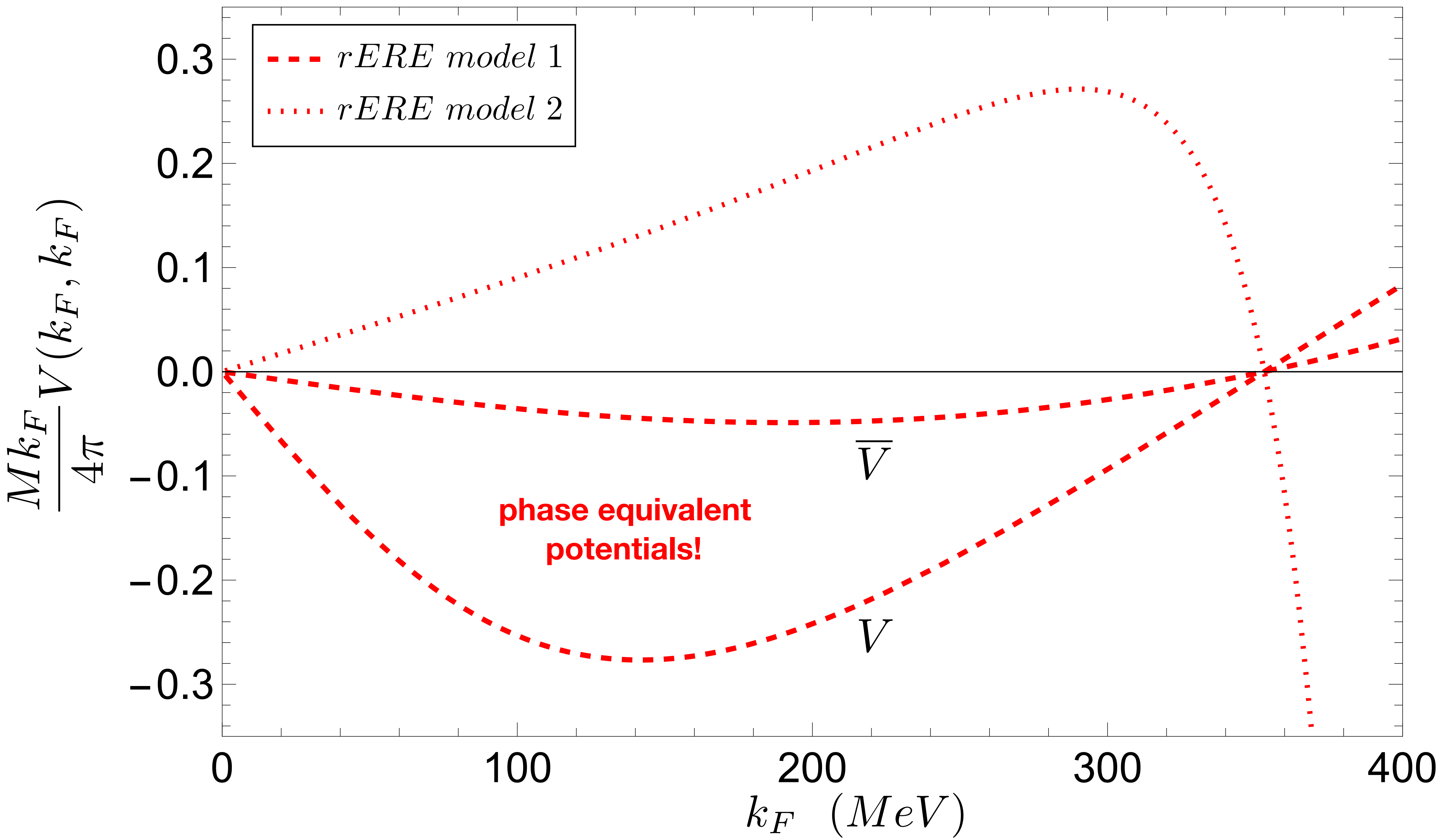
$$\gamma_1 = 212.19 \text{ MeV} \quad , \quad \gamma_2 = 618.74 \text{ MeV} \quad , \quad \gamma_3 = -7.93 \text{ MeV}$$

$$\overline{V}\left(k,k'\right) = c_0 \sqrt{\frac{k_*^2 - k^2}{\gamma_2^2 + k^2}} \sqrt{\frac{k_*^2 - k'^2}{\gamma_2^2 + k'^2}}$$

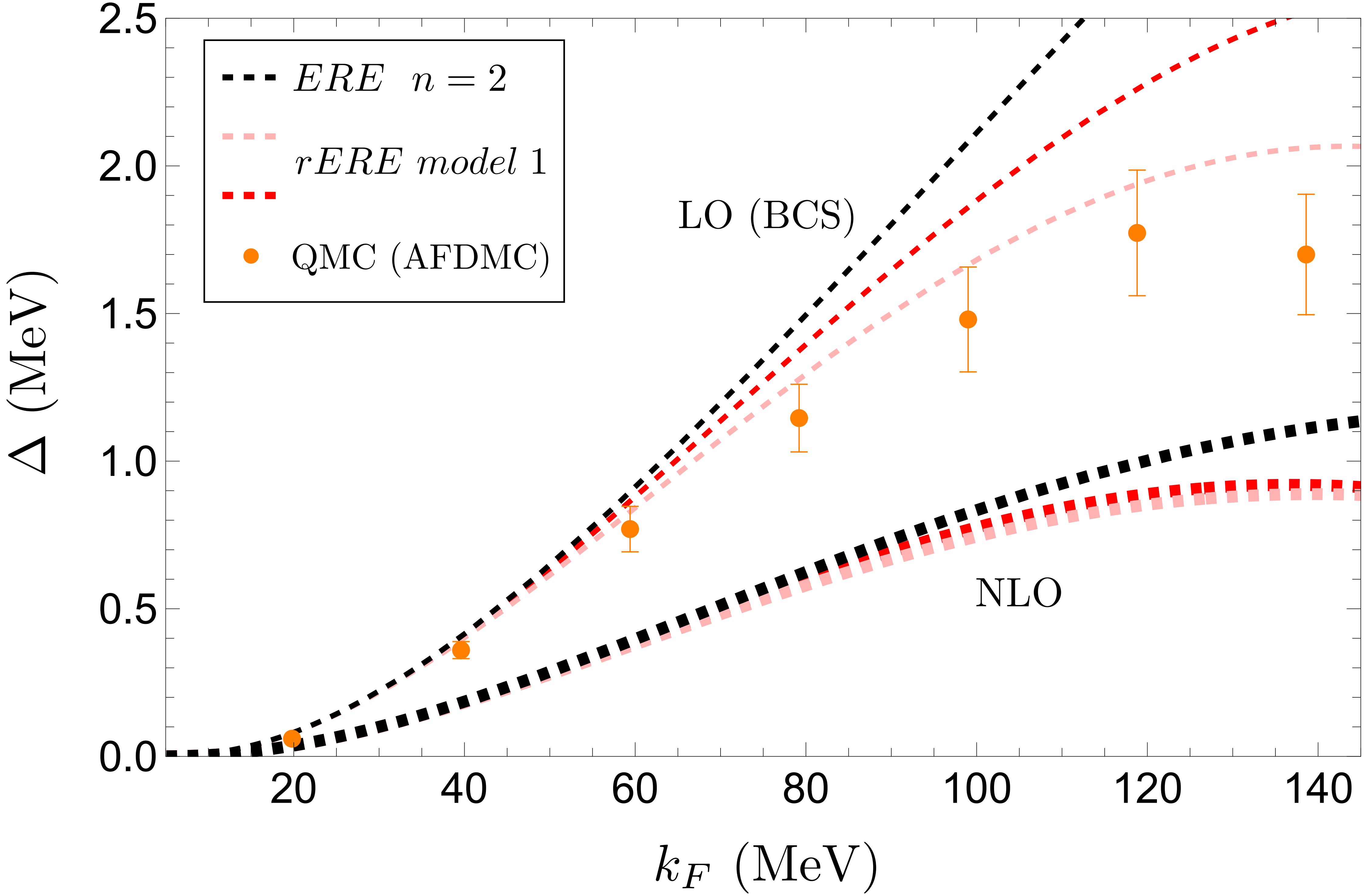
phase equivalence!

$$V\left(k,k'\right) = \frac{4\pi}{M}\mathbf{e}_1^{(4)} \sqrt{\frac{k_*^2 - k^2}{(\gamma_1^2 + k^2)(\gamma_2^2 + k^2)}} \sqrt{\frac{k_*^2 - k'^2}{(\gamma_1^2 + k'^2)(\gamma_2^2 + k'^2)}}$$

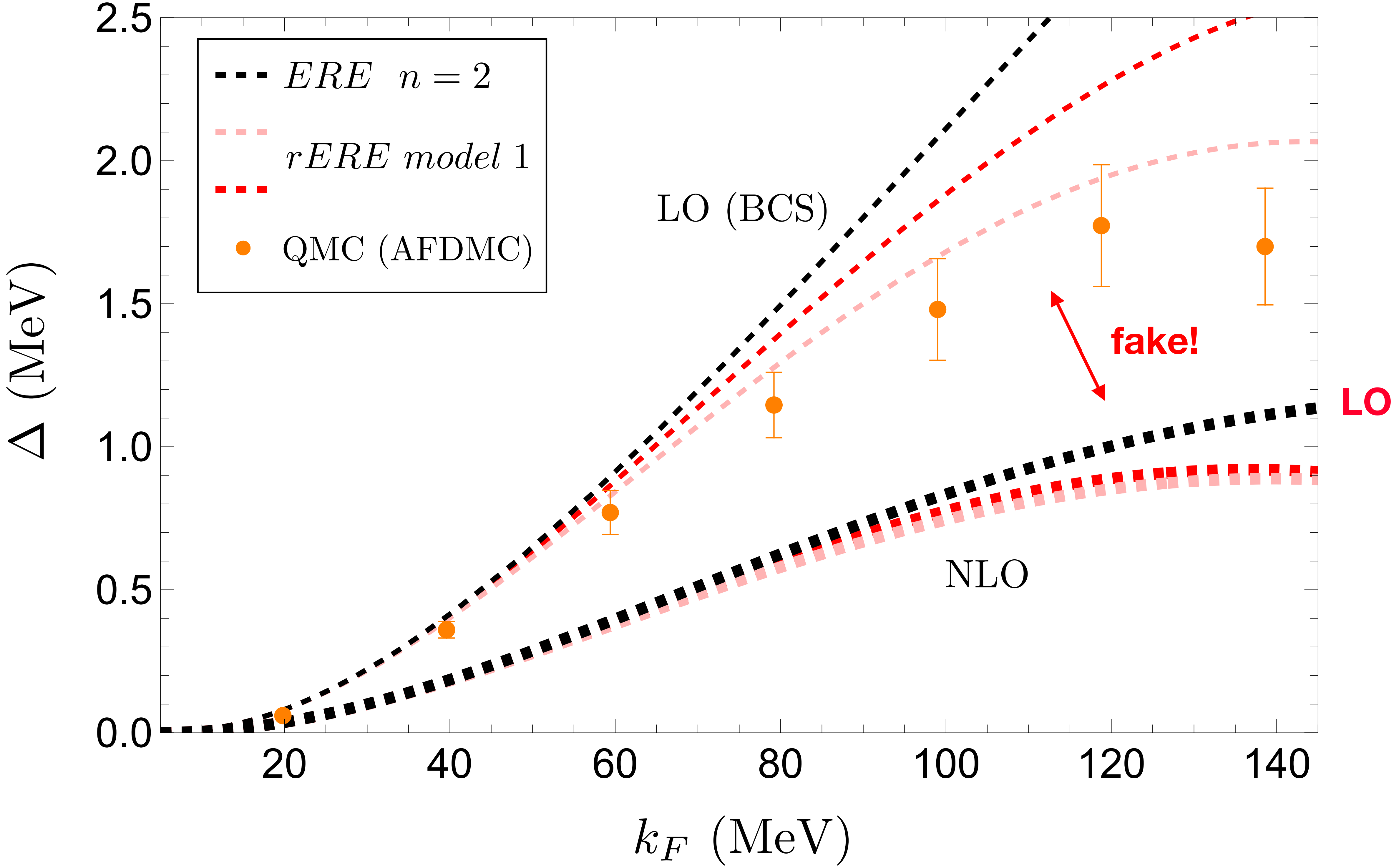
$$\gamma_1 = 37.04{+}425.89i \text{ MeV} \; , \; \gamma_2 = 37.04{-}425.89i \text{ MeV} \; , \; \gamma_3 = 168.76 \text{ MeV} \; , \; \gamma_4 = -7.93 \text{ MeV}$$



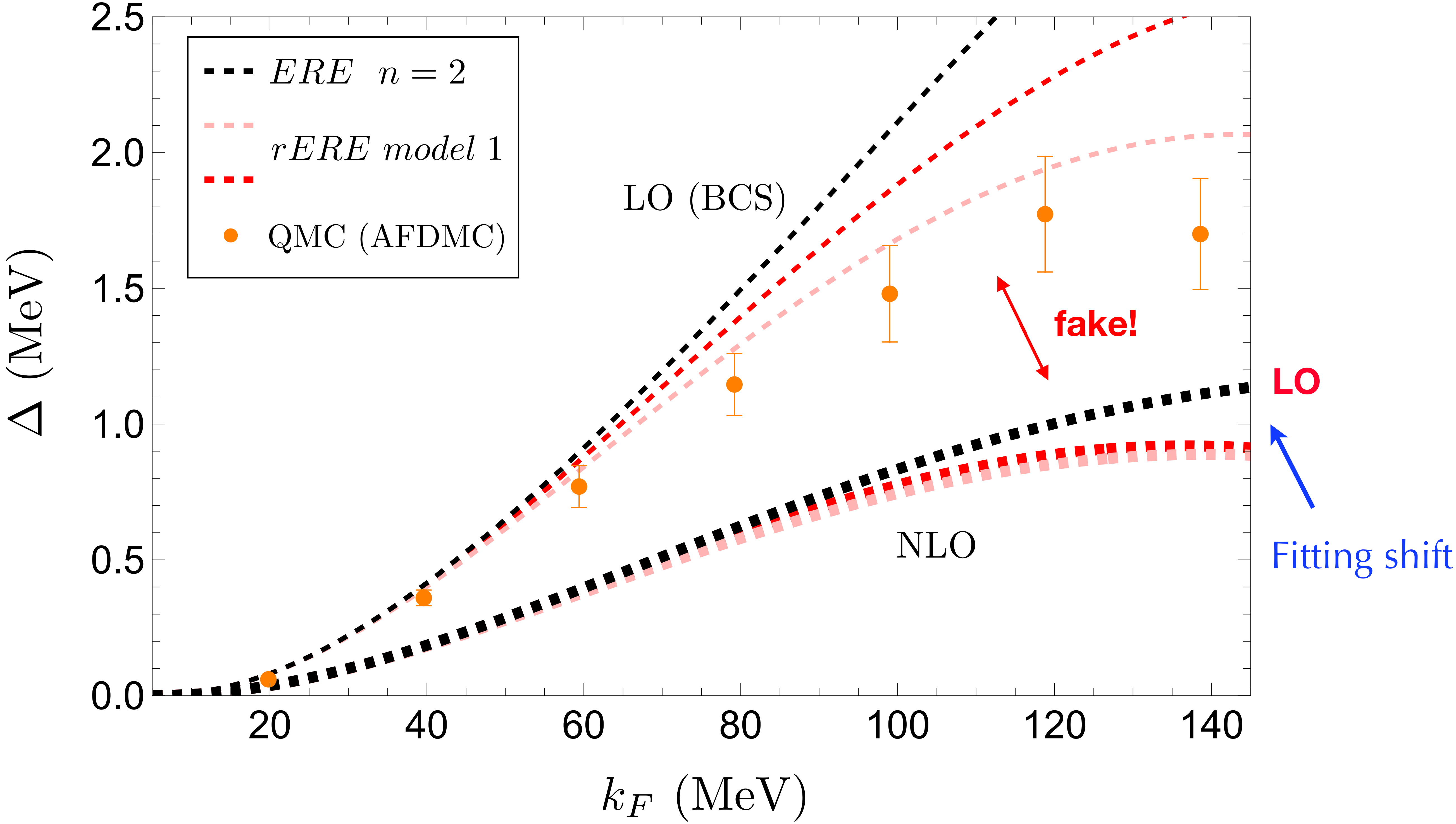
rERE model 1
rERE model 2
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0
100
200
300
400
$k_F \ (MeV)$
$\frac{Mk_F}{4\pi}V(k_F,k_F)$
$\overline{V}$
V
phase equivalent
potentials!



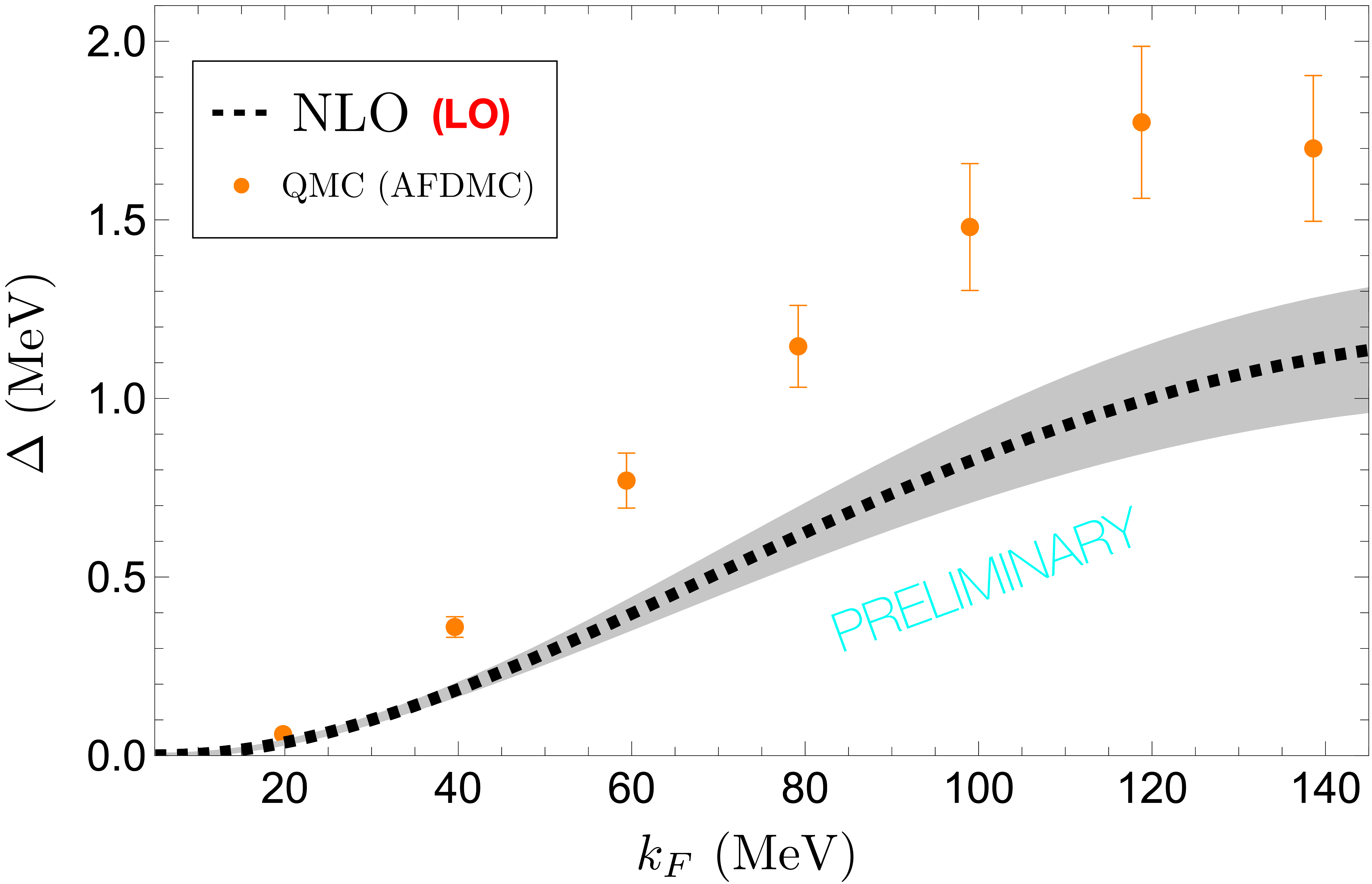
2.5
2.0
1.5
1.0
0.5
0.0
20
40
60
80
100
120
140
Δ (MeV)
k_F (MeV)
$ERE \quad n = 2$
$rERE \; model \; 1$
QMC (AFDMC)
LO (BCS)
NLO



$ERE \quad n=2$
$rERE \; model \; 1$
QMC (AFDMC)
LO (BCS)
fake!
LO
NLO
Δ (MeV)
k_F (MeV)
0.0
0.5
1.0
1.5
2.0
2.5
20
40
60
80
100
120
140

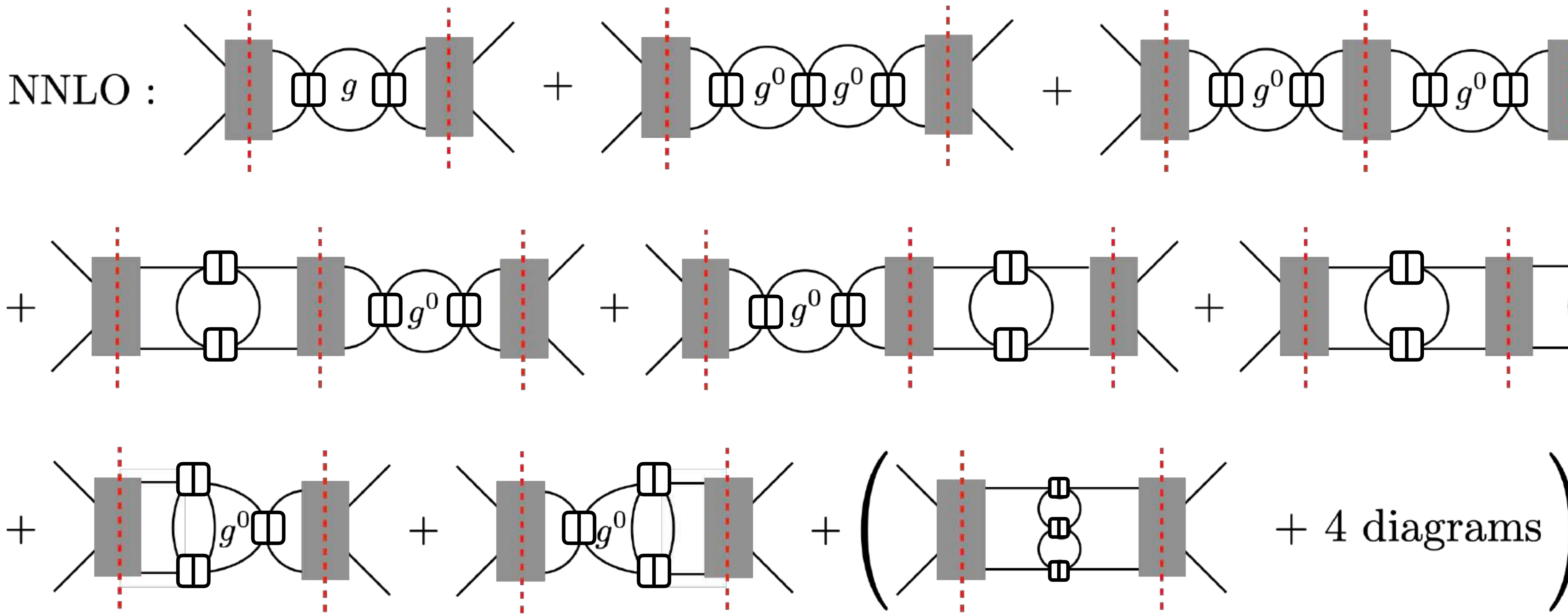


2.5
2.0
1.5
1.0
0.5
0.0
20
40
60
80
100
120
140
Δ (MeV)
k_F (MeV)
$ERE \quad n = 2$
$rERE\ model\ 1$
QMC (AFDMC)
LO (BCS)
fake!
LO
NLO
Fitting shift

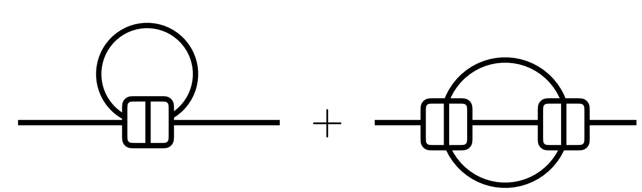


NLO (LO)
QMC (AFDMC)
PRELIMINARY
2.0
1.5
1.0
0.5
0.0
Δ (MeV)
20
40
60
80
100
120
140
k_F (MeV)

NNLO in progress



Self-energy corrections



Summary

- A systematic power counting for computing the superfluid gap has been developed. This method applies to free-space interactions that are weak in units of the Fermi momentum.
- Corrections to the Gor'kov,Melik-Barkhudarov effect in the case of a momentum-independent contact interaction have been computed and are found to further suppress the gap.
- The LO gap in the pionless effective field theory has been computed. There is tension with QMC simulations. NLO is coming soon.

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