Universitat d'Alacant

GOBIERNO DE ESPAÑA | MINISTERIO DE CIENCIA E INNOVACIÓN

GENERALITAT VALENCIANA
Conselleria de Educación, Cultura, Universidades y Empleo

Origin and Dynamics of Magnetar Magnetic Fields: Insights from 3D Magneto-Thermal Simulations

MATINS
MAgneto-Thermal evolution of Isolated Neutron Stars

December 12, 2024
IReNA-INT Workshop

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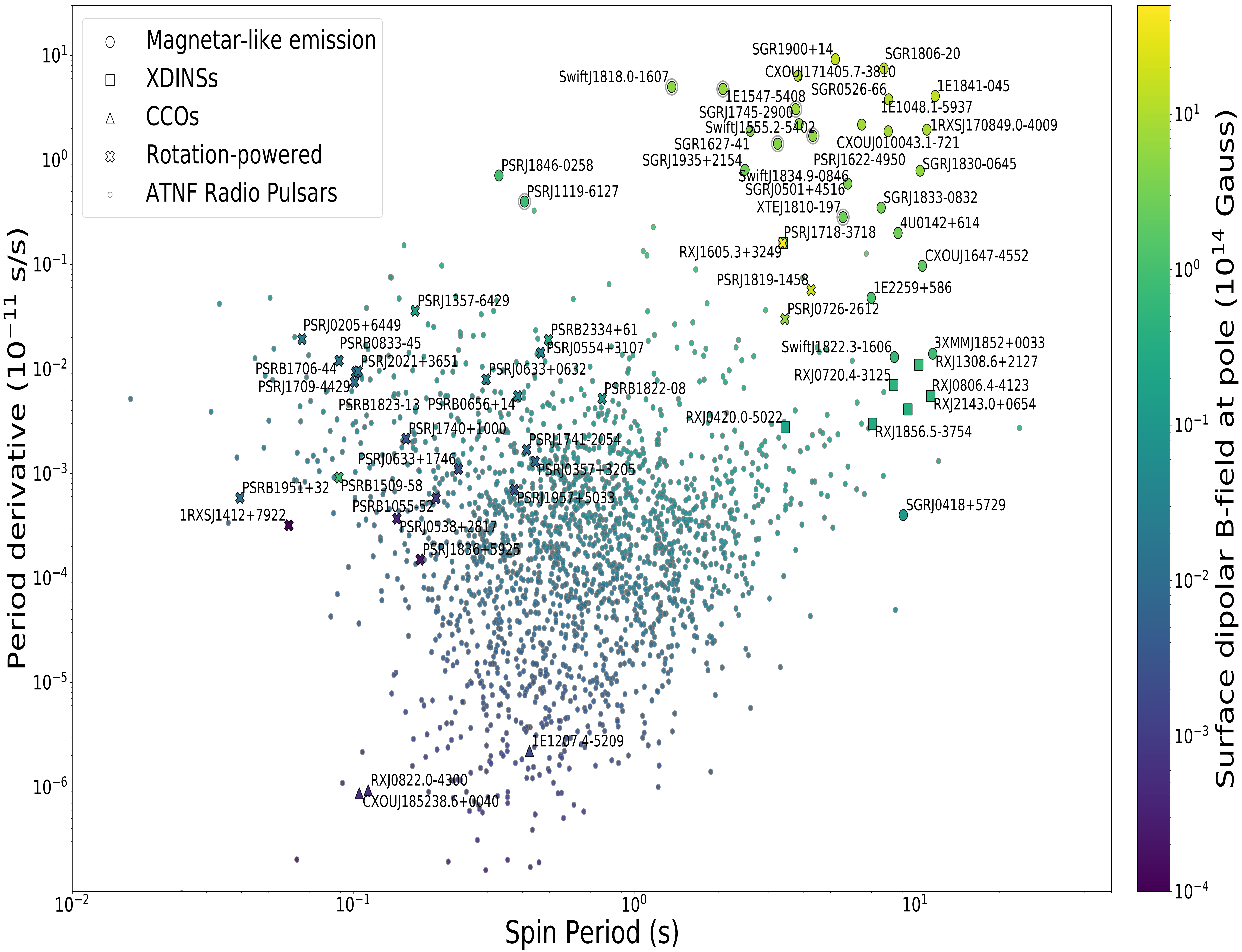
P-Pdot diagram for isolated neutron stars



Dipolar magnetic field at the pole of the star:

$$B_{dip} = 6.4 \times 10^{19} \sqrt{P\dot{P}}$$

Characteristic age:

$$\tau_c = \frac{P}{2\dot{P}}$$

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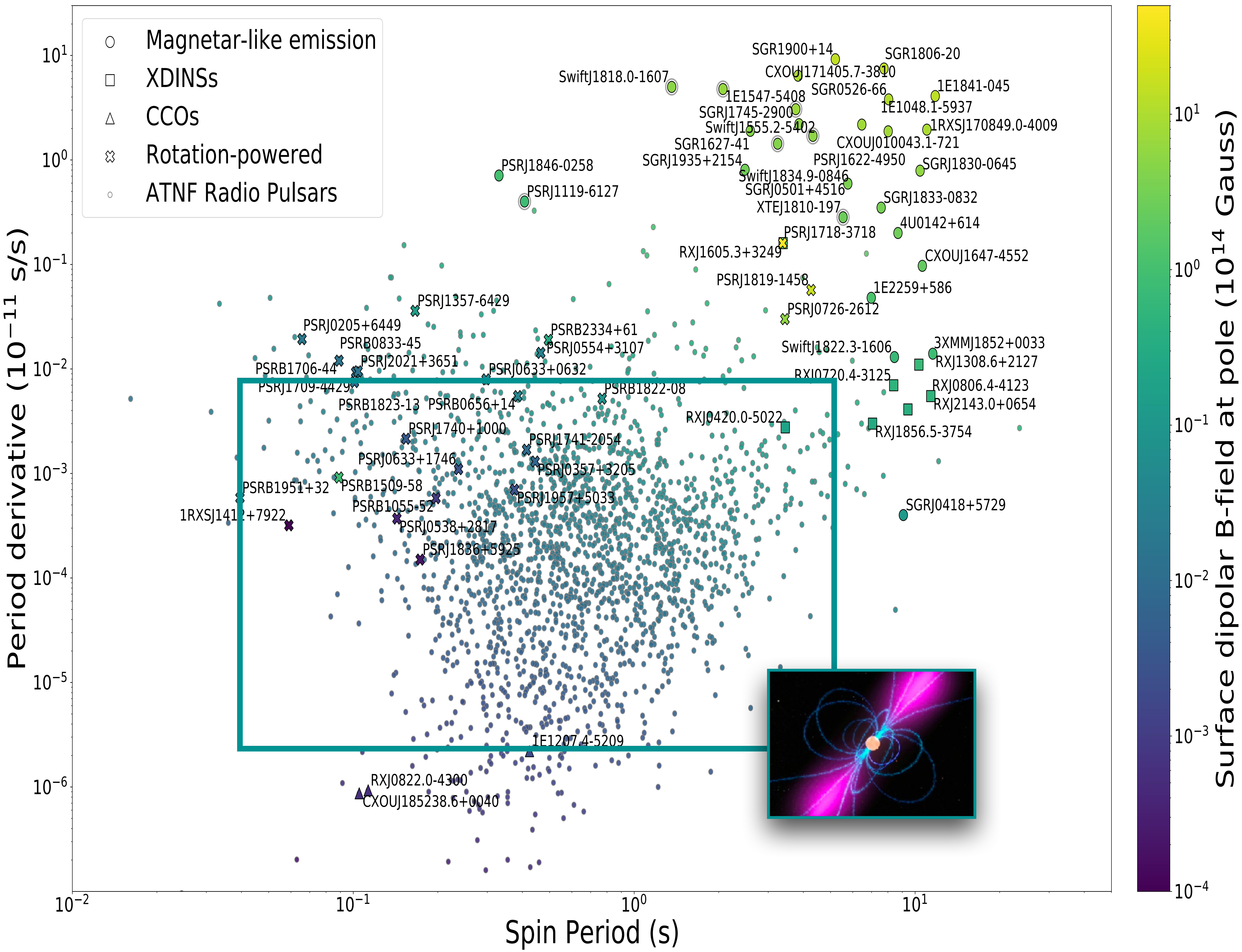

Rotational-Powered Pulsars

Powered by rotational energy. Typically emitting in radio.

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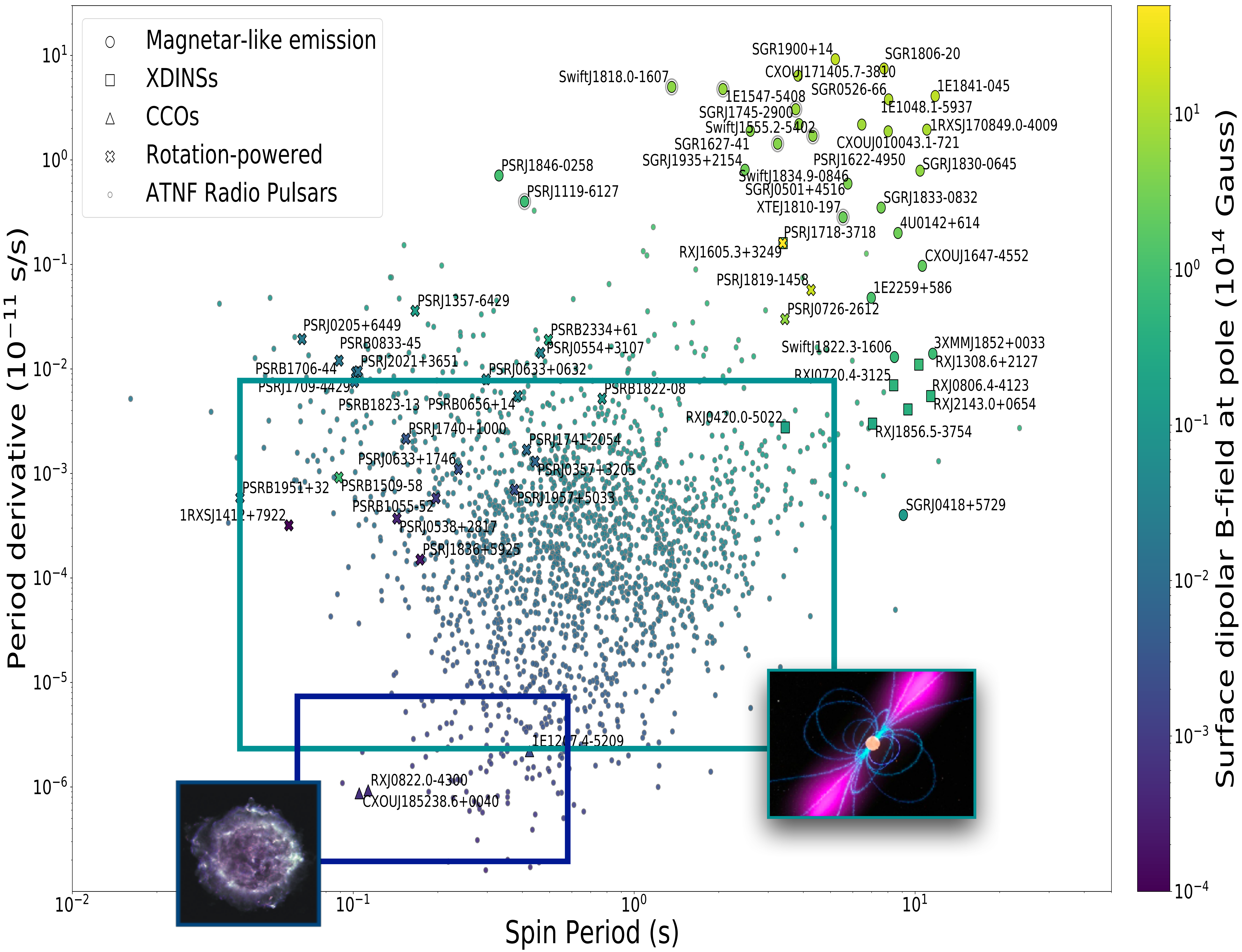

Central Compact Objects

Powered by magnetic energy. Young with bright SNRs. Typically emitting in X-rays

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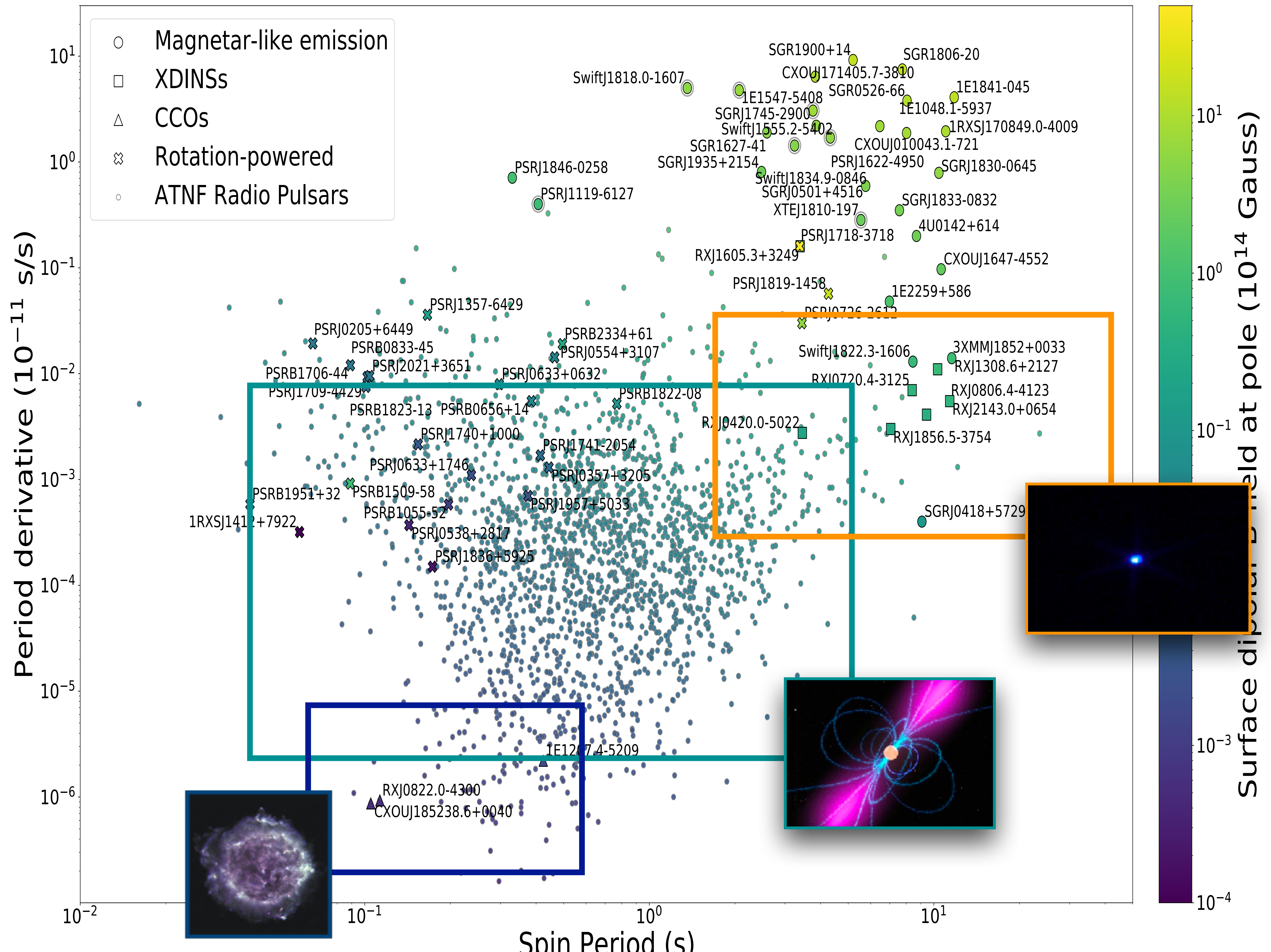

X-ray Dim Isolated Neutron Stars

Powered by magnetic energy. Old, almost pure blackbodies. Typically emitting in X-rays

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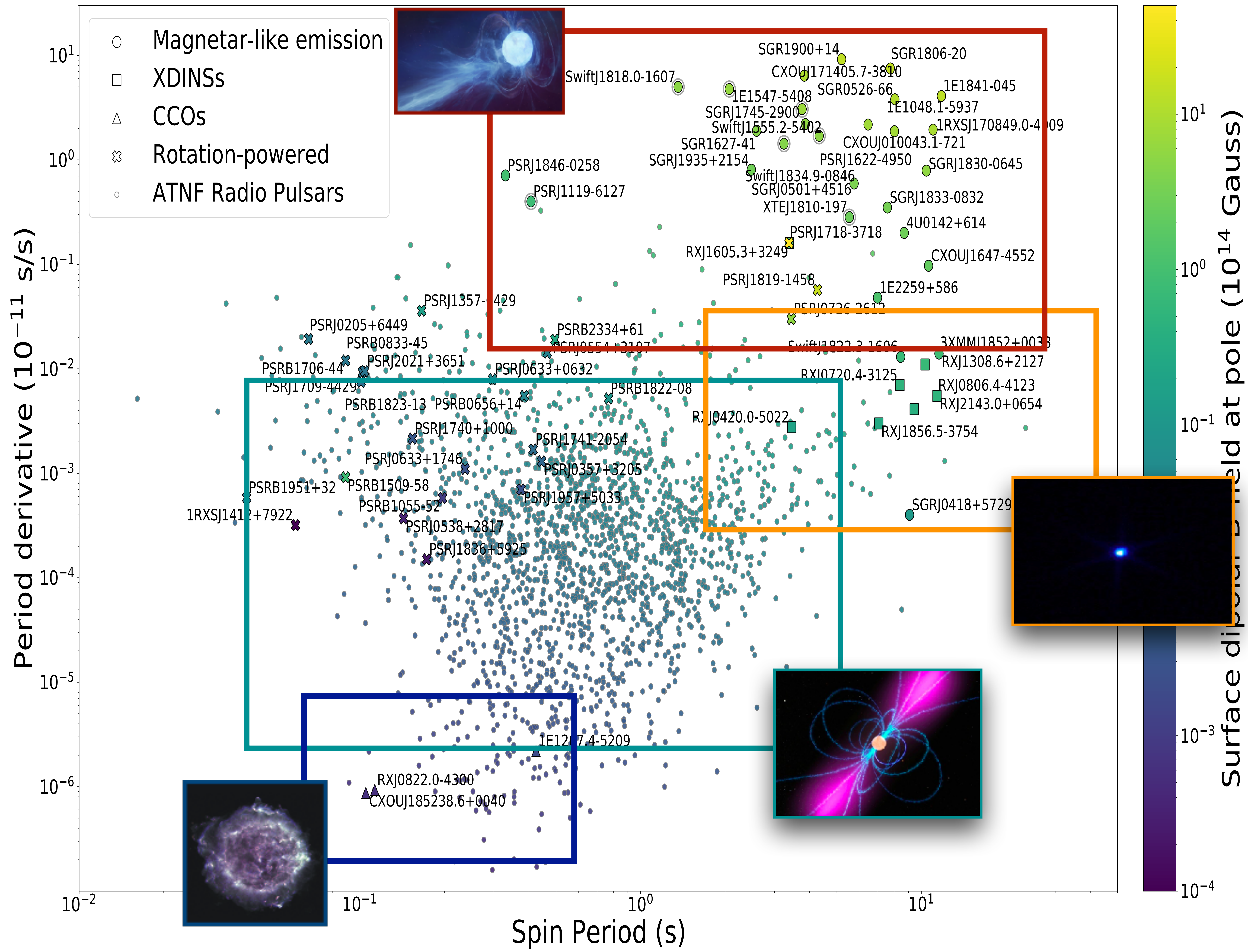

Magnetars

Powered by magnetic energy.
Characterised by outbursts and flares.
Typically emitting in X-rays

X-ray Dim Isolated Neutron Stars

Powered by magnetic energy. Old, almost pure blackbodies. Typically emitting in X-rays

Central Compact Objects

Powered by magnetic energy. Young with bright SNRs. Typically emitting in X-rays

Rotational-Powered Pulsars

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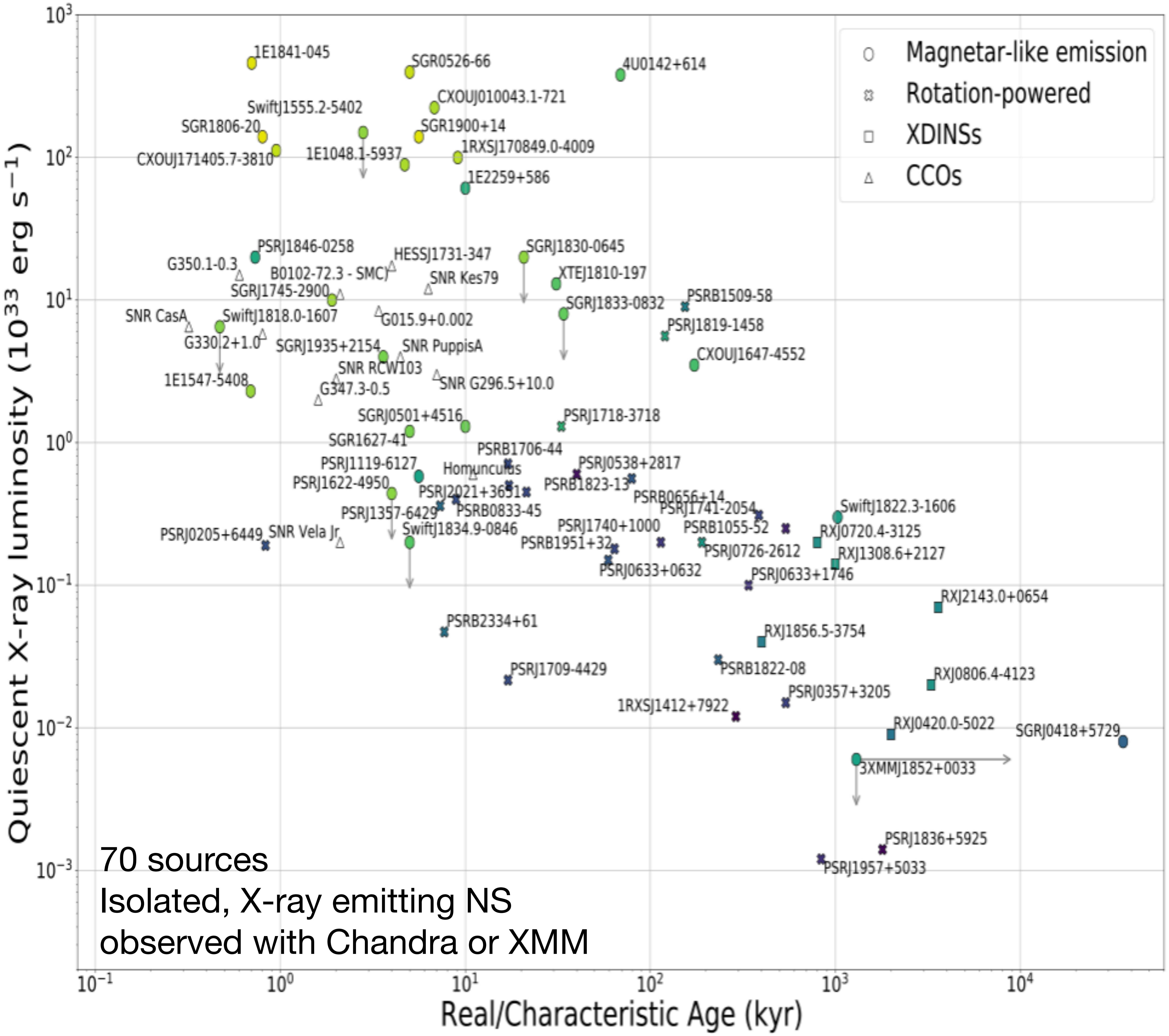
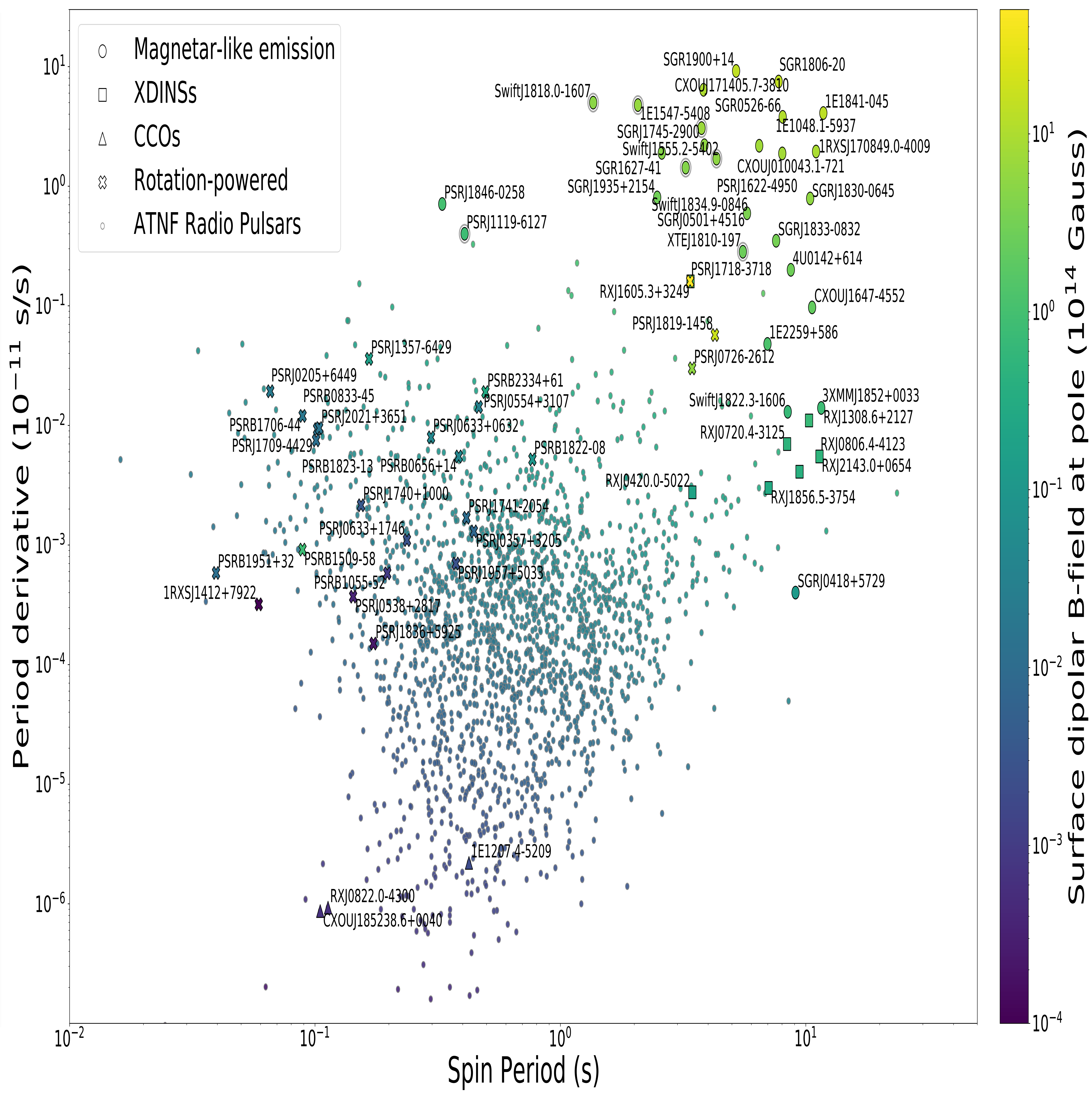
Thermal X-ray luminosity for isolated neutron stars





Four independent measured parameters.

"Need for coupled magneto-thermal simulations to explain the luminosity and the evolution path of different isolated neutron stars" — [Viganò et al. 2023].

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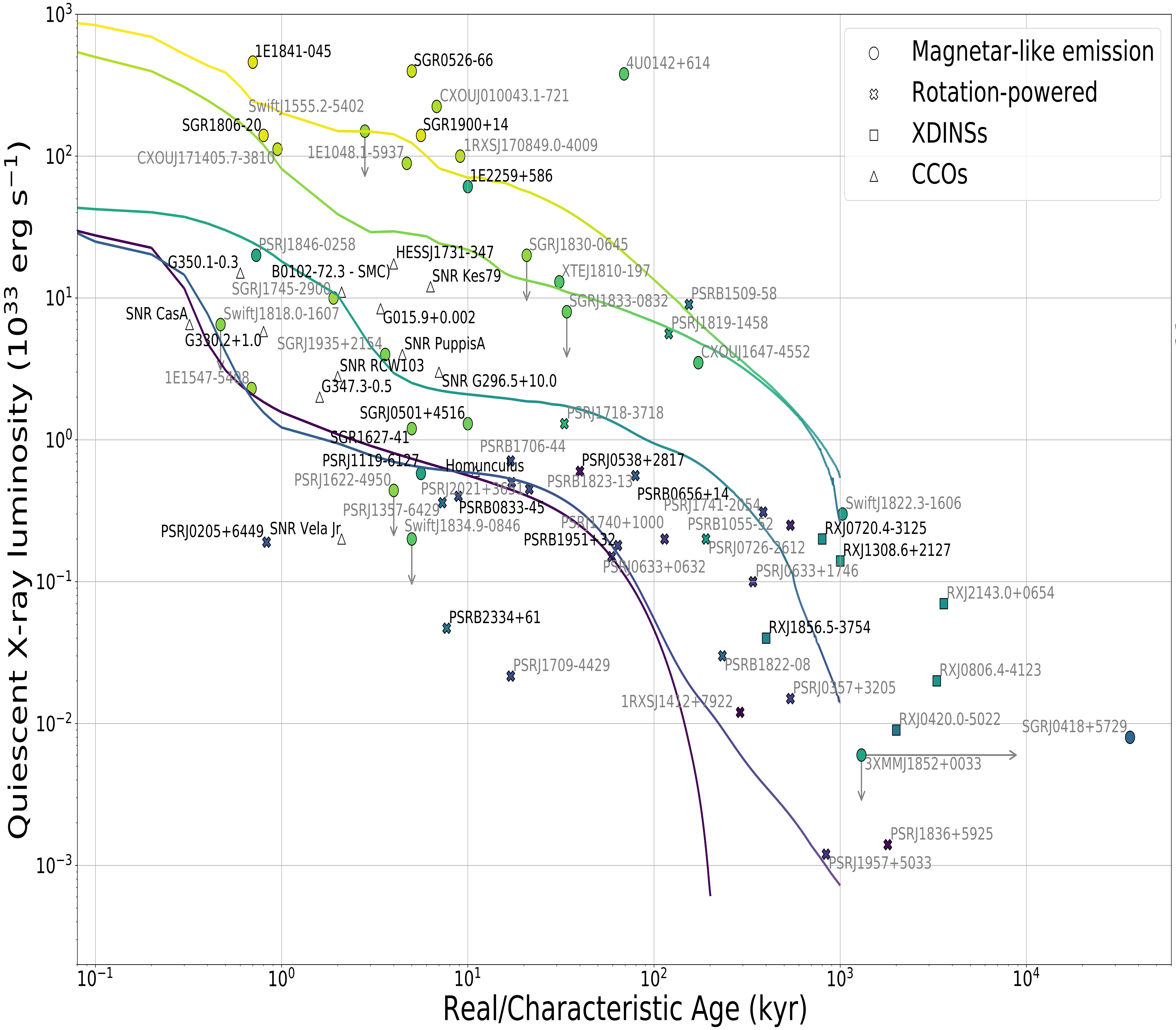
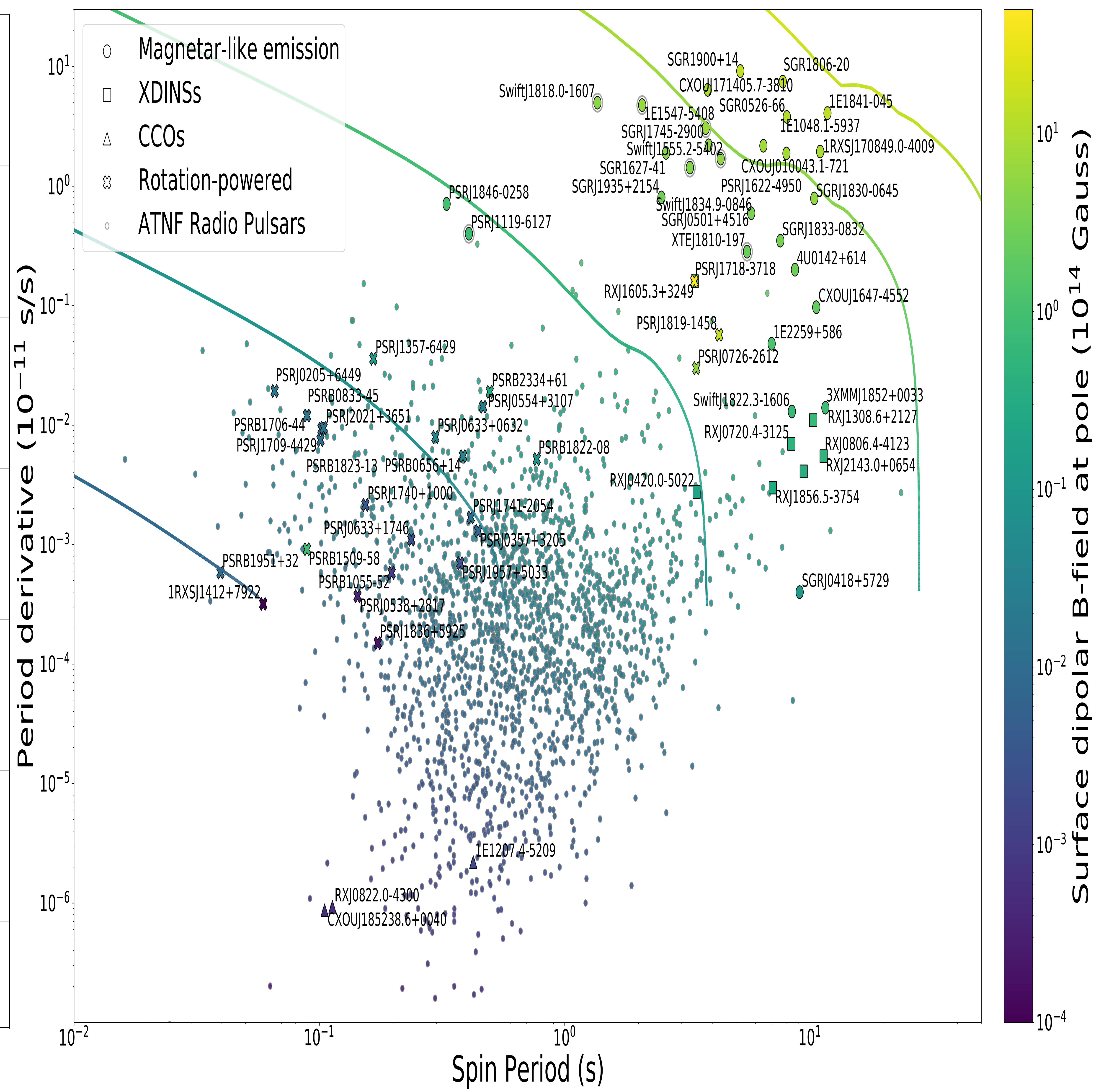
Why we need 3D magneto-thermal models?





- Realistic magnetic topology: complex and non-axisymmetric
- The need to model cooling curves, that depend on the 3d configuration

[Dehman, Marino, kovlakas, Rea et al. in prep.]

"New 3D code MATINS"

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Thermal evolution & cooling curves

$$c_V(T)\frac{\partial\left(Te^{\nu}\right)}{\partial t} = \vec{\nabla}\cdot\left(e^{\nu}\hat{\kappa}\cdot\vec{\nabla}(e^{\nu}T)\right) + e^{2\nu}(Q_J - Q_\nu)$$

(See S. Ascenzi's talk)

Ingredients:

- Neutron star model: EoS + central pressure —> star structure & composition (fixed)
- Heat capacity $C_V(\rho, T)$: main contribution by neutrons in the core
- **Thermal conductivity** $\boldsymbol{\kappa}(\rho, T, \boldsymbol{B})$ very large (star core rapidly isothermal), dominated by electrons, becomes **anisotropic** in presence of magnetic field
- Neutrino emissivity $Q_\nu(\rho, T, \boldsymbol{B})$
- Sources of internal heat Q_j: nuclear reactions, **Ohmic dissipation**, accretion…
- Hydrostatic equilibrium models of envelope (i.e., liquid outermost 100 m), that due to its stronger gradients of density and temperature has much faster timescales than the interior
- Emission model (atmosphere, blackbody, condensed surface...)

[Potekhin et al. 2015, review]

http://www.ioffe.ru/astro/conduct/
https://compose.obspm.fr/

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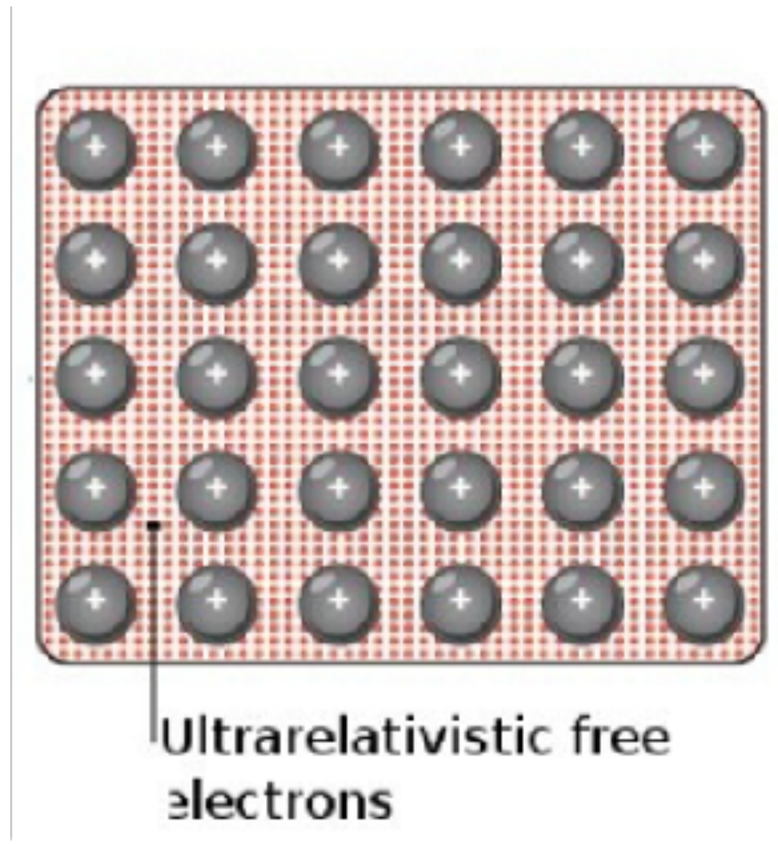
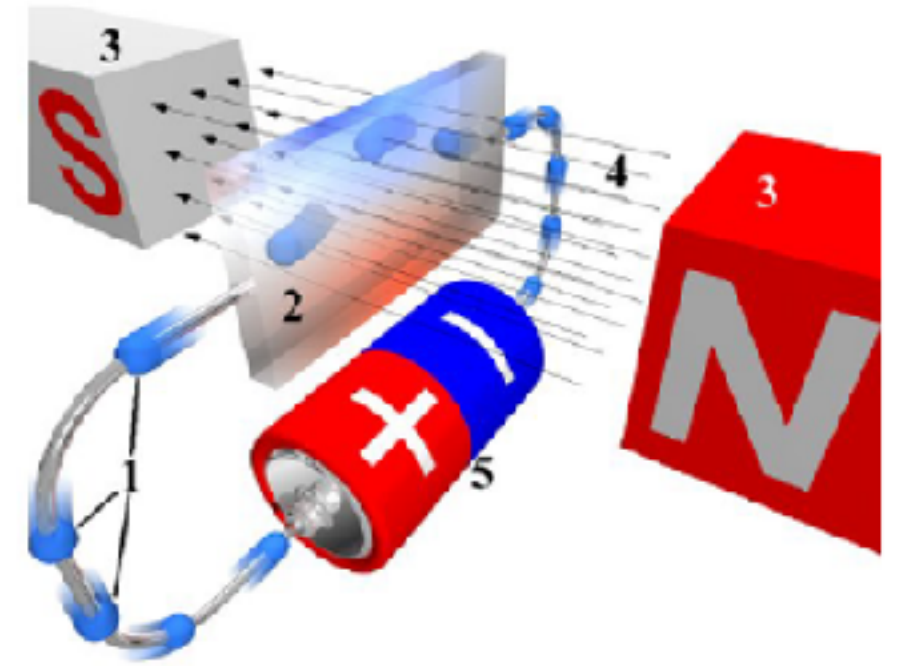
Magnetic field evolution

— Hall MHD limit —





- Neutron stars interior ⟶ complex multi-fluid system
- A solid crust is formed soon after birth ⟶ restricted nuclei mobility ⟶ conduction governed by electrons
- Core: full multi-fluid system
- Approximation: electrons MHD limit in the crust (eMHD)

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left[\frac{c^2}{4\pi\sigma_e}\nabla \times (e^{\nu}\boldsymbol{B}) + \frac{c}{4\pi e n_e}[\nabla \times (e^{\nu}\boldsymbol{B})] \times \boldsymbol{B}\right]$$

Ohmic dissipative term: the magnetic resistivity is very sensitive to temperature evolution and electron density

Hall drift term: It naturally creates magnetic discontinuity and transfers energy between different scales.

- Crustal-confined (perfect conductor at the crust-core interface).
- Potential boundary conditions (i.e. no current, $\nabla \times \boldsymbol{B} = 0$) - better force-free magnetosphere.
- Divergence-free magnetic field $\nabla \cdot B = 0$.

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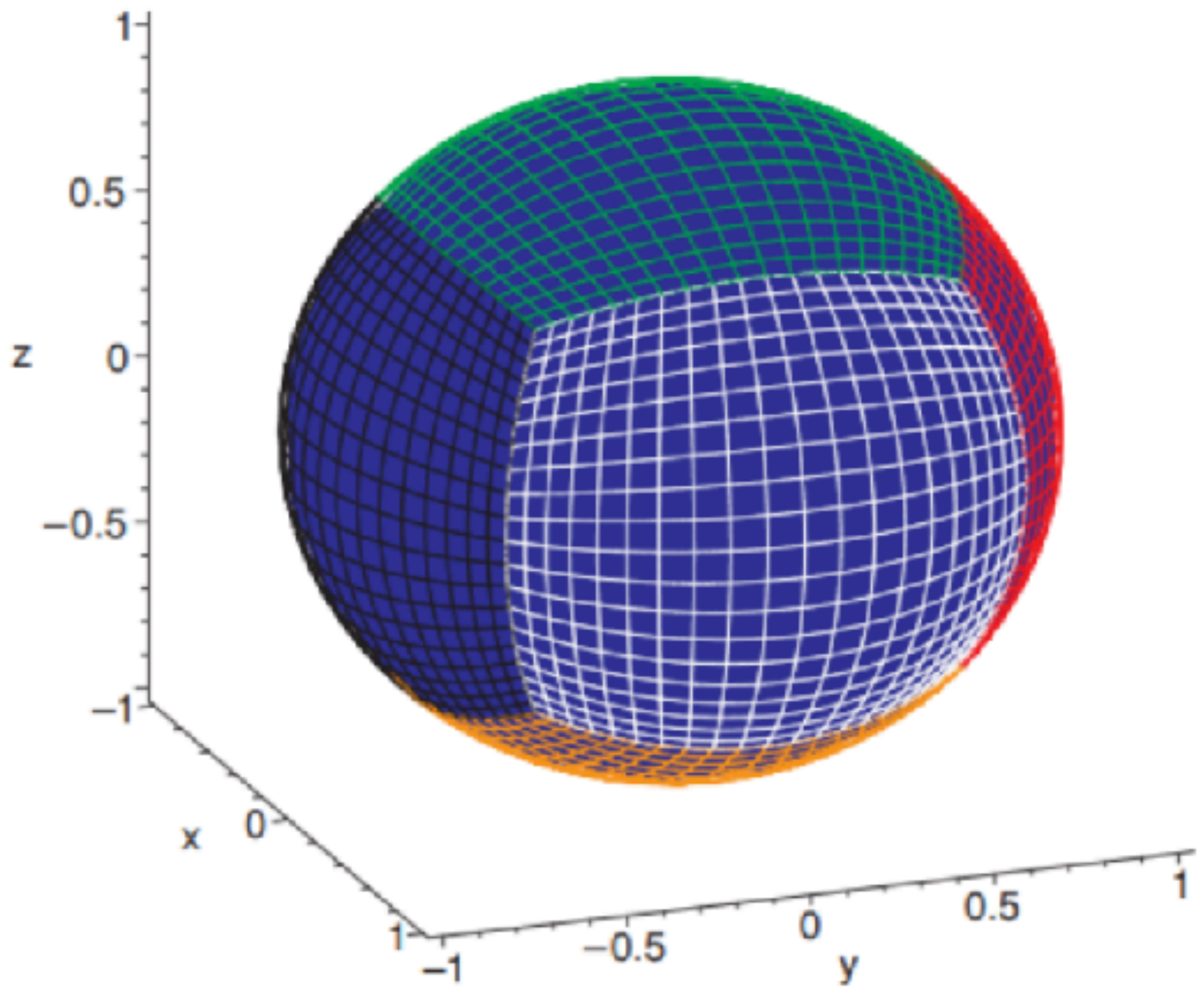
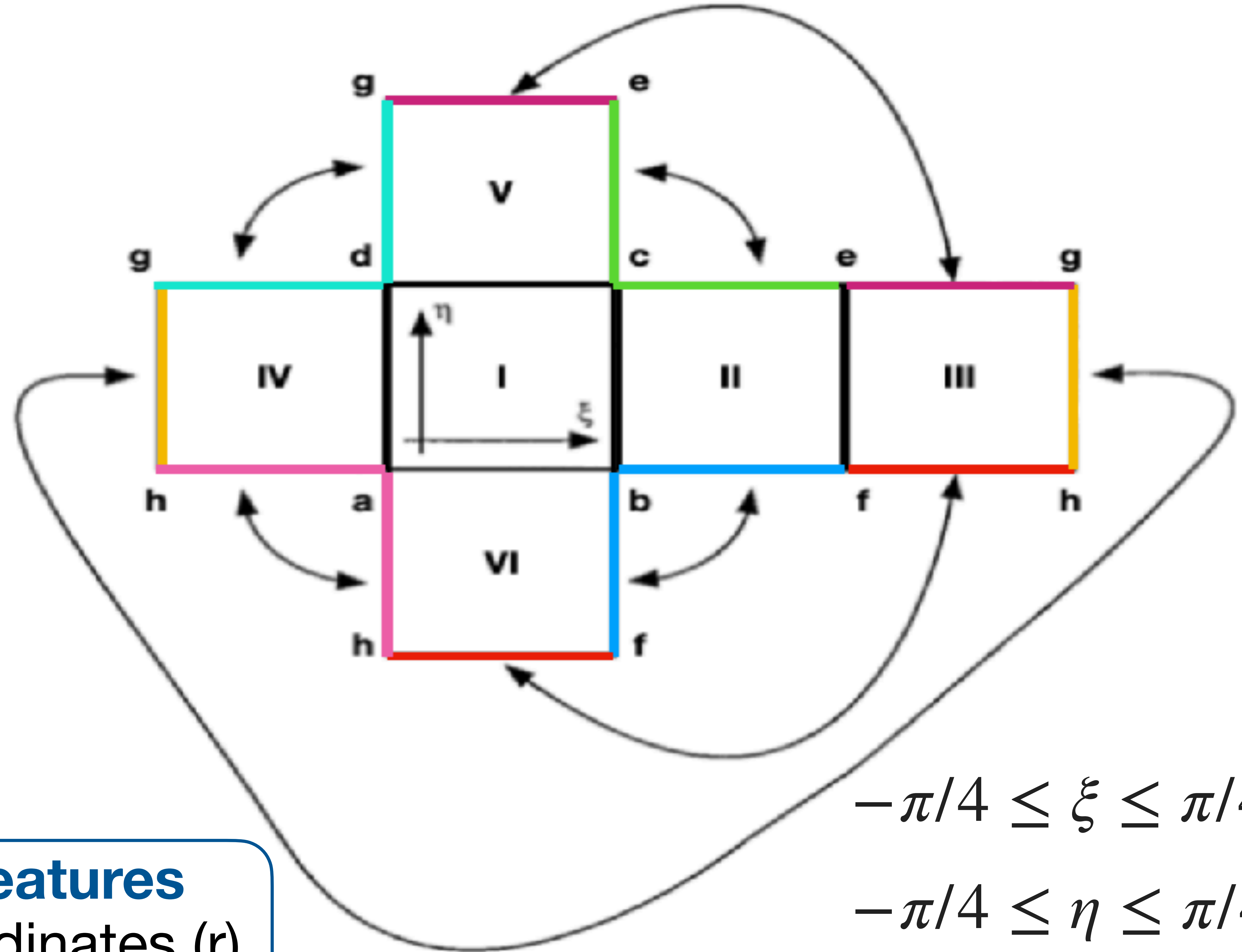
[Pons & Viganò 2019, living review]

Schwarzschild cubed-sphere

In 3D spherical coordinates if you want to use **finite-volume**/difference methods, the axis is a singularity. The cubed sphere coordinates are a widely used solution, used in climate and atmospheric simulations

[Ronchi et al. 1996]





$$-\pi/4 \leq \xi \leq \pi/4$$

$$-\pi/4 \leq \eta \leq \pi/4$$

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} \\ 0 & -\frac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} & 1 \end{pmatrix}$$

non-orthogonal coordinate system

Desirable features
- Radial coordinates (r)
- No axis-singularity
- GR correction

[Dehman et al. 2022 MNRAS]

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MATINS the brand new 3D code

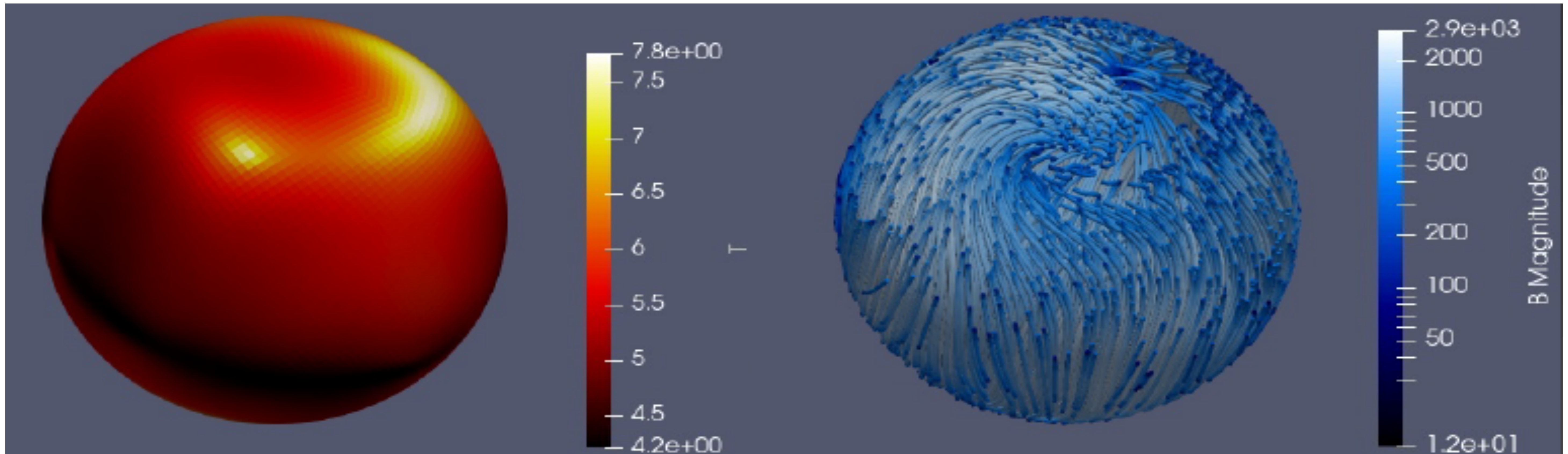
Dehman**, Viganò, Pons & Rea 2022, MNRAS* (DOI: **10.1093/mnras/stac2761**): ***Cubed-sphere grid + Magnetic formalism

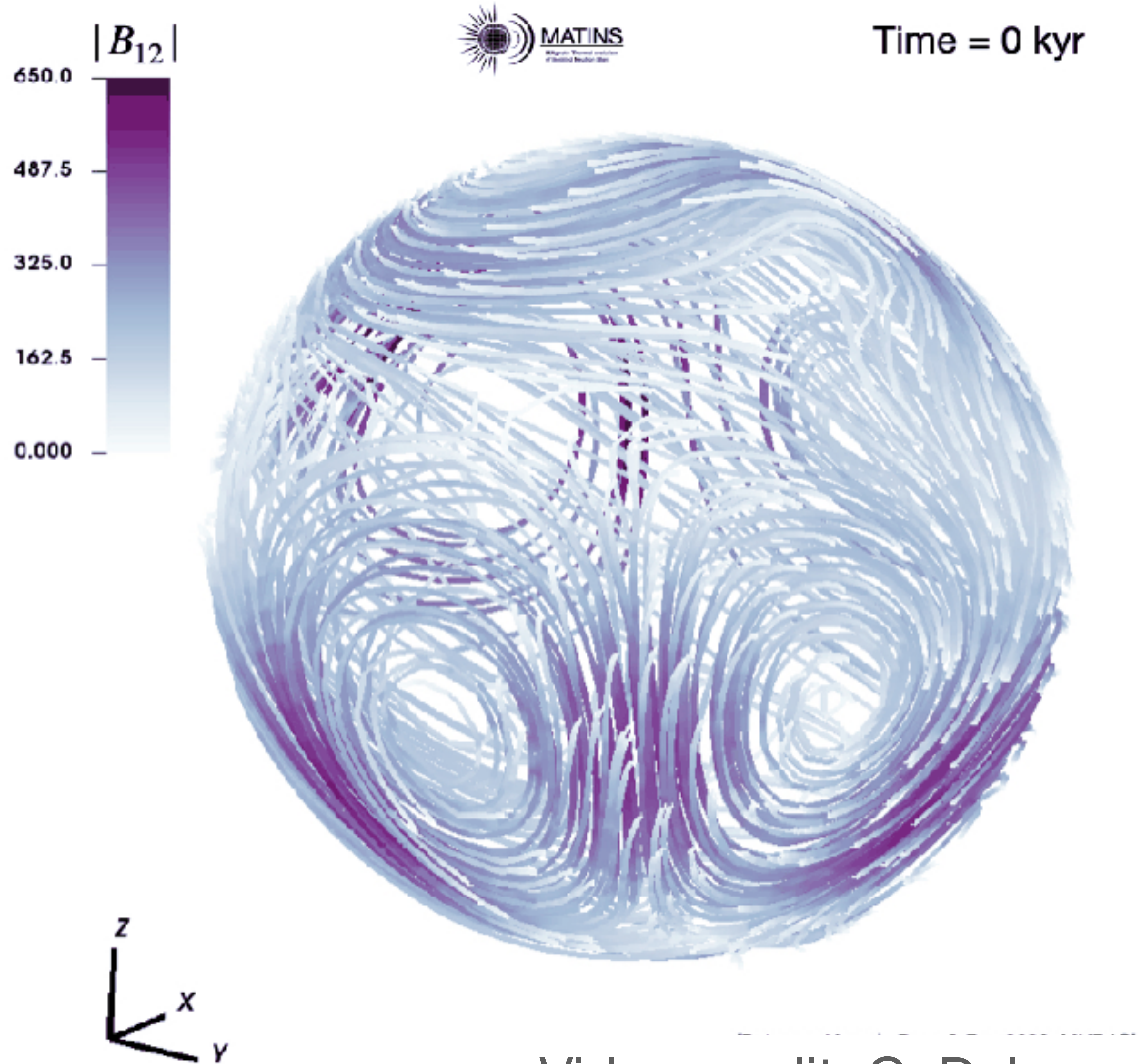
Dehman**, Viganò, Ascenzi, Pons & Rea 2023, MNRAS* (DOI: 10.1093/mnras/stad1773): **First *3D magneto-thermal simulation

*Ascenzi, Viganò, **Dehman**, Pons & Rea, Perna 2024, MNRAS* (DOI: **10.1093/mnras/stae1749**)*:* ***Thermal formalism***

Soon to be public.
What's better than 2D:
- Simulation of 3D magnetic modes, hotspots, and light curves
- Better documentation, **use of novel coordinates (cubed-sphere)**
- Optimization and use of OpenMP

Advance obtained (only another 3D code was existing so far):
- **Realistic 3D evolution and topology, appearance of hotspots**
- **State-of-the-art microphysics and realistic structure**
- **Numerical scheme to better capture non-linear dynamics**
- **General relativistic correction**
- **State of art envelope model**
- **Flexibility in implementing new physics**
- **Documentation and modularity (for public)**





Video credit: C. Dehman

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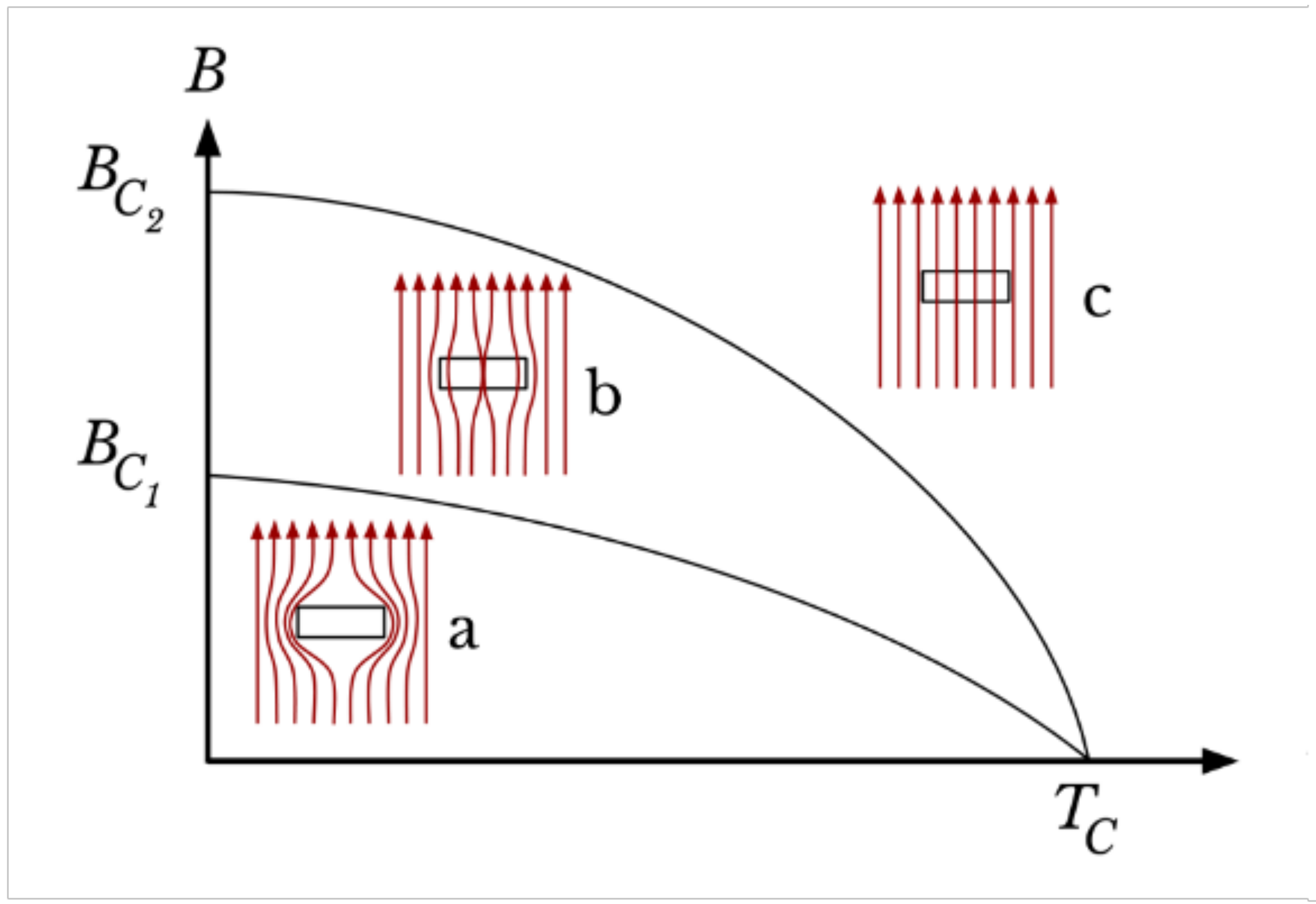
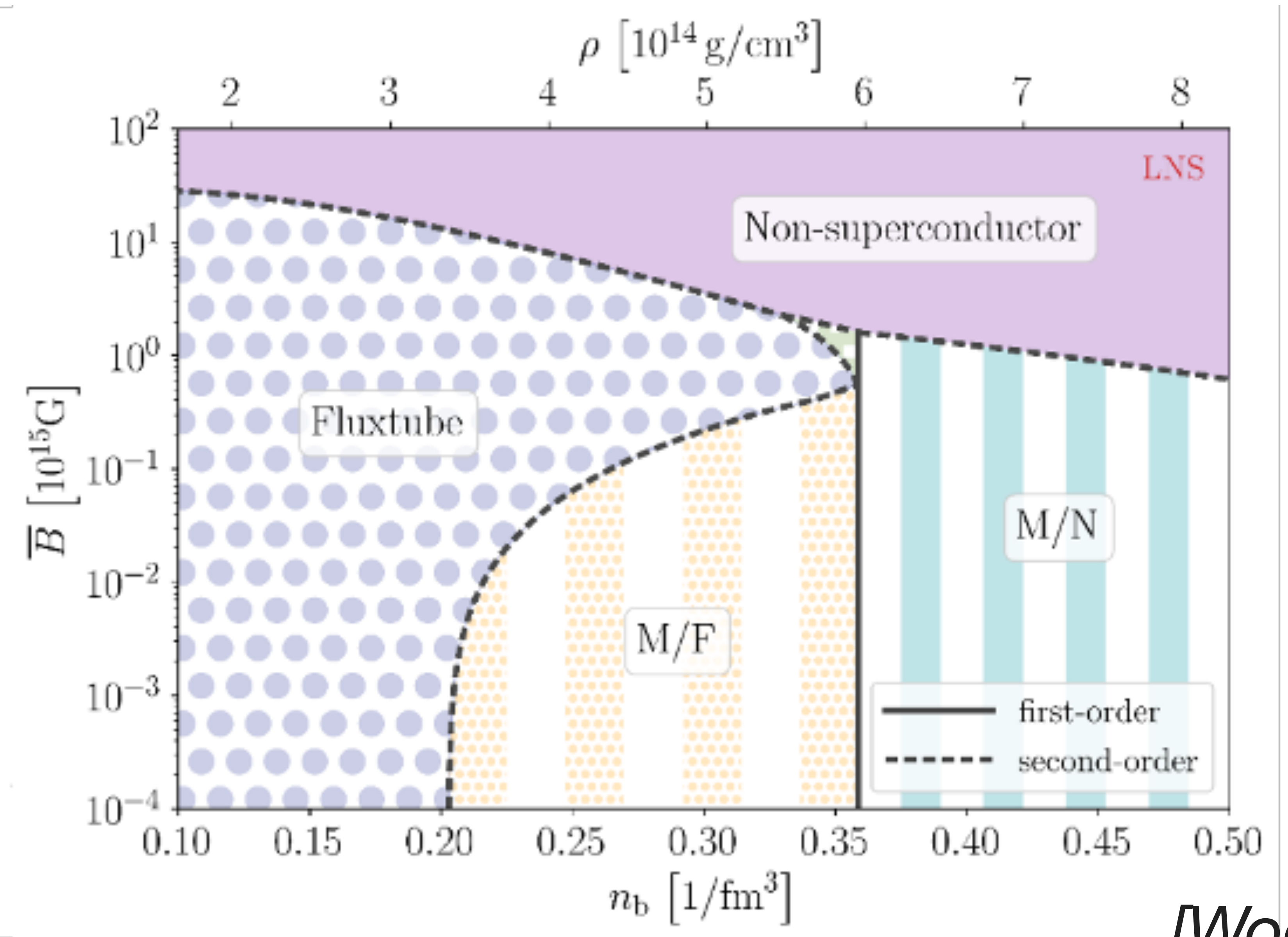
What physics is still missing?

I- Magnetic field evolution in the core

II- Force-free magnetosphere coupled with the interior (already done in 2D but not yet in 3D)

III- Initial conditions: How the large scale field dipolar field is formed?





[Wood & Graber 2022]

In the core, physics is more complex. A mixture of neutrons, protons, electrons, muons (and others?) is expected: multi-fluid MHD -> ambipolar diffusion.

Moreover: superfluidity (neutrons) and superconductivity (protons) of type II (b), with the magnetic flux contained in tiny vortexes (fluxtubes); for lower B, Meissner effect (type I, (a)).

Multi-spatial/timescale problem extremely challenging! (Gusakov et al. 2020 for a review of the current debate, and Igoshev & Hollerbach 2022 for the most recent results –no SC, only ambipolar diffusion-).

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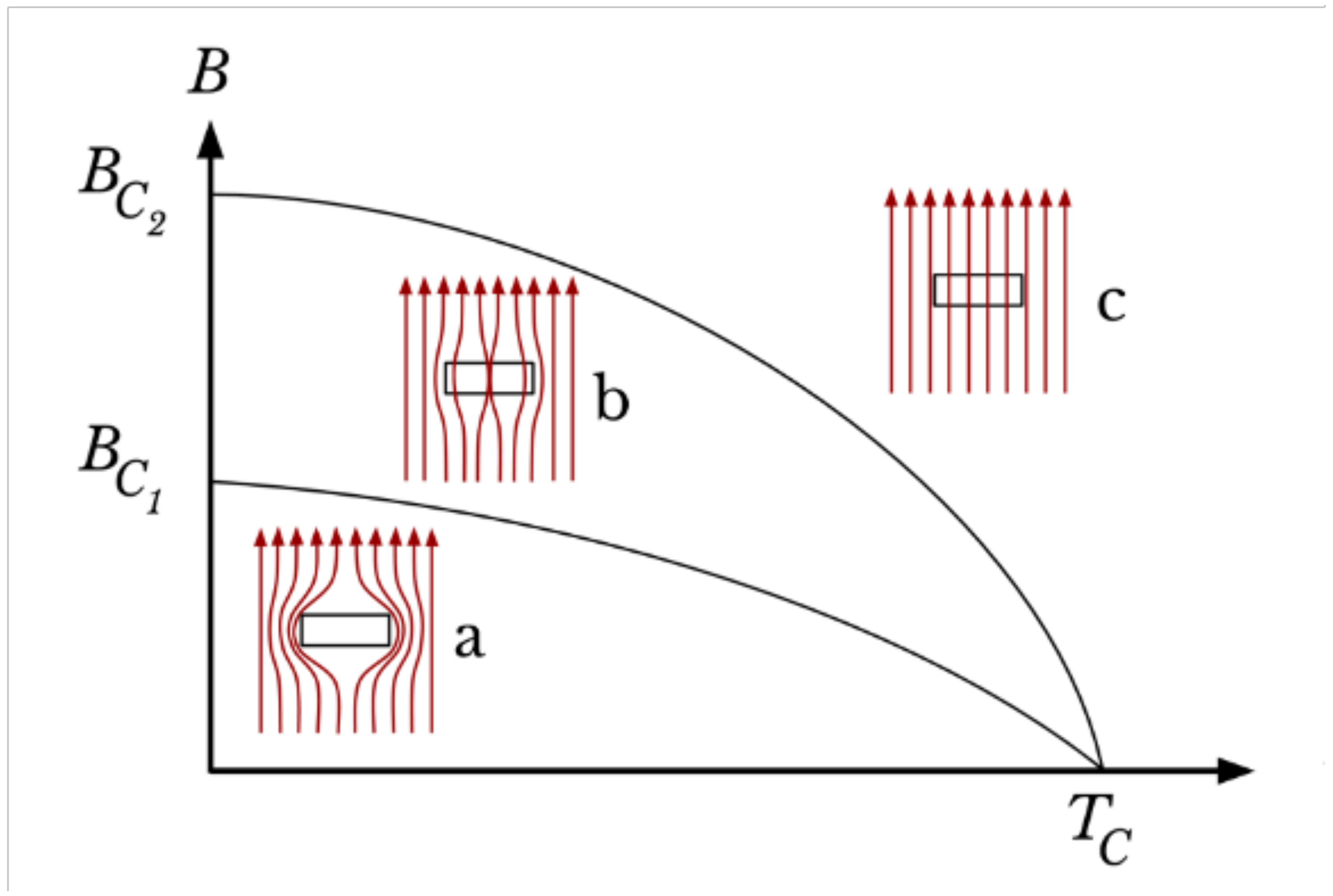
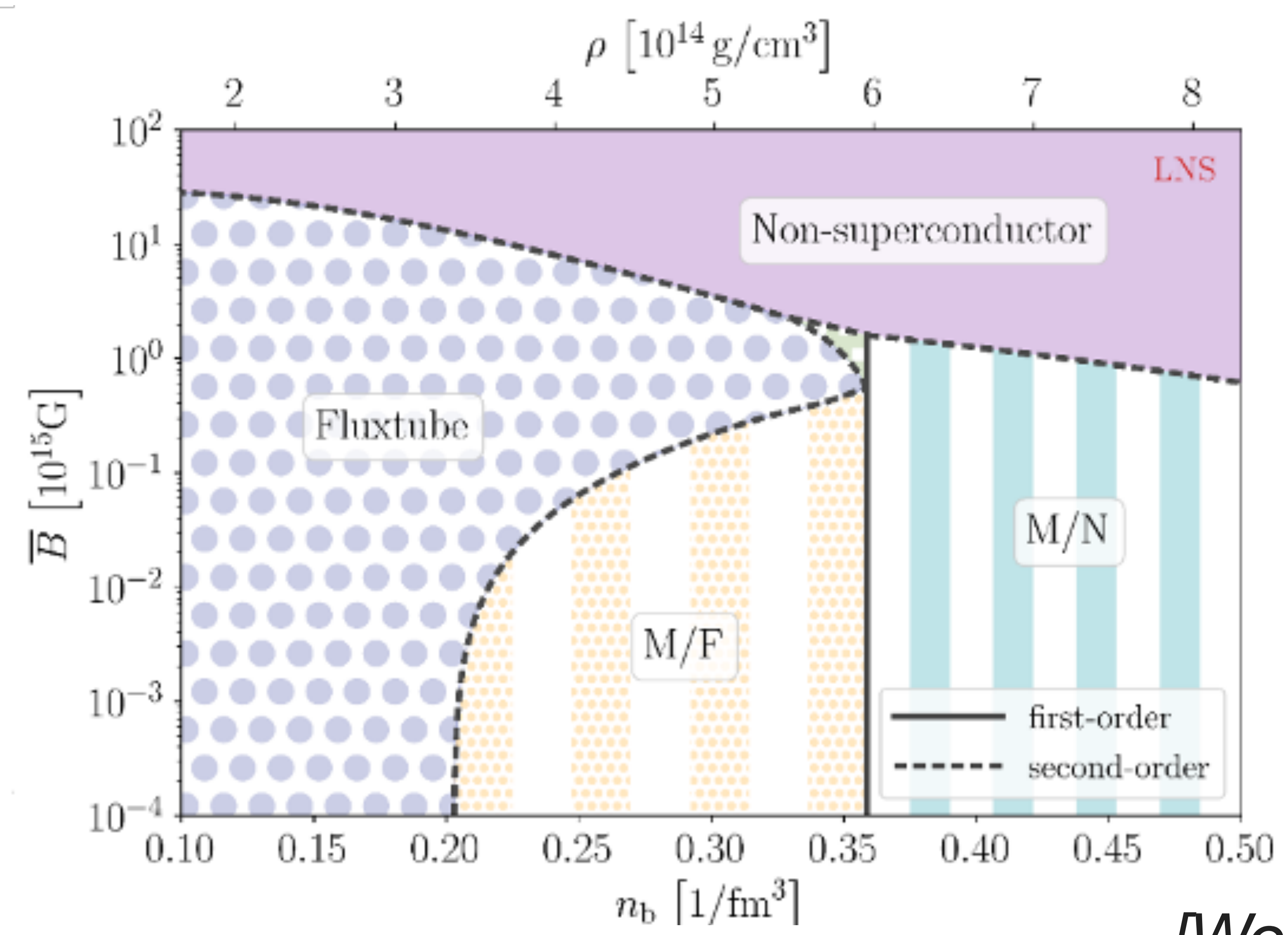
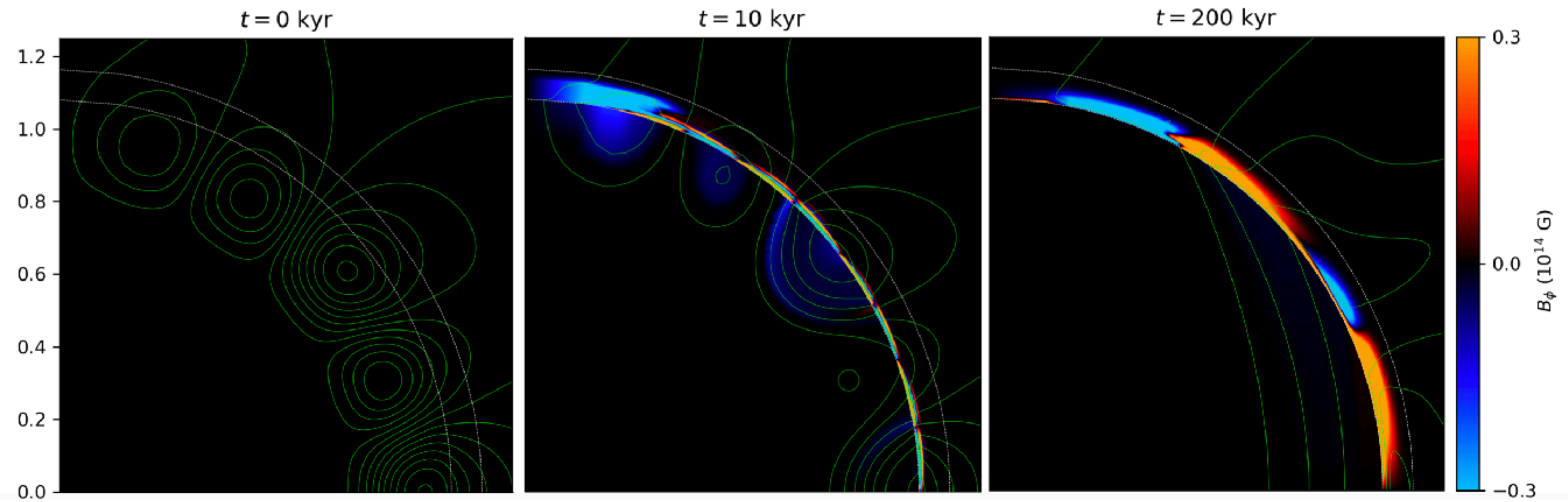




[Wood & Graber 2022]



[Bransgrove, Levin & Beloborodov 2024]

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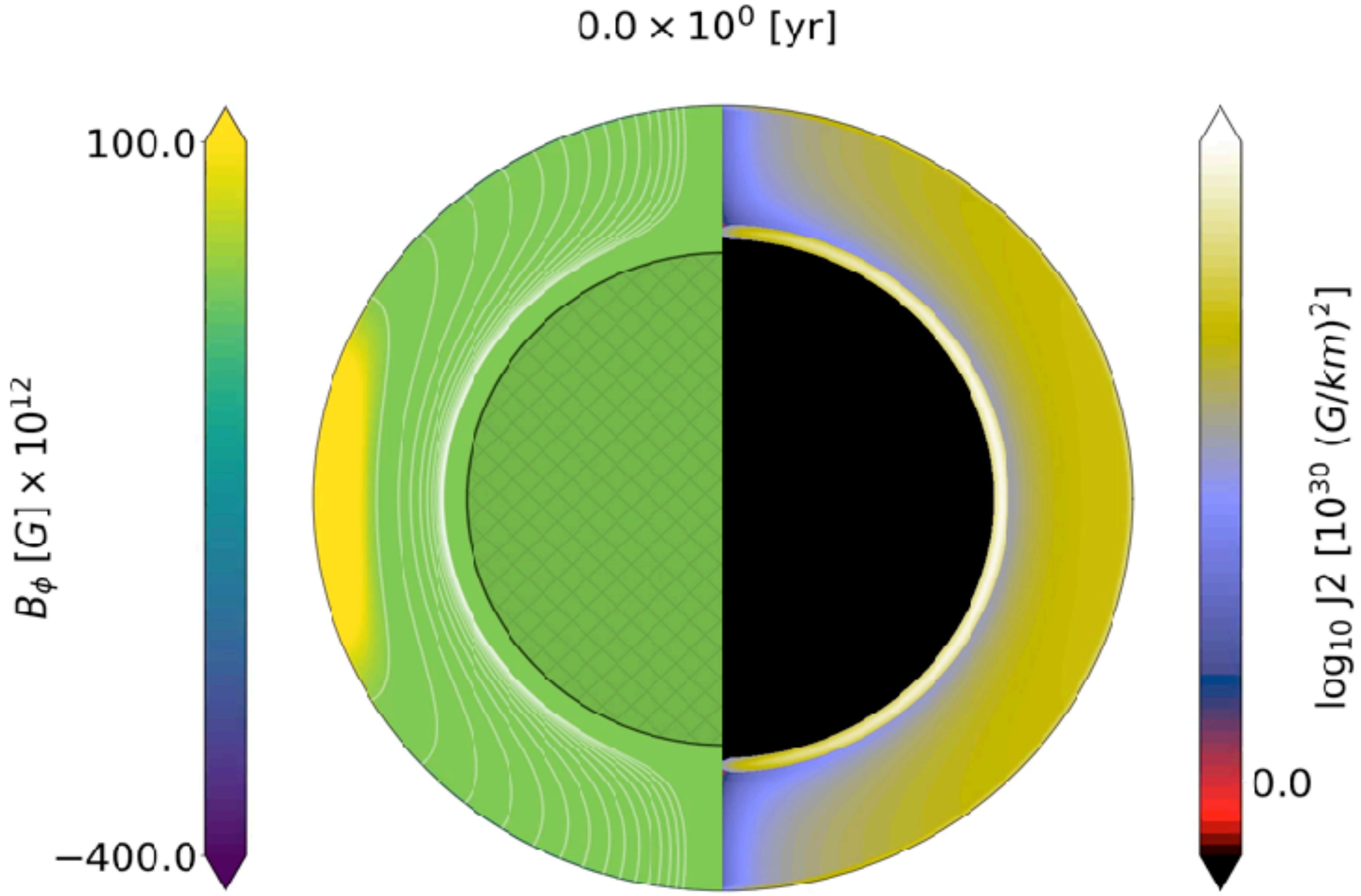
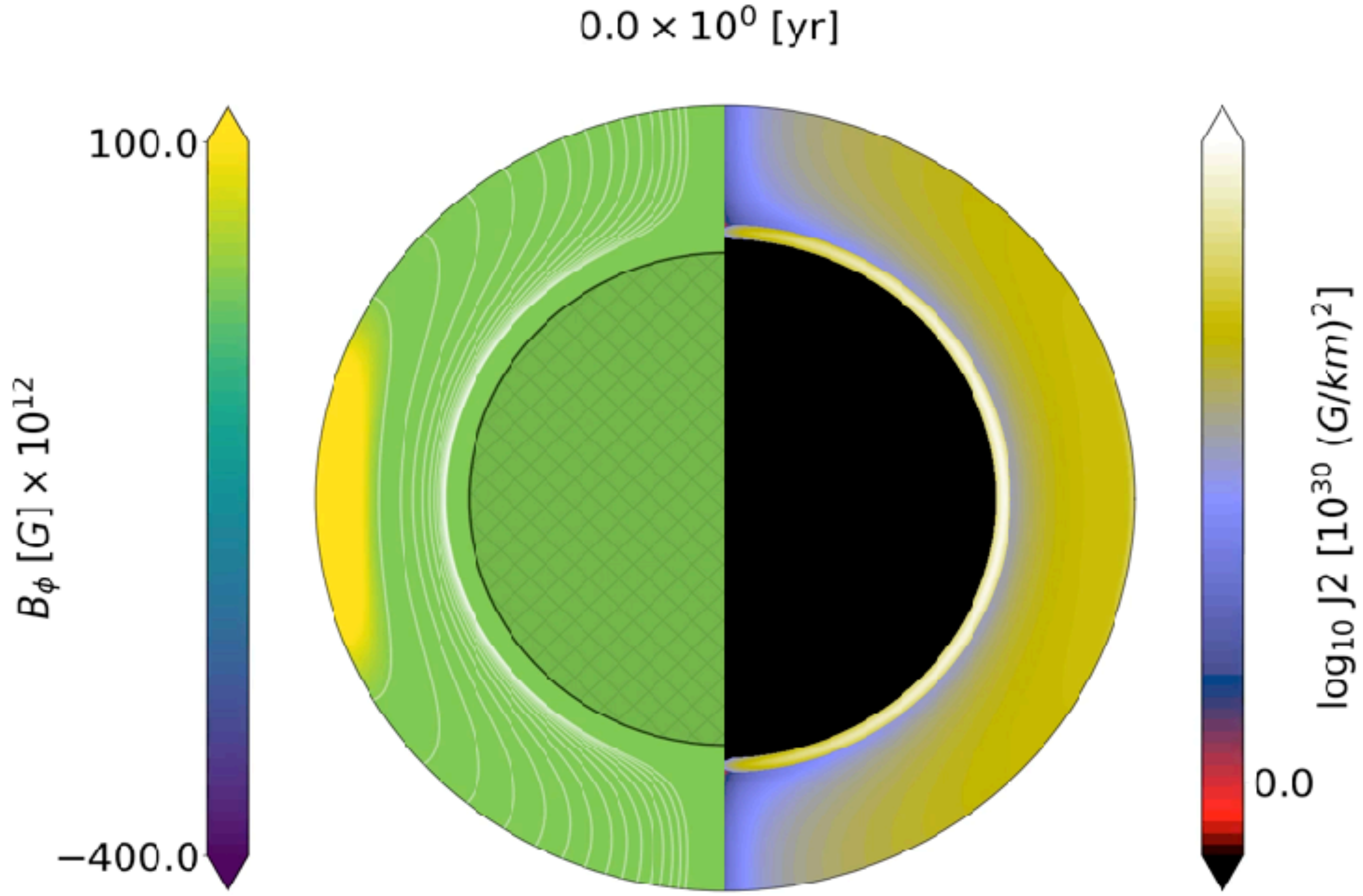
Force-free Magnetosphere



Vacuum Magnetosphere



The currents can flow in the magnetospheric $\left(\vec{\nabla} \times \vec{B} = \alpha \vec{B}\right)$.
Toroidal field does not vanish at the surface
$\mathcal{T}(\mathcal{P}) = s_1\mathcal{P} + s_2\mathcal{P}^2$

→ Crust-magnetosphere coupling affects the interior field evolution

Left hemisphere:
Field lines: Poloidal field.
Colorbar: Toroidal field.

Right Hemisphere:
Electric current.

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[Urban, Stefanou, Dehman & Pons 2023, MNRAS]

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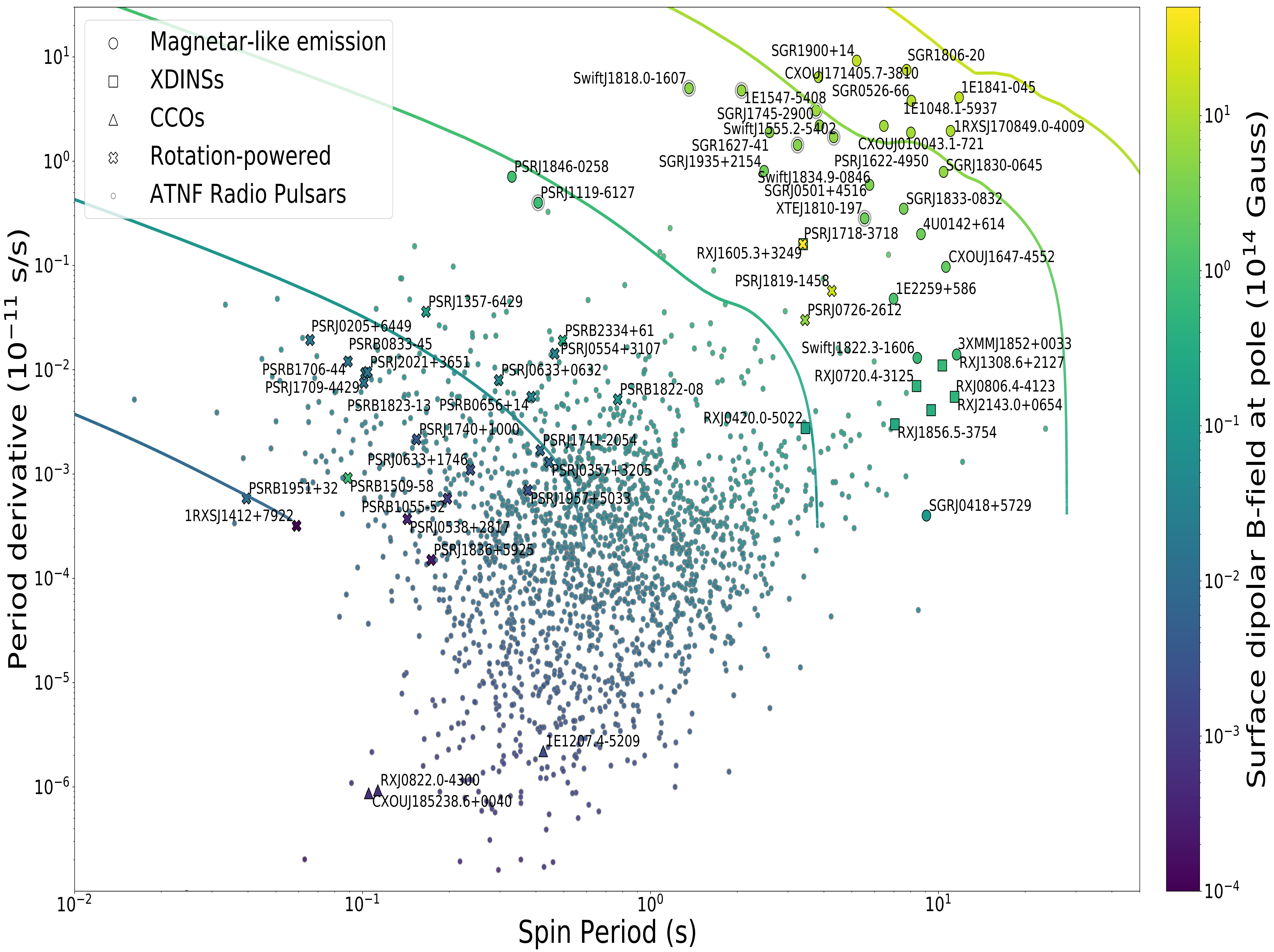

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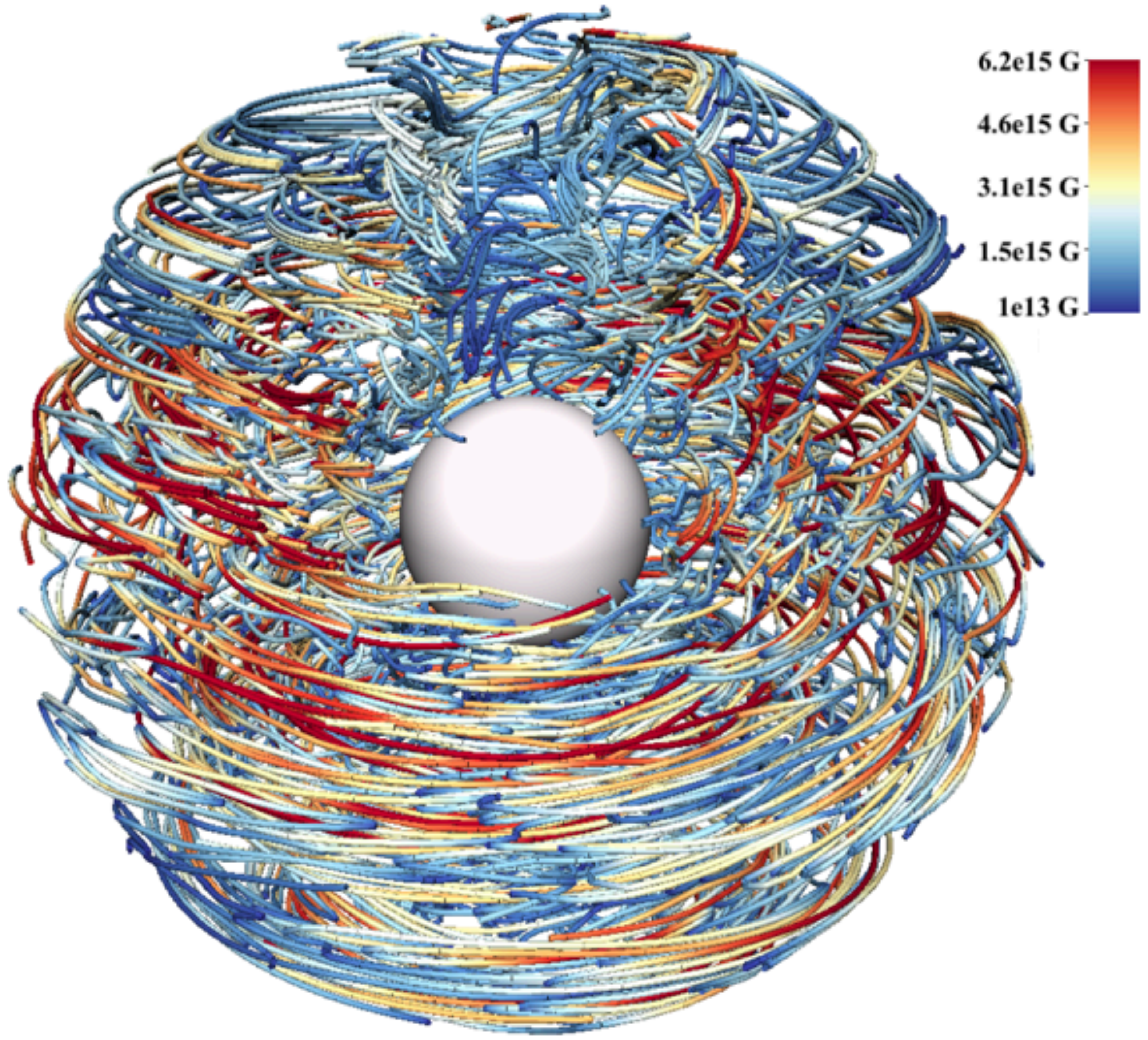
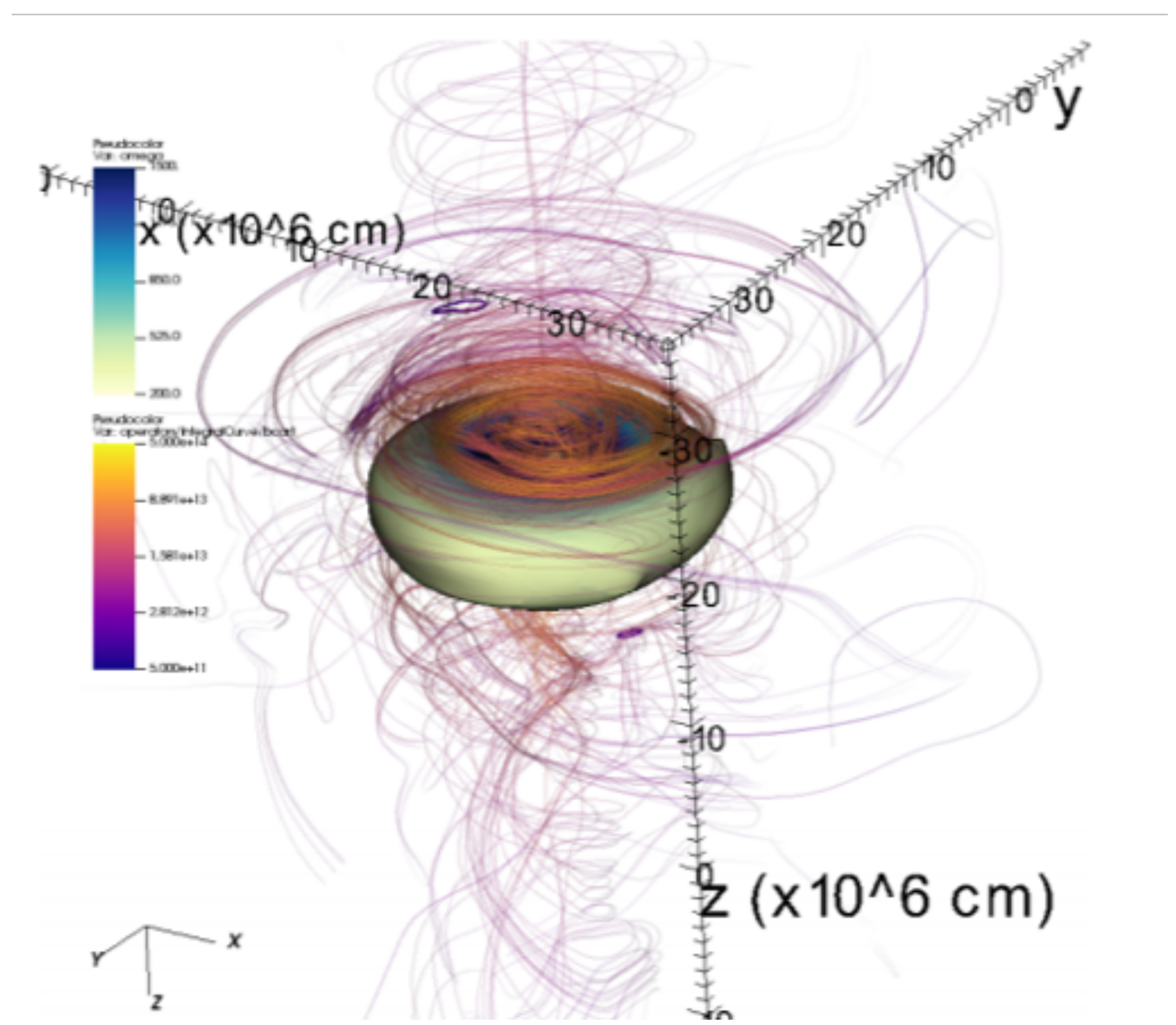
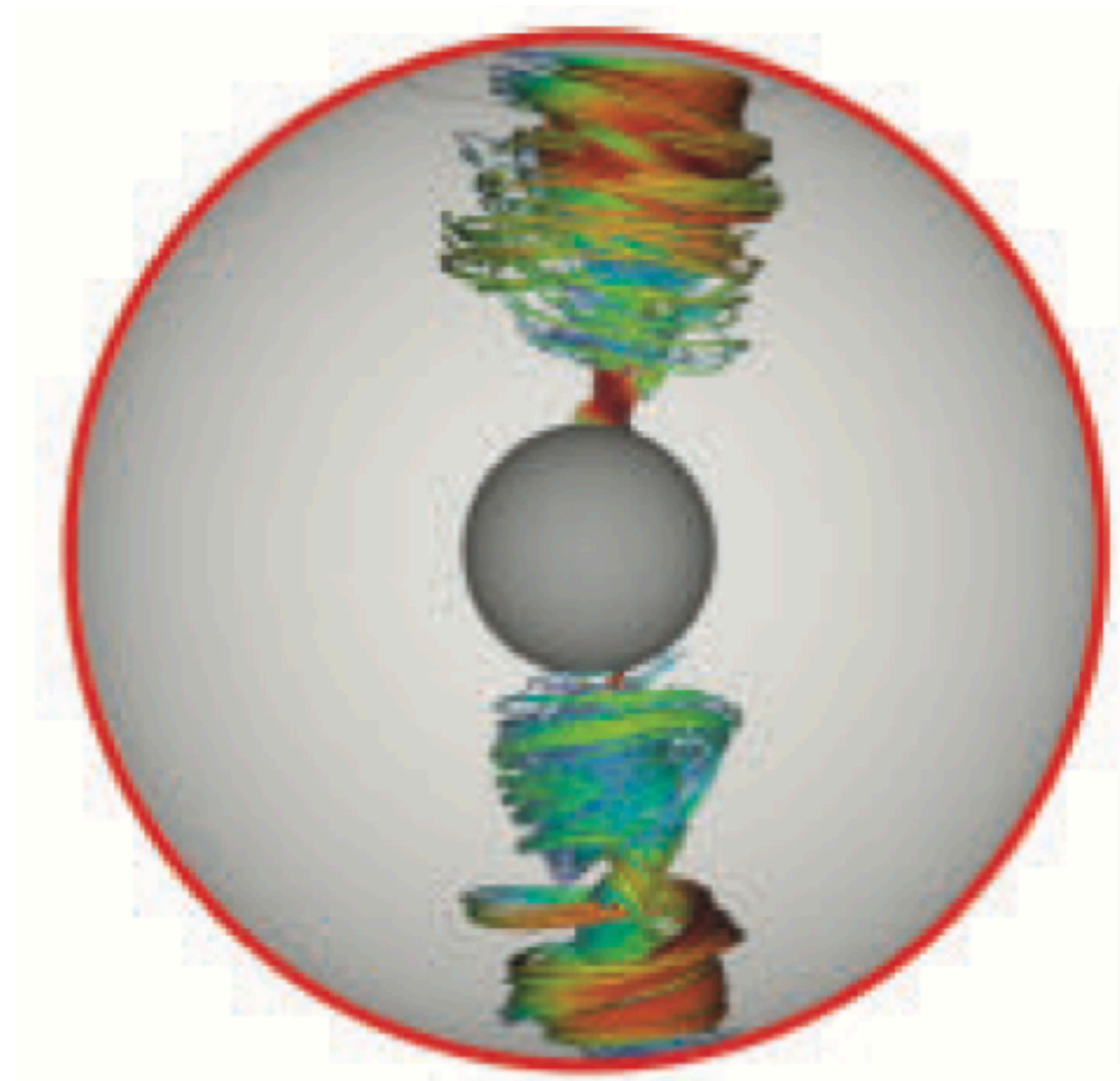
Magnetic field at birth: core-collapse simulations

After the **core-collapse supernova**, during the **proto neutron star**, different dynamo mechanism take place: MRI-driven dynamo, Taylor-Spruit dynamo, leading to complex topology. A new-born, rigidly rotating neutron star should have:

1. MHD equilibrium -> constraints on the magnetic topology (but infinite solutions)
2. The environment around the star is very dirty and it's very likely that currents circulate in the magnetosphere: no vacuum around the star
3. The axial symmetry is unlikely to hold (asymmetry in the explosion, turbulence…)
4. Large-scale structures (dipoles) can be formed, but most magnetic energy is stored in smaller scales







Reboulz-Salze et al. 2021
Aloy & Obergaulinger 2020
P. Barrère et al. 2024

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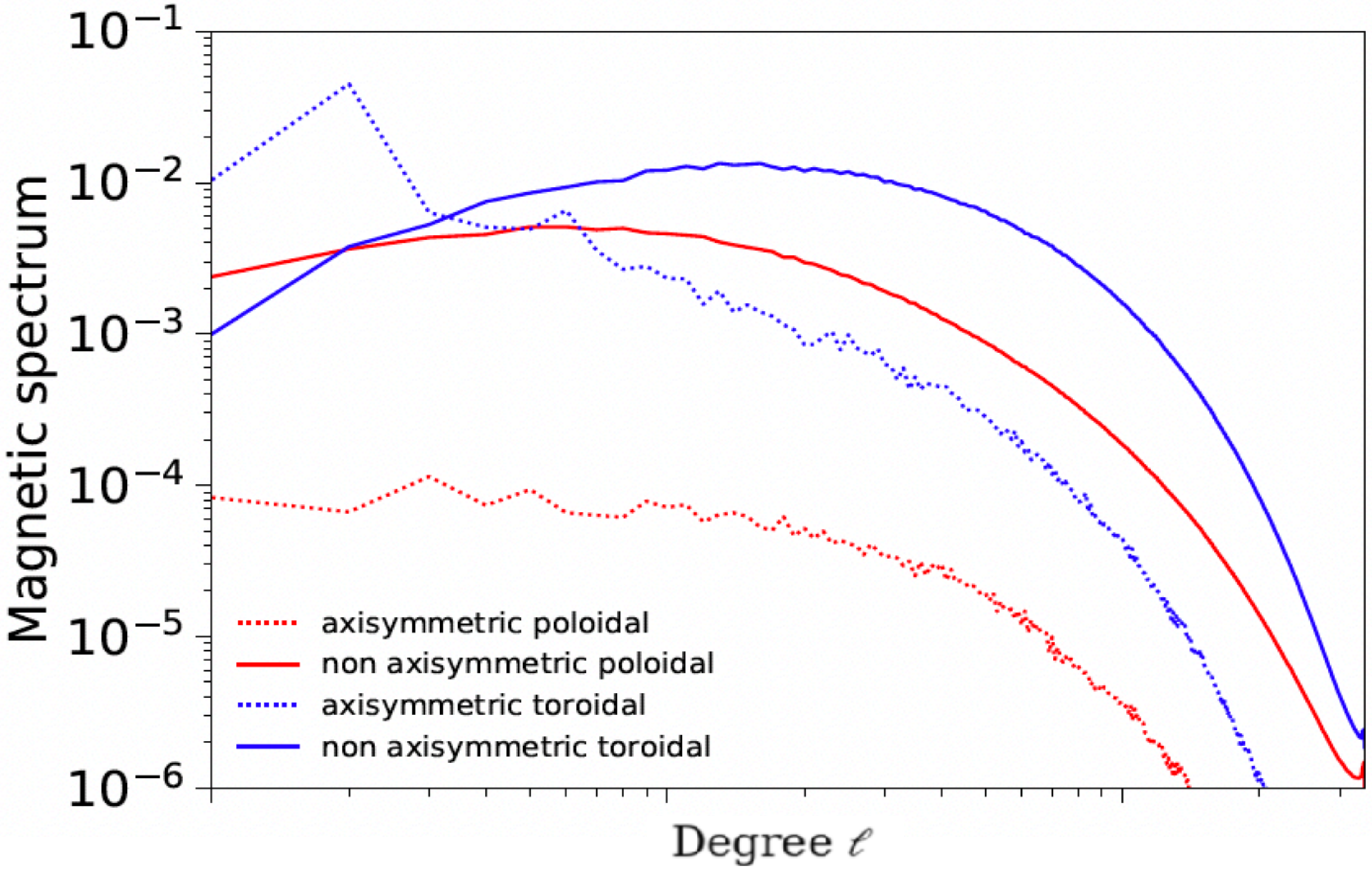
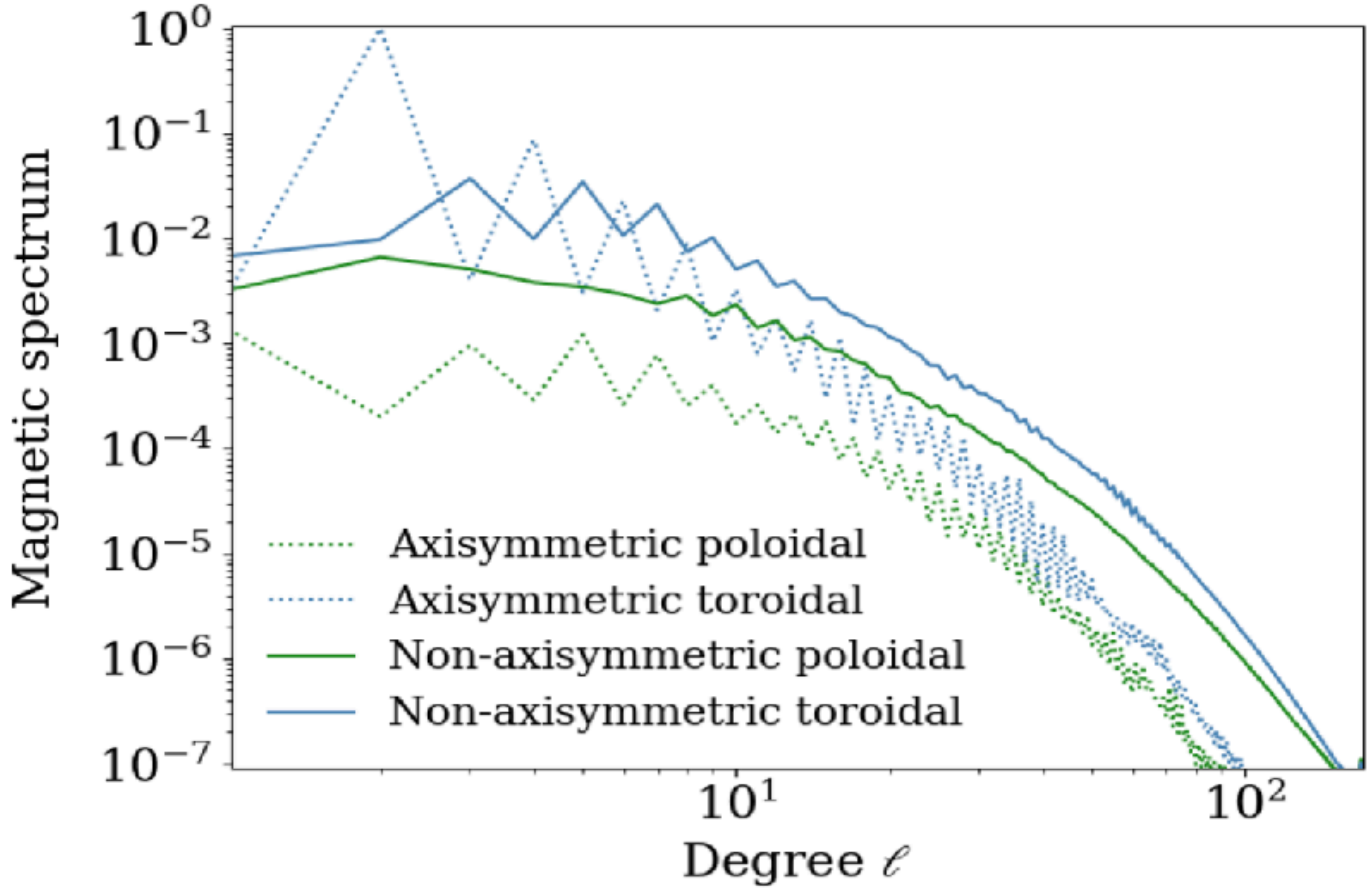
Magnetic field at birth: Magnetic spectrum after core-collapse





Magnetic energy is distributed across a broad range of scales.

Toroidal axisymmetric quadrupole and non-axisymmetric components dominate.

Dipolar field accounts for less than 5% of the total magnetic energy.

From post-collapse to neutron star phase, plenty of MHD timescales to approach an equilibrium, with dynamo still going on and at the same time dissipating the smallest scales.

But: no way to leave only a dipole or axisymmetric configurations!

[Reboul-Salze et al. 2021]
[Barrère et al. 2023]

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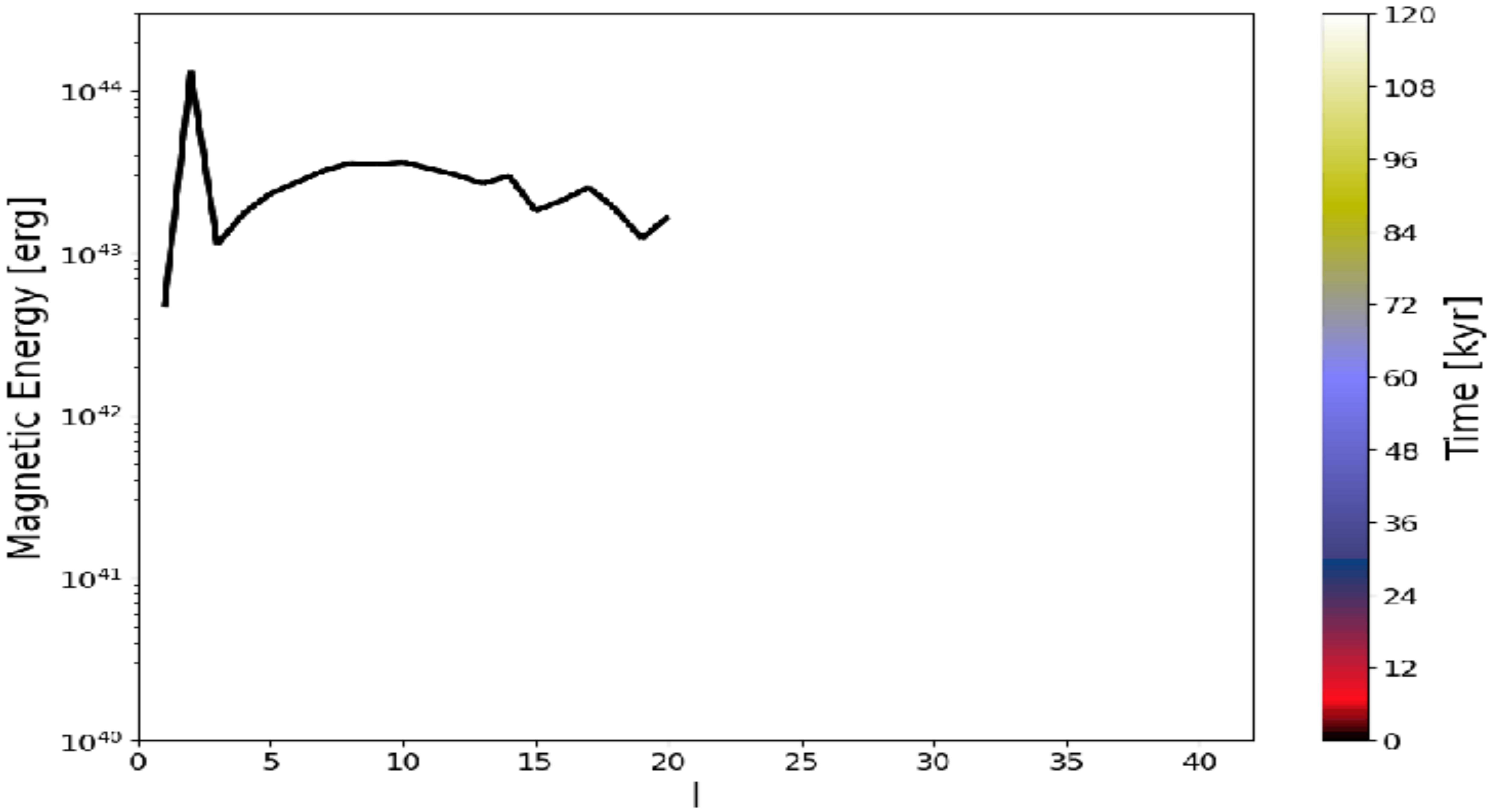
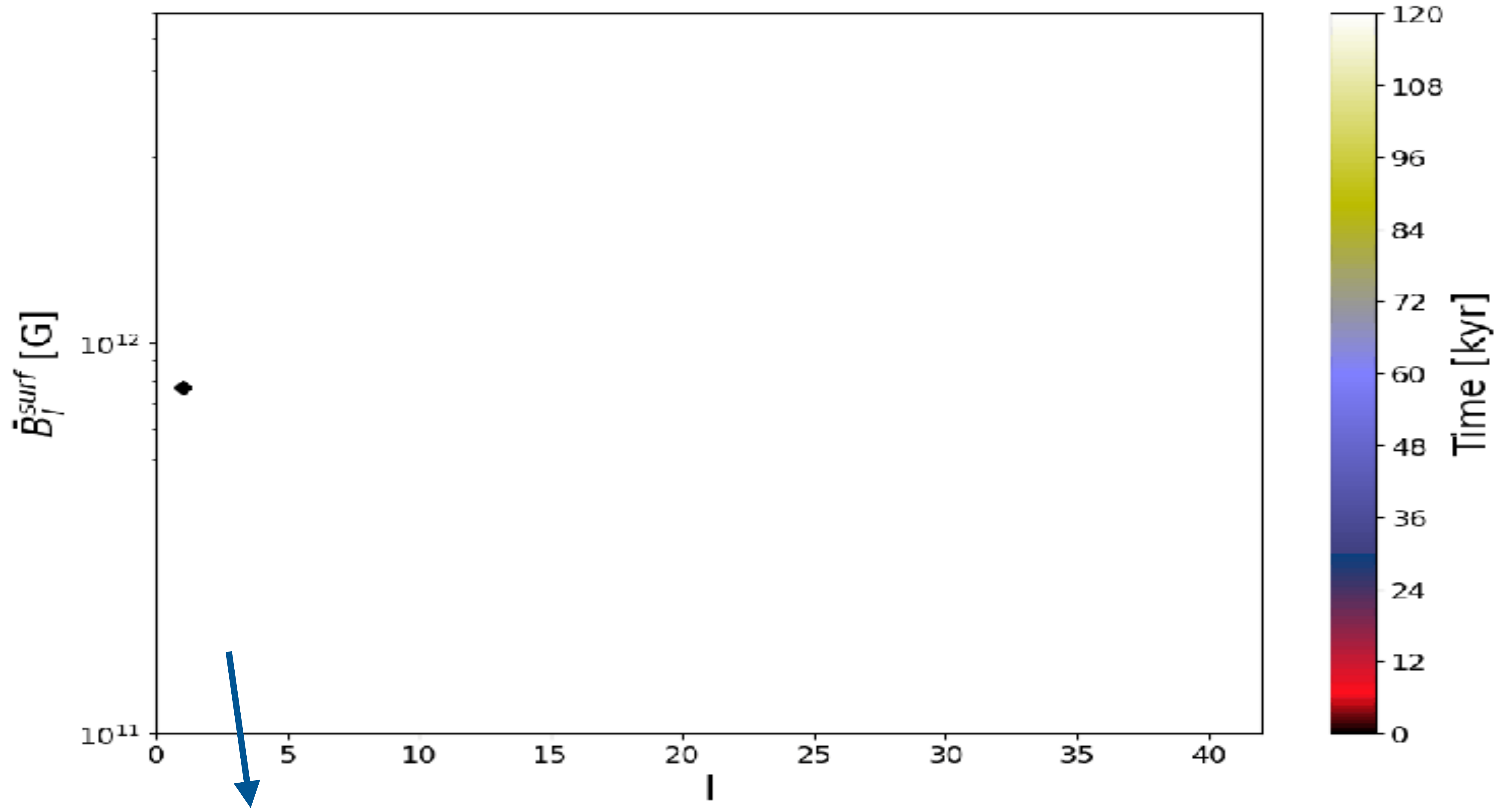
First 3D coupled magneto-thermal evolution

— Initial magnetar-like magnetic field strength —





$l = 1 \rightarrow$ **dipole**

Small-scale field $\sim 10^{15} \ldots 10^{16}\,\mathrm{G}$
Dipolar field $\sim 10^{12} \ldots 10^{13}\,\mathrm{G}$

[Dehman et al. 2023b, MNRAS]

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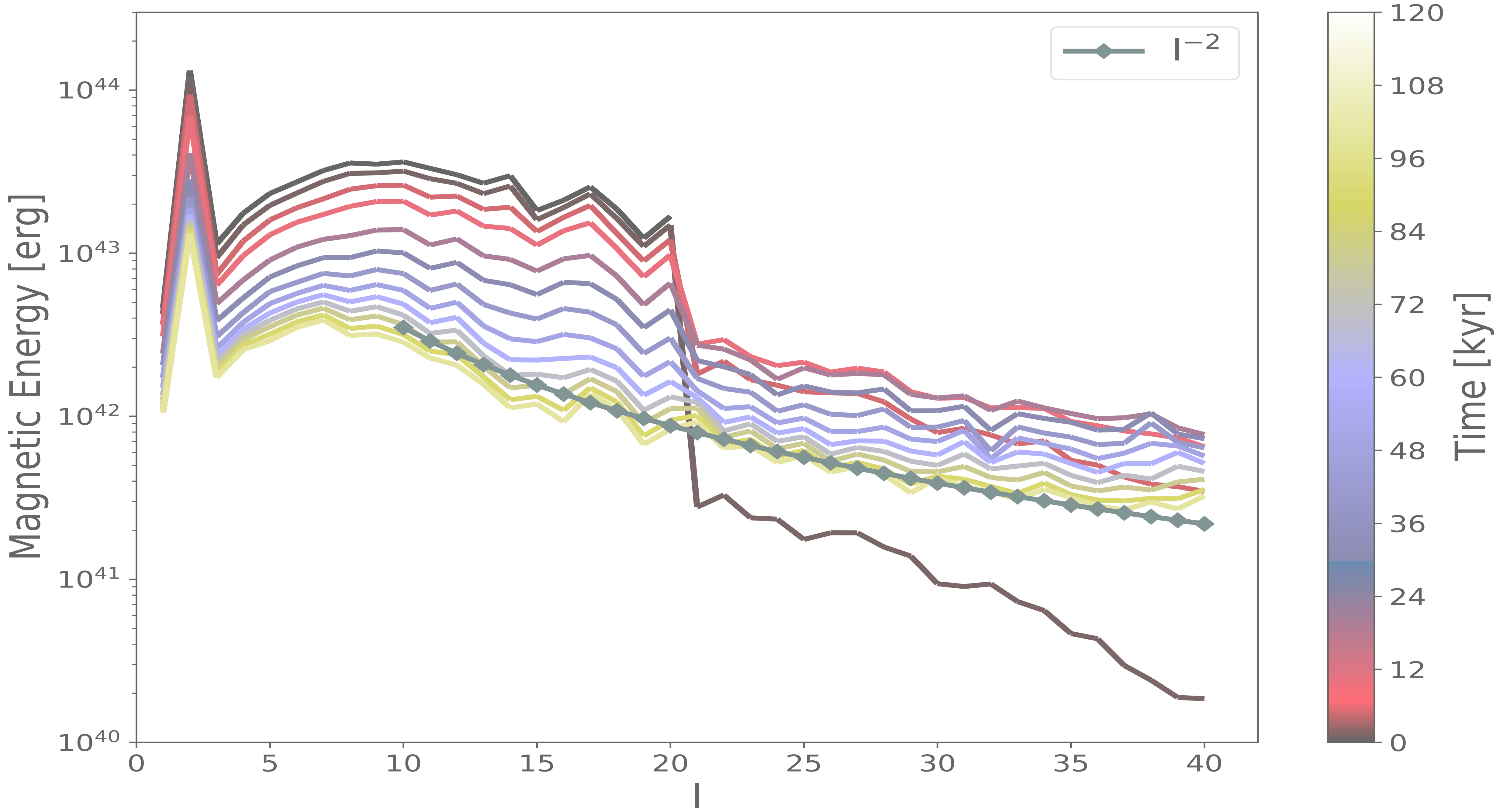

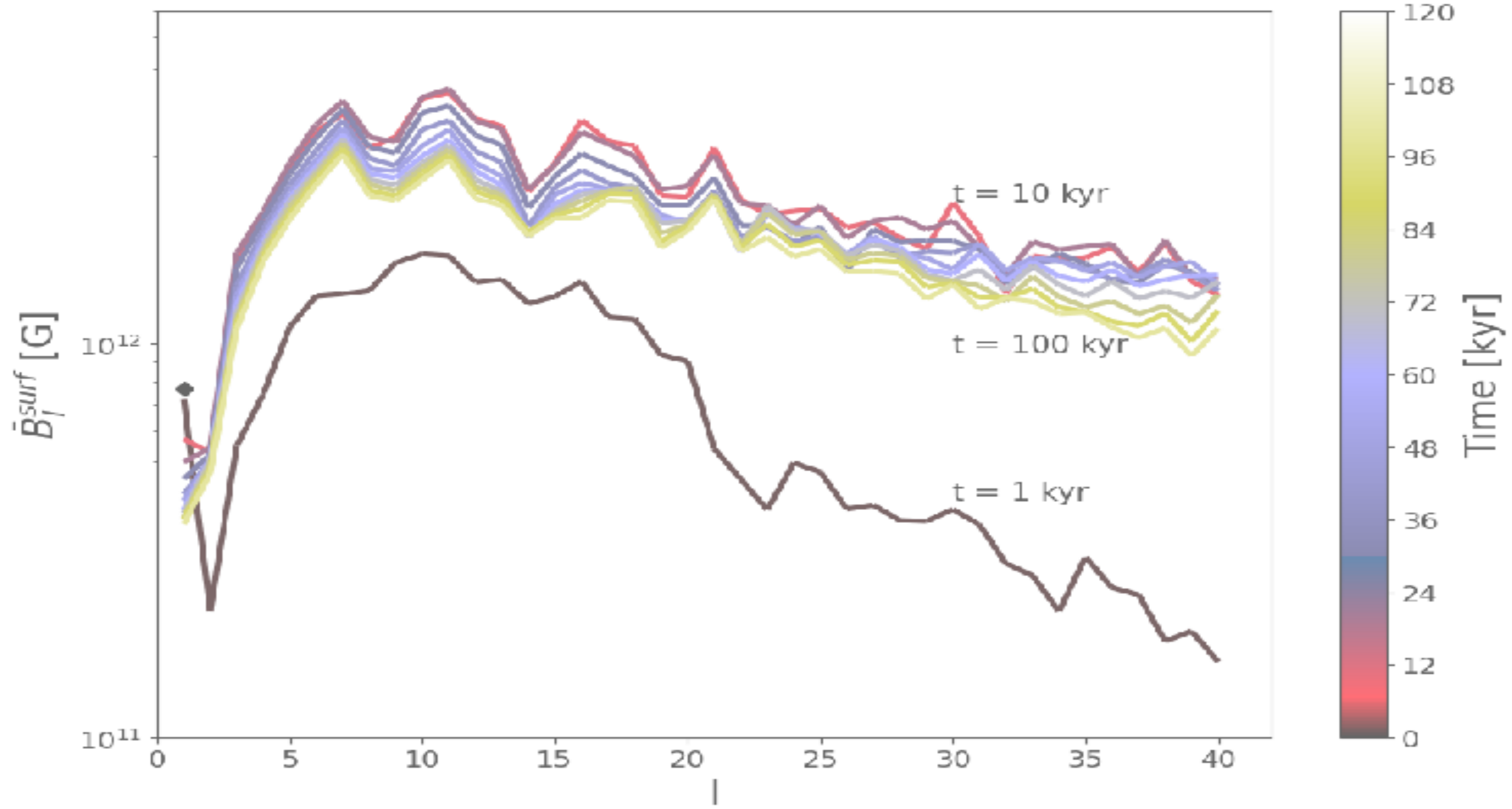

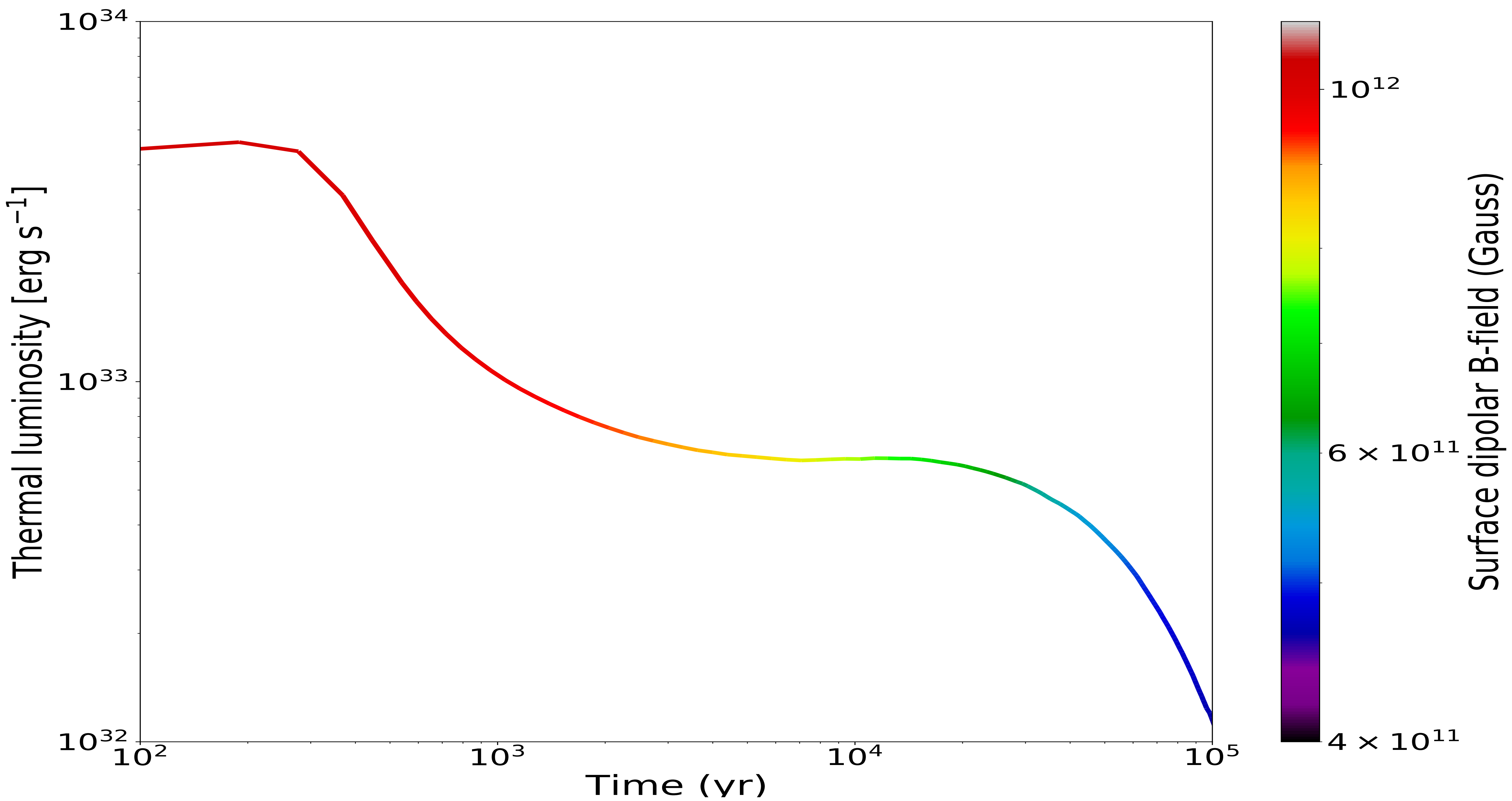

Small-scale field $\sim 10^{15} \ldots 10^{16}\,\mathrm{G}$
Dipolar field $\sim 10^{12} \ldots 10^{13}\,\mathrm{G}$

Thermal luminosity suitable to describe CCOs & low-field magnetars.

"A similar study by Igoshev (2024, unpublished yet) using the Taylor-Spruit initial field yields comparable results: strong small-scale structures and a weak dipolar field."

[Dehman et al. 2023b, MNRAS]

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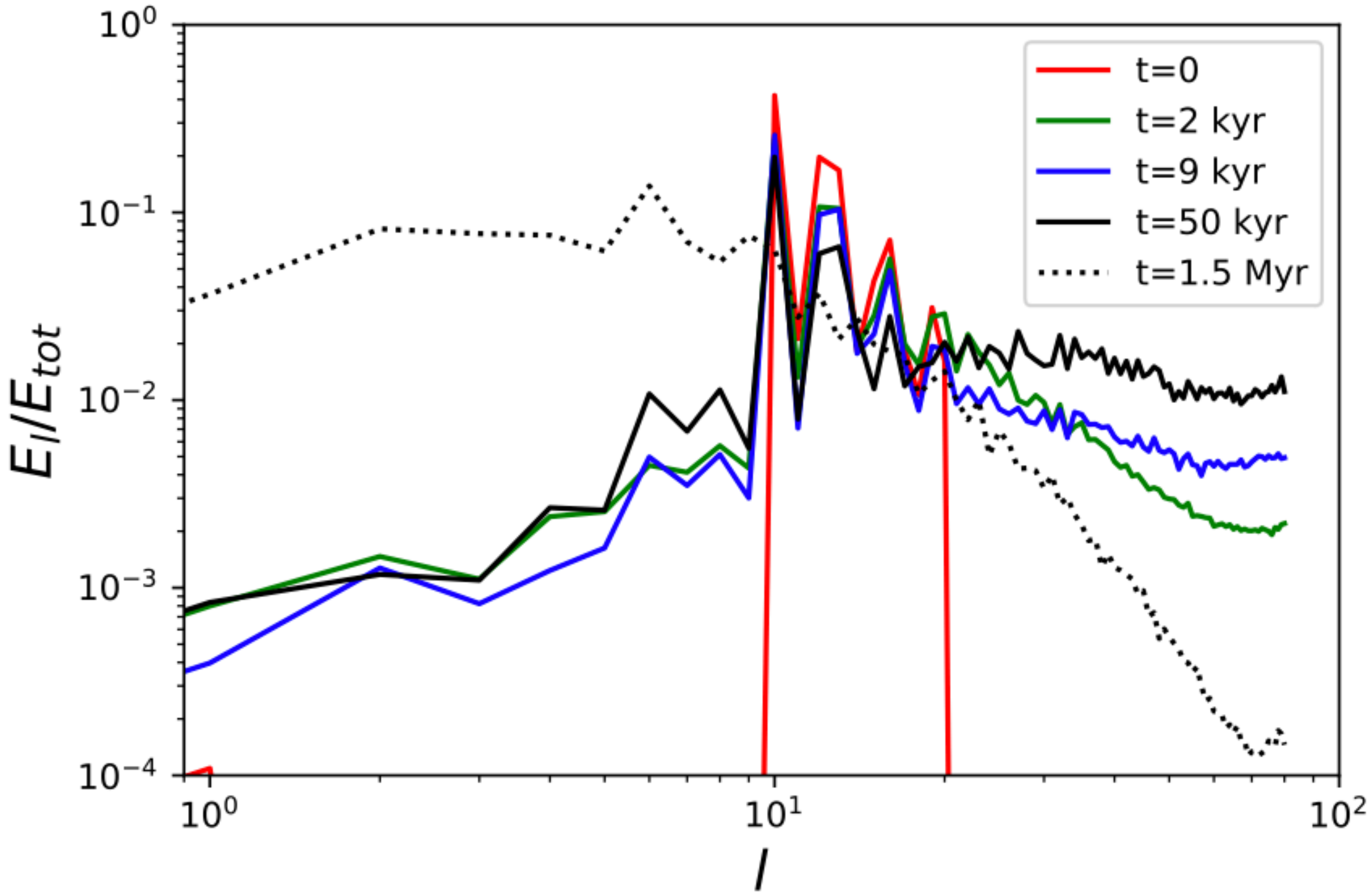
Inverse cascade in neutron star crusts

—a shift in the energy spectrum peak from smaller to larger scales—



- Initial magnetic in $\ell = 10 \ldots 20$, with minimal or zero contribution of other multipoles.
- Mode coupling, driven by the non-uniformity of coefficients and Hall nonlinearity, redistributed energy to neighbouring multipoles, resulting in a shallow energy spectrum dominated by the initial scales.
- No true sign of inverse cascade.
- Late time: smaller-scale structures dissipated as Hall drift became less efficient, shaping their final energy spectra.

[Gourgouliatos et al. 2020, MNRAS]

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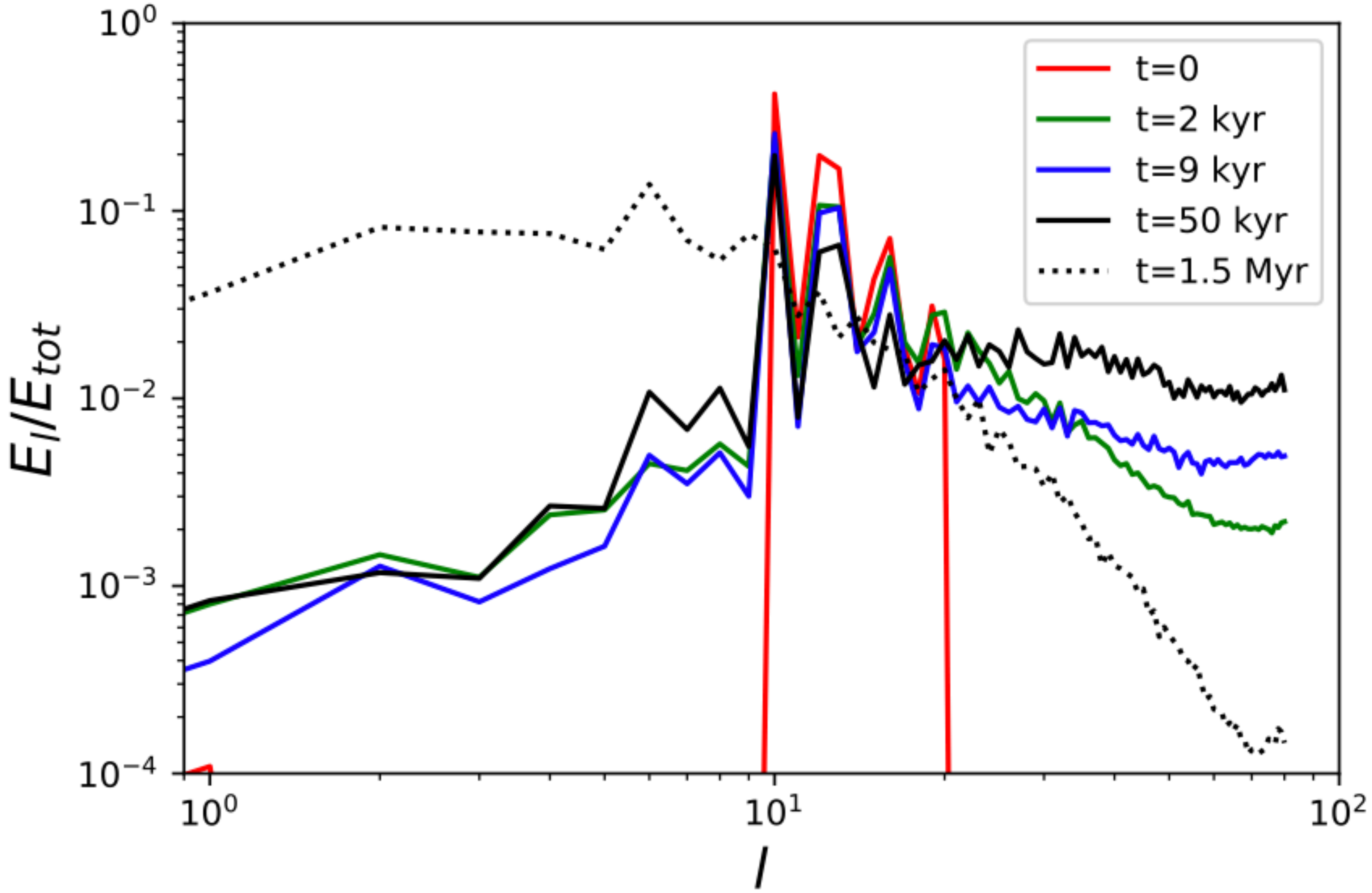

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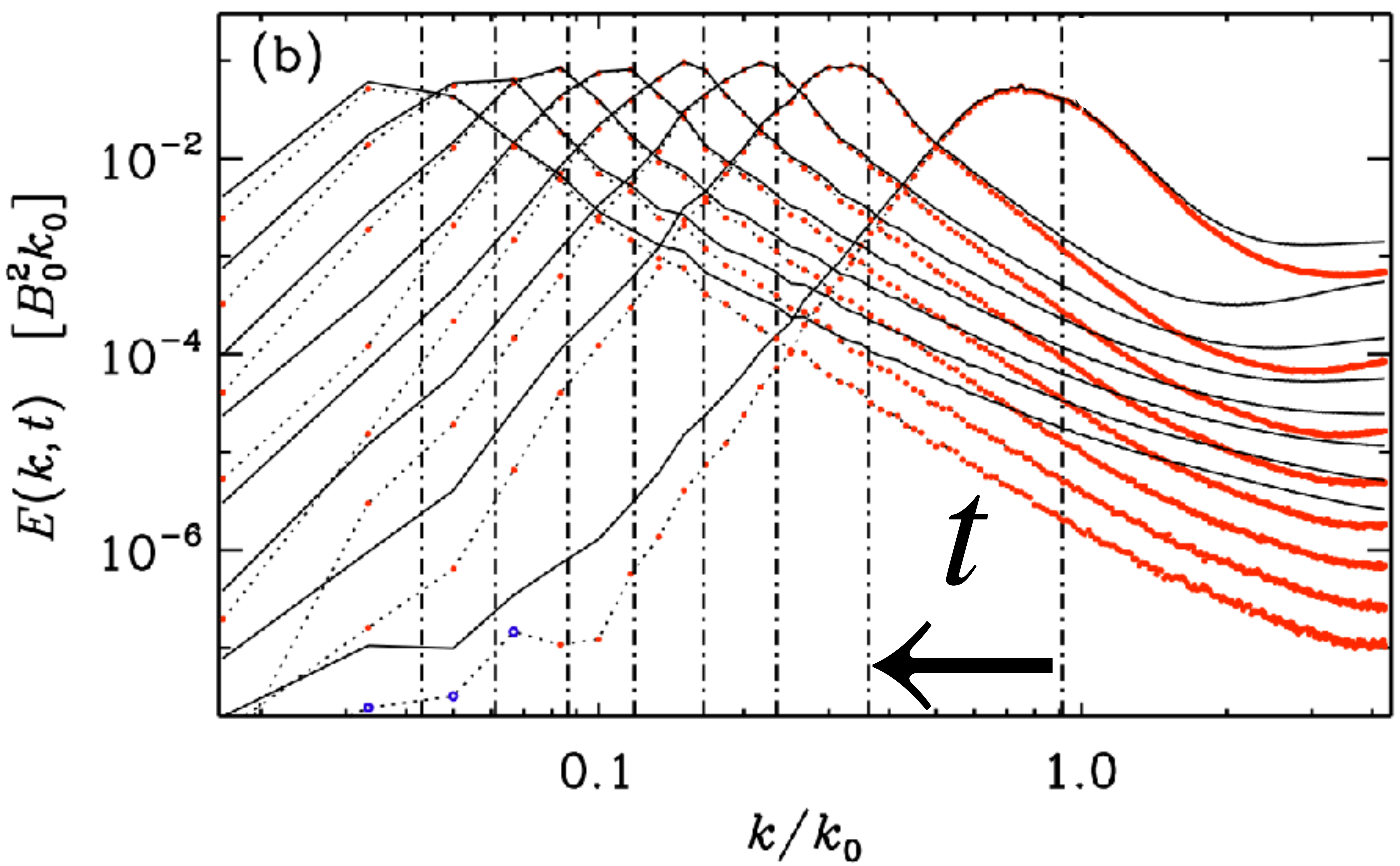
[Gourgouliatos et al. 2020, MNRAS]

A scenario for generating a large-scale dipolar field is through an inverse cascade, starting with an initial helical magnetic field.

$k\,|H_M(k)|/2E_M(k) \leq 1$



Local Hall-MHD simulations in a cubic Cartesian domain and period boundary conditions.

[Brandenburg 2020, ApJ]

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Reality of inverse cascade in neutron star crusts

Initial field:

Helical magnetic field. $k\,|H_M(k)|/2E_M(k) \leq 1$

Random initial field peaking at $l_0 \sim 100$.

Causal spectrum as used in the cosmological context.

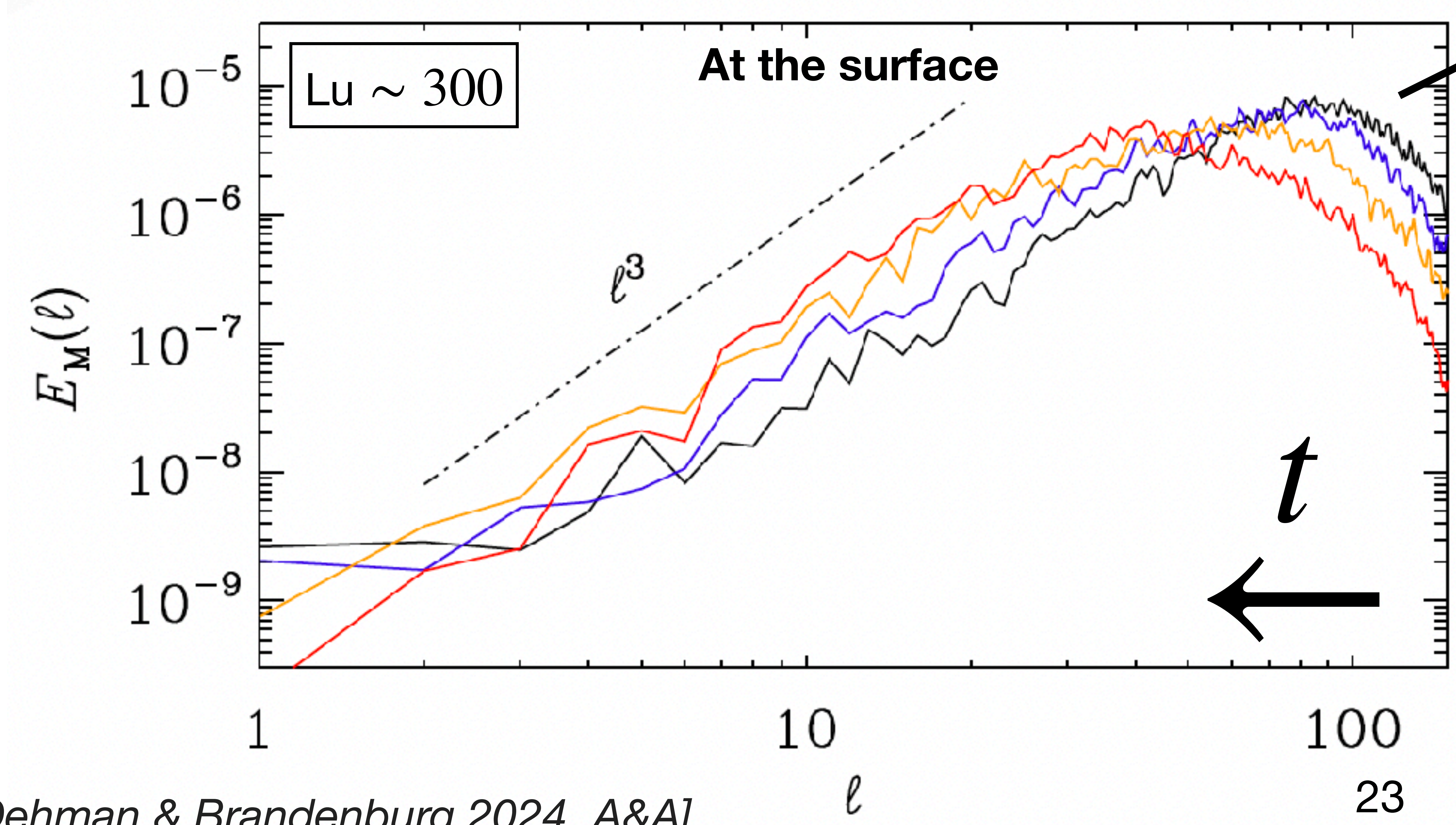
Correct aspect ratio of the NS crust.

Inverse Cascade occurs!

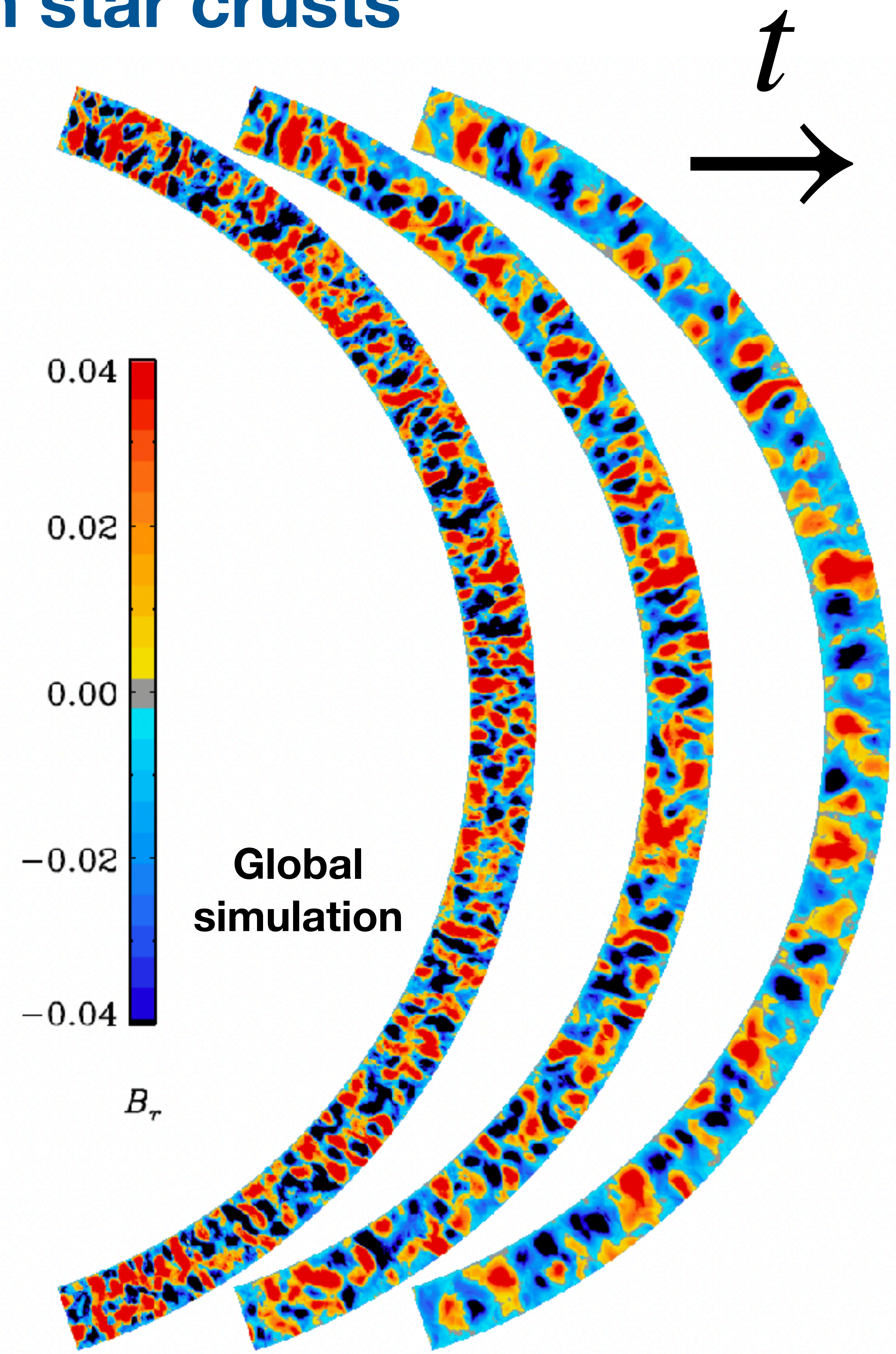
Energy transferred from small to large-scale multipoles.

Not observed in previous neutron star simulation studies.

Extreme aspect ratio (1:30)—thin crust— limits the inverse cascade.



$t = 0$



Pencil Code

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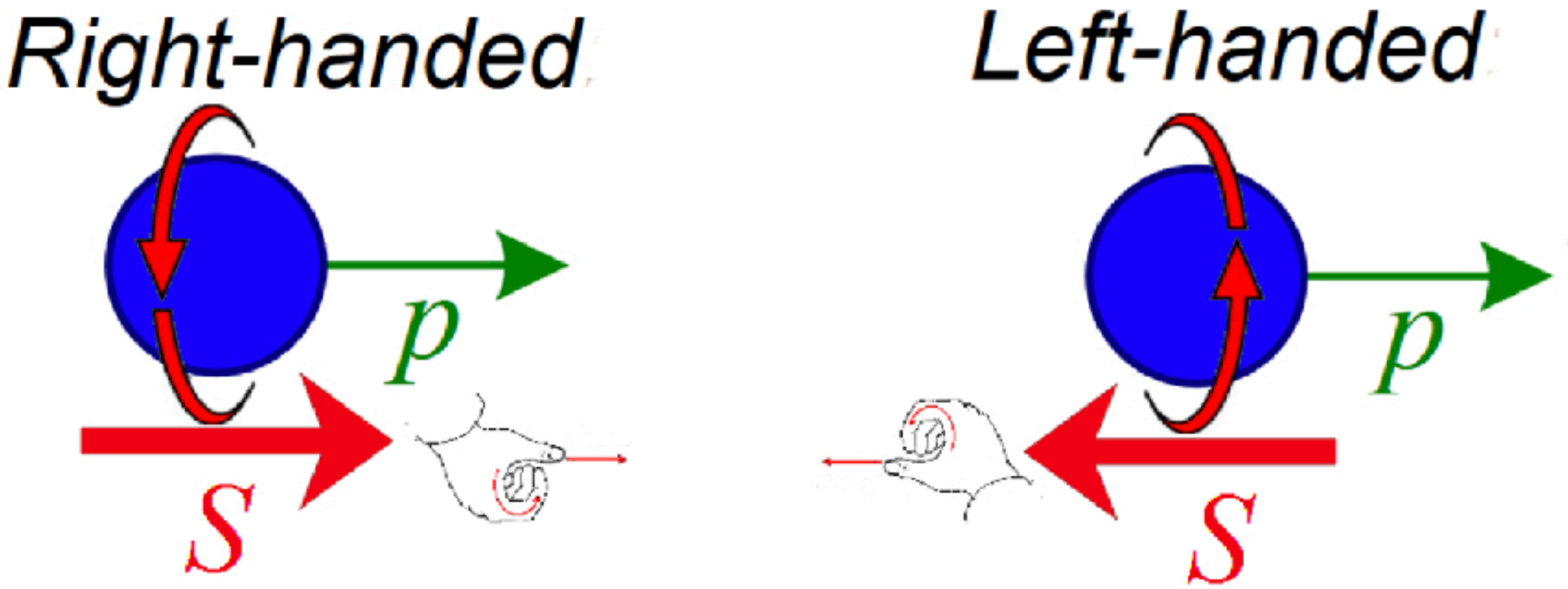
[Dehman & Brandenburg 2024, A&A]

Microphysics-based dynamo mechanism

— Chiral anomaly —



A particle is moving with momentum p represented by the green arrow.

The chirality of a particle is the projection of the spin along the direction of movement: it is right-handed if it is parallel to the movement; left-handed otherwise.

Plasma of charged particles with spin 1/2, e.g., neutron star interior:

$e_L + A \rightarrow A' + \nu_L$ ⟶ **Loosing a left handed electron from the system** (Γ_w)

$A \rightarrow A' + e_L + \bar{\nu}_R$ ⟶ **Gaining a left handed electron from the system** $(\Gamma_w^{\rm inv})$

$$\Gamma_w^{\rm eff} = \Gamma_w\left(1 - \exp(-\delta\mu/k_b T)\right)$$

β-equilibrium:

The two reactions happen at the same rate.

$$\delta\mu = \mu_p + \mu_e - \mu_n = 0, \quad \Gamma_w^{\rm eff} = 0$$

Deviation from β-equilibrium :

Chiral chemical potential $\mu_5 = \dfrac{\mu_R - \mu_L}{2} > 0$

$$\Gamma_w^{\rm eff} \neq 0$$

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Modified magnetic evolution equation in the crust

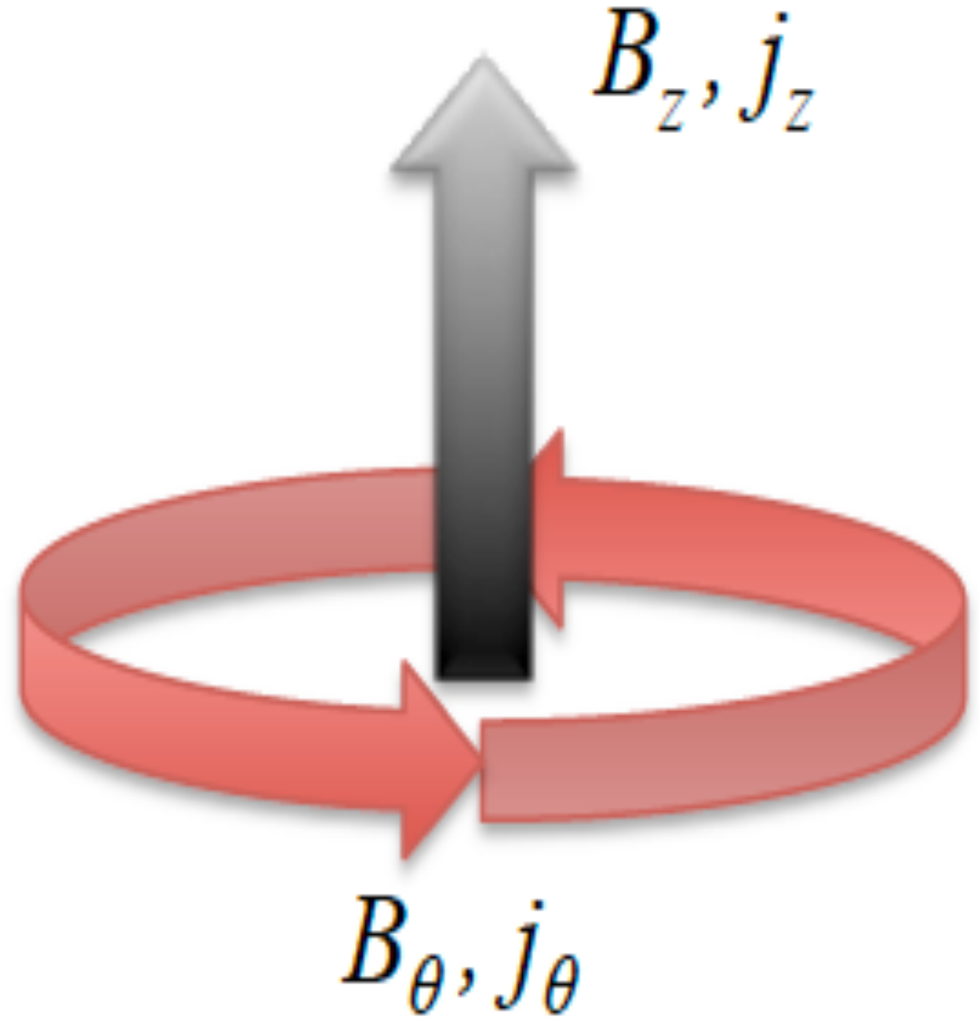

Image credit : Ohnishi

Non-vanishing chiral chemical potential: $\mu_5 = \frac{\mu_R - \mu_L}{2}$

$$\boldsymbol{J}_5 = \frac{2\alpha\mu_5}{\pi\hbar}\boldsymbol{B} \qquad \left(\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}\right)$$

Chiral electric current density in the direction of the magnetic field

Total electric current density:

$$\boldsymbol{J}_{tot} = \frac{c}{4\pi}(\nabla \times \boldsymbol{B}) - \boldsymbol{J}_5$$

Magnetic field evolution (described by Faraday law):

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E} \longrightarrow \frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \underbrace{\left\{\eta\nabla \times \boldsymbol{B} - 2\eta k_5 \boldsymbol{B} + f_h(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}\right\}}_{c\boldsymbol{E}} \qquad \left(k_5 = \frac{4\alpha\mu_5}{\hbar c}\right)$$

Chiral number density evolution $n_5 \approx \mu_e^2\mu_5/\pi^2(\hbar c)^3$ **:**

$$\frac{\partial n_5}{\partial t} = \frac{2\alpha}{\pi\hbar}\boldsymbol{E}\cdot\boldsymbol{B} + n_e\Gamma_w^{\mathrm{eff}} - n_5\Gamma_f \xrightarrow{\Gamma_f t \gg 1}$$

Quasi-steady state:

$$n_5(t) = \frac{2\alpha}{\pi\hbar}\frac{\boldsymbol{E}\cdot\boldsymbol{B}}{\Gamma_f} + n_e\frac{\Gamma_w^{\mathrm{eff}}}{\Gamma_f}$$

Γ_w^{eff} ***is the rate of depletion of the electron fraction due to electron capture via weak interaction.***

$\boldsymbol{E}\cdot\boldsymbol{B}$ ***is the anomalous depletion rate of*** n_5 ***due to the conversion of*** n_5 ***into magnetic field due to CMI.***

Γ_f ***is the flip term (electromagnetic reactions): electron scattering on nuclei*** $(e_R + A \rightarrow e_L + A)$

[A. Vilenkin 1980]

[Rogachevskii et al. 2017]

[Dehman & Pons 2024, under revision]

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Rates in the crust: electron capture and flip term

$$\mu_5 = \frac{\hbar c}{4\alpha} k_5 = \frac{\pi \hbar c}{2\alpha\lambda_5}$$

Rate of electron capture on nuclei: *[Fuller et al. 1985]* *[Langanke et al. 2003]*

$$\Gamma_w = \frac{\ln(2)M}{K}\left(\frac{T}{m_e c^2}\right)^5 [F_4(\zeta) - 2\chi F_3(\zeta) + \chi^2 F_2(\zeta)]$$

Leading order: $F_4(\zeta) \propto \mu_e^5$

Γ_w **is nearly temperature independent.**

High electrons degeneracy $\mu_e \gg 1$ MeV —> the Fermi energy of the degenerate electrons surpasses the electron capture energy thresholds *[W. J. Ong et al. 2020].*

The **Q-value** dependence is relevant primarily at low densities ($\rho \lesssim 10^{11}$ g/cm^3) *[Langanke et al. 2003]. They found agreement to better than a factor 2 between these rates and the diagonalization shell model rates at stellar conditions* ($T \lesssim 0.8\,\mathrm{MeV}$)*, consistent with NS temperatures.*

Effective net weak interaction rate:

$$\Gamma_w^{\mathrm{eff}} = \Gamma_e(1 - \exp(-\delta\mu/k_b T))$$

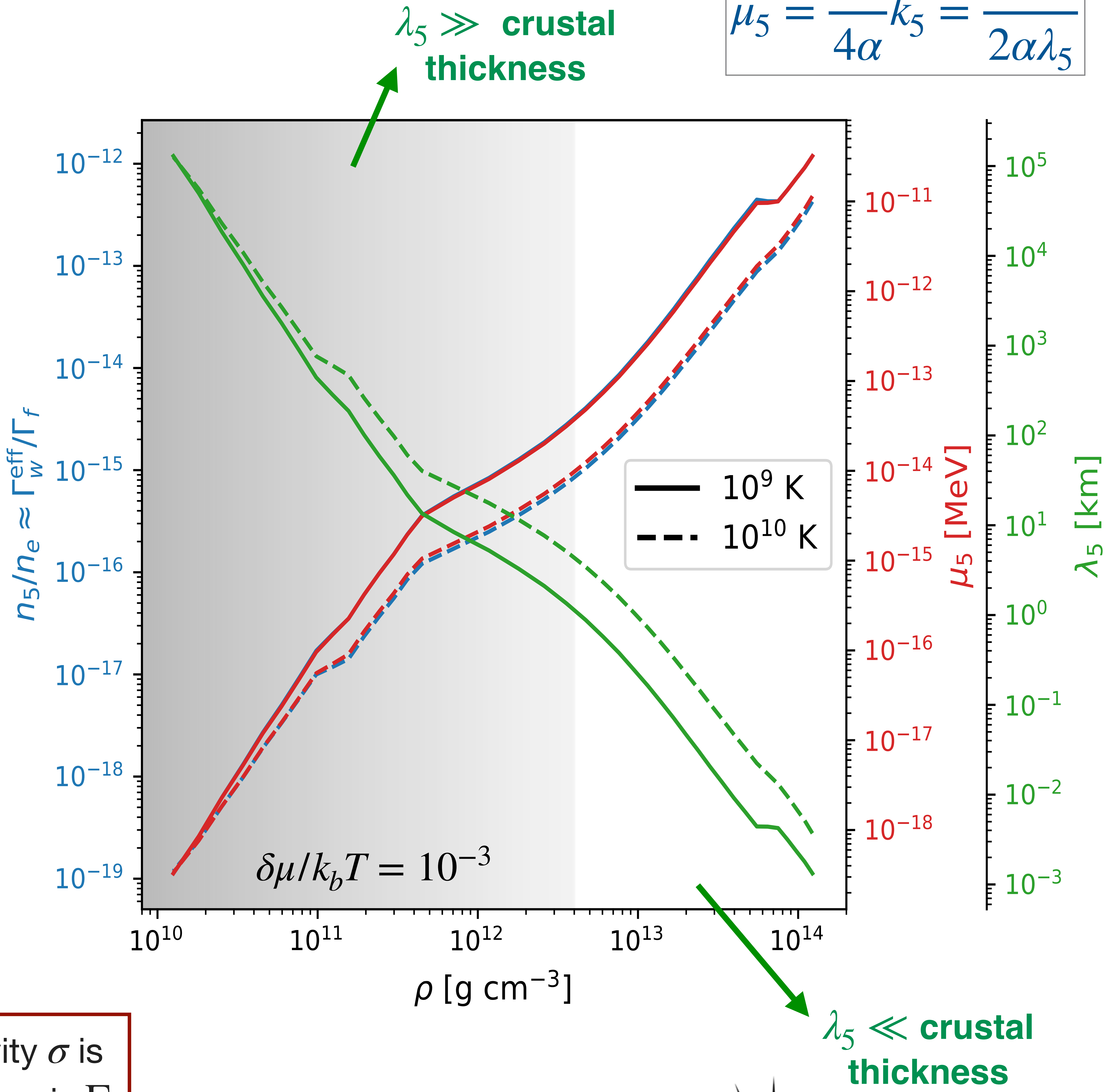
Flip rate (Rutherford scattering for degenerate electrons):

$(e_R\ +\ A \rightarrow e_L + A)$

$$\Gamma_f = \left(\frac{m_e}{\mu_e}\right)^2 \nu_{\mathrm{coll}}, \quad \nu_{\mathrm{coll}} = w_p^2/\sigma$$

$$w_p = \mu_e\sqrt{4\alpha/(3\pi)}$$

The electrical conductivity σ is temperature dependent, so is Γ_f



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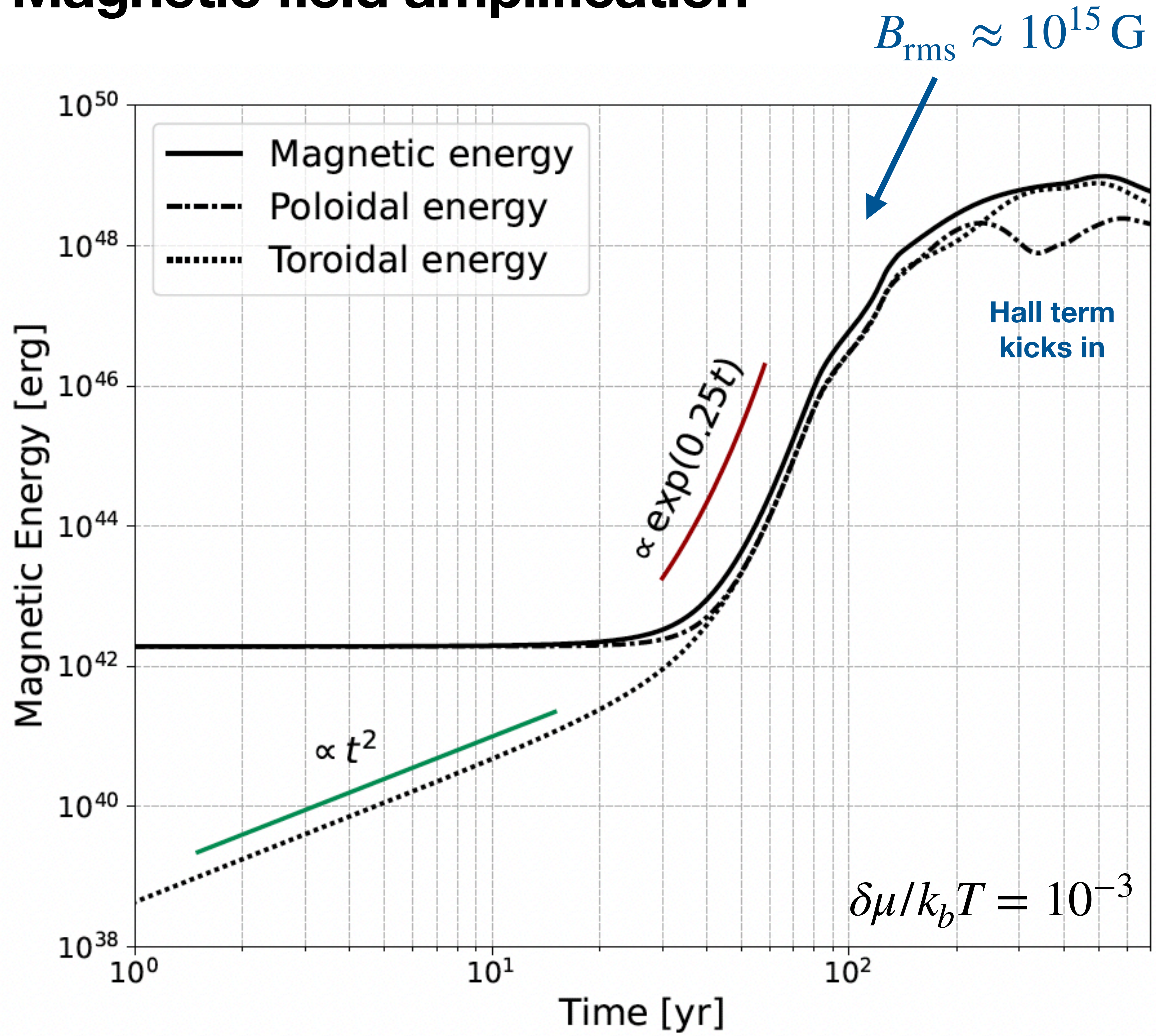
Chiral Magnetic Effect: Magnetic field amplification

Initial pure dipolar poloidal field of a few $10^{12}\,\mathrm{G}$

First Stage:

- Seed Poloidal field remains almost constant.
- Linear growth of the toroidal field.
- Due to magnetic helicity generation.



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[Rogachevskii et al. 2017]

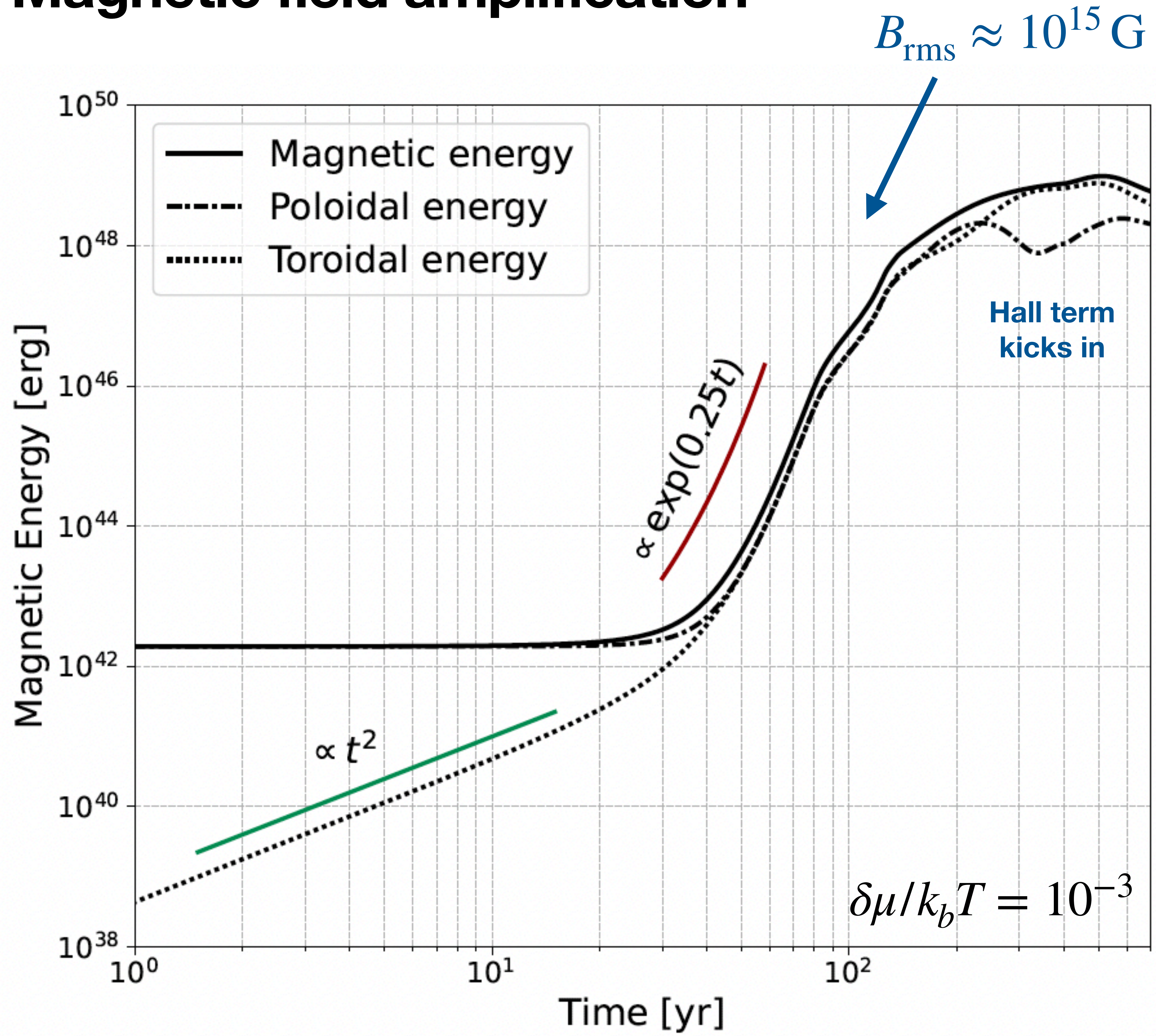
Axial charge: $Q_5 = \int d^3\boldsymbol{x}\, n_5$

Integrating of space, the variation of the axial charge Q5 over time is:

$$\frac{dQ_5}{dt} = \frac{\alpha}{\pi\hbar}\int d^3\boldsymbol{x}\,\boldsymbol{E}\cdot\boldsymbol{B} = -\frac{\alpha}{\pi\hbar c}\frac{d}{dt}\int d^3\boldsymbol{x}\,\boldsymbol{A}\cdot\boldsymbol{B}$$

Total helicity is conserved:

$$\frac{d}{dt}\left(Q_5 + \frac{\alpha}{\pi\hbar c}\chi_m\right) = 0$$



> ***"Changes in magnetic helicity can create or destroy chirality in the fermion state of the system. Vice versa, a change in the occupation of right-chiral or left-chiral fermions leads to the generation or decay of magnetic helicity"***

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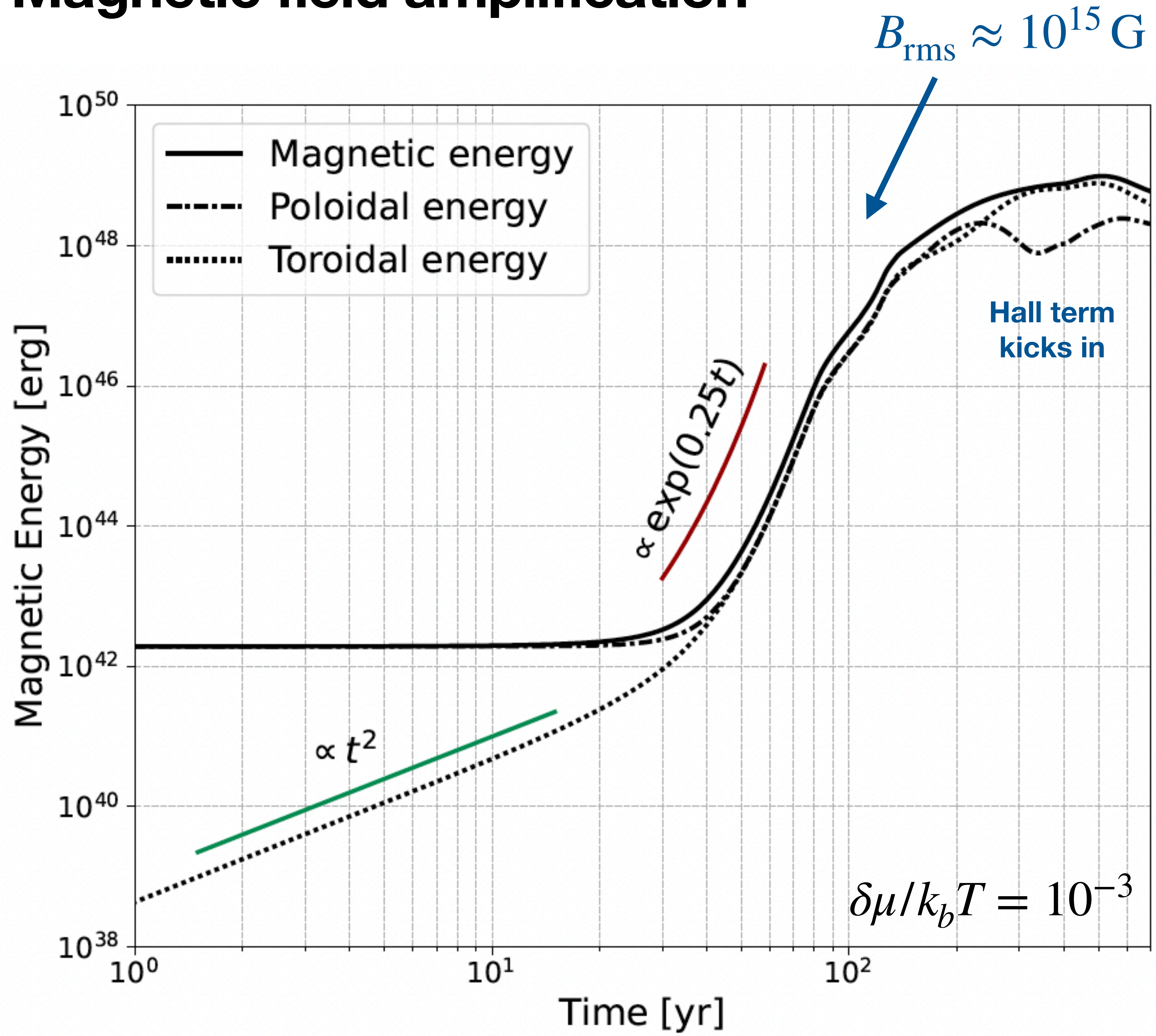
Second Stage:

- Poloidal and toroidal fields become comparable.
- CMI induces exponential growth of both components.
- Equipartition of energy between poloidal and toroidal.
- Magnetic energy grows as $\exp(0.25\,t)$.
- $B_{\rm rms} \approx 10^{15}\,\mathrm{G}$ by the end of this stage (~100 yrs).



$\delta\mu/k_bT$ larger —> faster field amplification

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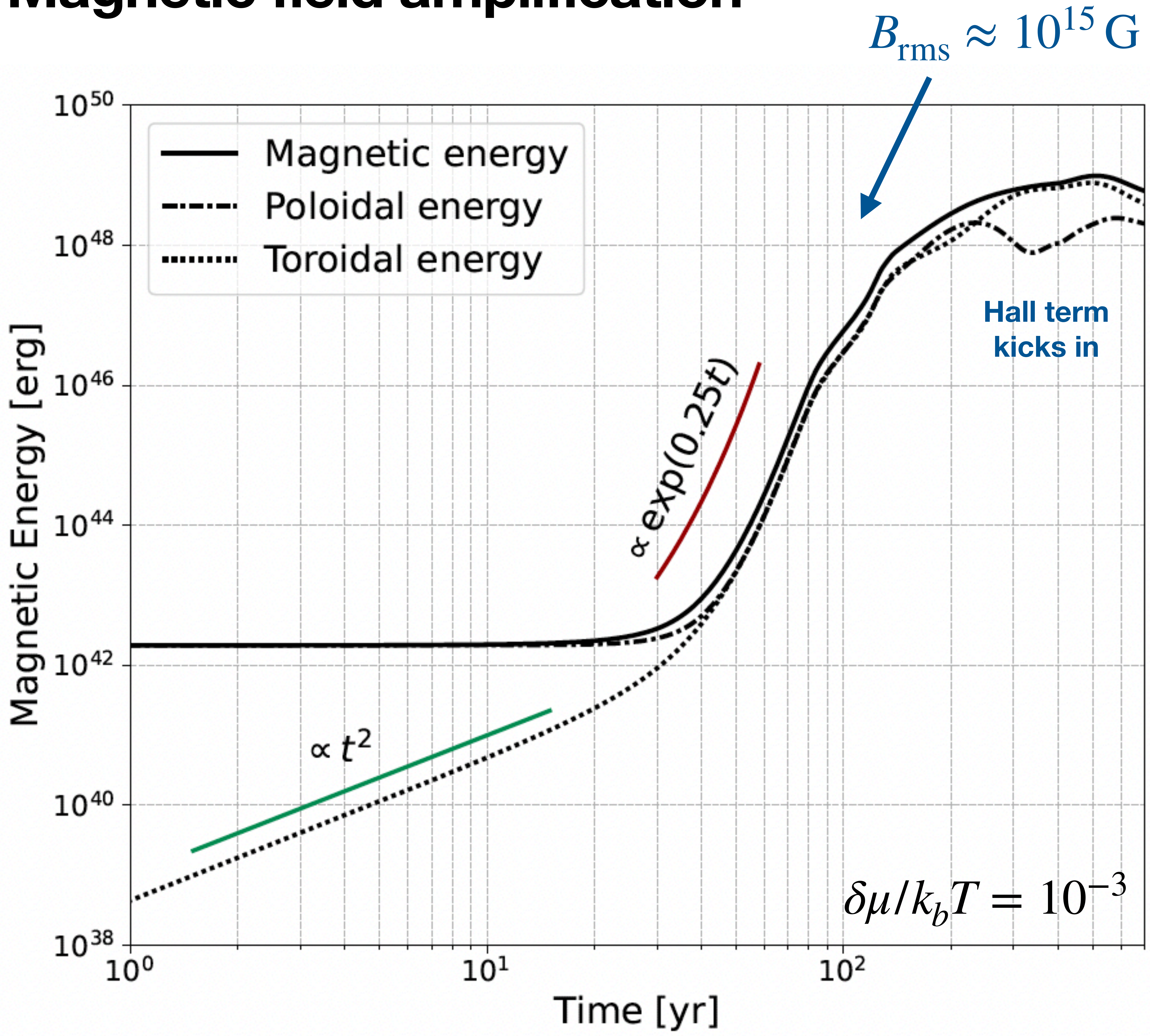
Third Stage:

- The Hall non-linear term is now important.
- $\boldsymbol{B}$ increases, $\boldsymbol{E}\cdot\boldsymbol{B}$ increases, n_5 decreases.
- Saturation and change of exponential growth due to the combined Hall and CMI effects.
- Energy transfer between poloidal and toroidal fields.
- Saturation of at $B_{\mathrm{rms}} \approx 10^{16}\,\mathrm{G}$ after about 200 yrs.

[Dehman & Pons 2024, under revision.]



$\delta\mu/k_bT$ larger —> faster field amplification

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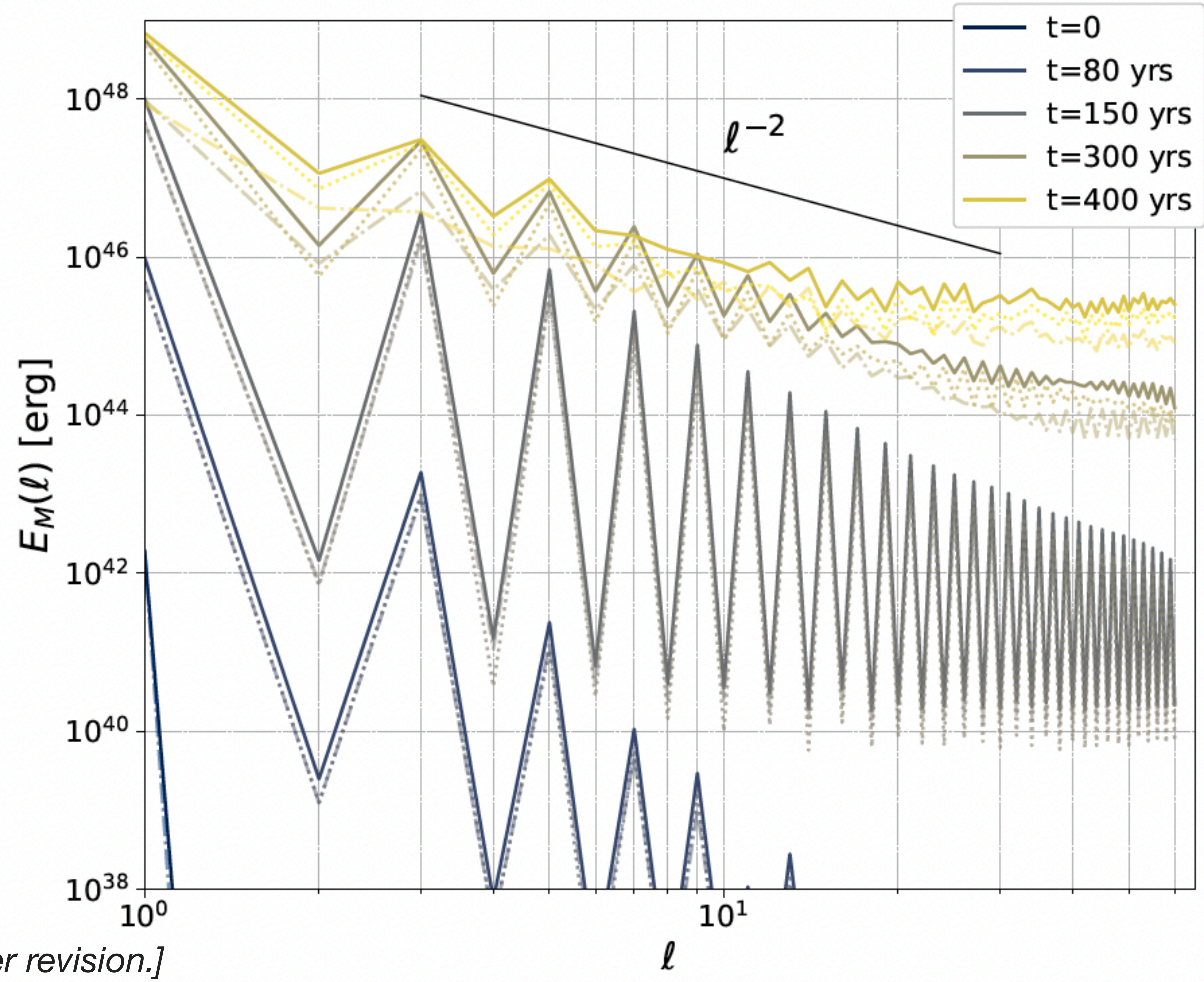
Mode growth and energy spectrum

$$\frac{\partial \Phi_{\ell m}}{\partial t} = \eta\left(\frac{1}{r}\frac{\partial^2(r\Phi_{\ell m})}{\partial r^2} - \frac{\ell(\ell+1)}{r^2}\Phi_{\ell m}\right) + D_{\ell m} + \boxed{2\eta k_5 \Psi_{\ell m}}$$

$$\frac{\partial \Psi_{\ell m}}{\partial t} = \eta\left(\frac{1}{r}\frac{\partial^2(r\Psi_{\ell m})}{\partial r^2} - \frac{\eta}{r}\frac{\partial \sigma}{\partial r}\frac{\partial(r\Psi_{\ell m})}{\partial r} - \frac{\ell(\ell+1)}{r^2}\Psi_{\ell m}\right) + C_{\ell m} \boxed{- 2\eta k_5\left(\frac{1}{r}\frac{\partial^2(r\Phi_{\ell m})}{\partial r^2} - \frac{\ell(\ell+1)}{r^2}\Phi_{\ell m}\right)}$$

Following Geppert & Wiebickie 1991 formalism



[Dehman & Pons 2024, under revision.]

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Summary & Conclusions

MATINS a new 3D code for magneto-thermal evolution in isolated neutron star crust.

"To be public soon"

Long-term evolution (10^6 yr) with a strong magnetic field $\sim 10^{14} \ldots 10^{15}\,\mathrm{G}$.

Proper treatments of microphysics, envelope models, axial singularity, field topology, temperature, etc.

Hall cascade, Inverse Hall cascade, outburst, etc.

MATINS limitations: No core field evolution and Vacuum magnetosphere.

Origin of magnetic field in magnetars:

- Dynamo simulations in core-collapse simulations and proto-neutron stars.
- Inverse cascade in neutron star crust.
- Chiral anomaly in neutron star crust.

A lot more can be explored !!

Questions?

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