Nuclear Physics at $m_\pi = 82$ MeV

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.

INT workshop on Chiral EFT: New Perspectives,
Seattle, 18/03/25

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- Why should we care?
- m_π dependence of interactions in ChiEFT - can we make reliable estimates?
- m_π dependence of the neutron matter EOS and axion condensation in neutron stars.
- Conclusions.

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Junior Fellow
U of Washington

Vincenzo Cirigliano
Senior Fellow
U of Washington

QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f} \left(i\gamma^\mu (\delta_{\alpha\beta}\partial_\mu - g\ (T_a G^a_\mu)_{\alpha\beta}) + m_f \right) \psi_{\beta f} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

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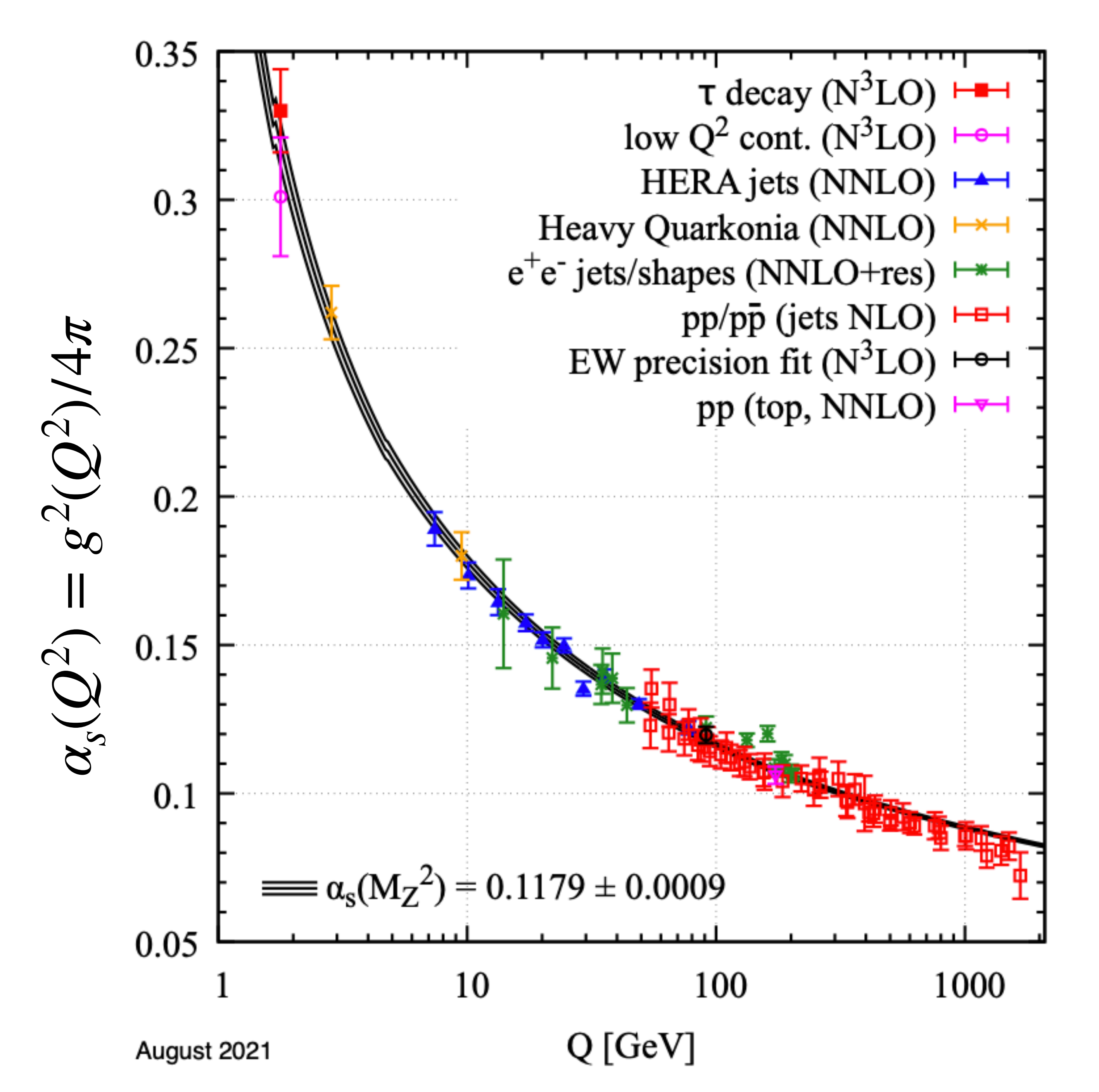
Running Coupling



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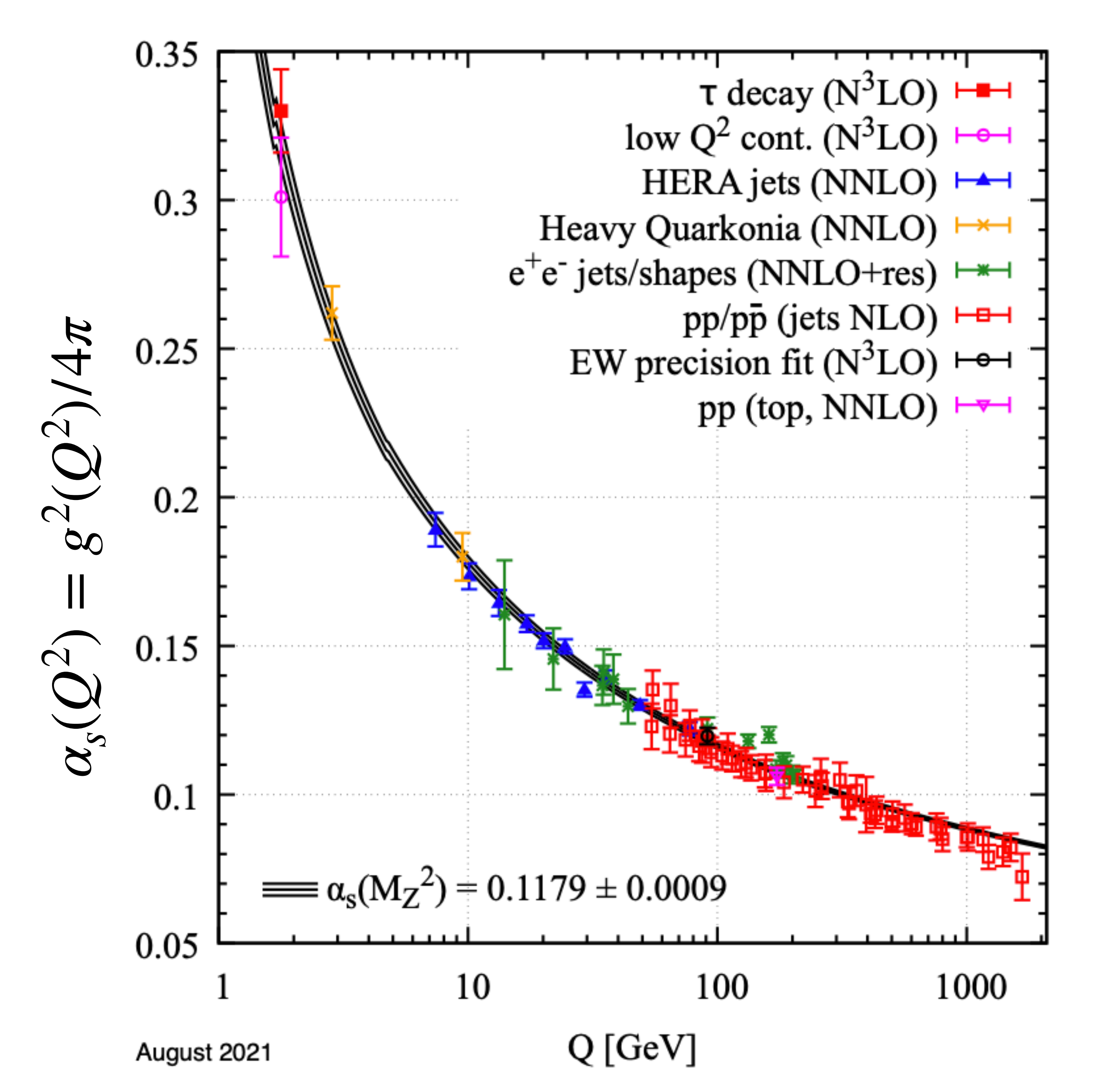

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Quark Mass Matrix

$$\begin{pmatrix} m_u \approx 2.5\ \text{MeV} & 0 & 0 \\ 0 & m_d \approx 5\ \text{MeV} & 0 \\ 0 & 0 & m_s \approx 100\ \text{MeV} \end{pmatrix}$$

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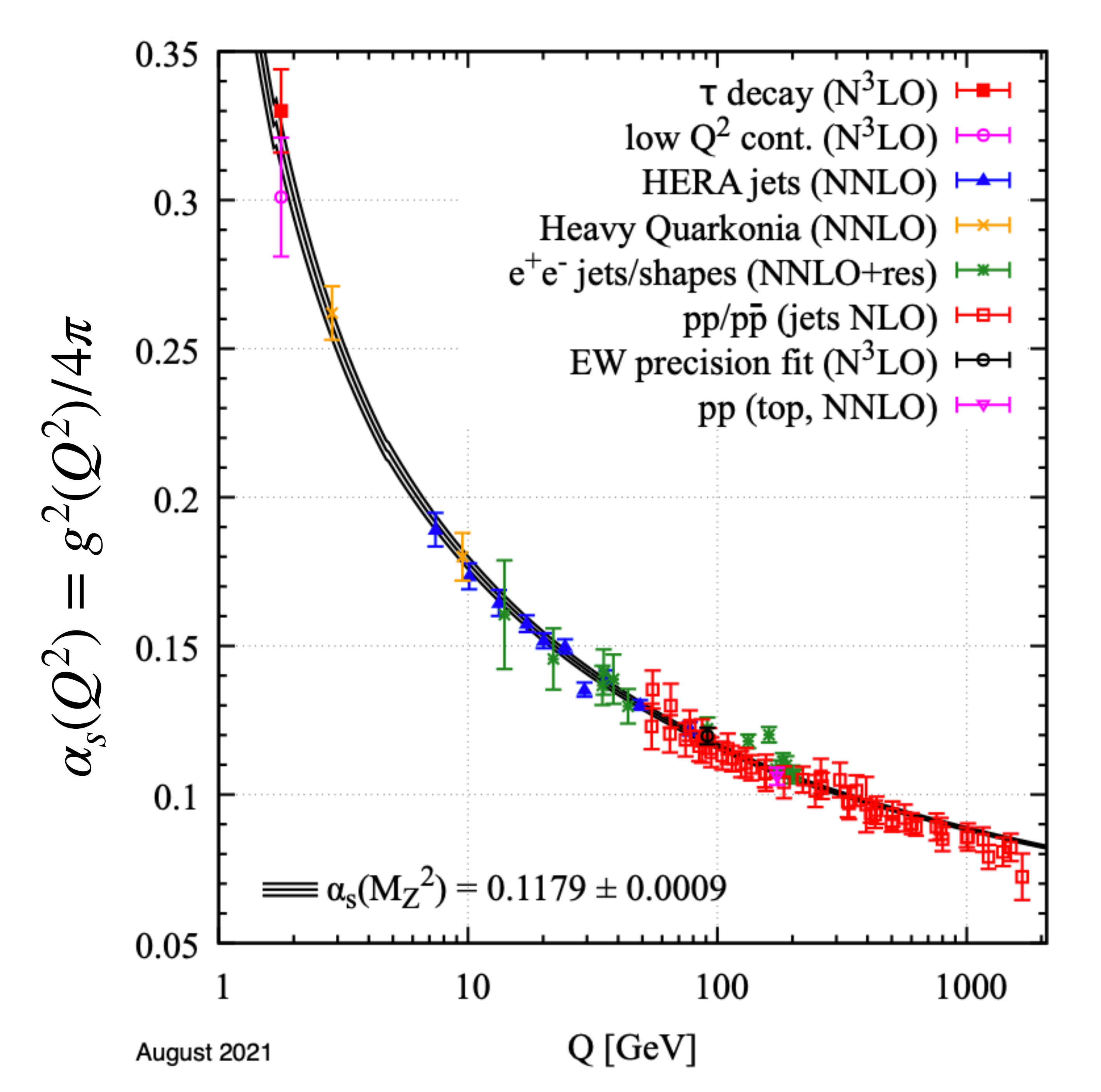

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θ-QCD

- Source of CP violation
- Induces neutron EDM:

  $d_n \approx 3 \times 10^{-16}\ \theta$ e cm
- Experimental bound:

  $d_n \lesssim 10^{-26}$ e cm

  or $\theta \lesssim 10^{-10}$

θ and Axions

$$\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

To explain $\theta < 10^{-10}$, θ was promoted to a dynamical quantity - a new field that relaxes to zero to minimize the free energy:

$$\theta = \frac{a}{f_a}$$

Axion field (a)

A new high energy scale (f_a)

The axion is a pseudo-scalar particle that arises as a Goldstone boson from the breaking of a new U(1) symmetry introduced by Pecci and Quinn.

R. Peccei and H. R. Quinn (1977), S. Weinberg (1978), F. Wilczek (1978)

Axion Mass and Energy

The axion coupling to gluons can be elimanted by a transformation of the quark mas matrix M_q

$$M_q \to M_q \ \exp\left(i\frac{a}{f_a}\,Q_a\right) \qquad \text{where} \qquad Q_a = \frac{M_q^{-1}}{\mathrm{Tr}\ M_q^{-1}} = \frac{1}{m_u + m_d}\begin{pmatrix} m_d & & \\ & m_u & \\ & & 0 \end{pmatrix} + \mathcal{O}[m_u/m_s, m_d/m_s]$$

This leads to an axion mass which can be calculated from Chiral Perturbation Theory

$$m_a^2 = \frac{f_\pi^2}{f_a^2}\left(\frac{m_u m_d}{(m_u + m_d)^2}\right) m_\pi^2$$

And a corresponding contribution to the energy density or an axion potential

$$V\left(\theta = \frac{a}{f_a}\right) = f_\pi^2 m_\pi^2 \left[1 - \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2}\sin^2\left[\frac{\theta}{2}\right]}\right] = \frac{1}{2} f_a^2 m_a^2 \theta^2 + \cdots$$

Which is minimized at $\theta = 0$.

At $\theta = \pi$

$$\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

At $\theta = \pi$

$$\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \longrightarrow \begin{pmatrix} -m_u & 0 \\ 0 & m_d \end{pmatrix}$$

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Energy cost associated with the axion field:

$$V(\theta = \pi) = f_\pi^2 m_\pi^2 \left[1 - \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2}} \right] \simeq \frac{2}{3} f_\pi^2 m_\pi^2$$

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$$\frac{m_\pi^2(\theta = \pi)}{m_\pi^2(\theta = 0)} = \frac{m_d - m_u}{m_d + m_u} \approx \frac{1}{3} \longrightarrow m_\pi(\theta = \pi) \simeq 82 \text{ MeV}$$

How do (light) quark masses affect low-energy nuclear physics?

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2}\left(m_u + m_d\right) + \cdots \quad \Rightarrow \quad K_{m_\pi} = \frac{m_q}{m_\pi}\frac{\delta m_\pi}{\delta m_q} \simeq 0.5$$

$$m_n = m_0 + \sigma_{\pi n}\frac{m_\pi^2}{(m_\pi^2)_{\text{phys}}} + \cdots \quad \Rightarrow \quad K_{m_n} = \frac{m_q}{m_n}\frac{\delta m_n}{\delta m_q} \approx \frac{m_\pi^2}{m_n}\frac{\delta m_n}{\delta m_\pi^2} \simeq 0.05$$

$\sigma_{\pi n} \simeq 50 \pm 10$ MeV

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), J. Donoghue (2006) , Beane and Savage (2003),

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Masses of heavier vector mesons are relatively insensitive to the quark mass.

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Mass of the scalar sigma meson

$$K_{m_\sigma} = \frac{m_q}{m_\sigma}\frac{\delta m_\sigma}{\delta m_q} \simeq 0.1$$

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), J. Donoghue (2006) , Beane and Savage (2003),

Hadrons at $\theta \neq 0$

Because $M_q \to M_q \; \exp\left(2i\theta \; Q_a\right)$

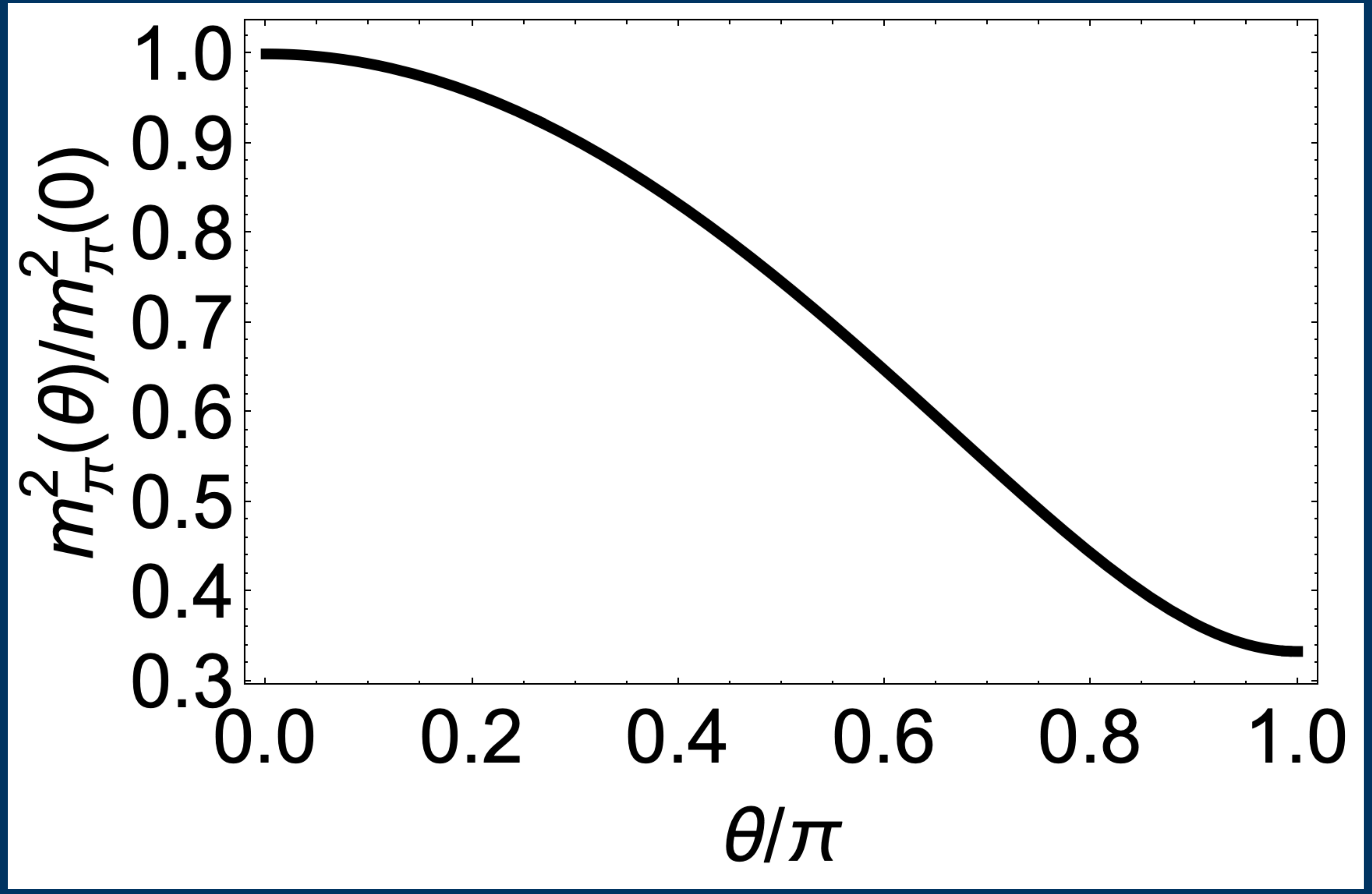
the pion mass decreases with θ:

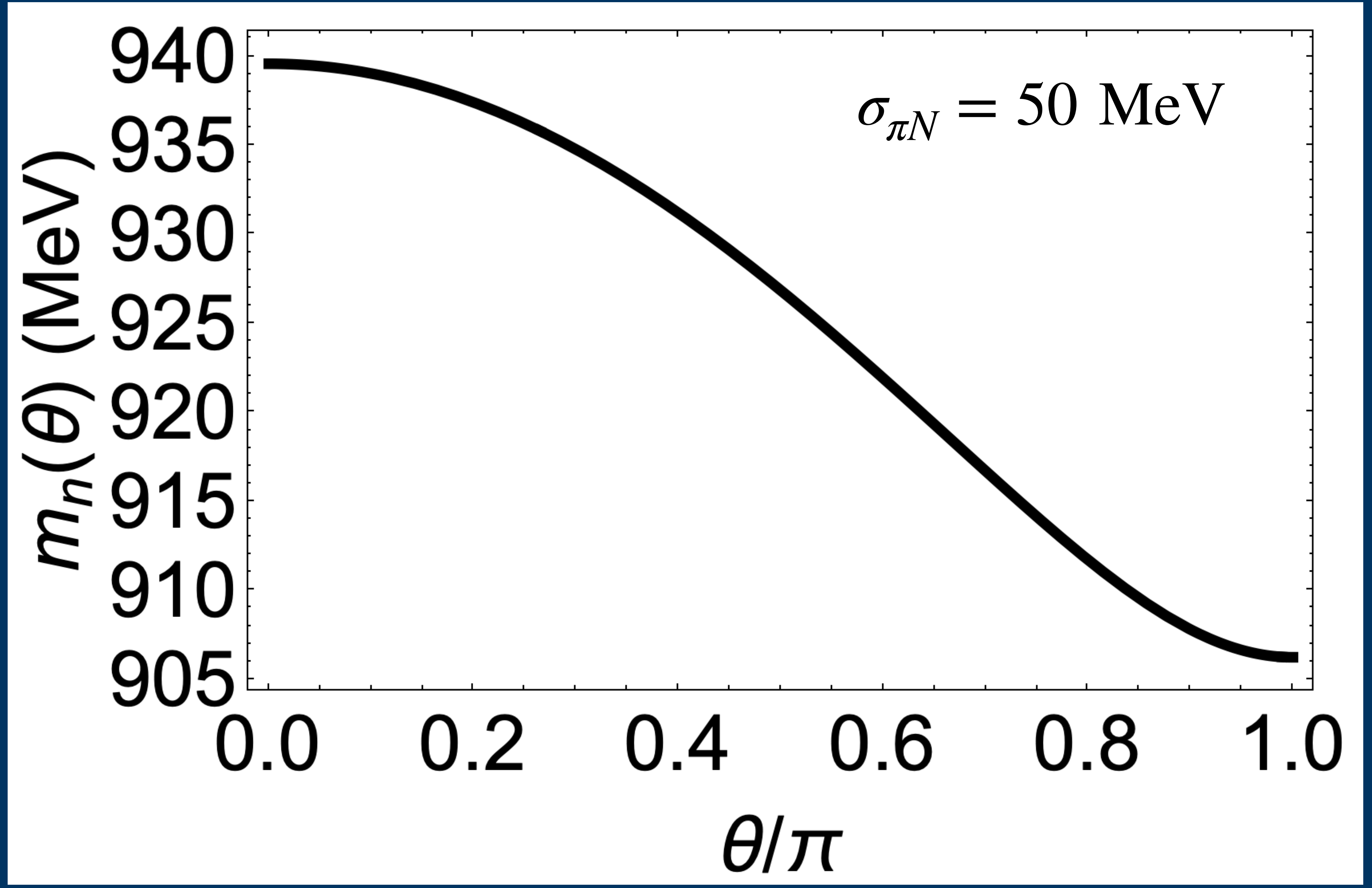
$$m_\pi^2(\theta) = m_\pi^2(\theta = 0)\sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2}\sin^2\left[\frac{\theta}{2}\right]}$$

$$\frac{m_\pi^2(\theta = \pi)}{m_\pi^2(\theta = 0)} = \frac{m_d - m_u}{m_d + m_u} \approx \frac{1}{3}$$

The resulting decrease in the nucleon mass

$$m_n(\theta) = m_0 + \sigma_{\pi n}\frac{m_\pi^2(\theta)}{m_\pi^2(\theta = 0)}$$





Axion Condensation

The decrease in the nucleon mass $m_n(\theta) = m_0 + \sigma_{\pi n} \frac{m_\pi^2(\theta)}{m_\pi^2(\theta = 0)} + \cdots$

favors a first-order transition to a ground state with $\theta = \pi$

Neglecting nuclear interactions the energy gain per nucleon is

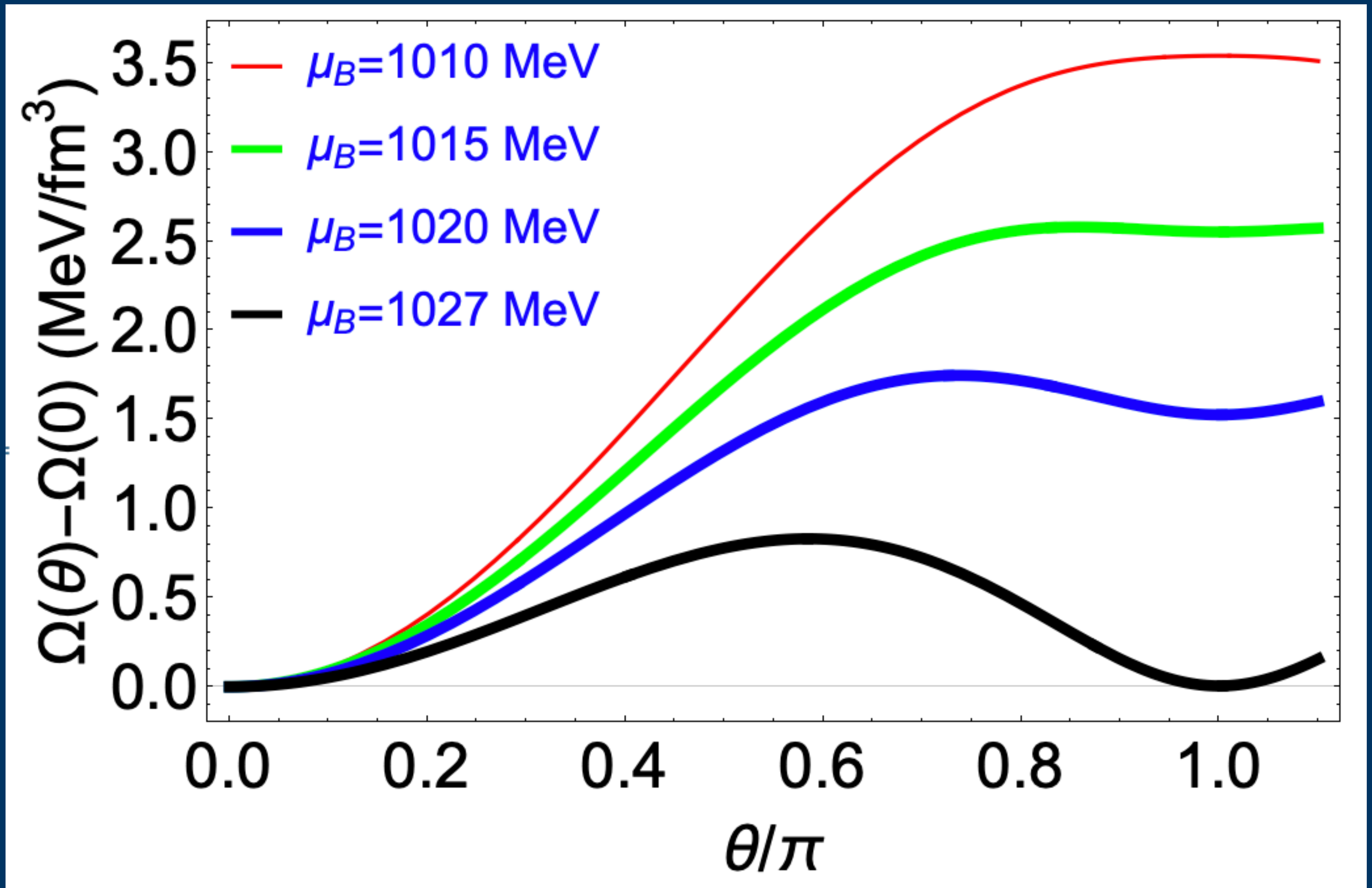
$$\Delta E \simeq \sigma_{\pi n} \left(1 - \frac{m_\pi^2(\theta = \pi)}{m_\pi^2(\theta = 0)} \right) \simeq \frac{2}{3} \sigma_{\pi n}$$

The energy cost (due to axion potential) per nucleon is

$$\Delta E = \frac{V(\theta = \pi)}{n_B} \simeq \frac{2}{3} \frac{f_\pi^2 m_\pi^2}{n_B}$$

Condensation occurs when $\sigma_{\pi N} n_B > f_\pi^2 m_\pi^2$

For $\sigma_{\pi n} = 50$ MeV → $n_B^c \simeq 2.6\ n_{\rm sat}$



A. Hook and J. Huang (2018)
R. Balkin, J. Serra, K. Springmann, and A. Weiler, (2020)
R. Kumamoto, J. Huang, C. Drischler, M. Baryakthar, and S. Redd (2024)

Interactions

Does the energy per particle in nuclear systems increase or decrease with m_π ?

If $\Delta E_{\text{int}} = E_{\text{int}}(m_\pi = 82\ \text{MeV}) - E_{\text{int}}(m_\pi^{\text{phys}}) < 0$ condensation is expected at $n_B^c < 2.6\ n_{\text{sat}}$

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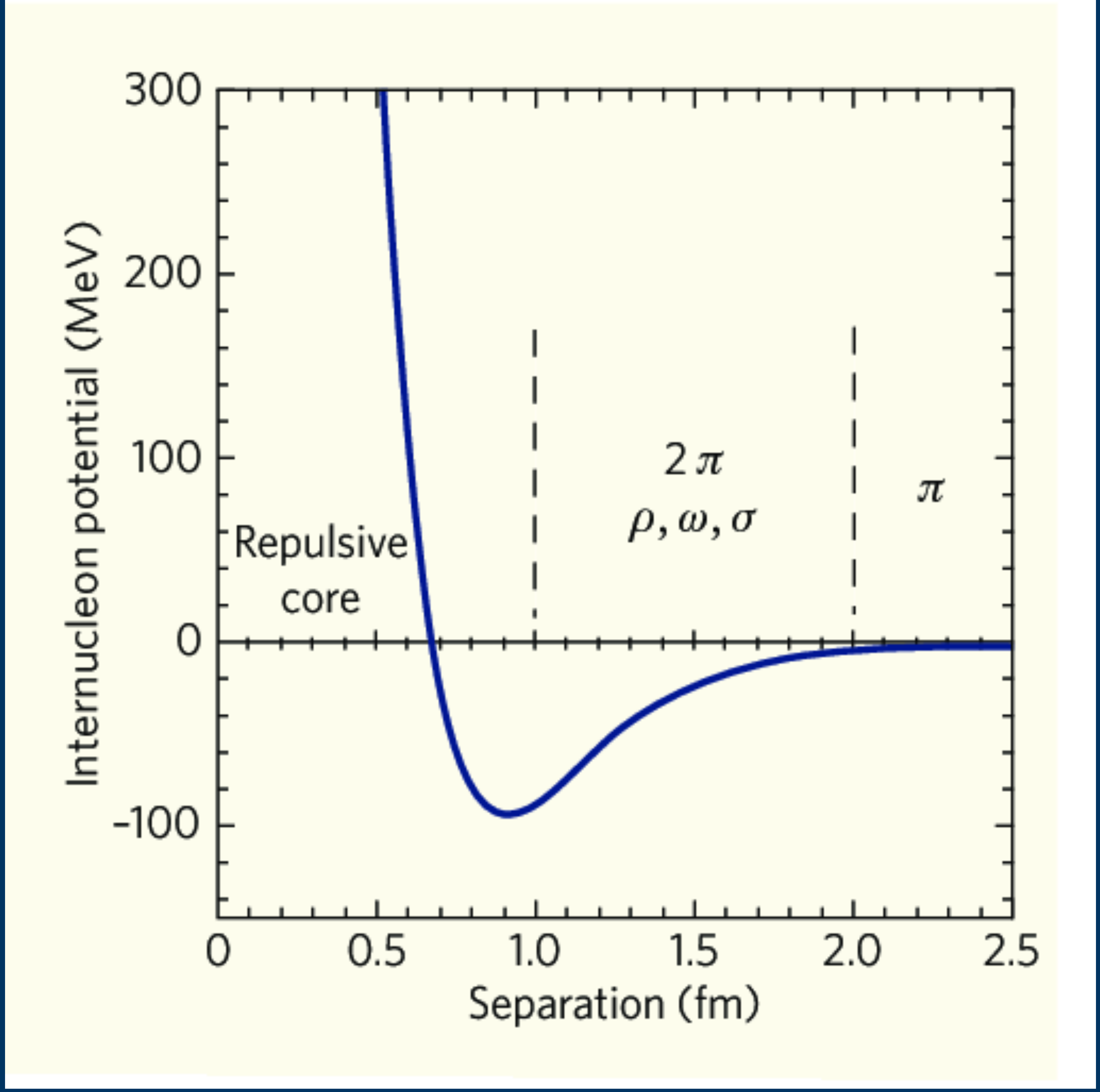
Can χEFT predict the sign of $\Delta E_{\rm int}(n_B \simeq n_{\rm sat})$?

How do quark masses affect nuclear interactions at low-energy?

Short answer: We do not really know.

- We can implement changing quark mass into pion-exchanges, but effects at short distances are not well understood.



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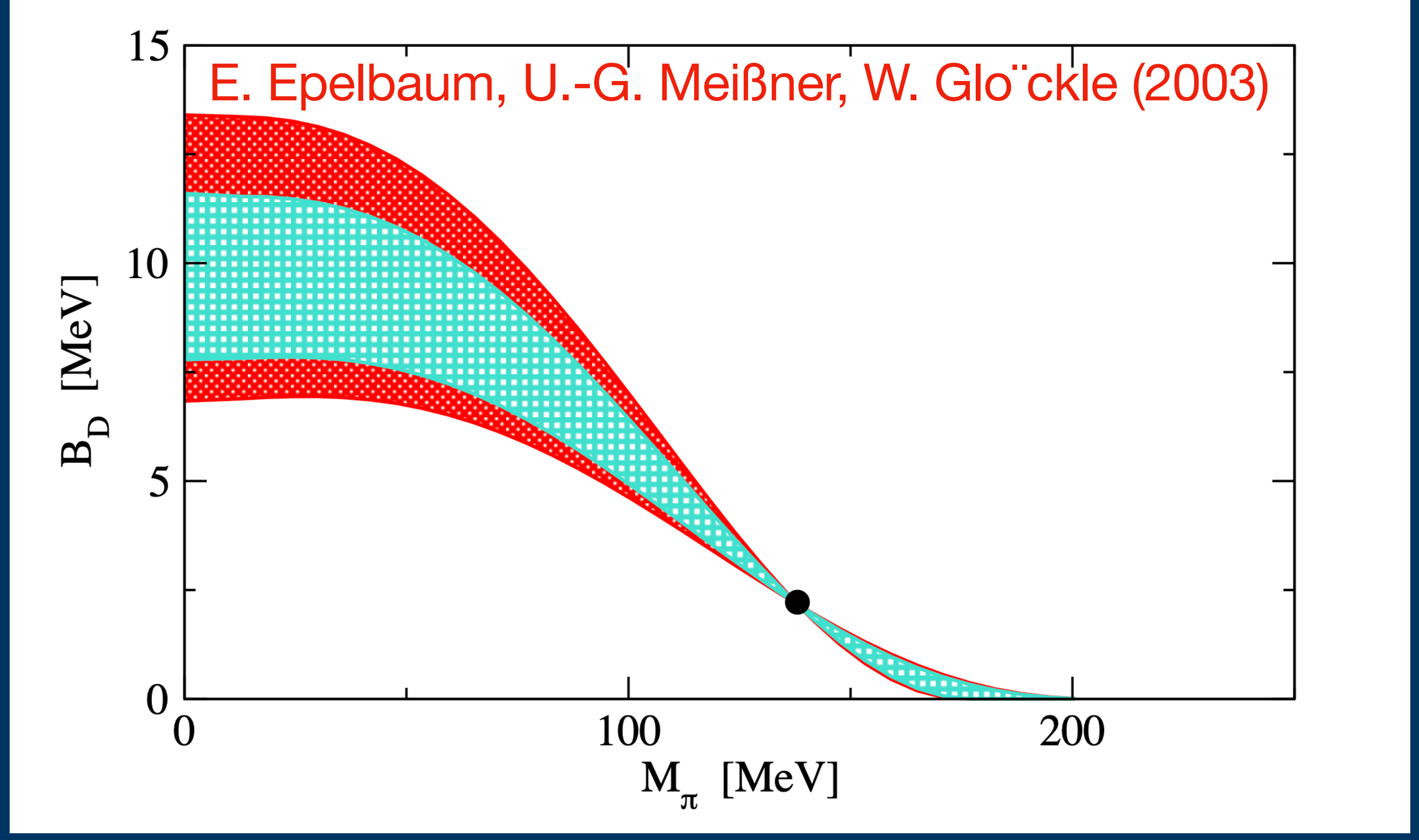

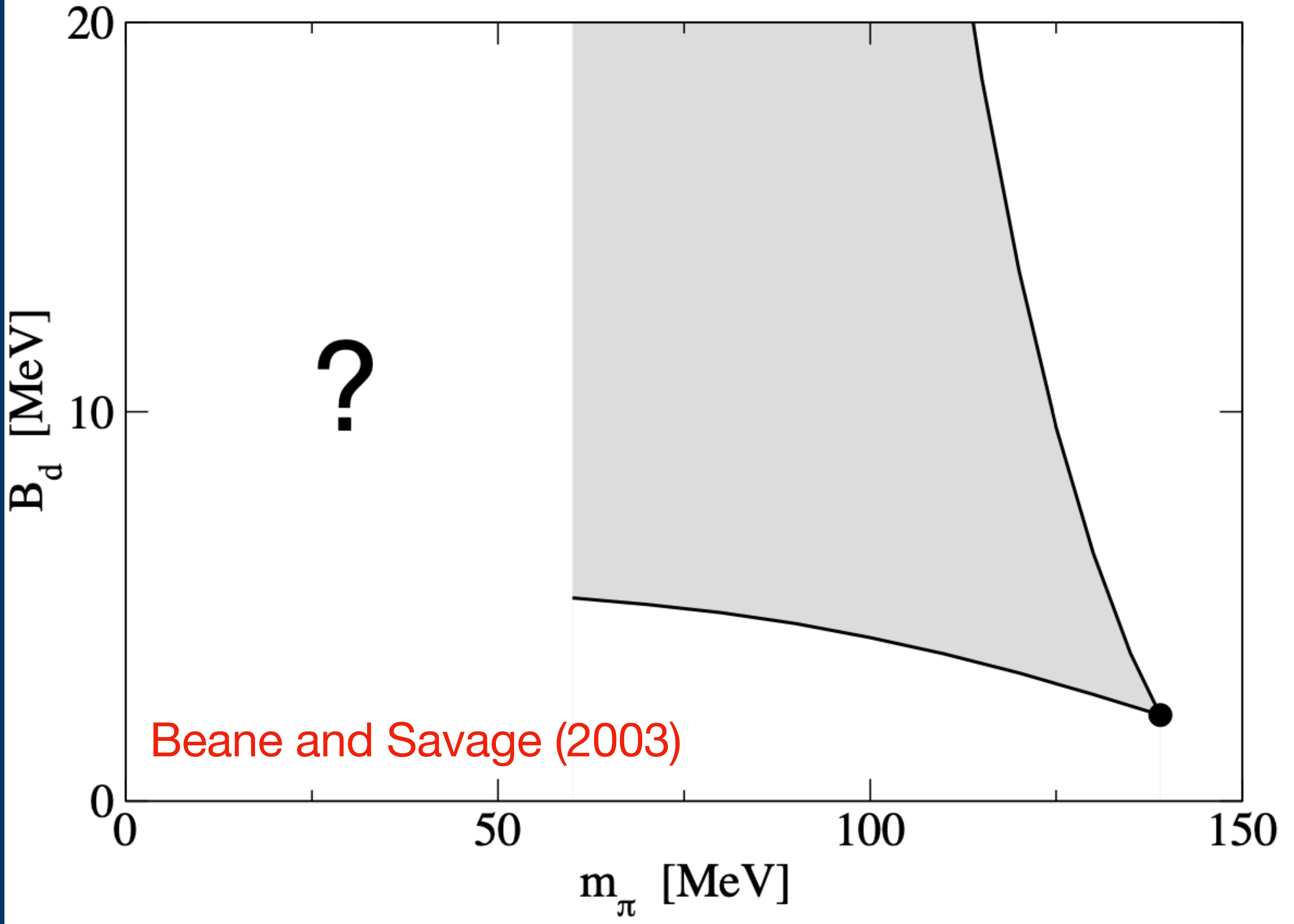

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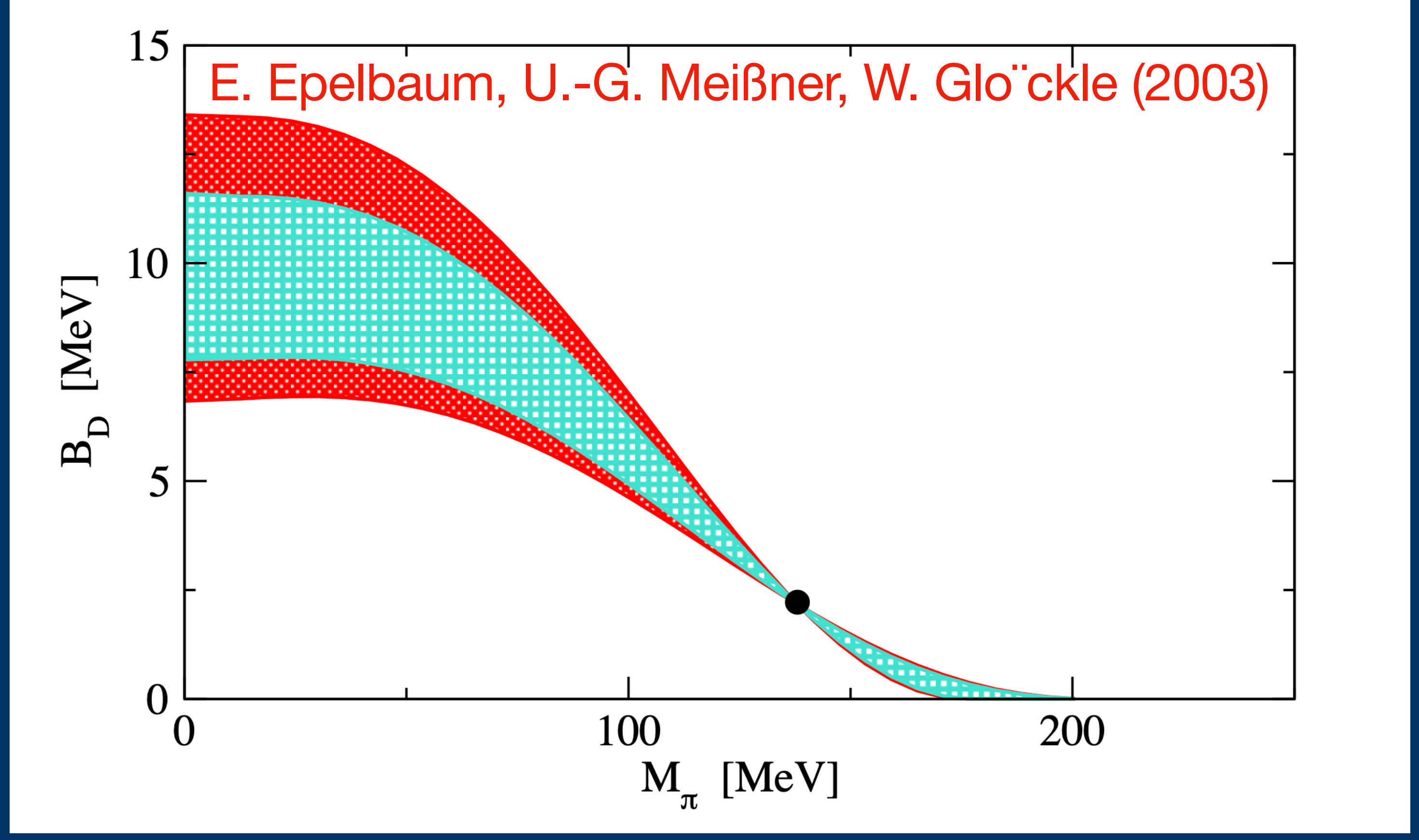

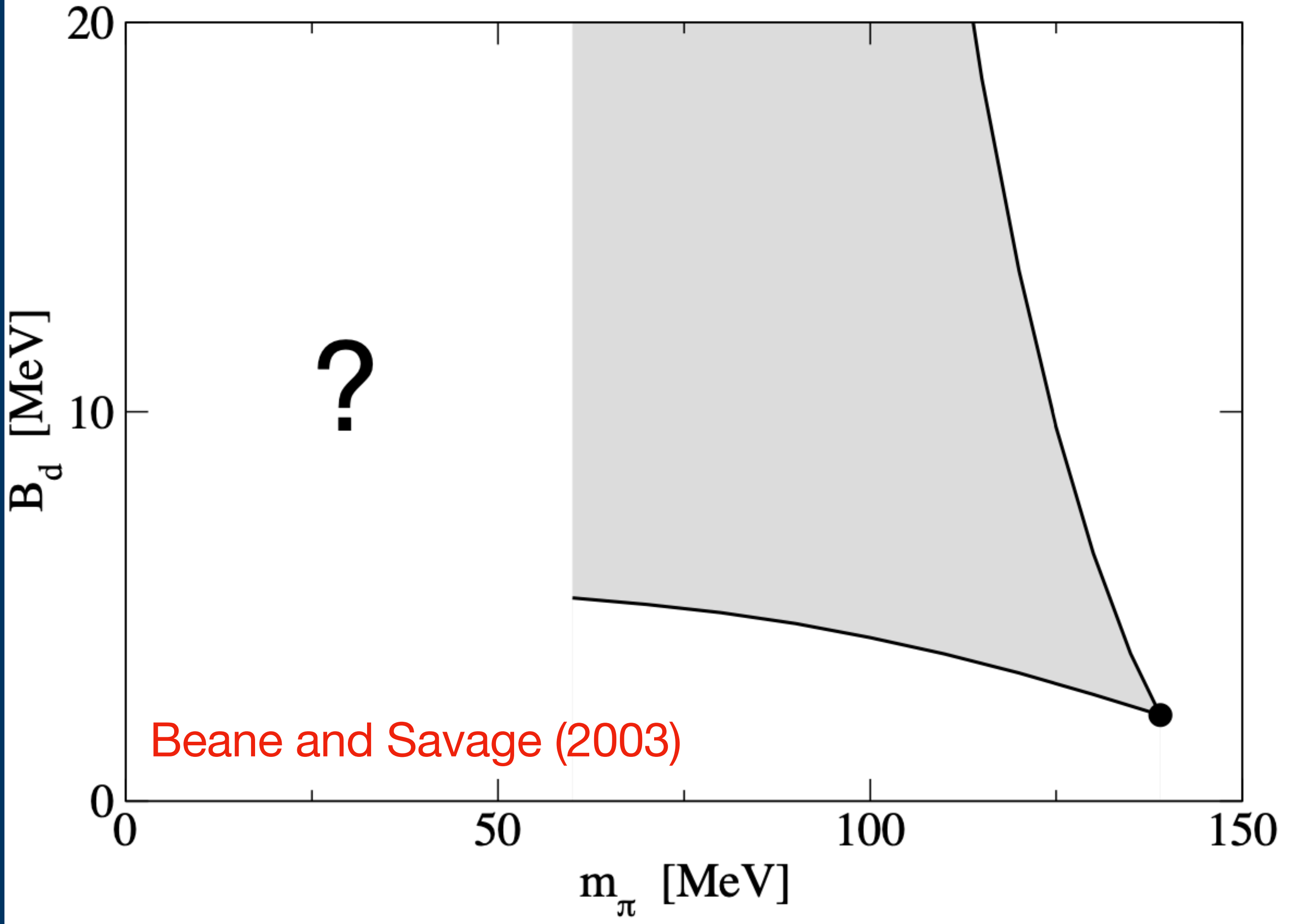

Effect of quark mass (pion mass) on the scattering length:

$$K_{a_s} = \frac{m_q}{a_s}\frac{\delta a_s}{\delta m_q} \simeq 2.4 \pm 3$$ J. C. Berengut, E. Epelbaum, et al. (2013)

$$\simeq 5 \pm 5$$ Beane and Savage (2003)

$$\simeq 2.3 \pm 1.9$$ E. Epelbaum, U.-G. Meißner, W. Glo¨ckle (2003)

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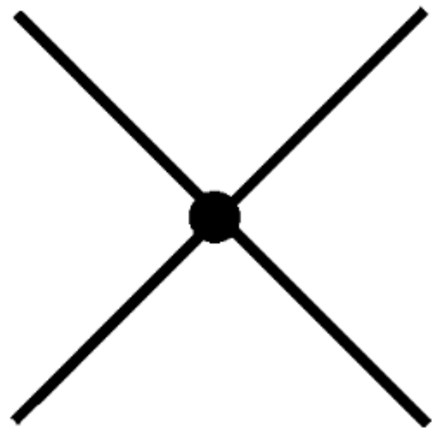
Quark (pion) mass-dependence of NN interaction in EFT



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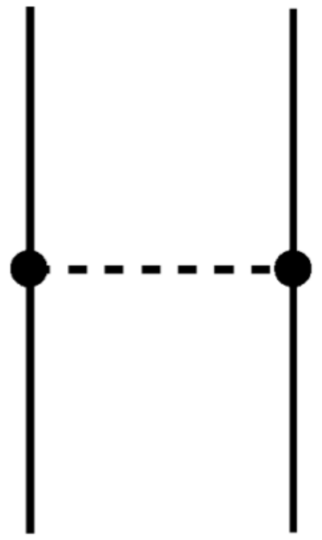

$$V_{\mathrm{LO}}(q) = C_0 + D_2\, m_\pi^2 + \frac{g_A^2}{4f_\pi^2}\,\frac{\sigma_1\cdot\mathbf{q}\;\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\,\tau_1\cdot\tau_2$$

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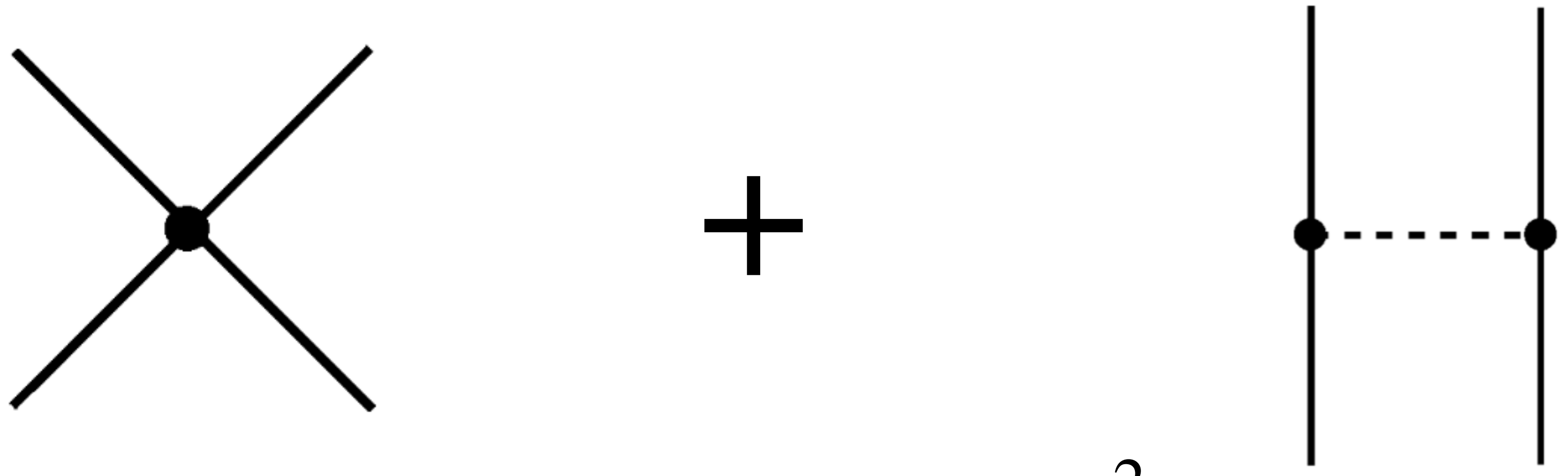

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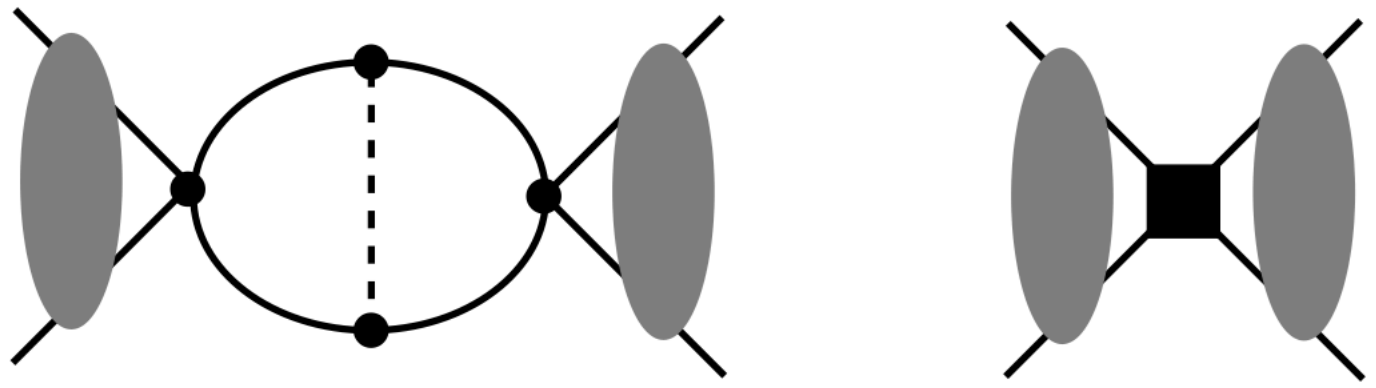
Renormalization requires D_2 :

Kaplan, Savage, Wise (1998)

To obtain a scattering amplitude that is independent of cut-off Λ requires:



$$\Lambda\frac{d}{d\Lambda}\left(\frac{D_2}{C_0^2}\right)_{\mathrm{KSW}} = \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \quad \Rightarrow \quad \frac{|D_2|}{C_0^2} \approx \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \simeq \frac{1}{4}$$

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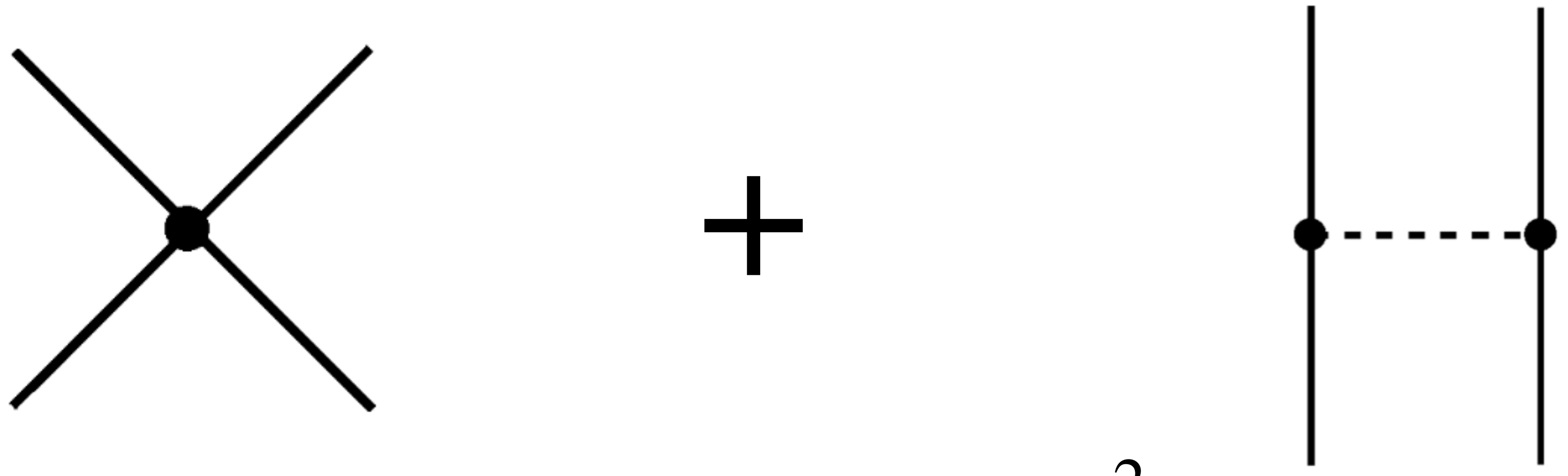

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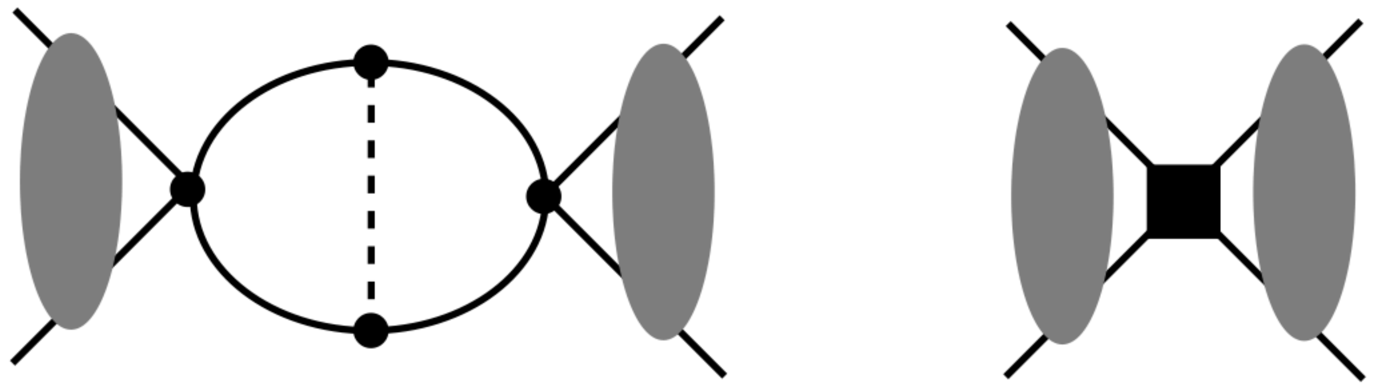

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Analysis of 2-nucleon scattering in Lattice QCD for different values m_π could, in principle, determine D_2 but systematics are too large at this time.

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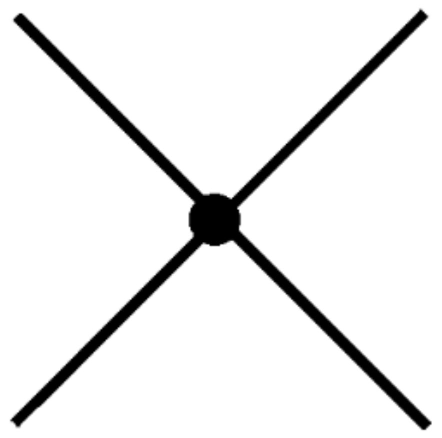

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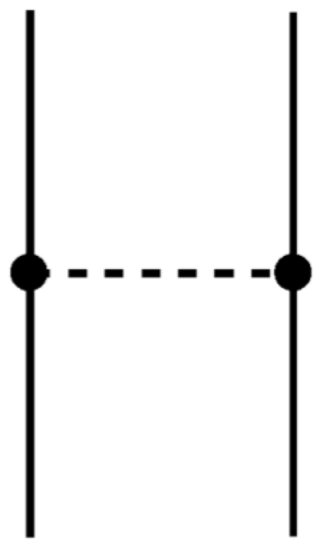

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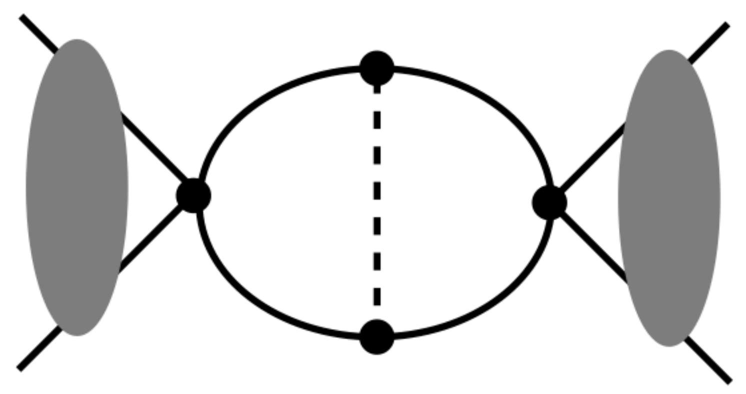

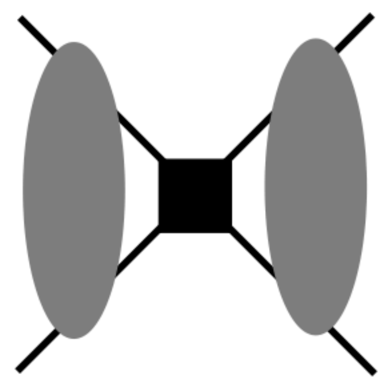

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Beane, Bedaque, Detmold, Savage (NPLQCD), Walker-Loud (Cal-Lat), Aoki, Hatsuda, Ishii (HAL QCD Collaboration), ….

D_2 can be important.

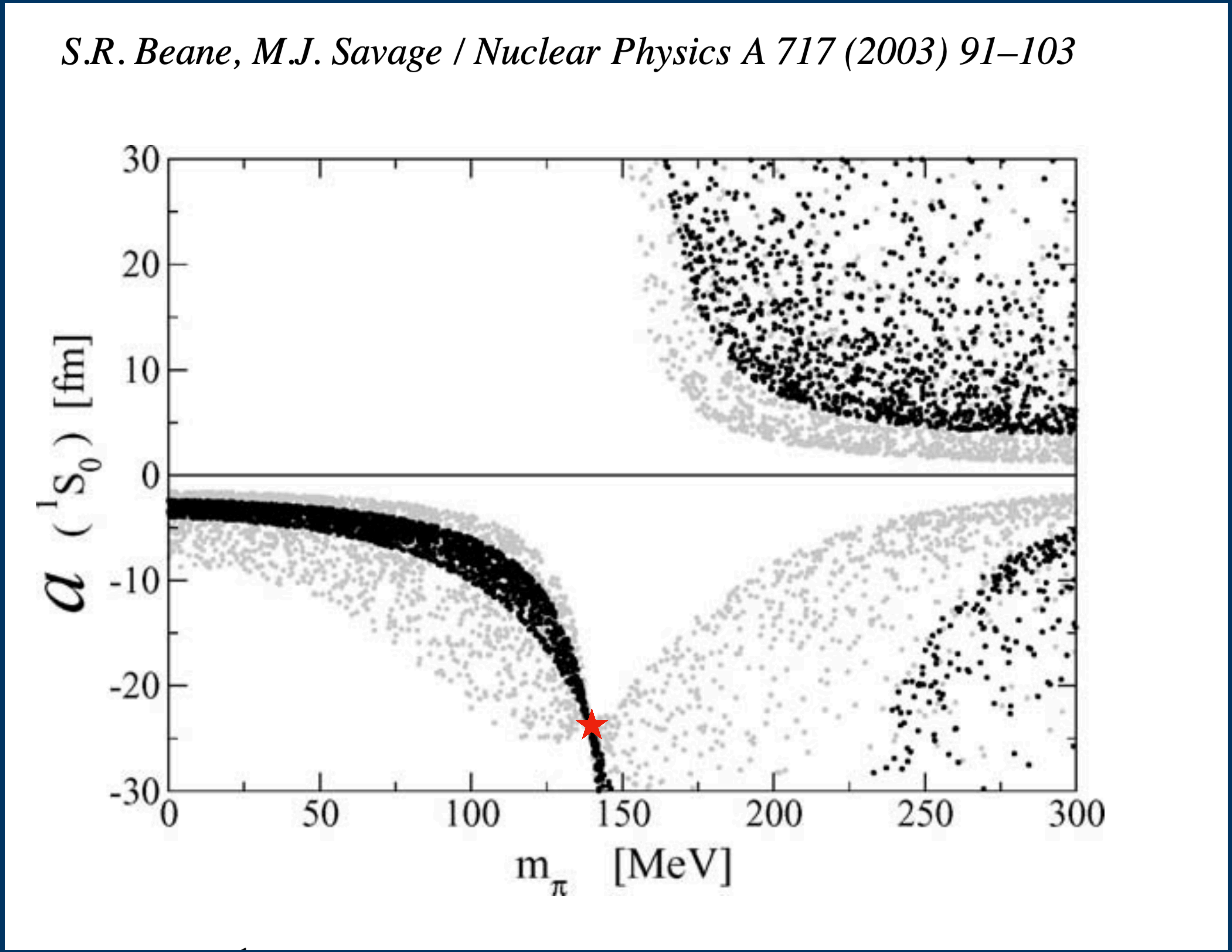
RG suggests:

$$|D_2| \simeq \frac{C_0^2}{4} \approx \frac{1}{5f_\pi^4}$$

Variation over a smaller range:

$$-\frac{1}{5} < \eta = \frac{D_2 m_\pi^2}{C_0^2} < \frac{1}{5}$$

has a significant impact on s-wave observables.



Pion Mass Dependence of Binding Energies

From resonance saturation and matching ChiEFT to the Bonn. potential model.

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013)

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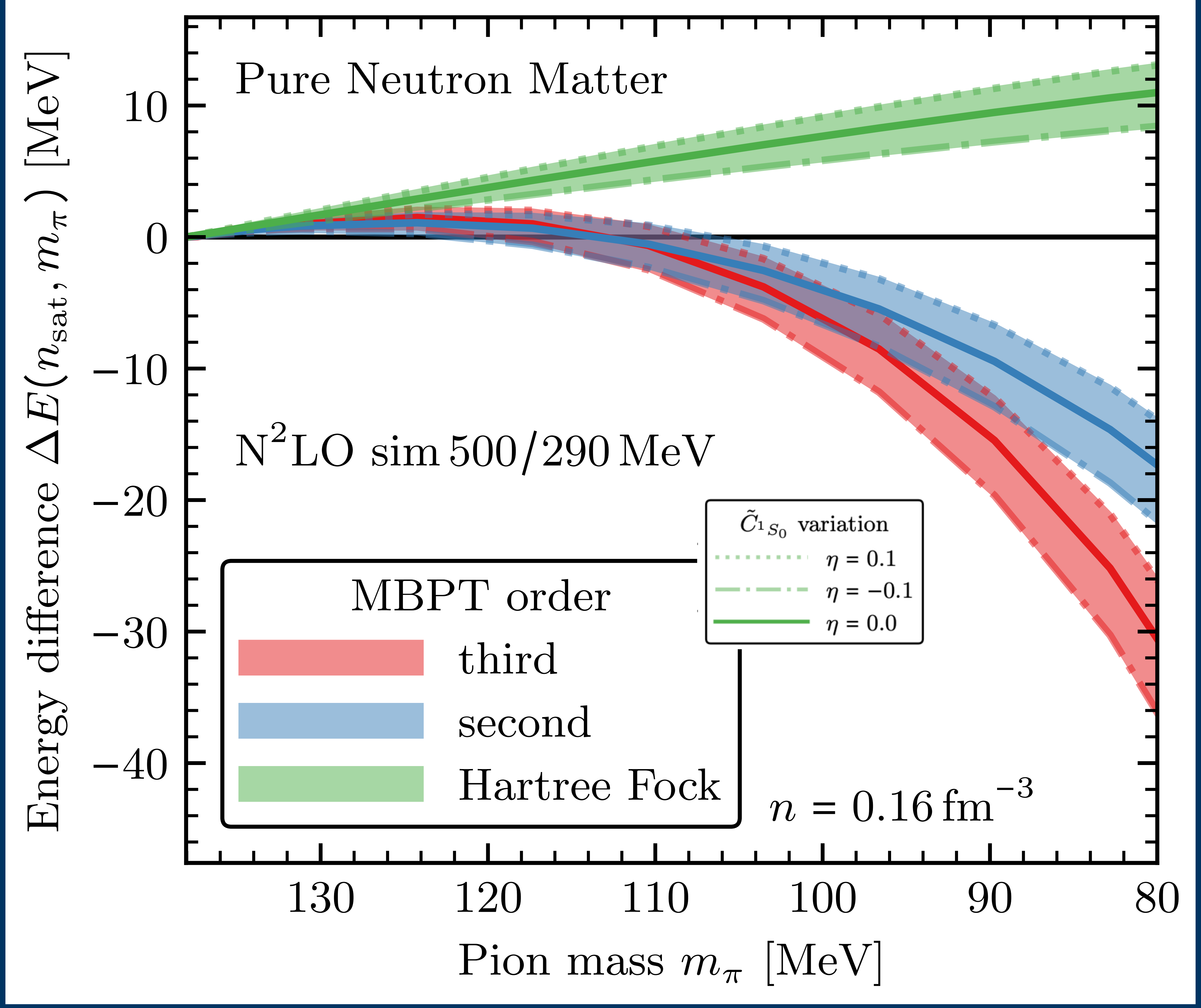
A modest increase in the binding of nuclear matter at $\theta = \pi$!

Can interactions favor $\theta = \pi$ in neutron matter ?

- ChiEFT at N²LO, with simple assumptions about short-distance forces.
- $$m_n = m_0 + \sigma_{\pi n} \frac{m_\pi^2}{(m_\pi^2)_{\text{phys}}}$$
- $g_A =$ constant and $f_\pi = f_0 \left(1 + l_4 \frac{m_\pi^2}{(4\pi f_0)^2} \right)$
- Variation of D_2 in the range
$$-0.1 < \eta = \frac{D_2 m_\pi^2}{\tilde{C}_{^1S_0}} < 0.1$$
- Cut-off variation is significant .. suggesting missing short-distance pion mass dependent corrections.

$$\Delta E_{\text{int}} = E_{\text{int}}(m_\pi) - E_{\text{int}}(m_\pi^{\text{phys}})$$



M. Kumamoto, J. Huang, C. Drischler, M. Baryakhtar, S. Reddy (2024)

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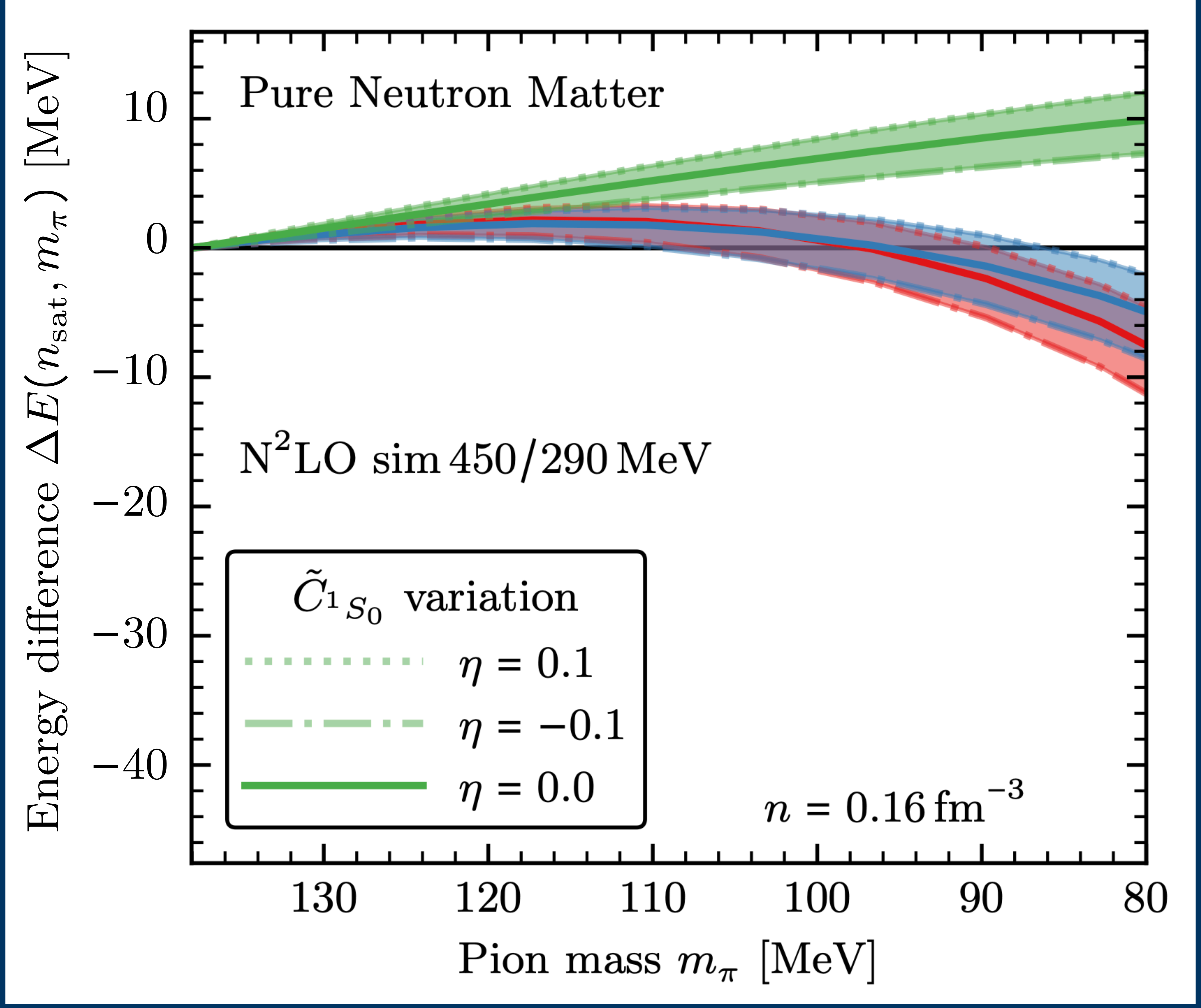

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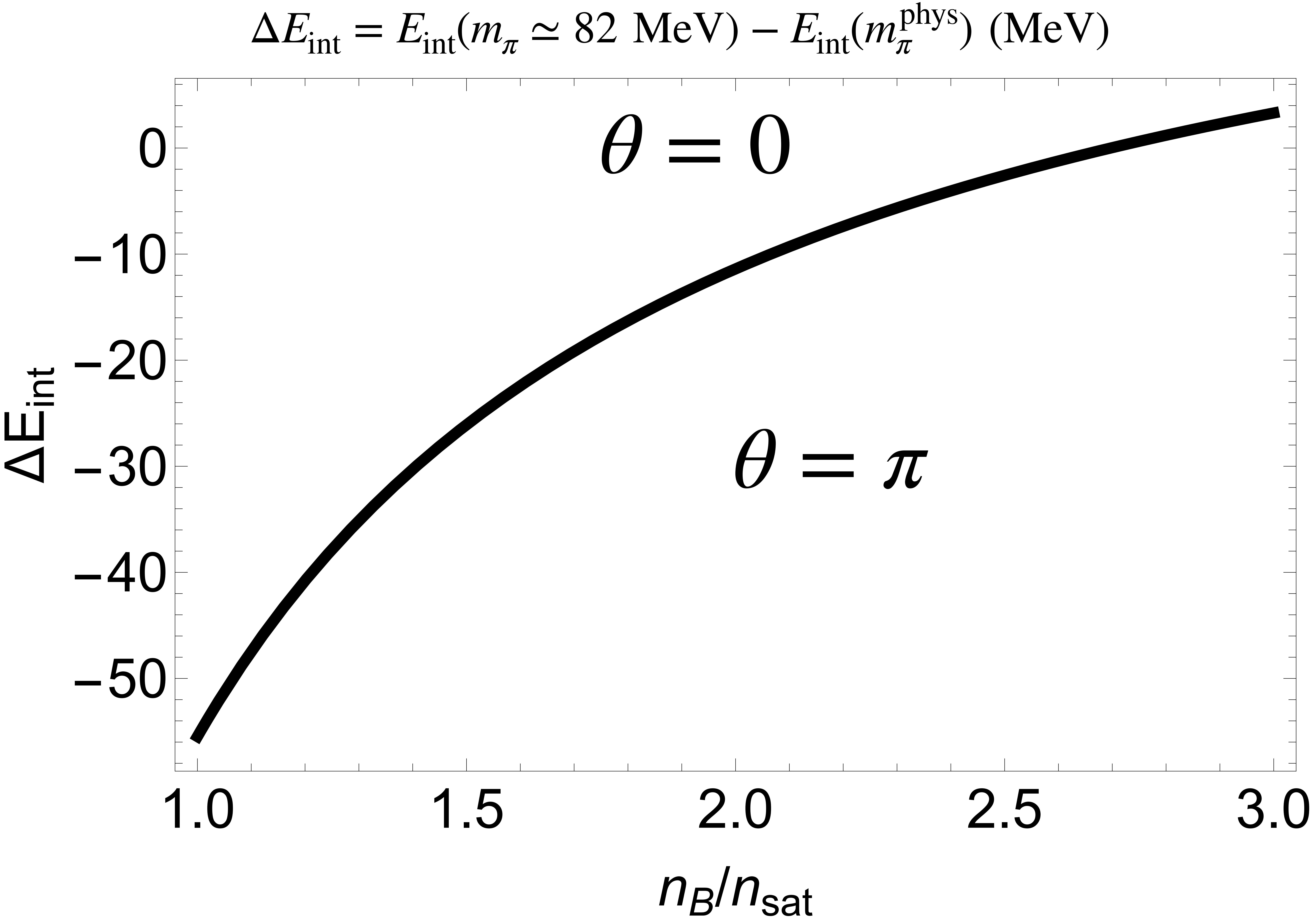
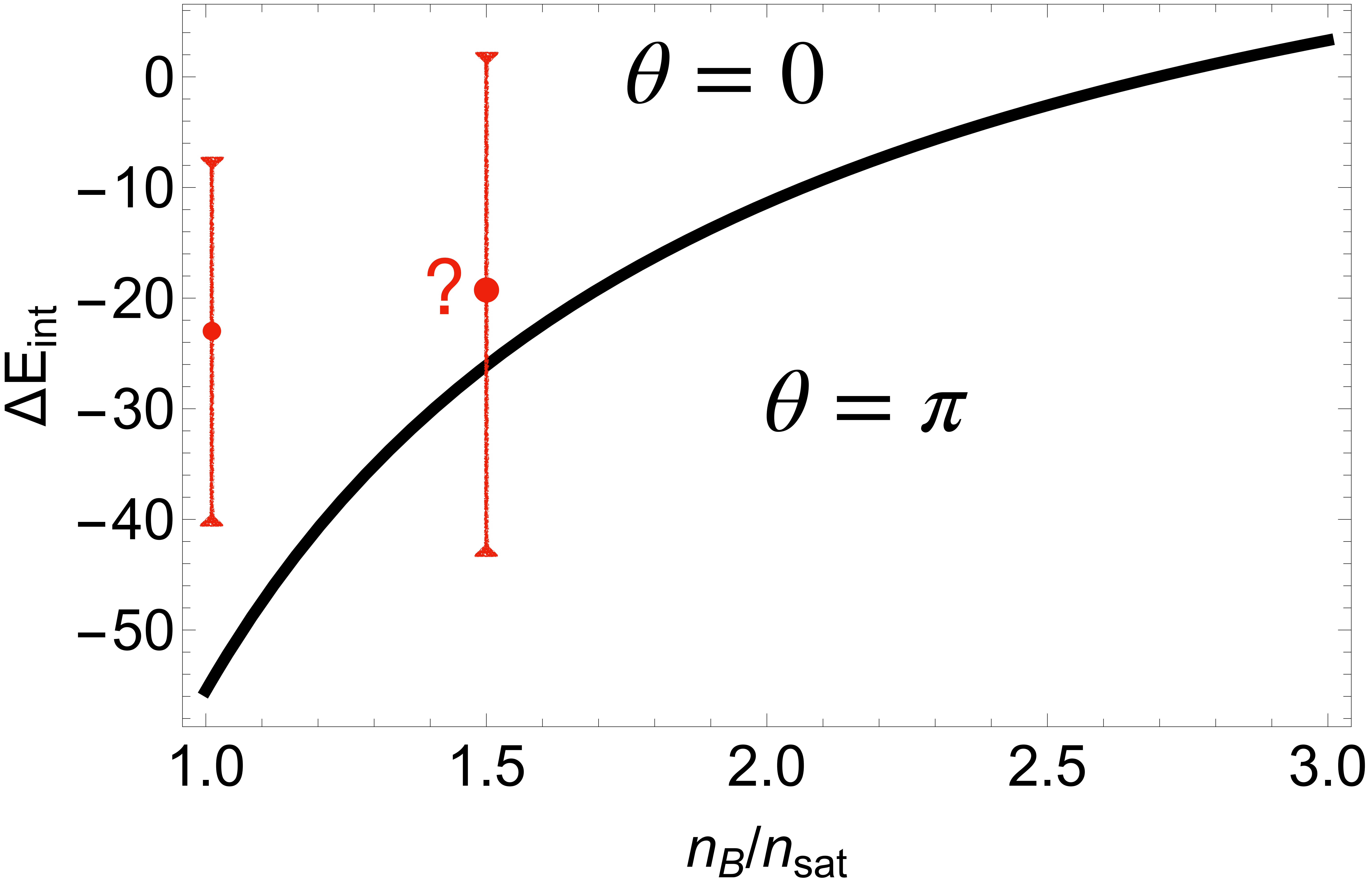
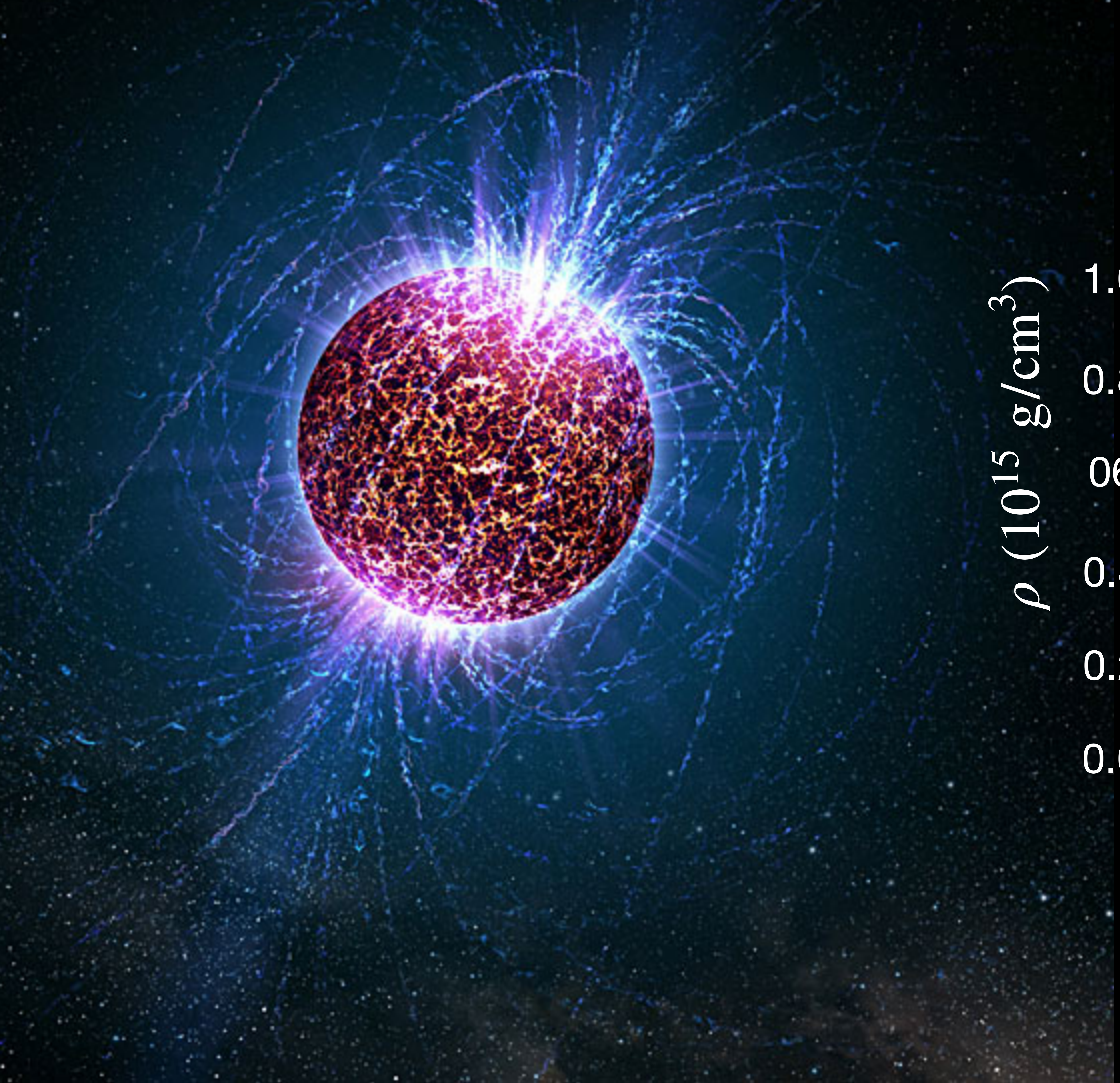


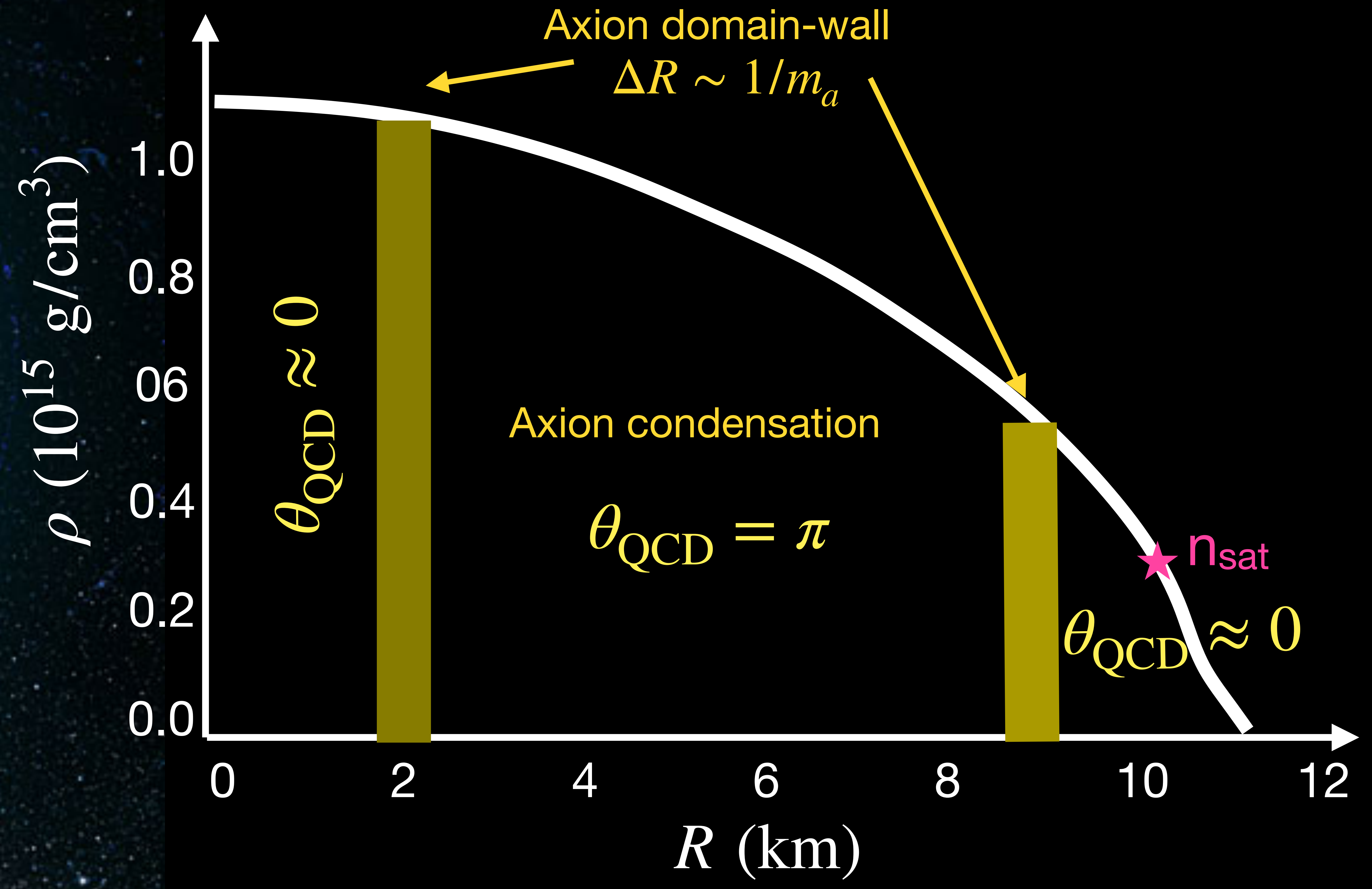
$\Delta E_{\rm int} = E_{\rm int}(m_\pi \simeq 82\ {\rm MeV}) - E_{\rm int}(m_\pi^{\rm phys})$ (MeV)
$\theta = 0$
$\theta = \pi$
0
−10
−20
−30
−40
−50
$\Delta E_{\rm int}$
1.0
1.5
2.0
2.5
3.0
$n_B/n_{\rm sat}$

$$\Delta E_{\rm int} = E_{\rm int}(m_\pi \simeq 82 \text{ MeV}) - E_{\rm int}(m_\pi^{\rm phys}) \text{ (MeV)}$$



Neutron Stars with an Axion Condensate: Pi in the Sky?



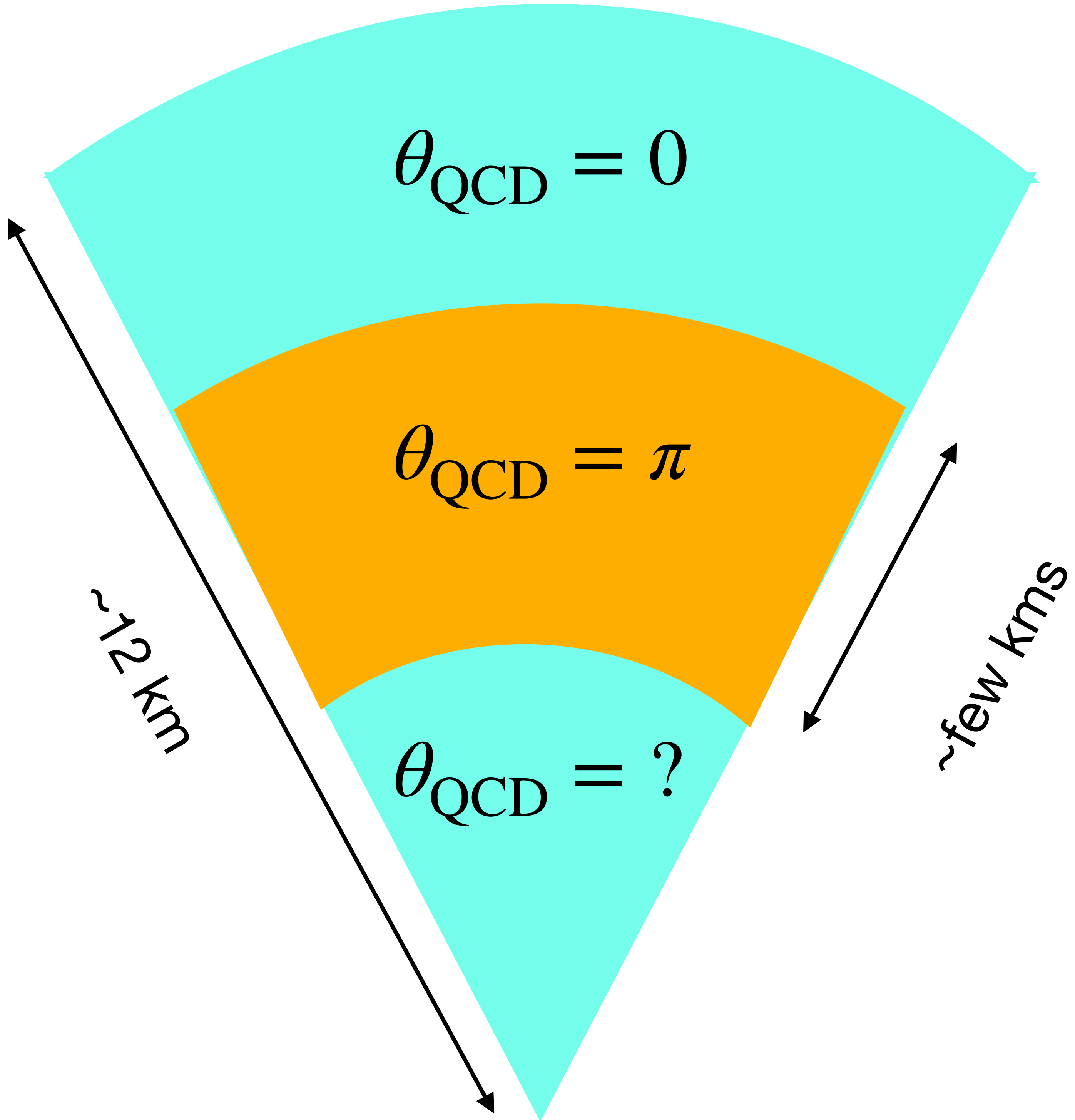


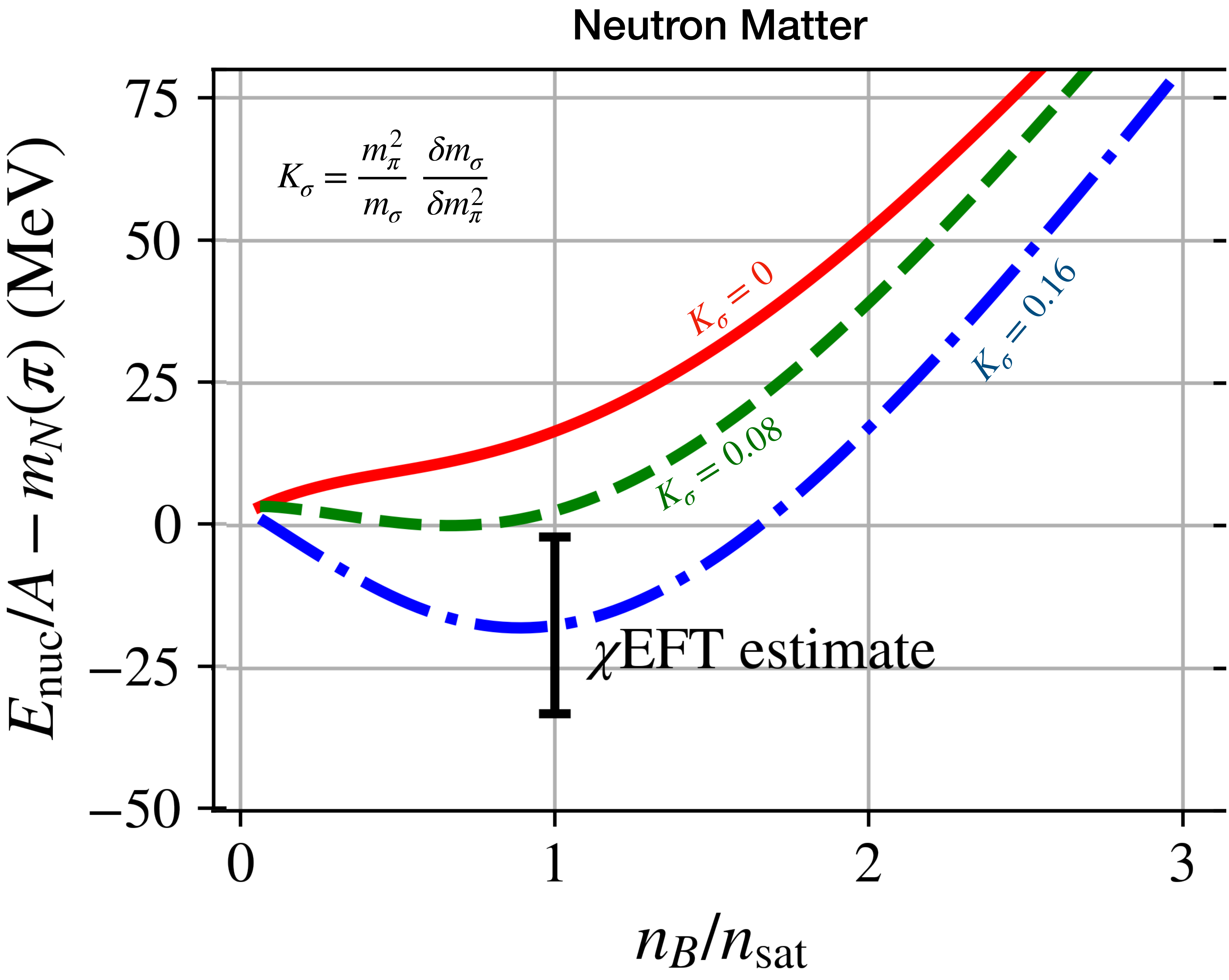
Axion Condensed Neutron Star Models

Condensation is realized in simpler mean field models of neutron-rich matter. Even a small increase in attraction at $\theta = \pi$ would favor axion condensation at

$$n_B^c \lesssim 2\ n_{\rm sat}$$

Mia Kumamoto





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Conclusions

- Understanding the quark or pion mass dependence of nuclear forces is important to address the possibility of axion condensation in neutron stars.
- If nuclear interactions favor axion condensation and we can identify robust neutron star observables, we can rule in or rule out the QCD axion — at any reasonable value of f_a !
- Studying the RG invariance of ChiEFT as a function of pion mass might help isolate new operators. (Ref. talk by Wouter Dekens).