Deconfinement Transition at high isospin chemical potential

Srimoyee Sen,
Iowa State University

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Outline

QCD phase diagram.

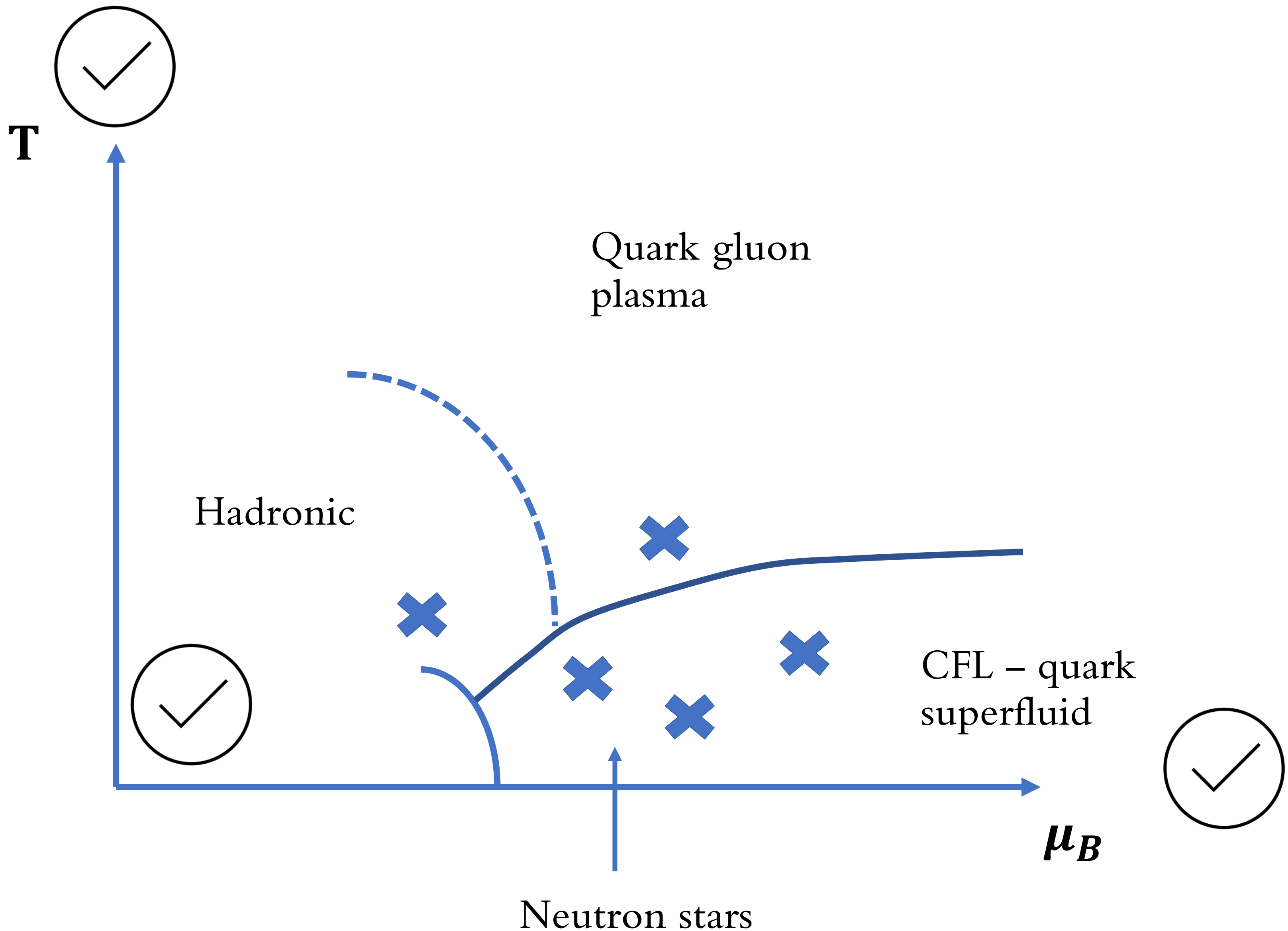
Why we should be interested in QCD with finite isospin chemical potential μ_I, what it can teach us about neutron star EOS

What we already know about this regime

Low energy effective theory

First order deconfinement phase transition

QCD phase diagram: the success so far



Finite Isospin regime: $\mu_I \neq 0, \mu_B = 0$

Advantages

- Learn all we can about QCD with chemical potential when finite μ_B regime is not accessible.
- Tractable using lattice → no sign problem.
- Nonzero μ_I present in neutron stars
- Recent work by Fujimoto & Reddy shows how finite isospin EOS can be used to constrain EOS at finite baryon density.

The limit considered in this talk

- QCD with two degenerate flavors of light quarks.

- Asymptotically high μ_I

- Set μ_B to zero.

Exciting features of this limit

- At low T, this limit of QCD is equivalent to $SU(3)$ Yang-Mills.

- Expected to undergo a first order deconfinement transition just like $SU(3)$ Yang-Mills with changing T.

- Scale of this deconfinement transition can be calculated using effective theory.

Exciting feature of this limit (contd.)

- The phase diagram at moderate isospin chemical potential is likely to have either a critical point or a triple point.

- Only a lattice calculation can settle this as this regime is beyond the reach of perturbative calculations.

Low Isospin

Lagrangian at low isospin with matrix pion fields:

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4}\text{Tr}\nabla_\nu\Sigma\nabla_\nu\Sigma^\dagger - \frac{m_\pi^2 f_\pi^2}{2}\text{ReTr}\Sigma.$$

where,

$$\nabla_i\Sigma = \partial_i\Sigma$$

$$\nabla_0\Sigma = \partial_0\Sigma - \frac{\mu_I}{2}(\tau_3\Sigma - \Sigma\tau_3)$$

For $\mu_I > m_\pi$, there is a charged pion condensate (Son, Stephanov 2001).

High isospin

- Fermi liquid of anti up quarks and down quarks.

- Attractive interaction at the Fermi surface leading to Cooper pair formation in the $\langle \bar{u}\gamma_5 d \rangle$ channel.

- The condensate has same quantum numbers as the pion condensate.

High isospin

- The condensate is color neutral → no Meissner screening for gluons.

- At temperatures below the gap no Debye screening either for the gluons.

- At low T the quarks are gapped → only pure gluodynamics $SU(3)$

High Isospin

- However, $L = \frac{F^2}{4g^2} + \cdots$ is not the complete picture.

- Despite being bound in color singlet Cooper pairs, the quarks can partially screen the gluons altering the chromo di-eletric constant ϵ of the system.

High Isospin

- ϵ and λ (chromoelectric permeability) can be calculated by integrating out the quarks around the Fermi surface.

- The deconfinement scale is related to ϵ

Derivation of the effective Lagrangian from the microscopic theory

Microscopic Lagrangian:

$$\mathscr{L} = \overline{\psi}(i\gamma^\mu D_\mu + \mu_I \gamma^0 \tau_3)\psi - \frac{1}{4}(F^a_{\mu\nu})^2$$

where $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ $\qquad D_\mu = \partial_\mu - igA^a_\mu t^a$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

Rewrite using $\tilde{\psi} \equiv \begin{pmatrix} u \\ \tilde{d} \end{pmatrix}$ $\qquad \tilde{d} = \gamma_5 d$

Find the full Fermion propagator using the gap equation

$$G = \begin{pmatrix} G^+ & \Sigma^- \\ \Sigma^+ & G^- \end{pmatrix}$$

where

$$G^-(k) = \sum_{e=\pm} \frac{-ik_0 + (\mu - ek)}{-k_0^2 - (\epsilon_k^e)^2} \lambda_{\mathbf{k}}^{-e} \gamma^0$$

$$G^+(k) = \sum_{e=\pm} \frac{-ik_0 - (\mu - ek)}{-k_0^2 - (\epsilon_k^e)^2} \lambda_{\mathbf{k}}^{+e} \gamma^0$$

$$\Sigma^-(k) = i \sum_{e=\pm} \frac{\Delta^e \lambda_k^e}{-k_0^2 - (\epsilon_k^e)^2}$$

$$\Sigma^+(k) = -i \sum_{e=\pm} \frac{\Delta^e \lambda_k^{-e}}{-k_0^2 - (\epsilon_k^e)^2}$$

$$(\epsilon_k^e)^2 = (|\mathbf{k}| - e\mu)^2 + (\Delta^e)^2, \qquad e = \pm, \qquad \lambda_{\mathbf{p}}^{\pm} \equiv \frac{1}{2}\left(1 \pm \gamma_0 \boldsymbol{\gamma} \cdot \hat{\boldsymbol{p}}\right)$$

Low energy effective action

The effective action:

$$S_{glue} = \sum_a \frac{1}{g^2} \int d^4q \left(\frac{\epsilon}{2} E_a^2 - \frac{1}{2\lambda} B_a^2 \right)$$

where $E_i^a = F_{0i}^a \qquad B_k^a = \epsilon_{ijk} F_{ij}$

The polarization tensor can be written as

$$\Pi_{ab}^{00}(q_0, \mathbf{q}) = -(\epsilon - 1)|\mathbf{q}|^2 \delta_{ab}$$

$$\Pi_{ab}^{ij} \frac{\delta^{ij}}{2} \frac{\delta_{ab}}{8} = -(\epsilon - 1) q_0^2$$

$$\Pi_{ab}^{ij}(q_0, \mathbf{q}) \left(-\delta^{ik} - \frac{q^i q^k}{|\mathbf{q}|^2} \right) = \left(\frac{1}{\lambda} - 1 \right) |\mathbf{q}|^2 \delta_{ab} \delta^{jk}$$

One loop polarization can be calculated as

$$\Pi^{ij}_{ab}(q) = g^2 T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \mathrm{Trace}\left[\gamma^i t_a G^+(k)\gamma^j t_b G^+(k-q) \right.$$
$$+\gamma^i t_a G^-(k)\gamma^j t_b G^-(k-q)$$
$$+\gamma^i t_a \Sigma^-(k)\gamma^j t_b \Sigma^+(k-q)$$
$$\left. +\gamma^i t_a \Sigma^+(k)\gamma^j t_b \Sigma^-(k-q)\right]$$

In the limit $T \to 0$ and around q and $|\mathbf{q}| \to 0$

$$\Pi^{00}_{aa} = -\frac{g^2\mu_I^2|\mathbf{q}|^2}{18\pi^2(\Delta)^2} \qquad \Pi^{ij}_{ab} = \frac{g^2\mu_I^2 q_0^2}{18\pi^2(\Delta)^2}\delta^{ij}\delta_{ab}$$

leads to $\lambda = 1$ $\qquad \epsilon = 1 + \frac{g^2\mu_I^2}{18\pi^2(\Delta)^2}$

The effective action can be recast as

$$S = -\frac{1}{4(g'')^2}\int d^4x''(F'')^2$$

using $t_0'' = \frac{t_0}{\sqrt{\epsilon}}$ $A_0^{a''} = \sqrt{\epsilon}A_0^a$ $g'' = \frac{g}{\epsilon^{1/4}}$

The coupling g'' runs like that of pure YM as the energy scale reached Δ from below at which point the coupling needs to be matched as follows

$$\alpha_s''(\Delta) = \frac{\alpha_s(\mu_I)}{\sqrt{\epsilon}}$$

Confinement

The new confinement scale can be found as

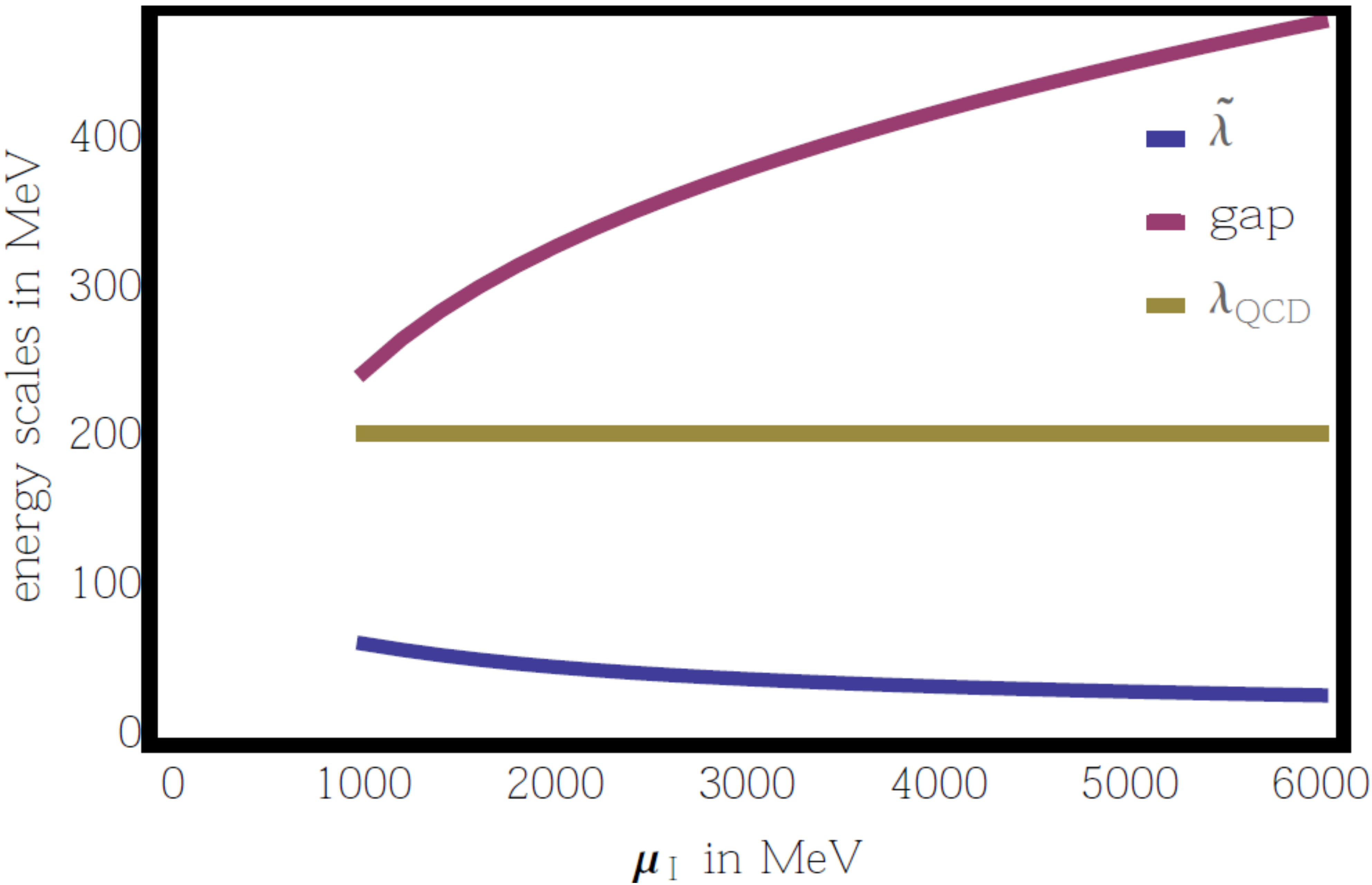
$$\tilde{\lambda} = \Delta \exp\left(-\frac{2\pi}{b_0 \alpha_s''(\Delta)}\right)$$
$$= \Delta \mathrm{Exp}\left[-2\sqrt{2\pi}\frac{\mu_I}{33\Delta\sqrt{\alpha_s(\mu_I)}}\right]$$

where for $SU(3)$ YM

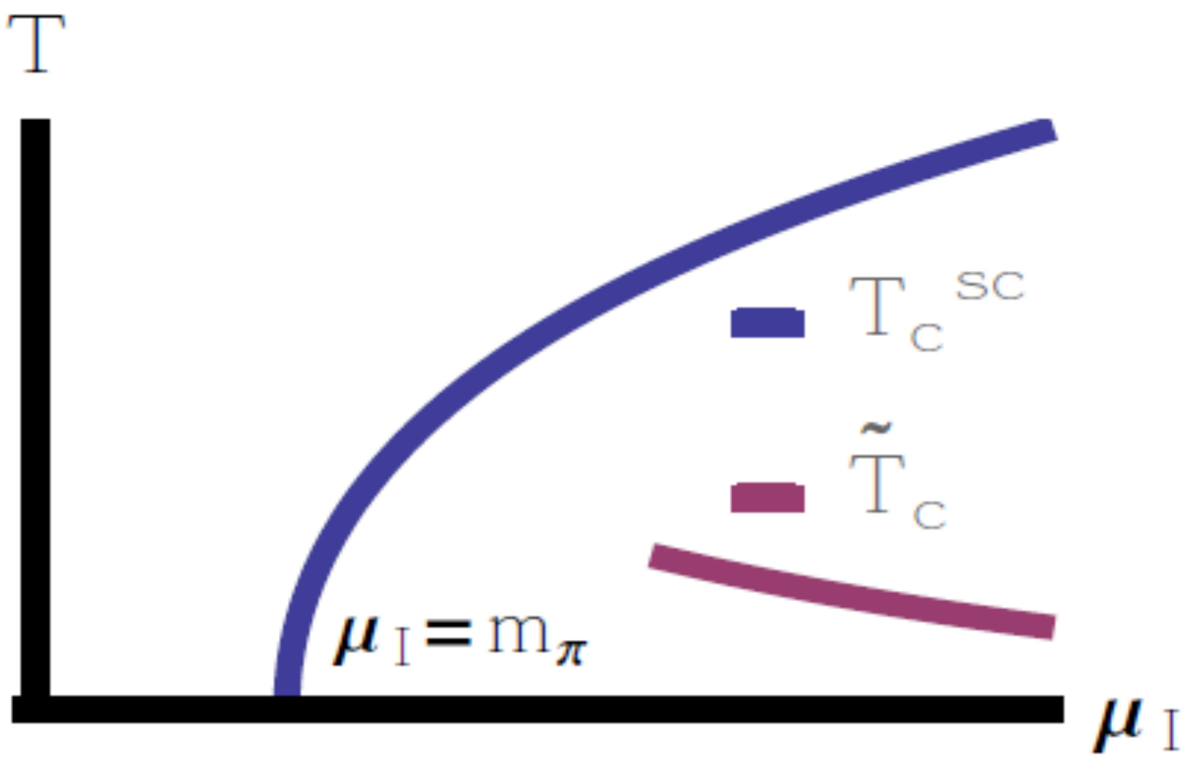
The gap is given by $\Delta = b|\mu_I| g(\mu_I)^{-5} \exp(-\frac{3\pi^2}{2g(\mu_I)})$

with $b = 10^4$

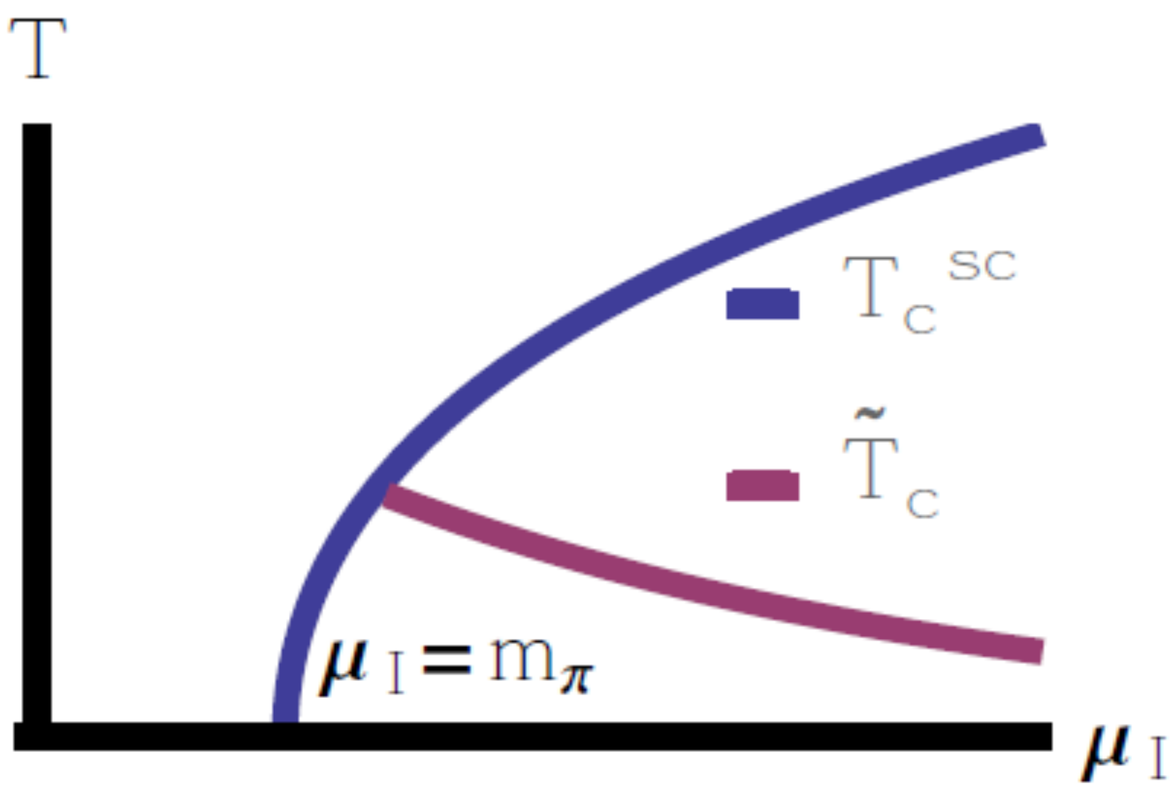
Different scales in the problem



Phase Diagram: Scenario 1



Phase diagram: Scenario 2



Summary/ future work

- QCD at very high isospin chemical potential undegoes a first order deconfinement transition with increasing temperature.
- We calculate the scale of this deconfinement transition.
- Does this transition exist in the $T-\mu_B$ phase diagram ($\mu_I = 0$)?
- If so, which neutron star observables can it affect?
- If existed in $T-\mu_B$ phase diagram ($\mu_I = 0$), there could be two critical points, one at low density, other at high density. How to test?
- Our prediction for this deconfinement scale as a function of μ_I should be tested using lattice.