Determining the leading-order contact term induced by sterile neutrinos in neutrinoless double β decay

Chiral EFT: New Perspectives @ INT

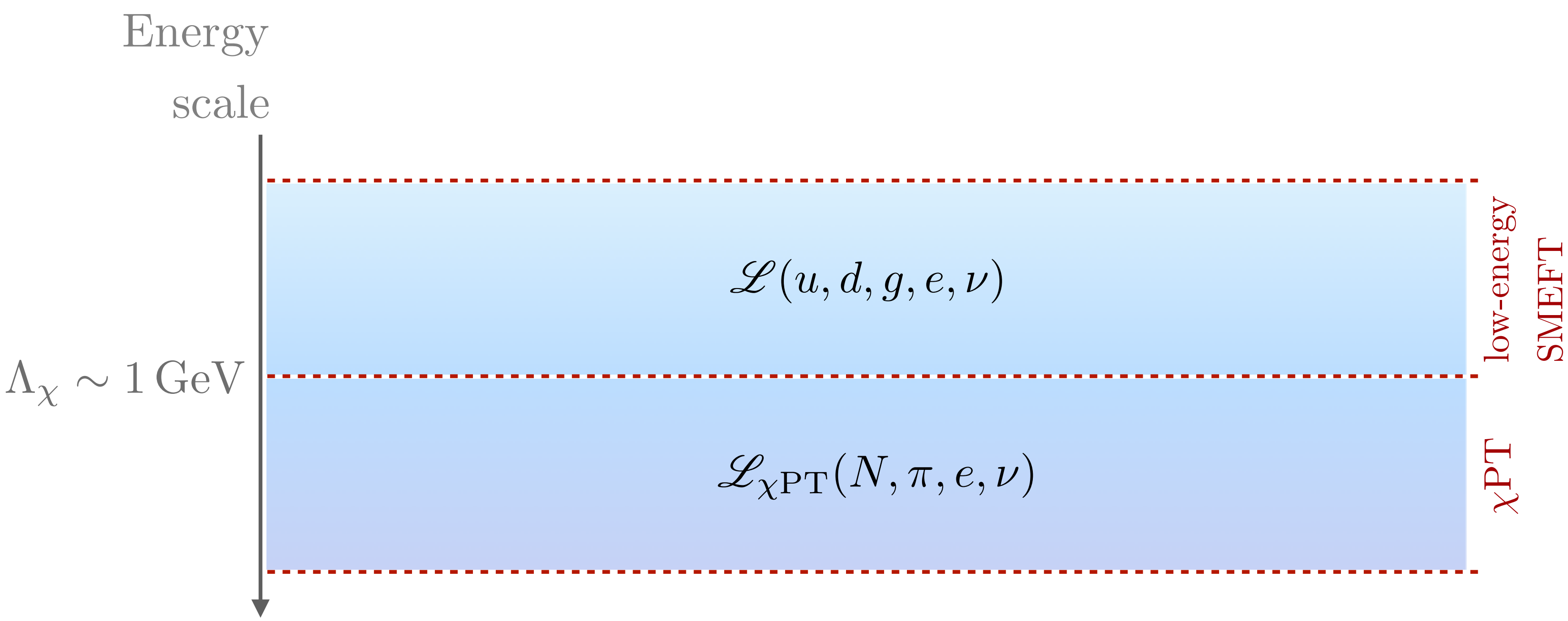
Mar 17 - 21, 2025

Sebastián Urrutia Quiroga 🇨🇱

suq90@uw.edu

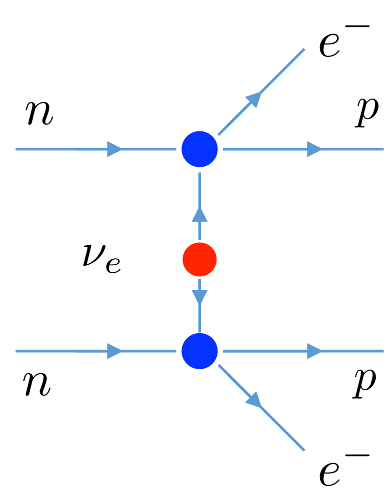
Based on: *V. Cirigliano, W. Dekens,* ***SUQ***, *arXiv:2412.10497*

$0\nu\beta\beta$: from quarks to nuclei using EFT



- ➡ The operators' structure is determined by chiral symmetry
- ➡ Each of them is accompanied by unknown constants (LECs)
- ➡ They are organized according to power-counting in Q/Λ_χ

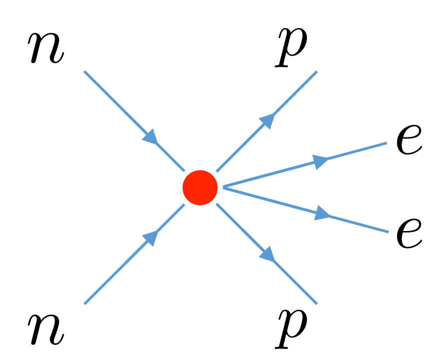
Leading order $0\nu\beta\beta$ operators



Hadronic input:
g_A

LEC:
$g_\nu^{NN} \sim 1/(4\pi F_\pi)^2$ in NDA/Weinberg

Sub-dominant?

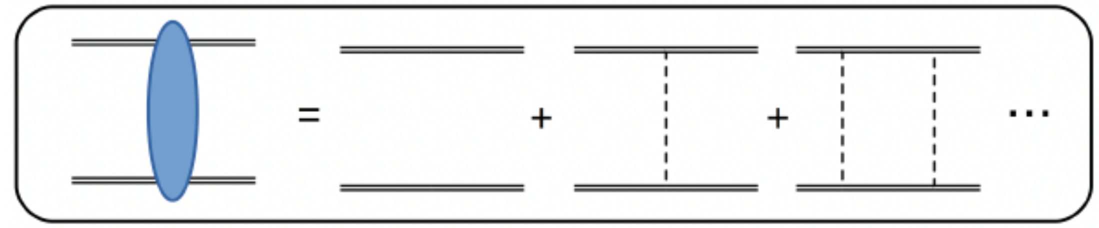


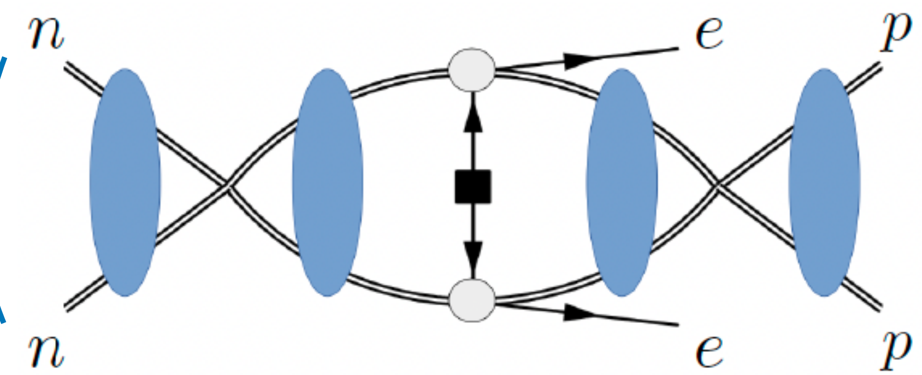
Tree-level exchange of Majorana neutrinos

Chiral symmetry also allows a contact term

Checking $nn \to ppe^-e^-$

$nn \to pp$ amplitude: LNV + strong interactions



Re()

$$= -\left(\frac{m_N^2}{4\pi}\right)^2 \frac{(1+2g_A^2)}{2}\left[\log\left(\frac{\mu_\chi^2}{|\mathbf{p}|_{\mathrm{ext}}^2}\right)+1\right] + \text{ finite}$$

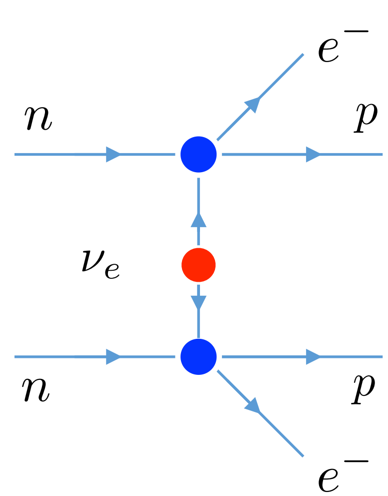
$V_S(\mathbf{p},\mathbf{p}') = C + (C_2/2)(\mathbf{p}^2 + \mathbf{p}'^2)$

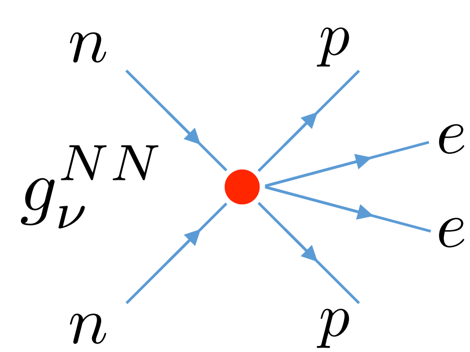
$|\mathbf{p}|_{\mathrm{ext}} \equiv |\mathbf{p}| + |\mathbf{p}'|$

- Some diagrams are divergent! Here μ_χ is the renormalization scale in $\overline{\mathrm{MS}}$
- New interaction is needed at **LO** to get physical amplitudes:

$$\mathcal{L}_{CT} = 2G_F^2 V_{ud}^2 m_{\beta\beta}\, g_\nu^{NN}\, \bar{p}n\, \bar{p}n\, \bar{e}_L C \bar{e}_L^T$$

$$V_{\Delta L=2} = V_\nu + V_{\nu,CT} =$$





g_ν^{NN} TBD from Lattice QCD; active area of research!

- Davoudi & Kadam '20, '21
- Feng et al. '20

Cirigliano et al. '18, '19

Challenge: determining g_ν^{NN}

Analytical Approach Cirigliano et al. '20, '21; Cirigliano, Dekens, SUQ, '24

- Analogy to the Cottingham approach for pion/nucleon mass differences

W. Cottingham '63; H. Harari '66

- $\Delta L = 2$ amplitudes controlled by neutrinoless effective action:

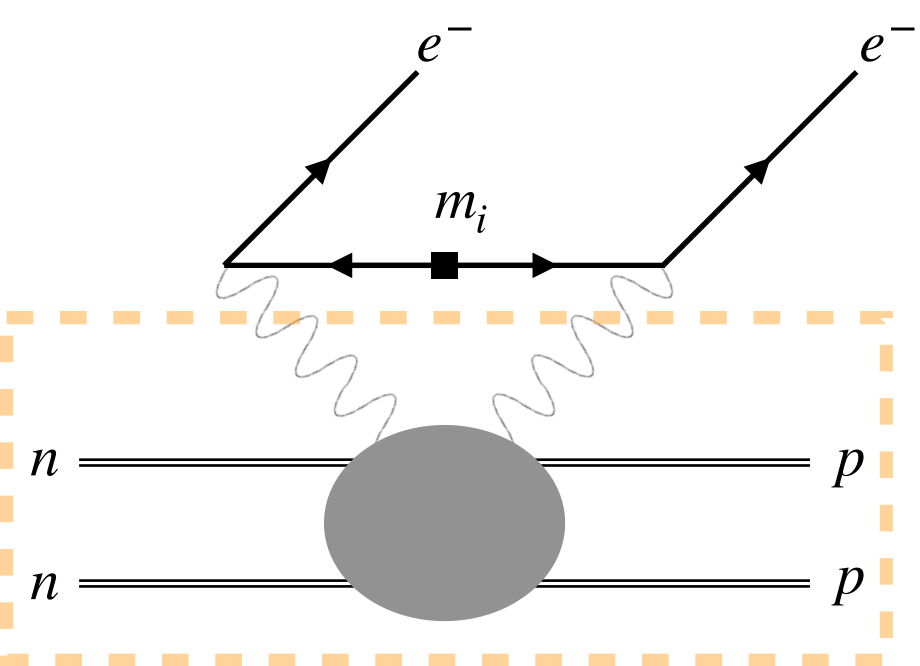
$$\langle e_1^- e_2^- pp \,|\, S_{\rm eff}^{\Delta L=2} \,|\, nn\rangle = (2\pi)^4 \,\delta^{(4)}(p_f - p_i)\Big(4G_F^2 V_{ud}^2\; \bar{u}_L(p_1) u_L^c(p_2)\Big) \times \sum_{i=1}^{3+n} U_{ei}^2\, m_i\, \mathcal{A}_\nu(m_i)$$

$$\mathcal{A}_\nu(m_i) \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2 - m_i^2 + i\epsilon} \int d^4x\; e^{ik\cdot x} \langle pp \,|\, T\{J_W^\mu(x)\, J_W^\nu(0)\} \,|\, nn\rangle$$

✓ For a generic Majorana neutrino (active or sterile) with mass $m_i \lesssim \Lambda_\chi$

✓ Results valid for limit $m_i \to 0$

Forward "Compton" amplitude



Challenge: determining g_ν^{NN}

Analytical Approach Cirigliano et al. '20, '21; Cirigliano, Dekens, SUQ, '24

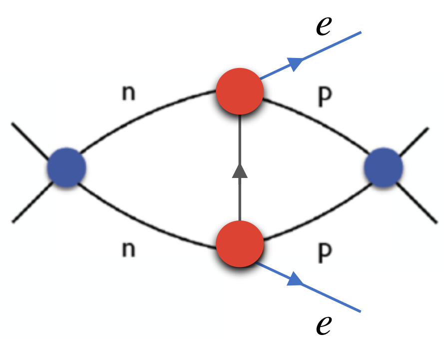
- Analogy to the Cottingham approach for pion/nucleon mass differences

W. Cottingham '63;
H. Harari '66

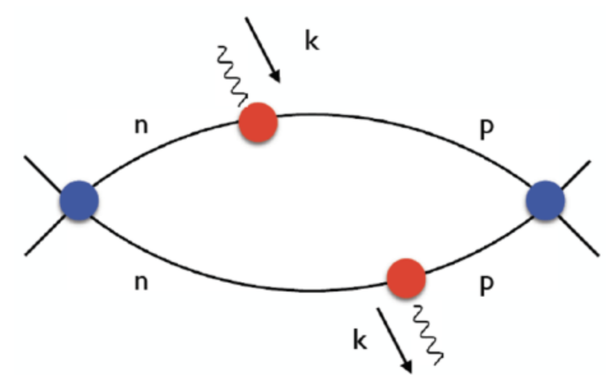
- Matching at the amplitude level:

$$\mathcal{A}_\nu^{\chi\mathrm{EFT}}(m_i) = \mathcal{A}_\nu^{\mathrm{full}}(m_i) \qquad \overline{\mathcal{A}}_X \equiv \left(\frac{4\pi}{m_N}\right)^2 \mathcal{A}_X$$

$\mathcal{A}_\nu^{\mathrm{full}}(m_i) =$



$$\propto \int d|\mathbf{k}|\, a(|\mathbf{k}|, m_i) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_i^2 + i\epsilon} \times$$



- Estimation of $\mathcal{A}_\nu^{\mathrm{full}}$ by modeling integrand:
  - The region $|\mathbf{k}| \ll \Lambda_\chi$ is determined by χEFT
  - The region $|\mathbf{k}| \gg \mathcal{O}(\mathrm{GeV})$ matches the OPE
  - The intermediate region is modeled using
    - Form factors
    - Off-shell effects from NN intermediate states

- Challenges of the $\mathcal{A}_\nu^{\chi\mathrm{EFT}}$ calculation:
  - The behavior of $g_\nu^{NN}(m_i)$ is expected to be corrected by terms of $\mathcal{O}(m_i/\Lambda_\chi)$ that we do not control

Modeling $\mathscr{A}_\nu^{\text{full}}$

- **High momentum region:** For $|\mathbf{k}| > \Lambda$ (scale at which the OPE becomes reliable),

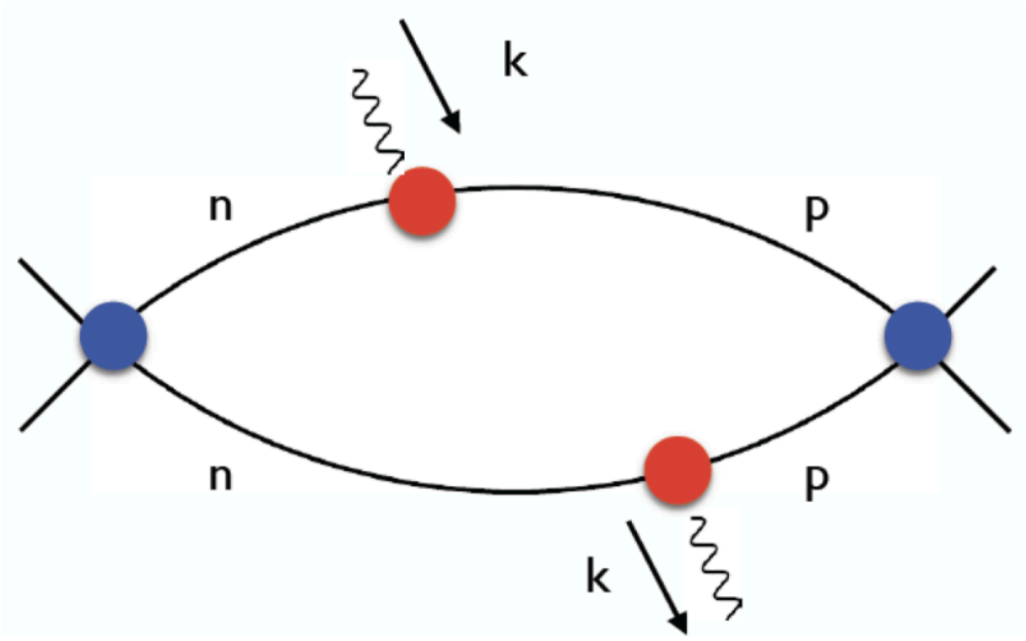
$$a_>(|\mathbf{k}|, m_i) = \frac{4\alpha_s}{\pi}\bar{g}_1^{NN} F_\pi^2 \frac{2|\mathbf{k}| + \omega_k}{\omega_k |\mathbf{k}|(\omega_k + |\mathbf{k}|)^2}$$

- $\omega_k \equiv \sqrt{\mathbf{k}^2 + m_i^2}$
- $\bar{g}_1^{NN} \sim \mathcal{O}(1)$ is proportional to the $nn \to pp$ matrix element of the local operator $\bar{u}_L \gamma^\mu d_L \bar{u}_L \gamma_\mu d_L$

- **Small and intermediate momentum region:** For $|\mathbf{k}| < \Lambda$, we use the χEFT representation

$$a_<(|\mathbf{k}|, m_i) = -8 g_{\text{full}}(\mathbf{k}^2) \frac{\mathbf{k}^2}{\mathbf{k}^2 + m_i^2} \mathrm{Re}\, I_C^<(|\mathbf{k}|)$$

- $g_{\text{full}}(\mathbf{k}^2) \equiv g_V^2(\mathbf{k}^2) + 2g_A^2(\mathbf{k}^2) + \dfrac{\mathbf{k}^2 g_M^2(\mathbf{k}^2)}{2m_N^2}$
- $I_C^<(|\mathbf{k}|) \equiv \displaystyle\int \frac{d^3\mathbf{q}}{(2\pi)^3}\, f_S(\mathbf{p}', \mathbf{q} + \mathbf{k}) \frac{1}{\mathbf{p}'^2 - (\mathbf{q} + \mathbf{k})^2 + i\epsilon} \frac{1}{\mathbf{p}^2 - \mathbf{q}^2 + i\epsilon} \boxed{f_S(\mathbf{q}, \mathbf{p})}$



Encodes the momentum-dependence of the short-range (not mediated by pions) ${}^1S_0$ scattering amplitude

Modeling $\mathcal{A}_\nu^{\text{full}}$: IR behavior

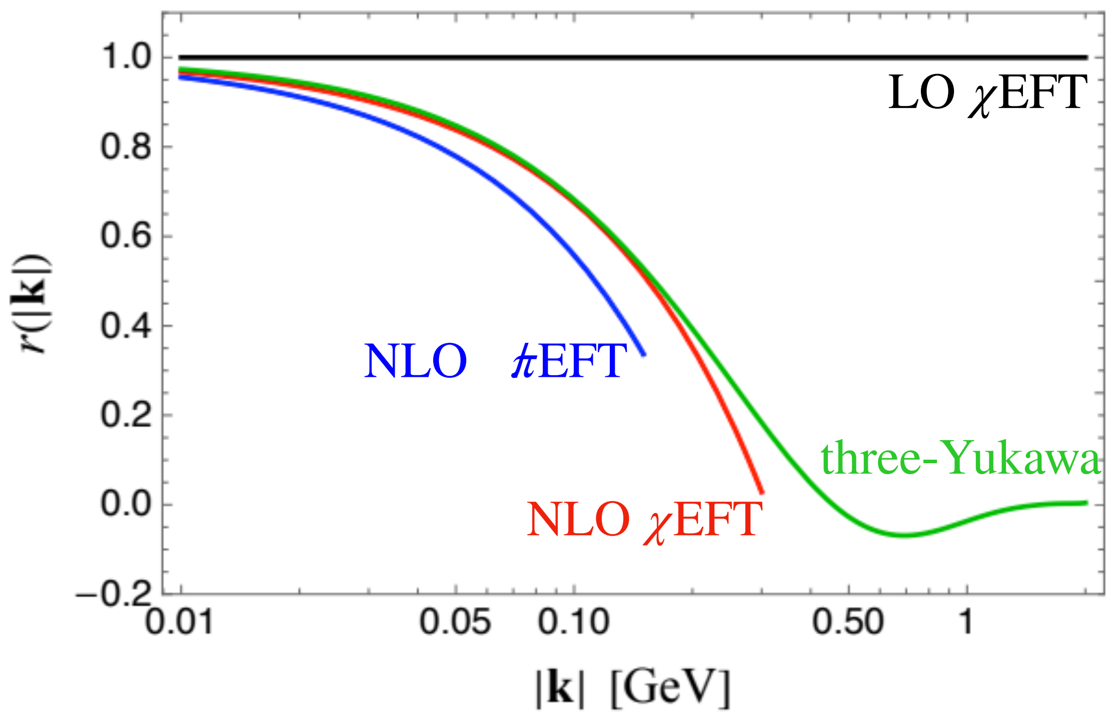
Introduce an auxiliary scale λ ($|\mathbf{p}|_{\text{ext}} \ll \lambda \lesssim m_\pi$) to isolate the external momenta dependence

$$\overline{\mathcal{A}}_<^{\text{full}}(m_i) = \left(\int_0^\lambda + \int_\lambda^\Lambda\right) d|\mathbf{k}|\ a_<(|\mathbf{k}|, m_i)$$

- Region $\lambda < |\mathbf{k}| < \Lambda$:

  - We introduce the ratio $r(|\mathbf{k}|) \equiv \dfrac{\operatorname{Re} I_C^<(|\mathbf{k}|)}{\operatorname{Re} I_C(|\mathbf{k}|)}$, where $\operatorname{Re} I_C(|\mathbf{k}|) \equiv \dfrac{\theta(|\mathbf{k}| - |\mathbf{p}|_{\text{ext}})}{8|\mathbf{k}|}$

$$\int_\lambda^\Lambda d|\mathbf{k}|\ a_<(|\mathbf{k}|, m_i) = -\int_\lambda^\Lambda d|\mathbf{k}|\ g_{\text{full}}(\mathbf{k}^2)\frac{|\mathbf{k}|\, r(|\mathbf{k}|)}{\mathbf{k}^2 + m_i^2}$$



Modeling $\mathscr{A}_\nu^{\text{full}}$: IR behavior

Introduce an auxiliary scale λ ($|\mathbf{p}|_{\text{ext}} \ll \lambda \lesssim m_\pi$) to isolate the external momenta dependence

$$\overline{\mathscr{A}}_<^{\text{full}}(m_i) = \left(\int_0^\lambda + \int_\lambda^\Lambda\right) d|\mathbf{k}|\; a_<(|\mathbf{k}|, m_i)$$

- Region $|\mathbf{k}| < \lambda$:
  - We can safely neglect the dipole effects in the form factors $\rightarrow g_{\text{full}}(\mathbf{k}^2) \approx g_{\text{full}}(0) = (1 + 2g_A^2)$
  - We can use the χEFT NLO expansion for $\mathrm{Re}\, I_C^<(|\mathbf{k}|)$, appropriate for $|\mathbf{k}| \lesssim \lambda$

$$\mathrm{Re}\, I_C^<(|\mathbf{k}|) \approx \mathrm{Re}\, I_C^<(|\mathbf{k}|)\Big|_{\text{NLO }\chi\text{EFT}} = \frac{1}{8|\mathbf{k}|}\Big[\underbrace{\theta(|\mathbf{k}| - |\mathbf{p}|_{\text{ext}})}_{\text{LO}} \underbrace{- d\,|\mathbf{k}|}_{\text{NLO}}\Big], \quad \left(d = \frac{8C_2}{m_N C^2}\right)$$

$$\int_0^\lambda d|\mathbf{k}|\; a_<(|\mathbf{k}|, m_i) = \frac{(1+2g_A^2)}{2}\left[2d\,\lambda - 2d\, m_i \tan^{-1}\left(\frac{\lambda}{m_i}\right) + \log\left(\frac{m_i^2 + |\mathbf{p}|_{\text{ext}}^2}{m_i^2 + \lambda^2}\right)\right]$$

$$m_i \tan^{-1}\left(\frac{\lambda}{m_i}\right) \xrightarrow{m_i \ll \lambda} \frac{\pi\, m_i}{2} - \frac{m_i^2}{\lambda} + \mathscr{O}(m_i^4)$$

➡ Explicit external momenta dependence

Chiral EFT amplitude $\mathcal{A}_\nu^{\chi\mathrm{EFT}}$: LO

- As argued before, the χEFT amplitude is divergent, requiring the introduction of a counter-term:

$$\overline{\mathcal{A}}_\nu^{\chi\mathrm{EFT}}(m_i) = \overline{\mathcal{A}}_\nu^{\mathrm{sing}}(\mu_\chi, m_i) + 2\,\bar{g}_\nu^{NN}(m_i)$$

$$g_X \equiv \left(\frac{m_N}{4\pi} C(\mu_\chi)\right)^2 \bar{g}_X$$

$C(\mu_\chi)$: leading non-derivative NN coupling in the ${}^1S_0$ channel

- At leading order, the singular amplitude takes the form:

$$\overline{\mathcal{A}}_\nu^{\mathrm{sing}}(\mu_\chi, m_i)\Big|_{\mathrm{LO}} = -\frac{(1+2g_A^2)}{2}\left[1 + \log\left(\frac{\mu_\chi^2}{m_i^2 + |\mathbf{p}|_{\mathrm{ext}}^2}\right)\right]$$

- ☑ Same $|\mathbf{p}|_{\mathrm{ext}}$ dependence as $\overline{\mathcal{A}}_\nu^{\mathrm{full}}$, so g_ν^{NN} is external-momenta independent
- ☐ Doesn't give the correct m_i dependence: $\bar{g}_\nu^{NN}(m_i) = \sum_{n=0} \bar{g}_\nu^{NN(2n)}\, m_i^{2n}$
  - Linear term in m_i mismatched

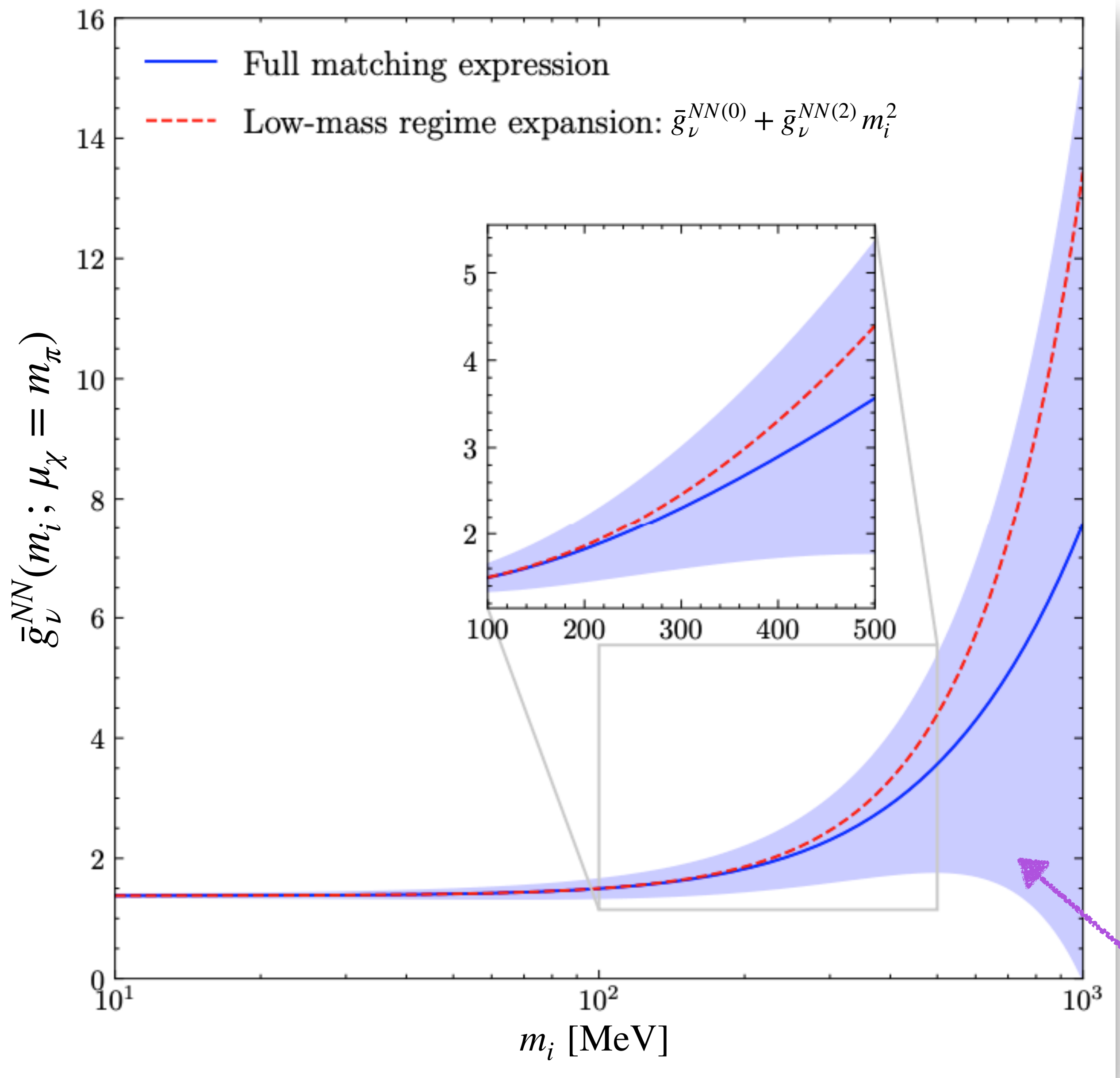
- The correct dependence on m_i arises at NLO:

$$\overline{\mathcal{A}}_\nu^{\mathrm{sing}}(\mu_\chi, m_i)\Big|_{\mathrm{NLO}} = -\frac{(1+2g_A^2)}{2}\left[1 + \log\left(\frac{\mu_\chi^2}{m_i^2 + |\mathbf{p}|_{\mathrm{ext}}^2}\right) + d\pi\, m_i\right]$$

Matching condition — Results

- Imposing the matching condition at the amplitude level, one gets:

$$\bar{g}_\nu^{NN}(m_i) = \frac{1}{2}\left[\int_0^\lambda d|\mathbf{k}|\ a_<(|\mathbf{k}|, m_i) + \int_\lambda^\Lambda d|\mathbf{k}|\ a_<(|\mathbf{k}|, m_i) + \int_\Lambda^\infty d|\mathbf{k}|\ a_>(|\mathbf{k}|, m_i) - \overline{\mathscr{A}}_\nu^{\mathrm{sing}}(\mu_\chi, m_i)\right]$$



Since the small-m_i behavior is phenomenologically interesting, we provide the explicit first few terms for the expansion in the IR limit of $m_i \ll \lambda < \Lambda$:

$$\bar{g}_\nu^{NN}(m_i\,;\,\mu_\chi = m_\pi) = 1.377 + \left(\frac{12.062}{\mathrm{GeV}^2}\right) m_i^2 + \left(\frac{-16.735}{\mathrm{GeV}^4}\right) m_i^4$$

$$\bar{g}_\nu^{NN}(m_i)\Big(1 \pm \mathcal{O}(m_i/\Lambda_\chi)\Big)$$

Example: A minimal ν_R scenario

- Add n singlets, ν_R, to the SM:

$$\mathscr{L}_{\nu_R} = \mathrm{i}\bar{\nu}_R \partial\!\!\!/ \nu_R - \frac{1}{2}\bar{\nu}_R^c M_R \nu_R - Y_D \bar{L}\tilde{H}\nu_R + \cancel{\mathscr{L}_{\nu_R}^{(6)} + \mathscr{L}_{\nu_R}^{(7)}}$$

- After EWSB,

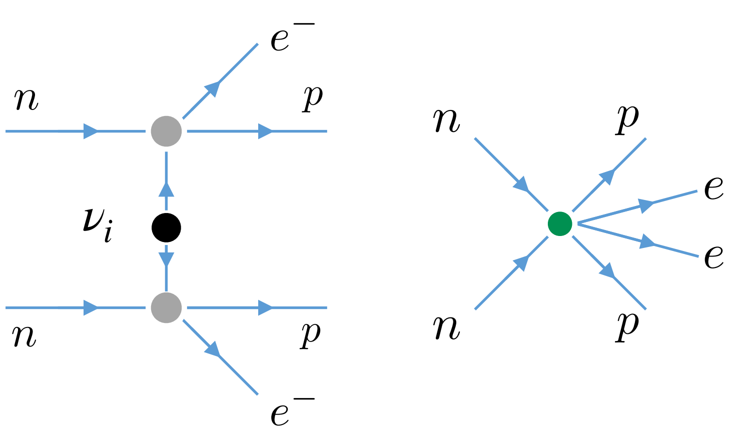
$$\mathscr{L}_{\text{mass}} = \frac{1}{2}\bar{N}^c M_\nu N \qquad N = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \qquad M_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}}Y_D \\ \frac{v}{\sqrt{2}}Y_D & M_R^\dagger \end{pmatrix} \qquad \nu_{\text{mass}} = U N_{\text{flavor}}$$

- $0\nu\beta\beta$ contributions:

\`Usual' contributions:

- Similar to $m_{\beta\beta}$ case

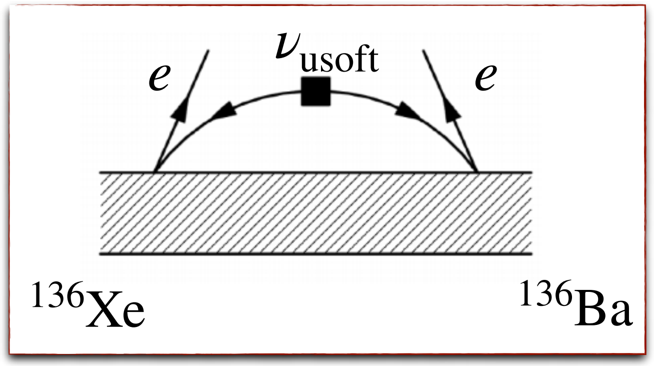
➡ NMEs and LECs are now m_i dependent



New: \`Ultrasoft' neutrinos:

- See the nucleus as a whole, have momenta $q^0 \sim |\vec{q}\,| \sim k_F^2/m_N \sim Q$

Cirigliano et al., '17; Castillo et al.,'23, '24



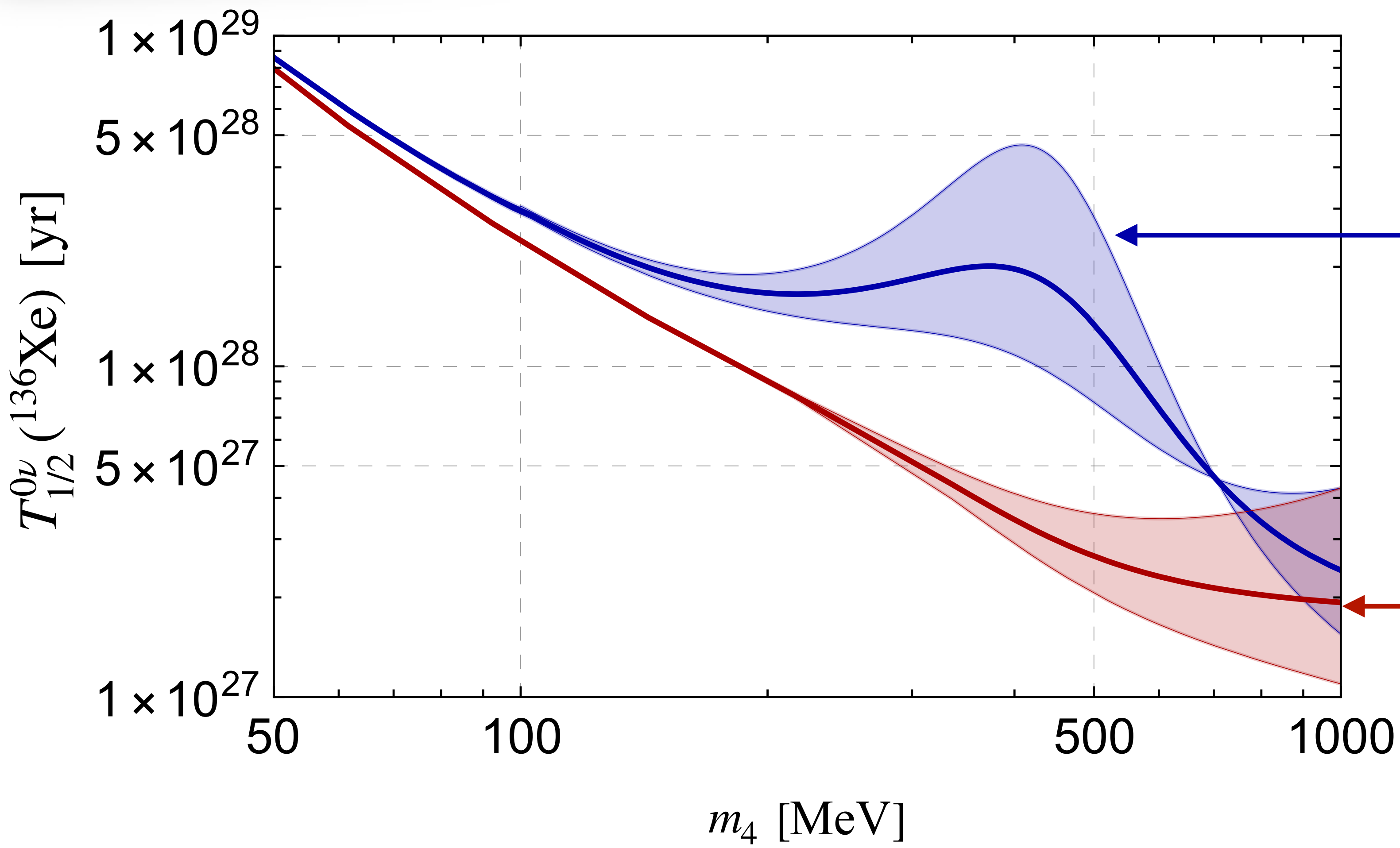
Example: Minimalistic minimal scenario (3 + 1)

- Add just **one** sterile neutrino to the SM
  - ➡ Assume a simple mass matrix

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & M_D \\ 0 & 0 & 0 & M_D \\ 0 & 0 & 0 & M_D \\ M_D & M_D & M_D & M_R \end{pmatrix}$$

Not realistic:
- It does not reproduce all neutrino masses/mixings

Simple scenario to test the impact:
- Similar features to more realistic cases



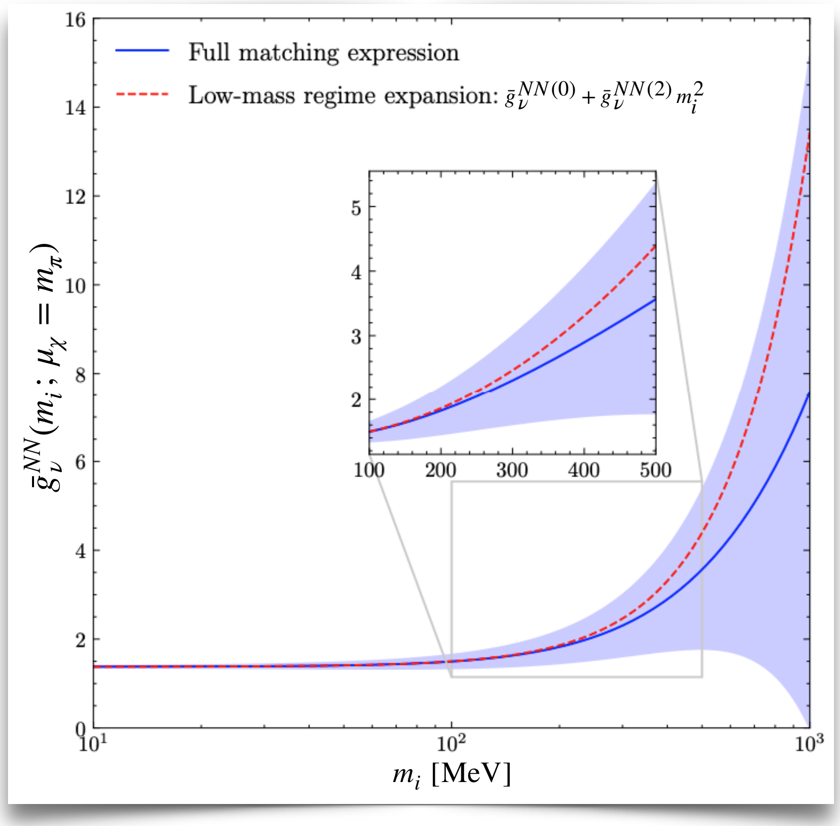
Adding $g_\nu^{NN}(m_4)$ to the interpolation formula

Interpolation formula between $g_\nu^{NN}(0)$ and $\sim m_i^{-2}$ scaling region

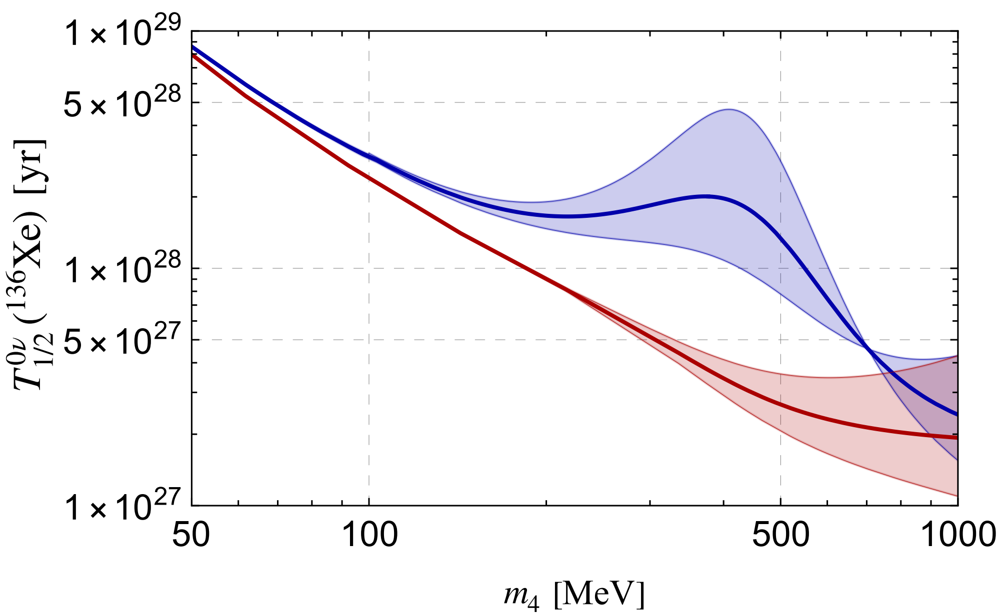
Dekens et al. [2402.07993]

Summary

- Combining EFTs and the Cottingham-like matching strategy successfully determined the mass dependence of the short-range $nn \to pp$ effective couplings
  - ✓ Generalizing the previous massless results
  - ✓ Subtleties in the matching analysis from the IR
  - ✓ Explicit expansion in powers of m_i



- Impact in $0\nu\beta\beta$ predictions from sterile neutrino models: νSM
  - ✓ Significant modifications by including $g_\nu^{NN}(m_i)$



Determining the leading-order contact term induced by sterile neutrinos in neutrinoless double β decay

Chiral EFT: New Perspectives @ INT

Mar 17 - 21, 2025

Sebastián Urrutia Quiroga 🇨🇱

suq90@uw.edu

Based on: *V. Cirigliano, W. Dekens,* ***SUQ***, *arXiv:2412.10497*