Nucleon tomography in the threshold limit

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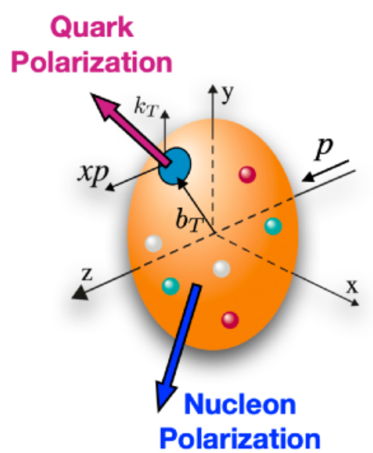
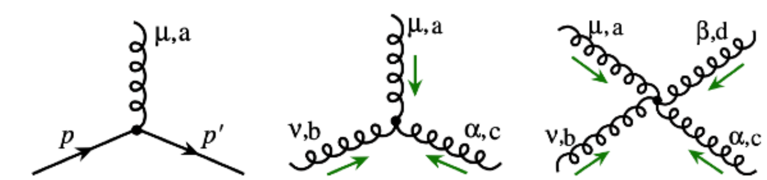
Introduction

Transverse momentum distributions (TMDs) encode the quantum correlations between hadron polarization and the motion and polarization of quarks and gluons inside it.



Imaging a hadron would provide insights on QCD

$$-\frac{1}{4}G^2_{\mu\nu,a}[A] + \sum_f \overline{\psi}_f \left(iD_\mu[A]\gamma^\mu - m_f\right)\psi_f$$



- **Both longitudinal and transverse motion**
- **Large Lorentz boost in longitudinal direction, but not in transverse momentum**
- **Correlation between hadron spin with parton(quark, gluon) orbital angular momentum**

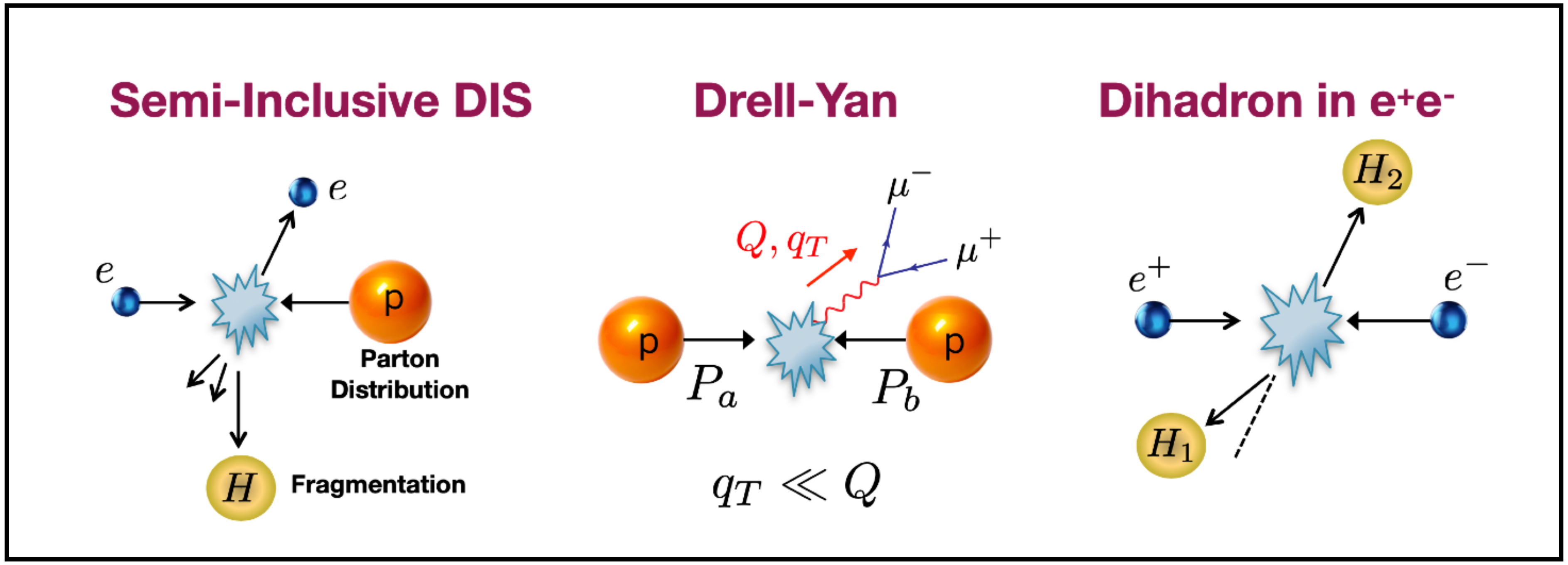
TMDs		Quark Polarization: Unpolarized (U)	Longitudinally polarized (L)	Transversely polarized (T)
Nucleon Polarization	U	f_1 unpolarized		$h_1^\perp$ Boer-Mulders
	L		g_{1L} helicity	$h_{1L}^\perp$ longi-transversity
	T	$f_{1T}^\perp$ Sivers	g_{1T} trans-helicity	h_1 transversity; $h_{1T}^\perp$ pretzelosity

Nucleon spin — Quark spin

Figure 2.5: The leading-twist quark TMD distributions.

Transverse momentum distributions of quarks

- **Three classical processes used to probe quark TMDs**



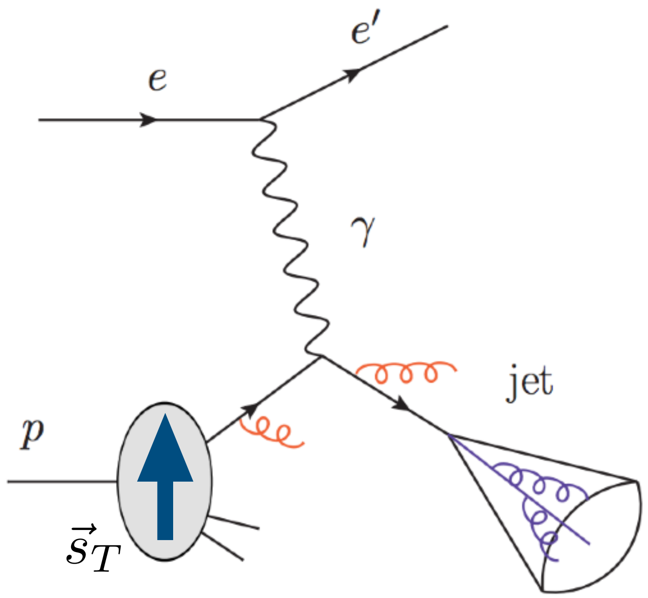
- **Typical "two-scale" problem:**

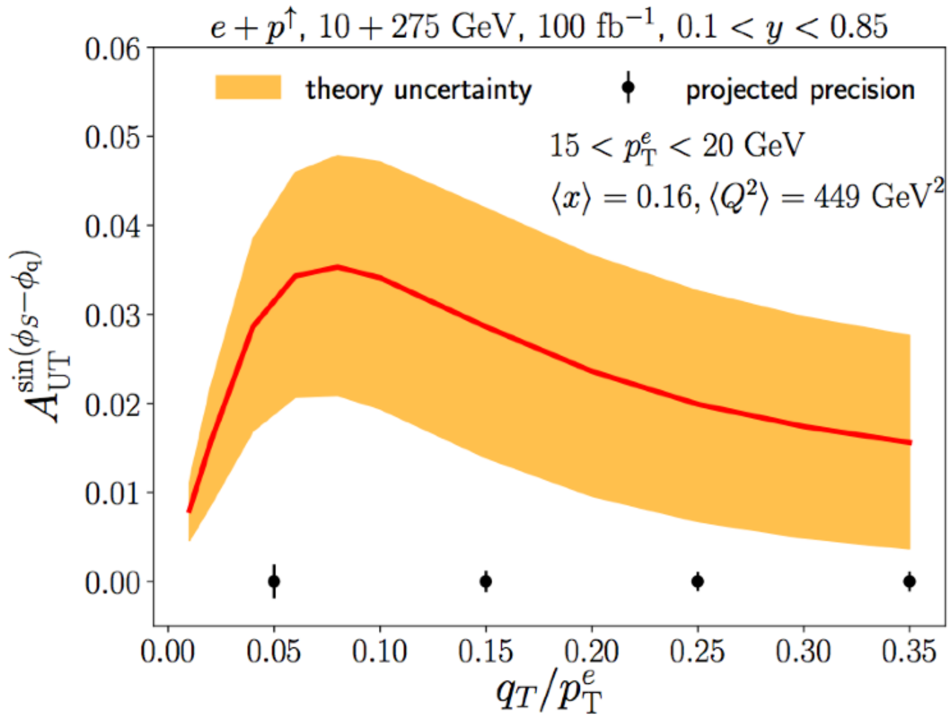
  transverse momentum of final particle (q_T) << scattering energy (Q)

- **Theory tools: TMD factorization theorem, effective field theory** Collins-Soper-Sterman, Ji-Ma-Yuan, Soft-Collinear Effective Theory ......

$$\frac{\mathrm{d}\sigma(ep \to ehX)}{\mathrm{d}Q\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\vec{q}_T} = H_{eq\to eq}(Q) F_q(\vec{q}_T, x) \otimes D_{q\to h}(\vec{q}_T, z)$$

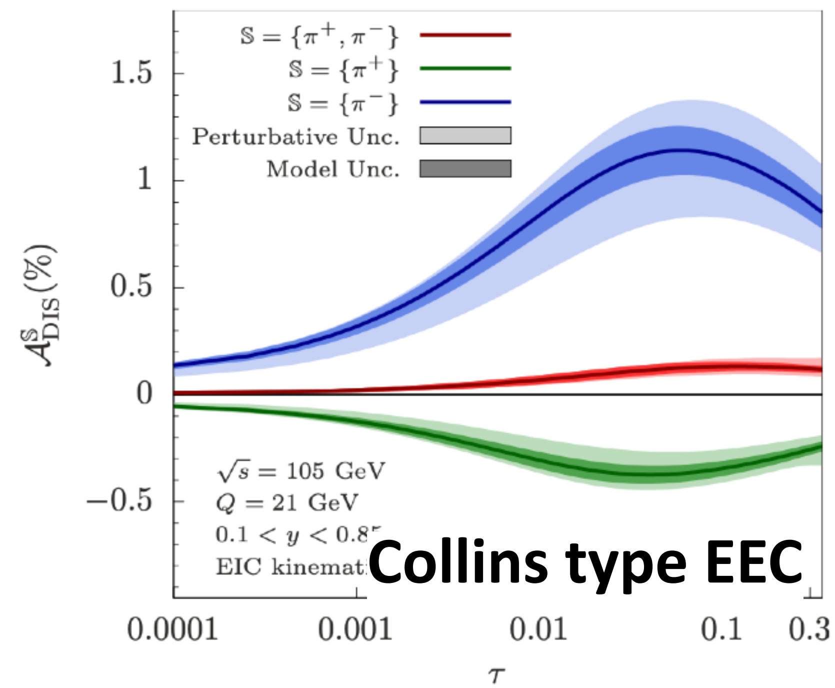
Jets and 3D imaging



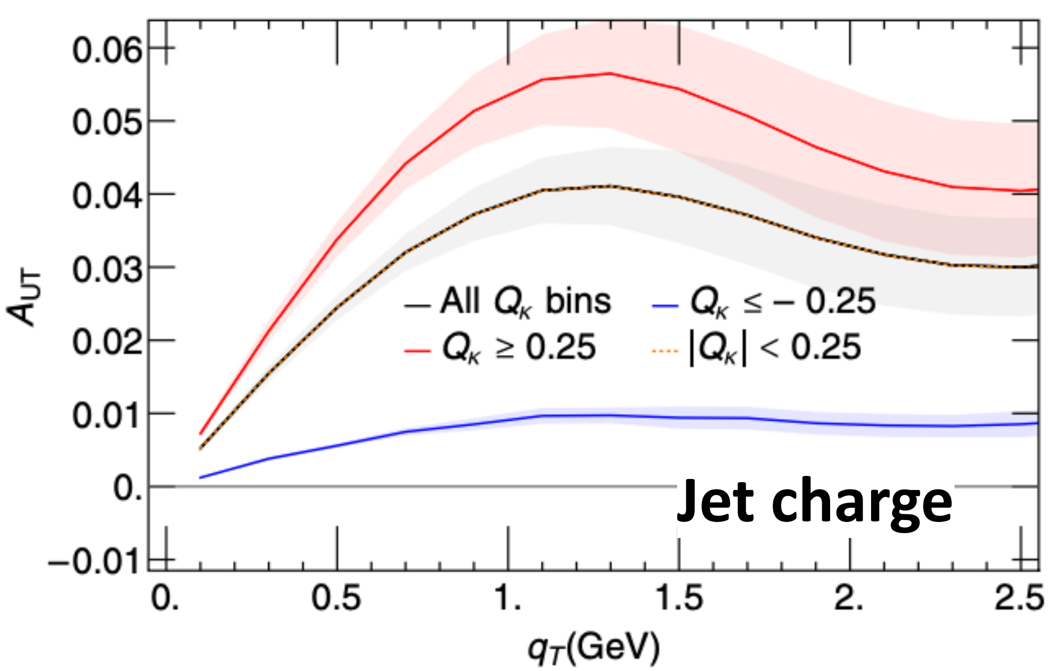


Arratia, Kang, Prokudin, Ringer ’19

Liu, Ringer, Vogelsang, Yuan ’19



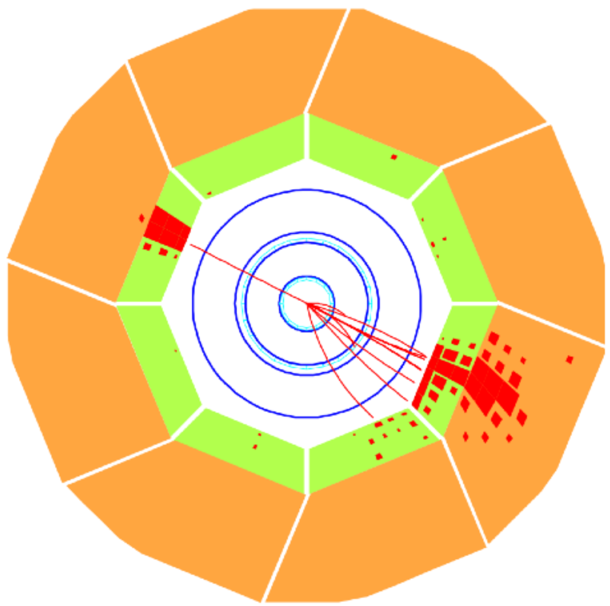
Kang, Lee, DYS, Zhao ’23 JHEP

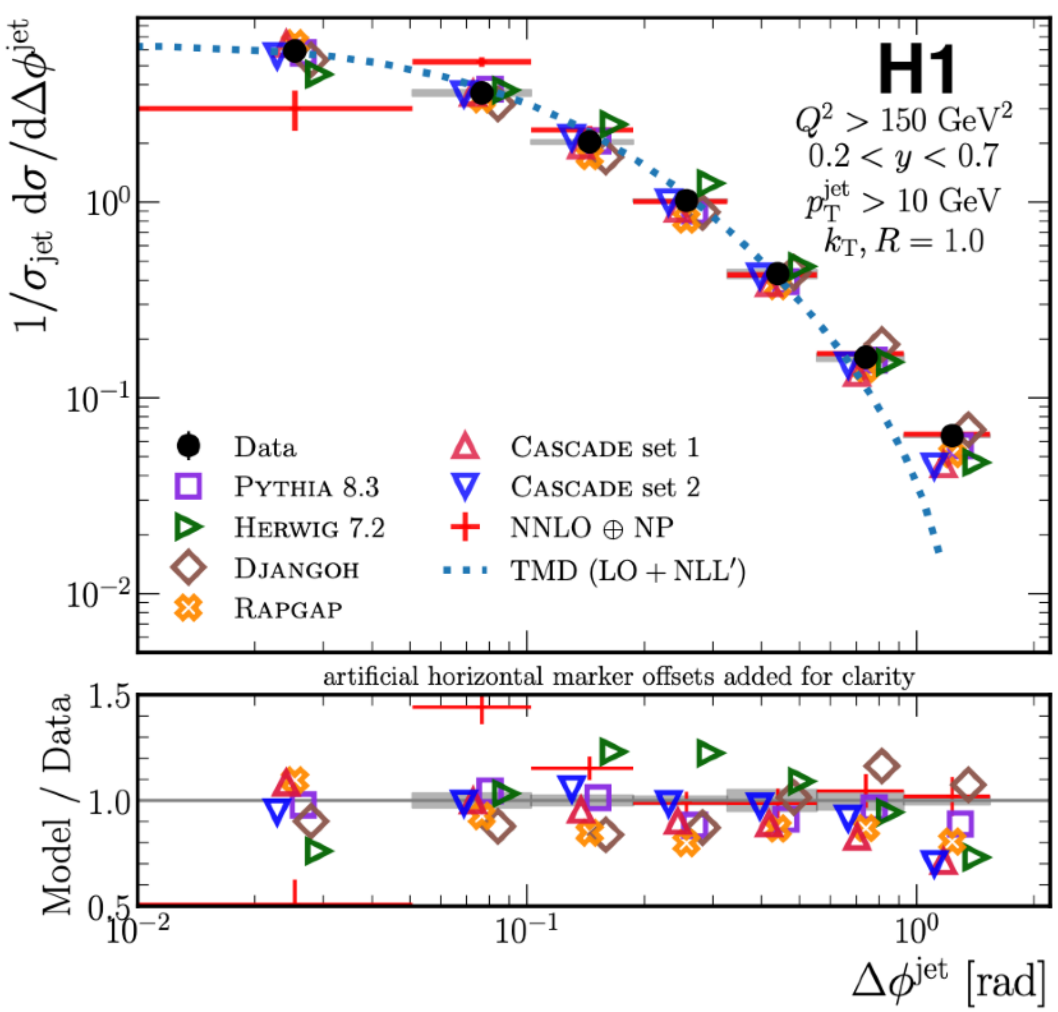


Kang, Liu, Mantry, DYS ’20 PRL

- Jets are complementary to standard SIDIS extractions of TMDs
- Jet measurements allow independent constraints on TMD PDFs and FFs from a single measurement
- Azimuthal correlation between jet and lepton sensitive to TMD PDFs

Jets and 3D imaging





H1 2108.12376

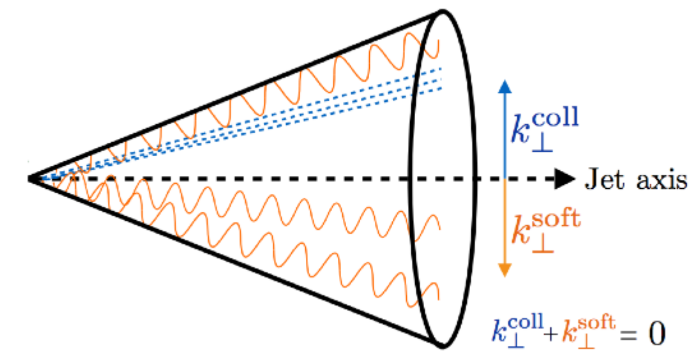
- NLL resummation result is consistent with HERA data
- Open questions:
  - Higher accuracy? SIDIS is known at N3LL' accuracy
  - Better angular resolution?
  - Reduce contamination from UE?
- One possible solution:
  - Recoil-free jet definition (Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi '18 '19)

E.g. anti-k_T clustering algorithm + p_T^n -weighted recombination scheme

Recoil-free jet and all-order structure

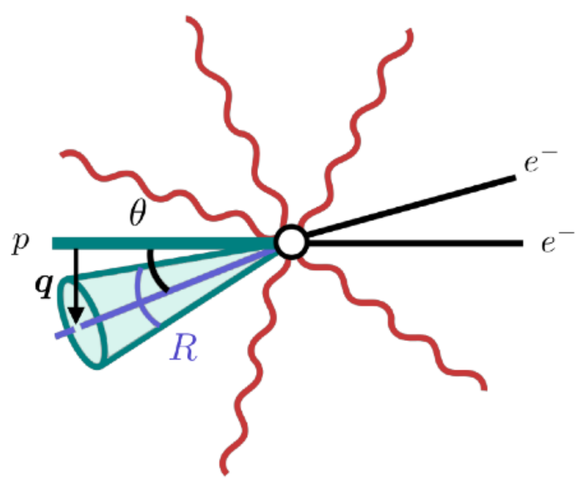
- **Recoil absent for the p_T^n-weighted recombination scheme** (Banfi, Dasgupta & Delenda '08)

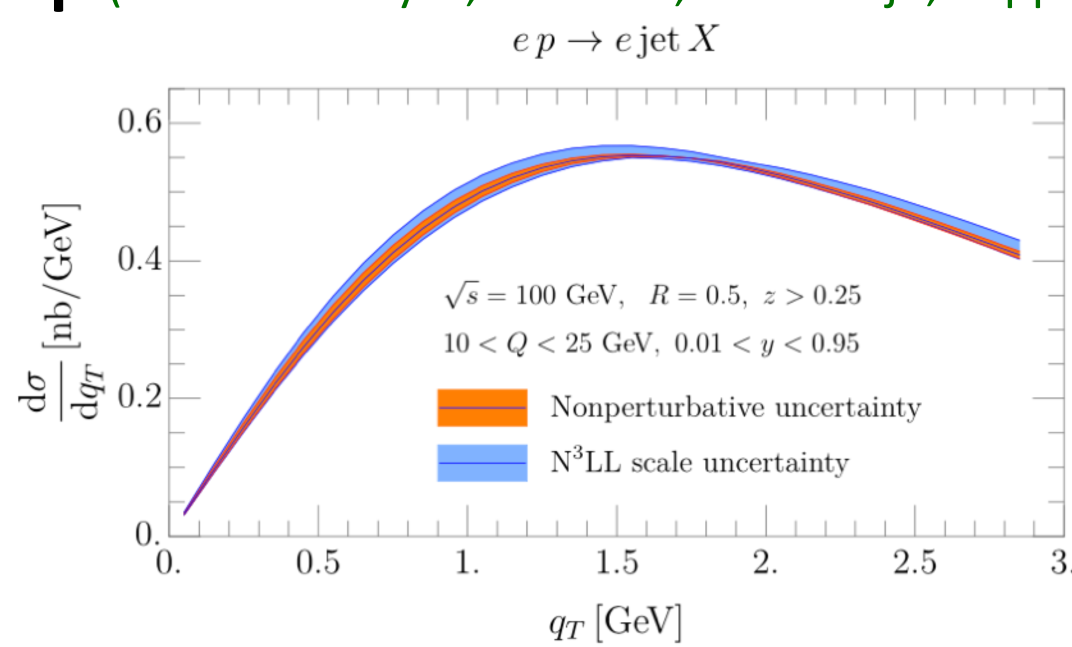
$$
\begin{aligned}
p_{t,r} &= p_{t,i} + p_{t,j}\,, \\
\phi_r &= (w_i\phi_i + w_j\phi_j)/(w_i + w_j) \\
y_r &= (w_i y_i + w_j y_j)/(w_i + w_j)
\end{aligned}
\qquad w_i = p_t^n
$$



$n \to \infty$ **Winner-take-all scheme** (Bertolini, Chan, Thaler '13)

- **N3LL resummation for jet q_T @ ee and ep** (Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi '18 '19)





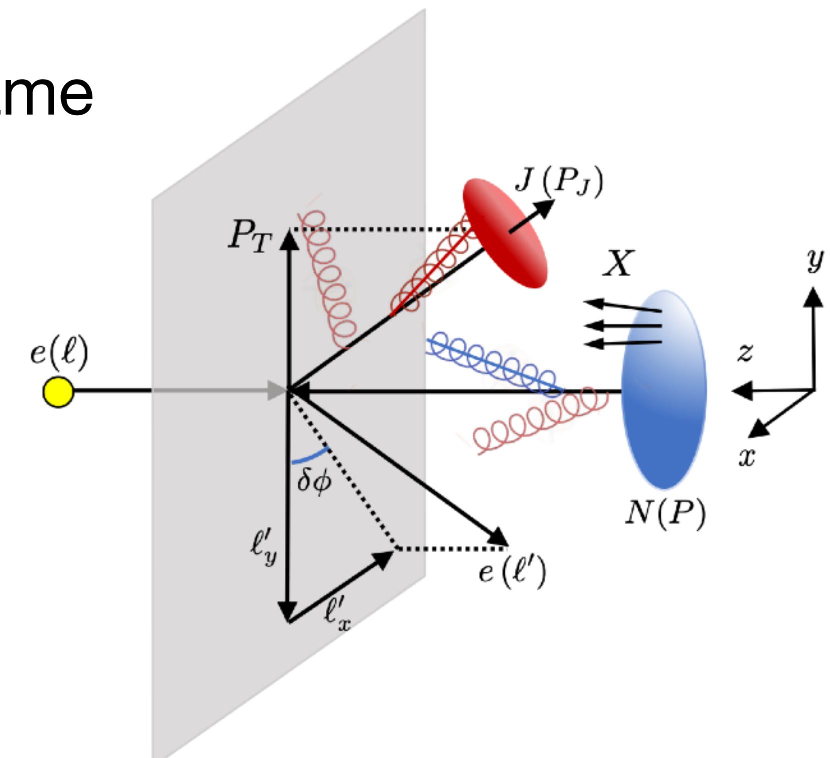
- **NNLL resummation for $\delta\phi$ @ pp** (Chien, Rahn, DYS, Waalewijn & Wu '22 JHEP + Schrignder '21 PLB)
- **NNLL resummation for $\delta\phi$ @ ep & eA** (Fang, Ke, DYS, Terry '23 JHEP)

Recoil-free azimuthal angle for electron-jet correlation

Fang, Ke, DYS, Terry ’23 JHEP

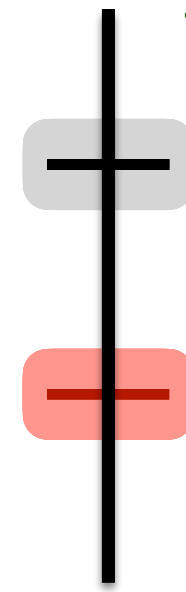
$$e(\ell) + N(P) \to e(\ell') + J(P_J) + X$$

Lab frame



Standard TMD in back to back limit: $Q >> q_T \sim l_T\, \delta\phi$

Chien, Rahn, DYS, Waalewijn & Wu ’22 JHEP + Schrignder ’21 PLB



$$p_h \sim Q(1,1,1)$$

$$p_{c_i}^{\mu} \sim l_T\left(\delta\phi^2, 1, \delta\phi\right)_{n_i \bar{n}_i}$$

$$p_s^{\mu} \sim l_T(\delta\phi, \delta\phi, \delta\phi)$$

Similar to Transverse-EEC in back to back

Following the standard steps in SCET and CSS, we obtain the following resummation formula

$$\frac{d\sigma}{d^2\ell'_T\, dy\, d\delta\phi} = \frac{\sigma_0\, \ell'_T}{1-y} H(Q,\mu) \int_0^\infty \frac{db}{\pi} \cos\left(b \ell'_T \delta\phi\right) \sum_q e_q^2\, f_{q/N}\left(x_B, b, \mu, \zeta_f\right) J_q\left(b, \mu, \zeta_J\right)$$

Hard factor

Fourier transformation in 1-dim

TMD PDF

Jet function

Predictions in e-p

Fang, Ke, DYS, Terry ’23 JHEP

TMD PDF (CSS treatment)

$$f_{q/N}(x_B,b,\mu,\zeta_f)=[C\otimes f]_{q/N}(x_B,b,\mu_f,\zeta_{fi})\,U^f_{\rm NP}(x_B,b,A,Q_0,\zeta_f)$$
$$\times\exp\left[\int_{\mu_f}^{\mu}\frac{d\mu'}{\mu'}\gamma^f_\mu(\mu',\zeta_f)\right]\left(\frac{\zeta_f}{\zeta_{fi}}\right)^{\frac{1}{2}\gamma^f_\zeta(b,\mu_f)},$$

Jet function

$$J_q(b,\mu,\zeta_J)=J_q(b,\mu_J,\zeta_{Ji})\,U^J_{\rm NP}(b,A,Q_0,\zeta_J)$$
$$\times\exp\left[\int_{\mu_J}^{\mu}\frac{d\mu'}{\mu'}\gamma^J_\mu(\mu',\zeta_J)\right]\left(\frac{\zeta_J}{\zeta_{Ji}}\right)^{\frac{1}{2}\gamma^J_\zeta(b,\mu_J)}$$

scale choice

$$\mu_H=Q\,,\qquad \mu_f=\mu_J=\sqrt{\zeta_{fi}}=\sqrt{\zeta_{Ji}}=\mu_b=2e^{-\gamma_E}/b$$

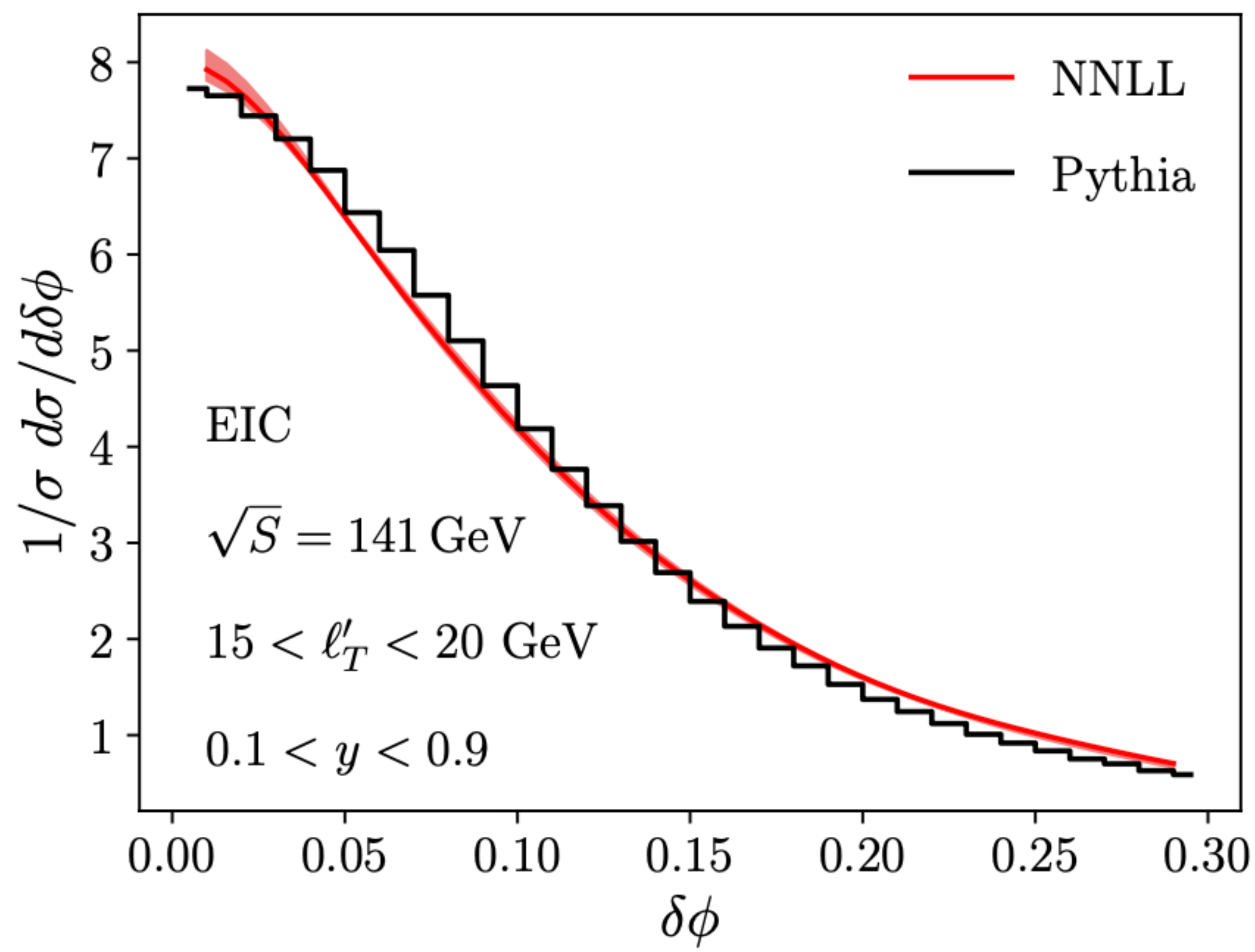
b*-prescription to avoid Landau pole

$$b_*=b/\sqrt{1+b^2/b^2_{\rm max}}\qquad \mu_{b_*}=2e^{-\gamma_E}/b_*$$

non-perturbative model

$$U^f_{\rm NP}=\exp\left[-g_1^f b^2-\frac{g_2}{2}\ln\frac{Q}{Q_0}\ln\frac{b}{b_*}\right]$$
$$U^J_{\rm NP}=\exp\left[-\frac{g_2}{2}\ln\frac{Q}{Q_0}\ln\frac{b}{b_*}\right]$$

Sun, Isaacson, Yuan, Yuan ‘14



μ_H varies between Q/2 and 2Q. μ_b is fixed

Predictions in e-A

Fang, Ke, DYS, Terry '23 JHEP

We apply nuclear modified TMD PDFs

$$g_1^A = g_1^f + a_N(A^{1/3} - 1) \quad a_N = 0.016 \pm 0.003 \text{ GeV}^2$$

Collinear dynamics (nPDF) using EPPS16

(Alrashed, Anderle, Kang, Terry & Xing, '22)

We include LO momentum broadening of the jet within SCET$_G$

$$J_q^A(b, \mu, \zeta_J) = J_q(b, \mu, \zeta_J) e^{\chi[\xi b K_1(\xi b) - 1]}$$

Opacity parameter $\chi = \frac{\rho_G L}{\xi^2} \alpha_s(\mu_{b_*}) C_F$

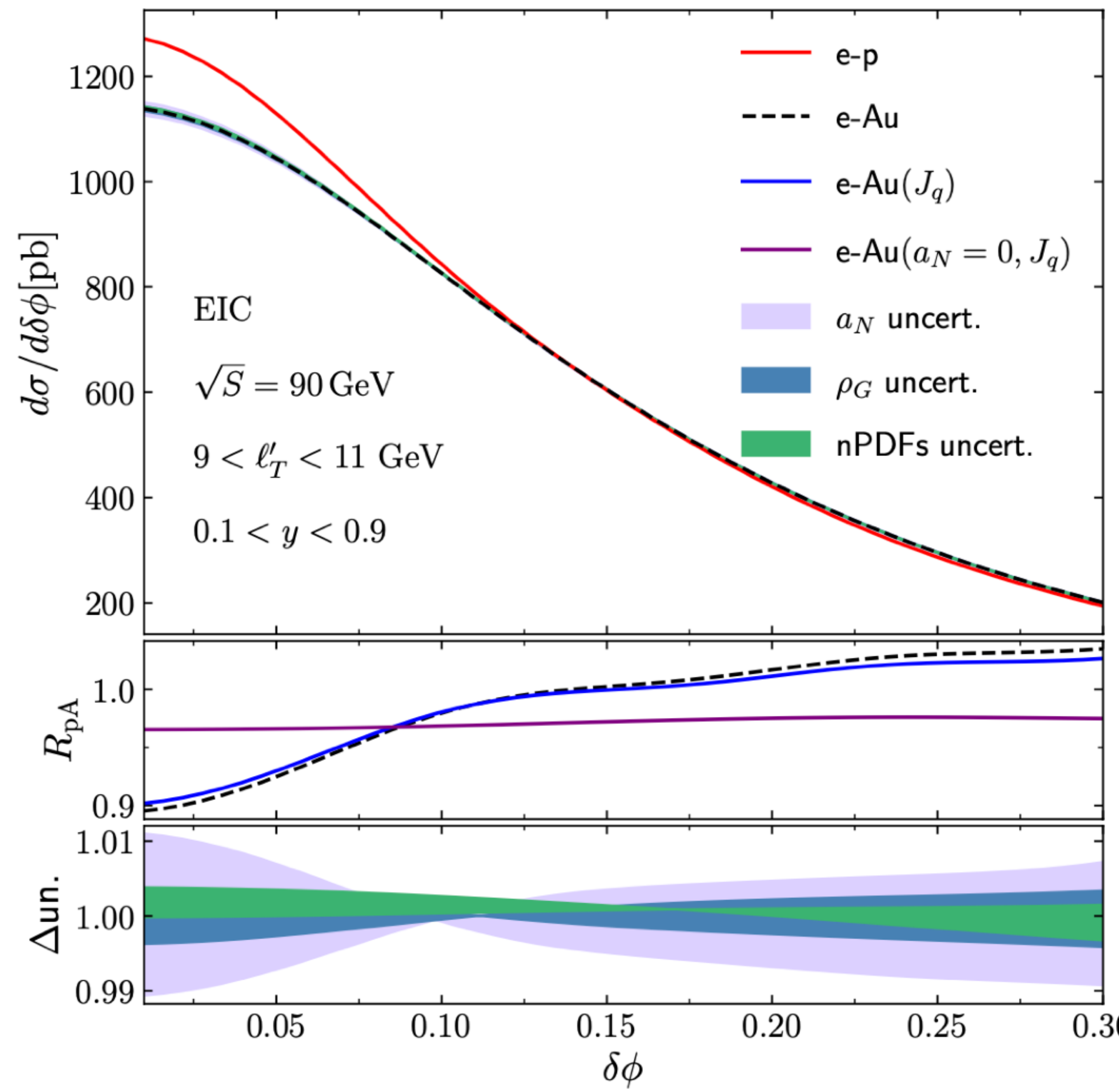
(Gyulassy, Levai, & Vitev '02)

ρ_G : density of the medium

ξ : the screening mass

L: the length of the medium

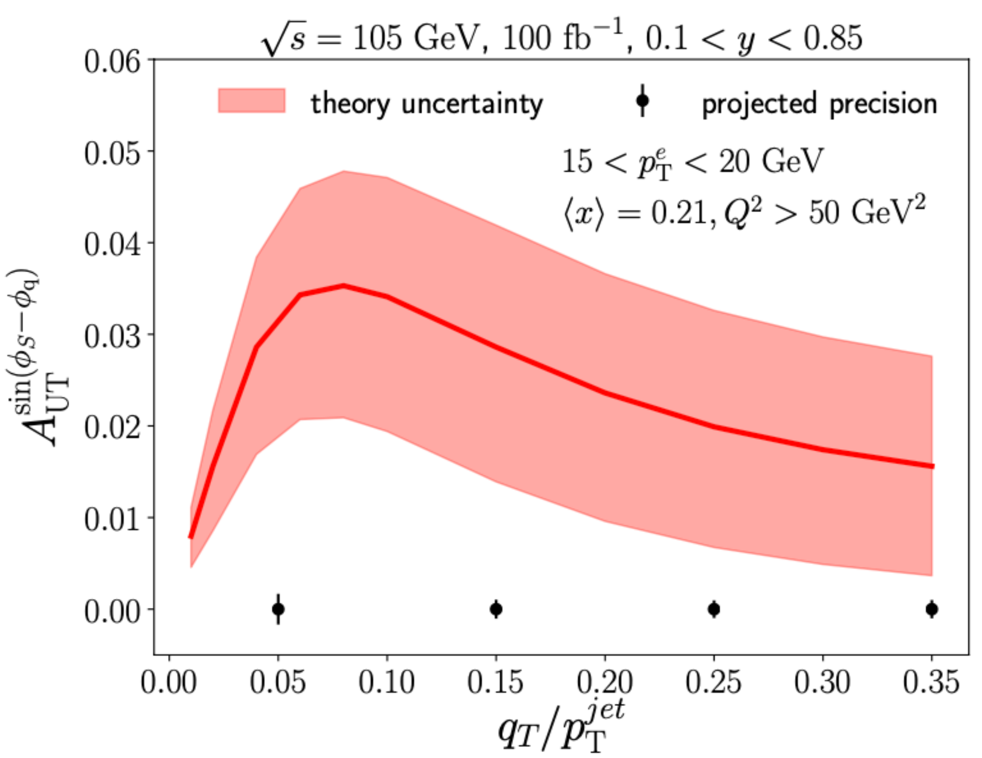
Parameter values are taken from a recent comparison between SCET$_G$ in e-A from the HERMES Ke and Vitev '23



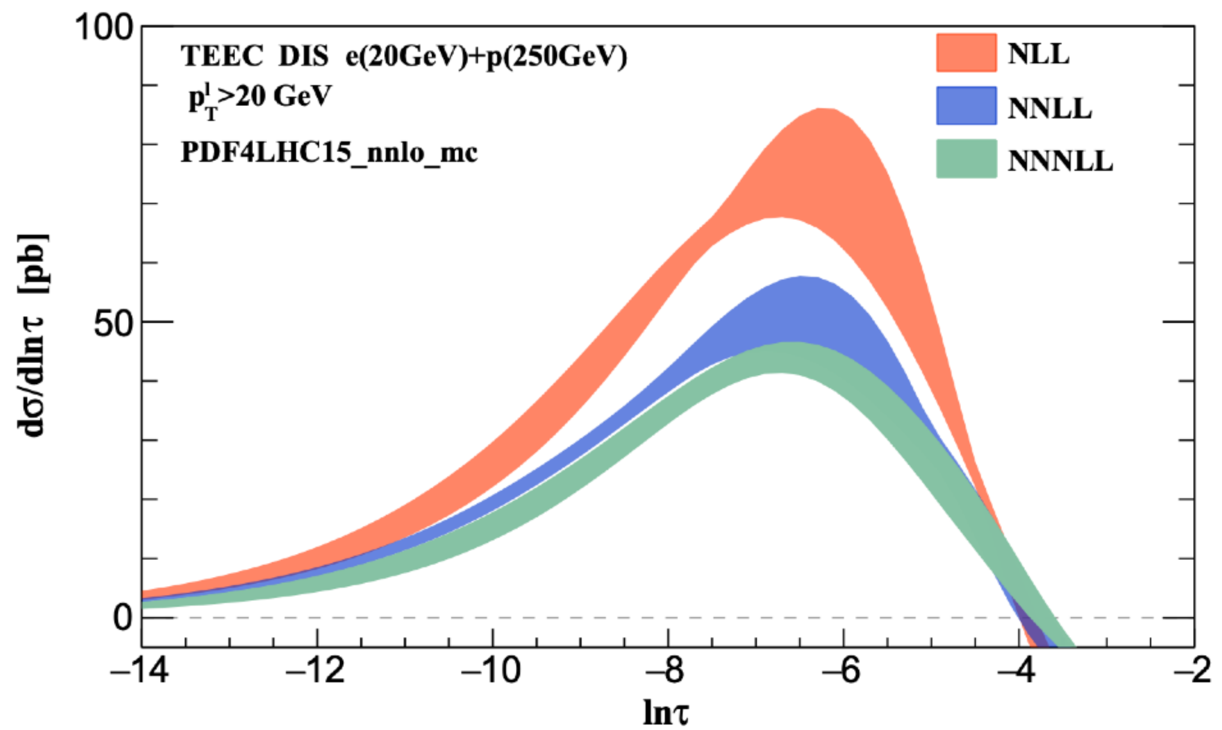
The process is primarily sensitive to the initial state's broadening effects, thereby serving as a clean probe of nTMD PDF

Precision calculation for jets in DIS

- **Precision calculations in DIS are essential for enhancing our understanding of partonic interactions and the internal structure of nucleons.**
- **The high-order calculation has reached N3LO accuracy for jet production in DIS** Currie, Gehrmann, Glover, Huss, Niehues, & Vogt '18
- **Several global event shape distributions in DIS are know at N3LL + $\mathcal{O}(\alpha_s^2)$**
  - **thrust** Kang, Lee, & Stewart '15
  - **(transverse) energy energy correlator** Li, Vitev, & Zhu '20, Li, Makris, Vitev '21
  - **1-jettiness** Cao, Kang, Liu & Mantry '23



Arratia, Kang, Prokudin, Ringer '19



Li, Vitev, & Zhu '20, Li, Makris, Vitev '21

$N^3LL + \mathcal{O}(\alpha_s^2)$ predictions on lepton jet azimuthal correlation in DIS

Fang, Gao, Li, DYS 2408.XXXXX

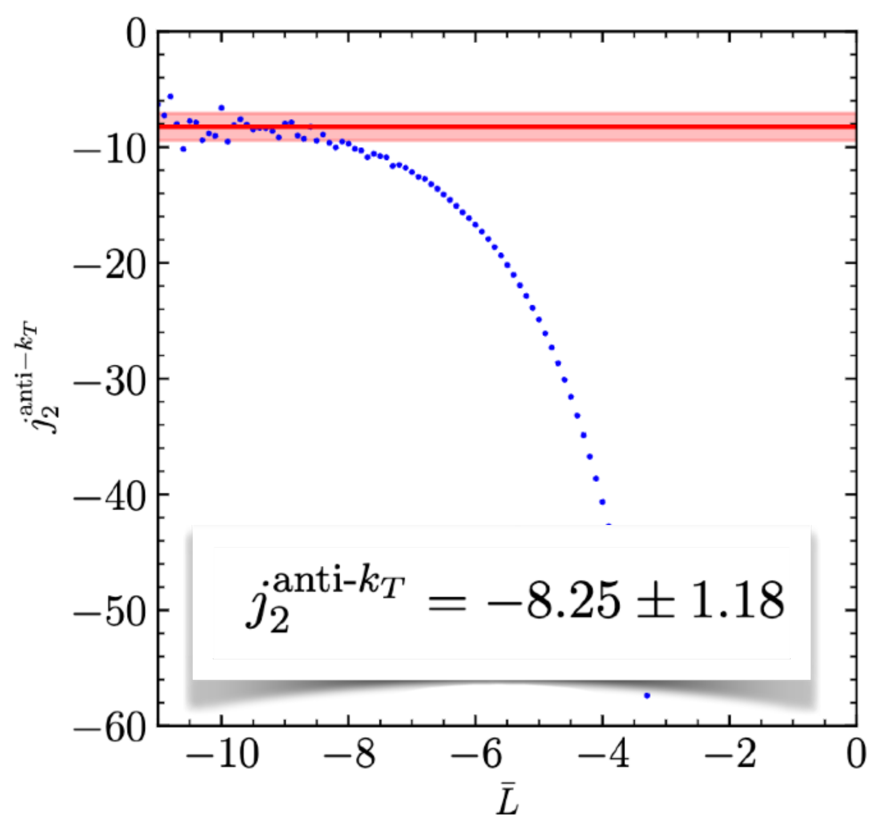
- **All ingredients are known at $N^3LL + \mathcal{O}(\alpha_s^2)$, except the two loop jet function j_2 .**
  - **It was extracted numerically from the Event2** (Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi '19)
  - **A preliminary numerical results are also calculated from SoftSERVE** (Brune SCET2023)
- **We study dijet production in e+e-, and compare two-loop singular cross section and $\mathcal{O}(\alpha_s^2)$ predictions from NLOJET++ generator to extract j2**

$$\frac{d\sigma}{dq_T} = \bar{\sigma}_0\, H(Q,\mu_h) q_T \int_0^\infty b_T\, db_T J_0(q_T b_T) J_q(b_T,\mu_h,\zeta_f) J_{\bar{q}}(b_T,\mu_h,\zeta_f)$$

Integrated cross section: $\sigma_L(Q_T) \equiv \int_0^{Q_T} dq_T \frac{d\sigma}{dq_T}$

Two-loop coefficient:

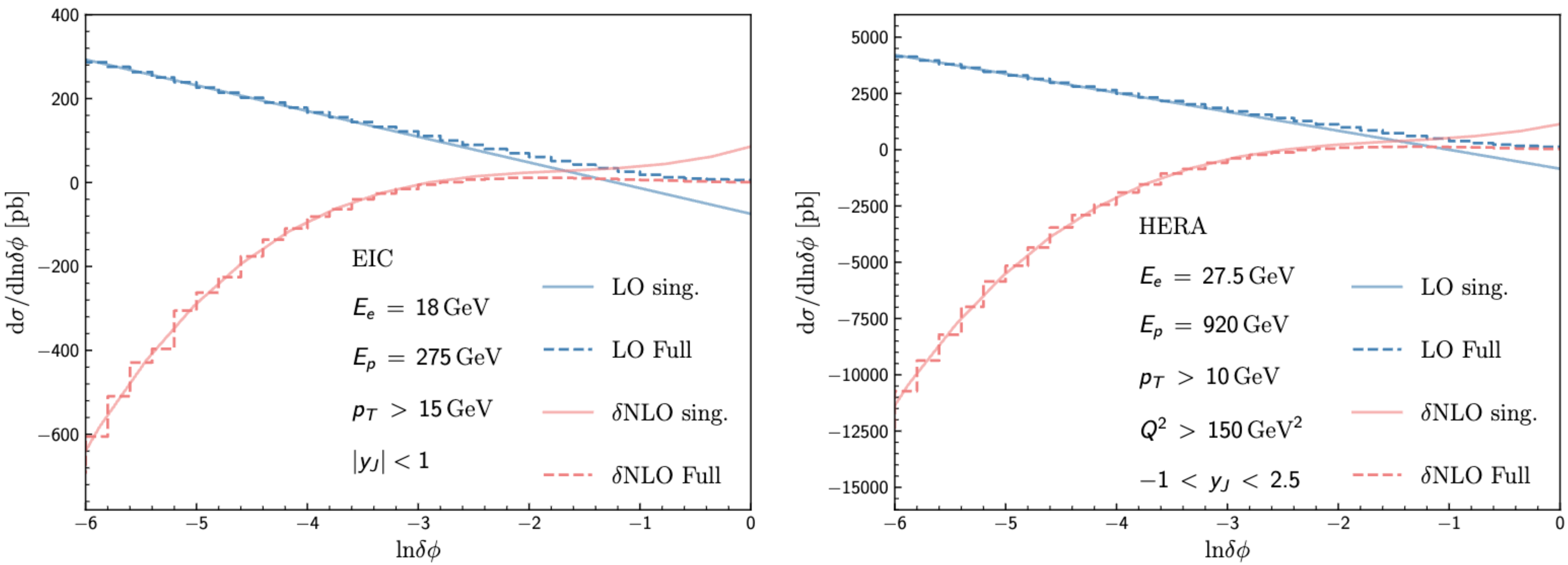
$$\begin{aligned} B = {} & C_F^2\left[\frac{\bar{L}^4}{2} + 3\bar{L}^3 + \bar{L}^2\left(\frac{11}{2} - \frac{\pi^2}{3} + 6\ln 2\right) + \bar{L}\left(\frac{9}{4} + 18\ln 2 - 4\zeta_3\right) - \frac{189}{16} + 5\pi^2 \right.\\ & \left. - \frac{173\pi^4}{720} + 27\ln 2 - \frac{9}{2}\pi^2\ln 2 + 9\ln^2 2 - 3\zeta_3\right] + C_F C_A\left[\frac{11\bar{L}^3}{9} + \bar{L}^2\left(-\frac{35}{36} + \frac{\pi^2}{6}\right)\right.\\ & \left. + \bar{L}\left(-\frac{57}{4} + \frac{11\pi^2}{18} + 11\ln 2 + 6\zeta_3\right) - \frac{51157}{1296} + \frac{1061\pi^2}{216} - \frac{2\pi^4}{45} + \frac{401\zeta_3}{18}\right]\\ & + C_F T_F n_f\left[-\frac{4\bar{L}^3}{9} + \frac{\bar{L}^2}{9} + \bar{L}\left(5 - \frac{2\pi^2}{9} - 4\ln 2\right) + \frac{4085}{324} - \frac{91\pi^2}{54} - \frac{14\zeta_3}{9}\right]\\ & + \frac{j_2}{2}, \end{aligned}$$



$N^3LL + \mathcal{O}(\alpha_s^2)$ predictions on lepton jet azimuthal correlation in DIS

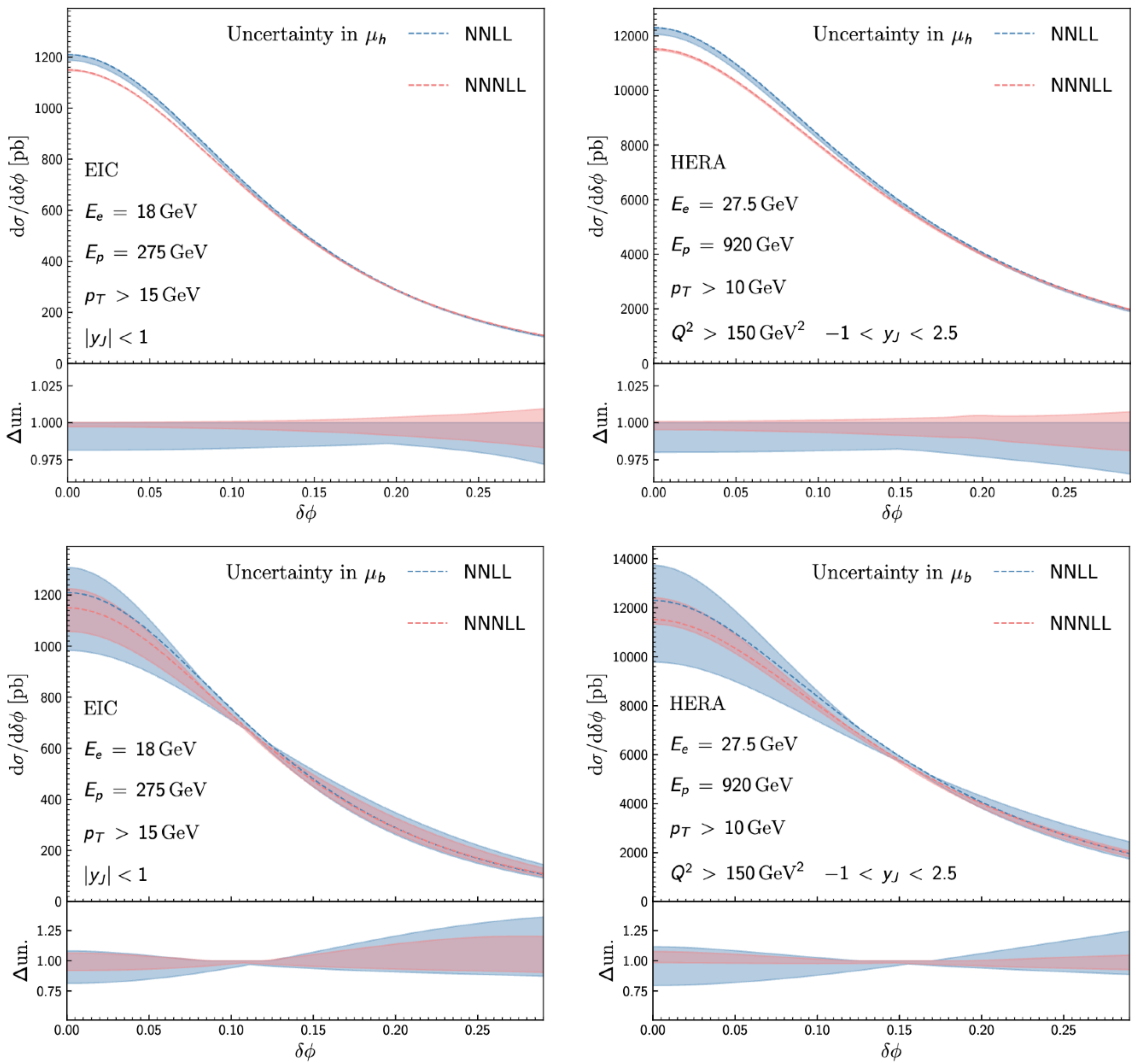
Fang, Gao, Li, DYS 2408.XXXXX

- **We also compare the resummation expanded singular contribution in DIS with the full prediction from NLOJET++ up to $\mathcal{O}(\alpha_s^2)$.**
- **Good agreement in the back-to-back limit ($\delta\phi$ —> 0) is observed.**
- **Matching corrections (Y term) are important in the large $\delta\phi$ region**



Comparison of resummation results at N2LL and N3LL

Fang, Gao, Li, DYS 2408.XXXXX



- **The uncertainty bands are narrower at N3LL (red) compared to NNLL (blue)**
- **At N3LL the dominant scale uncertainties are from μ_b variation**

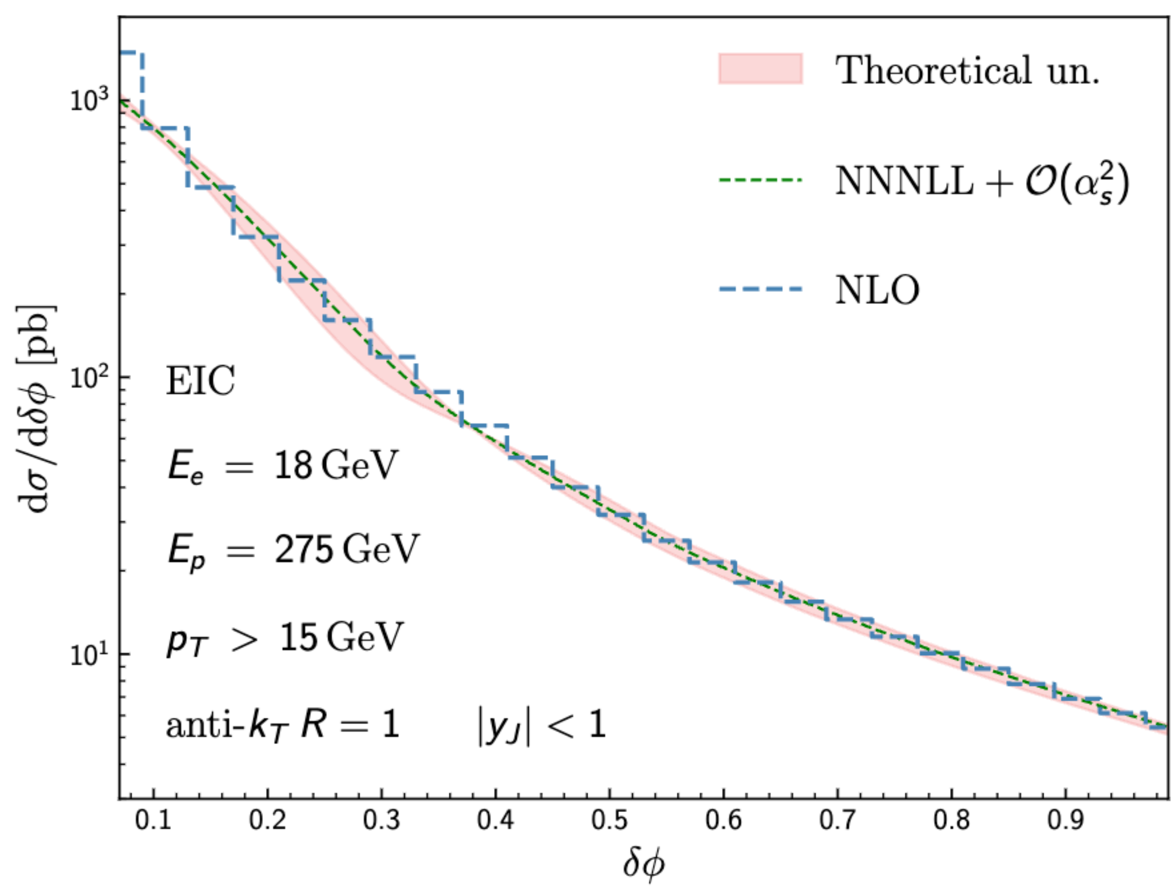
N3LL + $\mathcal{O}(\alpha_s^2)$ predictions on lepton jet azimuthal correlation in DIS

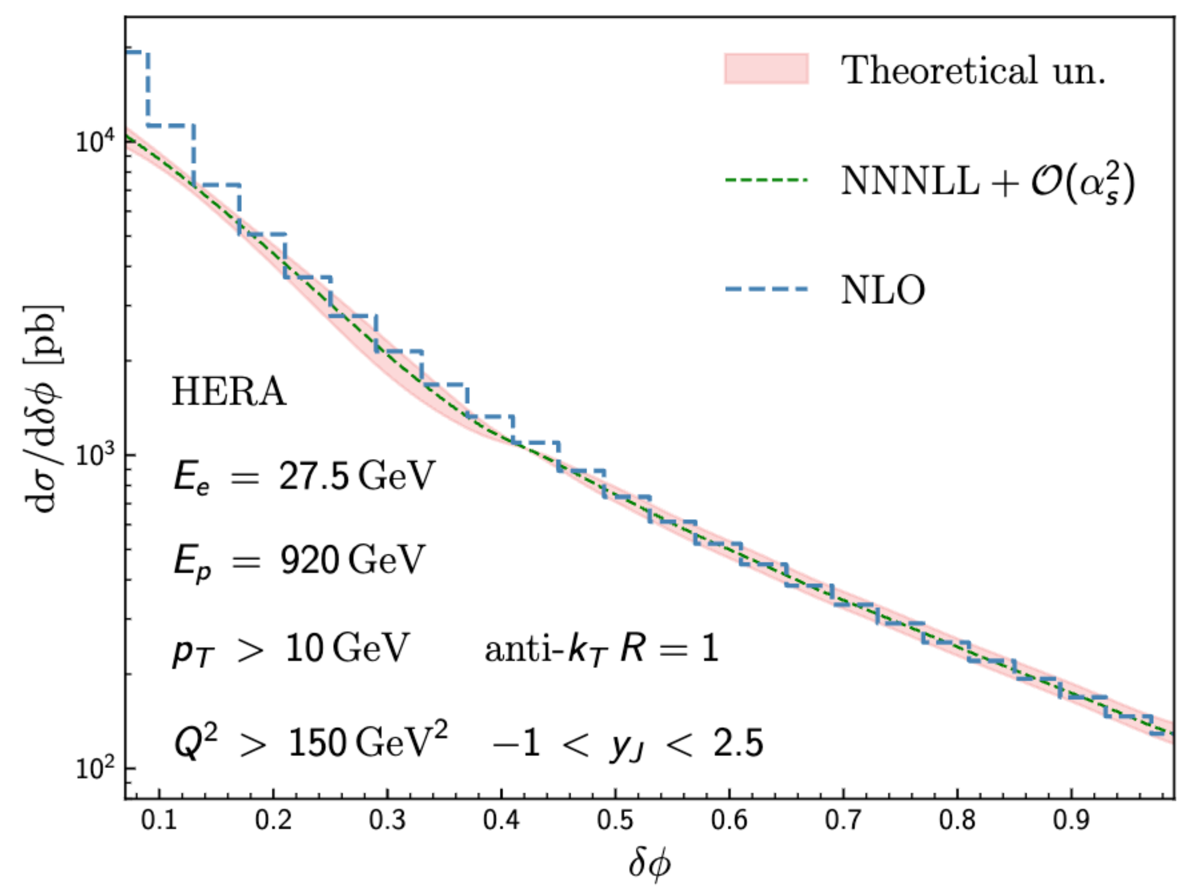
Fang, Gao, Li, DYS 2408.XXXXX

- **In the large $\delta\phi$ region the resummation formula receives significant matching corrections**
- **It is necessary to switch off the resummation and instead employ fixed-order calculations**

$$\mathrm{d}\sigma_{\mathrm{add}}\,(\mathrm{NNNLL}+\mathcal{O}(\alpha_s^2)) \equiv \mathrm{d}\sigma(\mathrm{NNNLL}) + \underbrace{\mathrm{d}\sigma(\mathrm{NLO}) - \mathrm{d}\sigma(\mathrm{NLO\ singular})}_{\mathrm{d}\sigma(\mathrm{NLO\ non\text{-}singular})}$$

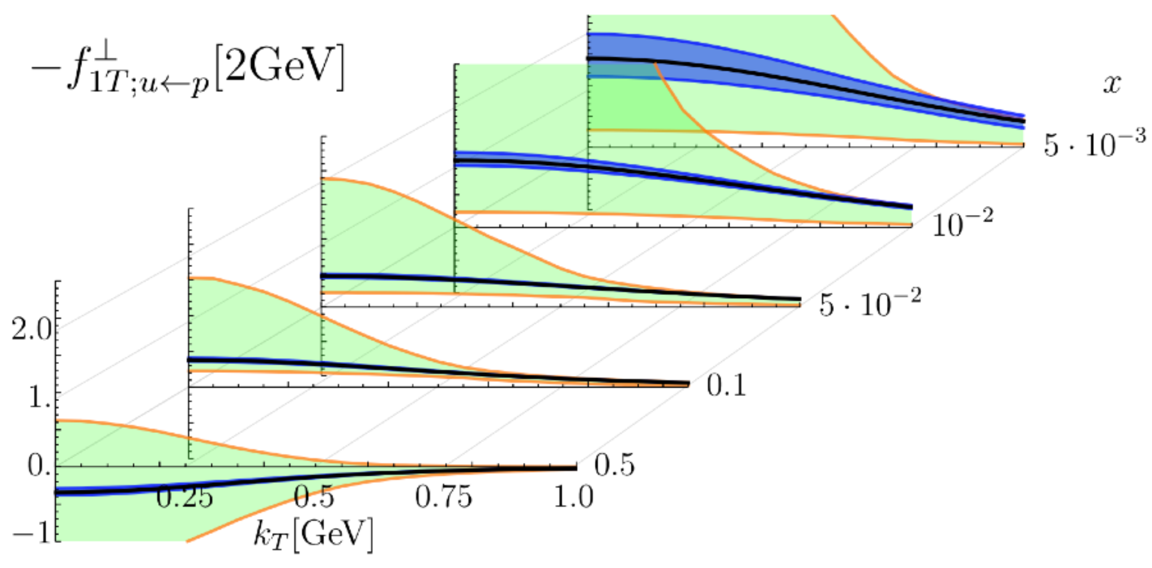
$$\mathrm{d}\sigma(\mathrm{NNNLL}+\mathcal{O}(\alpha_s^2)) = [1-t(\delta\phi)]\mathrm{d}\sigma_{\mathrm{add}}\,(\mathrm{NNNLL}+\mathcal{O}(\alpha_s^2)) + t(\delta\phi)\mathrm{d}\sigma(\mathrm{NLO})$$

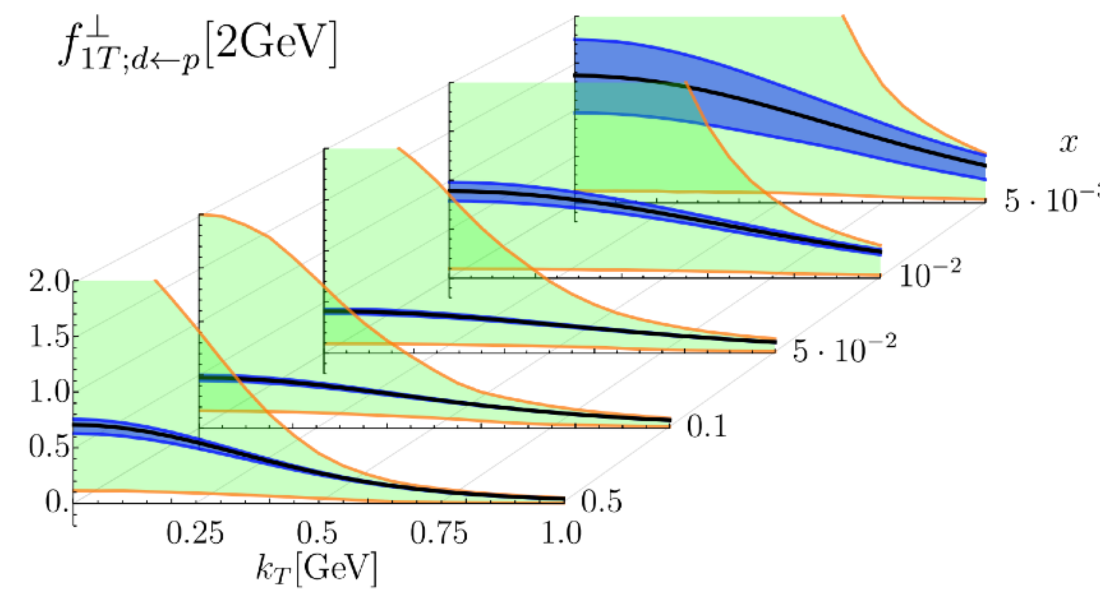




TMDs in the large-x limit

Up and down quark Sivers distributions as a function of the transverse momentum k_T for different values of x





EIC Yellow report '21

How to give a reliable framework for extracting TMDs at large x value ?

TMDs in the large-x limit

- **The usual TMD factorization is defined at moderate x value, i.e the x is not too low or too high**
- **TMDs at small x is important for gluon saturation in the Regge asymptotic of QCD, which was investigated in** Balitsky, Tarasov ’15, Zhou ’16, Xiao, Yuan, Zhou ’17

$$xG_{WW}(x,k_\perp,\zeta_c=\mu_F=Q) = -\frac{2}{\alpha_S}\int\frac{d^2v_\perp d^2v'_\perp}{(2\pi)^4}e^{ik_\perp\cdot r_\perp}\mathcal{H}^{WW}(\alpha_s(Q))e^{-\mathcal{S}_{sud}(Q^2,r_\perp^2)}\mathcal{F}^{WW}_{Y=\ln 1/x}(v_\perp,v'_\perp)$$

- **In the limit x->1 (threshold), the phase space of real radiations is restricted**

E.g. : One-loop quark TMDs in the parton model

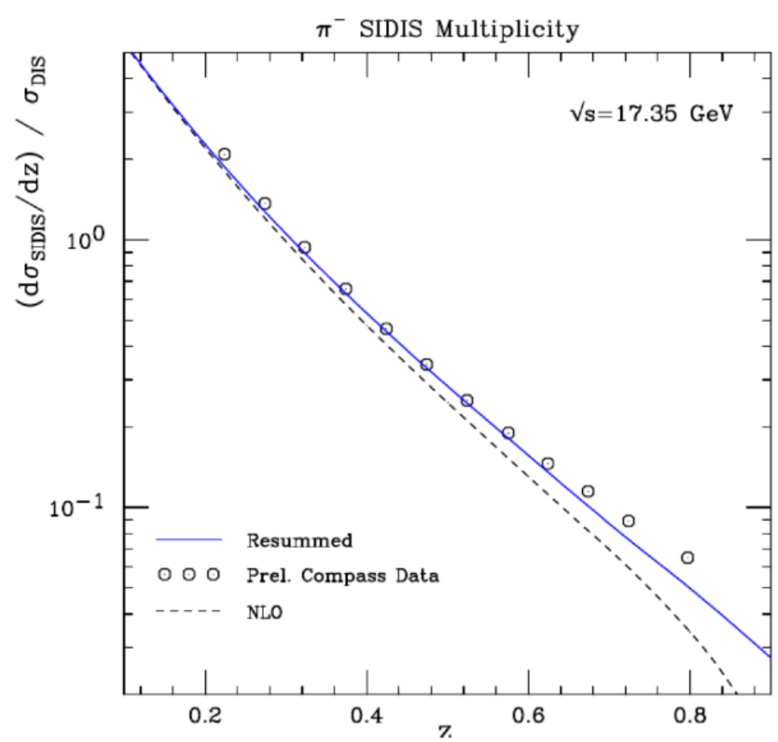
$$\tilde{f}^{(1)}_{q/q}(x,\mathbf{b}_T,\mu,\zeta) = \frac{\alpha_s(\mu)C_F}{2\pi}\left[-\left(\frac{1}{\epsilon}+L_b\right)[P_{qq}(x)]_+ + (1-x) - \frac{L_b^2}{2} + L_b\left(\frac{3}{2}+\ln\frac{\mu^2}{\zeta}\right) - \frac{\pi^2}{12}\right]$$

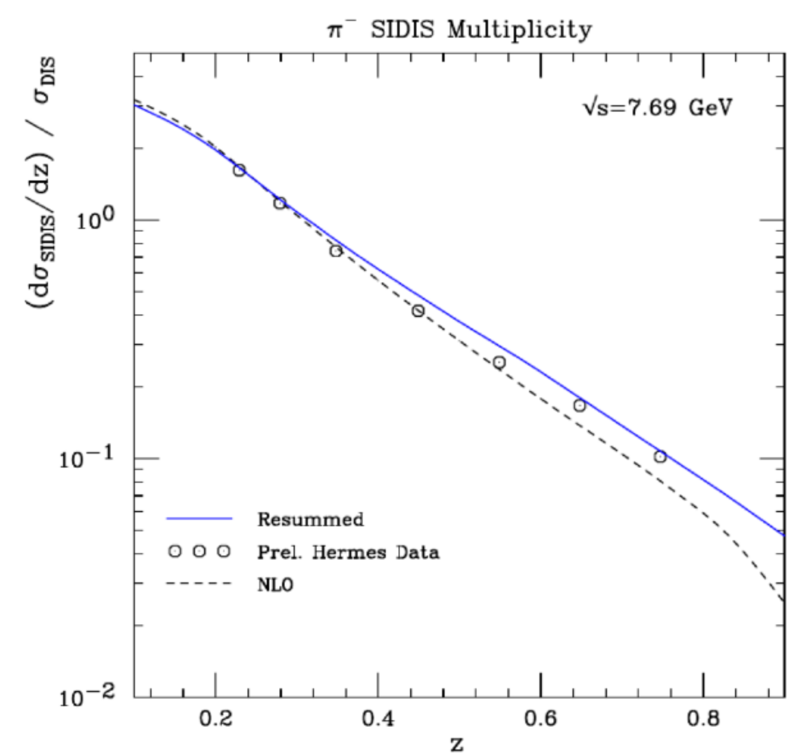
$$\boxed{P_{qq}(x) = \frac{1+x^2}{1-x}}$$

divergent as $x \to 1$

Threshold resummation

- **The threshold effect is important for a reliable theoretical prediction near the right edge of the phase space, e.g. threshold resummation of pion FFs** Anderle, Ringer, Vogelsang '13





- **The NLL joint resummation framework of threshold and TMD logarithms was first developed by** Laenen, Sterman & Vogelsang '00 …

- **A factorization formula based on SCET then was given by** Lustermans, Waalewijn, Zeune '16 . Y. Li, Neill, Zhu '16

- **We apply the joint threshold and TMD factorization theorem to introduce new threshold-TMDs — TTMDs** Kang, Samanta, Shao, Zeng '22 JHEP

From TMDs to TTMDs

- **Consider Drell-Yan process**

$$h_1(P_1) + h_2(P_2) \to \gamma^*(q) \to l^+ + l^- + X$$

- **As q_T << Q, we have the TMD factorization theorem**

$$\frac{\mathrm{d}^3\sigma^{\mathrm{DY}}}{\mathrm{d}^2\boldsymbol{q}_T\,\mathrm{d}\tau^{\mathrm{DY}}} = \sigma_0^{\mathrm{DY}} \int_{C_N} \frac{\mathrm{d}N}{2\pi i} \left(\tau^{\mathrm{DY}}\right)^{-N} \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{4\pi^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\, H^{\mathrm{DY}}(Q,\mu) \sum_q e_q^2 \tilde{f}_{q/h_1}^{\mathrm{TMD}}(N, b_T, \mu, \zeta) \tilde{f}_{\bar{q}/h_2}^{\mathrm{TMD}}(N, b_T, \mu, \zeta)$$

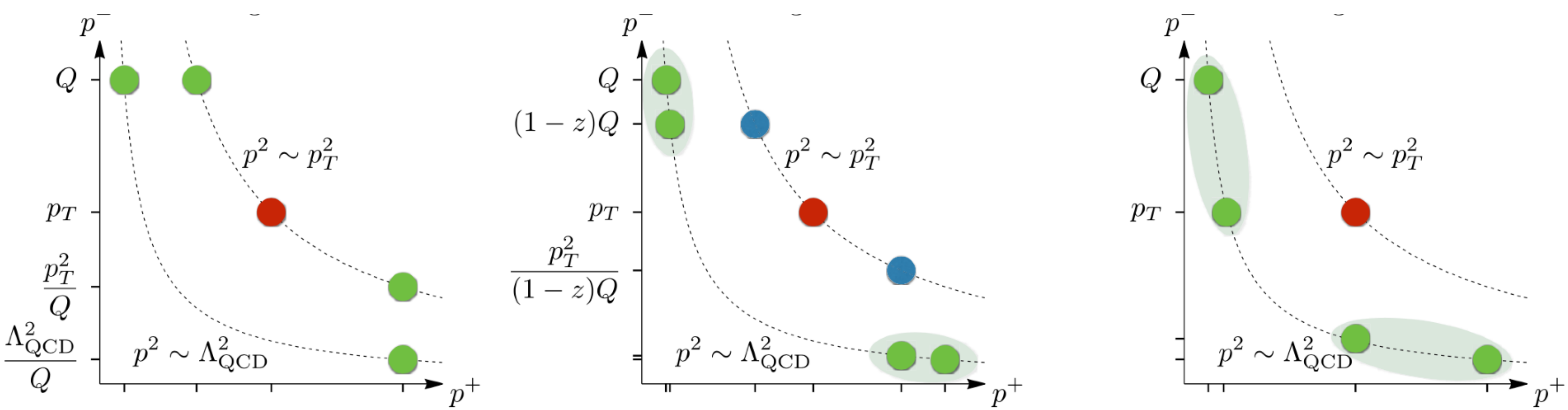
- **Threshold variables:** $\tau^{\mathrm{DY}} \equiv Q^2/S \qquad \hat{\tau}^{\mathrm{DY}} \equiv \tau^{\mathrm{DY}}/(x_1 x_2)$
- **TMDPDFs after Mellin transformation:** $\tilde{f}_{i/h}^{\mathrm{TMD}}(N, b_T, \mu, \zeta) \equiv \int_0^1 \mathrm{d}x\, x^{N-1} f_{i/h}^{\mathrm{TMD}}(x, b_T, \mu, \zeta)$
- **When the partonic threshold variable is close to 1, i.e.** $N \to \infty$ **, the above factorization is not complete**
- **To include both the TMD and threshold effects, we perform a re-factorization for the TMDPDF**

$$\tilde{f}_{i/h}^{\mathrm{TMD}}(N, b_T, \mu, \zeta) \xrightarrow{N\to\infty} \tilde{S}_c(N, b_T, \mu, \zeta) \tilde{f}_{i/h}(N, \mu)$$

EFT in the joint limit

In the threshold limit, the new scale hierarchy introduces an additional degrees of freedom known as collinear-soft degrees of freedom

$$k_{cs}^{\mu} \equiv (\bar{n} \cdot k_{cs},\, n \cdot k_{cs},\, k_{cs,\perp}) \sim \left(Q(1-\hat{\tau}),\, \frac{q_T^2}{Q(1-\hat{\tau})},\, q_T \right)$$



$Q \sim (1-z)Q \gg p_T$ $\quad$ $Q \gg (1-z)Q \gg p_T$ $\quad$ $Q \gg (1-z)Q \sim p_T$

TMD soft function is the same as usual, so the rapidity div. is not changed

Collinear-soft function

In the threshold limit, the new scale hierarchy introduces an additional degrees of freedom known as collinear-soft degrees of freedom

To validate the factorization theorem, we employ the threshold expressions of the perturbative matching coefficients to ascertain the three-loop collinear-soft function.

$$
\begin{aligned}
\tilde{S}_c^{(1)} &= 2C_F L_b L_\zeta , \\
\tilde{S}_c^{(2)} &= 2C_F^2 L_b^2 L_\zeta^2 + C_F C_A \left[\frac{11}{3} L_b^2 + \left(\frac{134}{9} - 4\zeta_2 \right) L_b + \left(\frac{404}{27} - 14\zeta_3 \right) \right] L_\zeta \\
&\quad + C_F T_F n_f \left(-\frac{4}{3} L_b^2 - \frac{40}{9} L_b - \frac{112}{27} \right) L_\zeta , \\
\tilde{S}_c^{(3)} &= \frac{4}{3} C_F^3 L_b^3 L_\zeta^3 + C_F^2 C_A \left[\frac{22}{3} L_b^3 + \left(\frac{268}{9} - 8\zeta_2 \right) L_b^2 + \left(\frac{808}{27} - 28\zeta_3 \right) L_b \right] L_\zeta^2 \\
&\quad + C_F C_A^2 \left[\frac{242}{27} L_b^3 + \left(\frac{1780}{27} - \frac{44}{3}\zeta_2 \right) L_b^2 + \left(\frac{15503}{81} - \frac{536}{9}\zeta_2 - 88\zeta_3 + 44\zeta_4 \right) L_b \right. \\
&\quad \left. + \left(\frac{297029}{1458} - \frac{3196}{81}\zeta_2 - \frac{6164}{27}\zeta_3 + \frac{88}{3}\zeta_2\zeta_3 - \frac{77}{3}\zeta_4 + 96\zeta_5 \right) \right] L_\zeta \\
&\quad + C_F C_A T_F n_f \left[-\frac{176}{27} L_b^3 + \left(-\frac{1156}{27} + \frac{16}{3}\zeta_2 \right) L_b^2 + \left(-\frac{8204}{81} + \frac{160}{9}\zeta_2 \right) L_b \right. \\
&\quad \left. + \left(-\frac{62626}{729} + \frac{824}{81}\zeta_2 + \frac{904}{27}\zeta_3 - \frac{20}{3}\zeta_4 \right) \right] L_\zeta \\
&\quad + C_F T_F^2 n_f^2 \left(\frac{32}{27} L_b^3 + \frac{160}{27} L_b^2 + \frac{800}{81} L_b + \frac{3712}{729} + \frac{64}{9}\zeta_3 \right) L_\zeta \\
&\quad + C_F^2 T_F n_f \left[-\frac{8}{3} L_b^3 L_\zeta^2 + \left(-4 - \frac{80}{9} L_\zeta \right) L_b^2 L_\zeta + \left(-\frac{224}{27} - \frac{110}{3} + 32\zeta_3 \right) L_b L_\zeta^2 \right. \\
&\quad \left. + \left(-\frac{1711}{27} + \frac{304}{9}\zeta_3 + 16\zeta_4 \right) L_\zeta \right] ,
\end{aligned}
$$

Collins-Soper scale in the joint limit

The Collins-Soper equation is the same as the one for TMD PDFs

$$\tilde{S}_c\,(N, b_T, \mu_b, \zeta_f) = \tilde{S}_c\,(N, b_T, \mu_b, \zeta_i)\left(\sqrt{\frac{\zeta_f}{\zeta_i}}\right)^{\kappa(b_T,\mu_b)}$$

where $\zeta_i = \mu_b^2 = b_0^2/b_T^2$ **and** ζ_f **is determined from RG consistency**

RG equations for the hard function, collinear-soft function and collinear PDFs read

$$\boxed{\begin{aligned}
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}H^{\mathrm{DY}}\,(Q,\mu) &= \Gamma^h(\alpha_s)H^{\mathrm{DY}}\,(Q,\mu)\,,\\
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\tilde{S}_c\,(N,b_T,\mu,\zeta_f) &= \Gamma^{\tilde{S}_c}(\alpha_s,\zeta_f)\tilde{S}_c\,(N,b_T,\mu,\zeta_f)\,,\\
\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\tilde{f}_q\,(N,\mu) &= \Gamma^{\tilde{f}_q}(\alpha_s)\tilde{f}_q\,(N,\mu)\,,
\end{aligned}}$$

with

$$\boxed{\begin{aligned}
\Gamma^h(\alpha_s) &= 2\,\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{Q^2}{\mu^2} + 2\gamma_V(\alpha_s)\,,\\
\Gamma^{\tilde{S}_c}(\alpha_s,\zeta_f) &= -\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{\zeta_f}{\mu^2} + \gamma_{\tilde{S}_c}(\alpha_s)\,,\\
\Gamma^{\tilde{f}_q}(\alpha_s) &= -2\,\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\bar{N} + 2\gamma_{\tilde{f}_q}(\alpha_s)\,,
\end{aligned}}$$

We find the Collins-Soper scale in the threshold limit is given by $\zeta_f^{\mathrm{TTMD}} = \frac{Q^2}{\bar{N}^2} \sim Q^2(1-\hat{\tau})^2$

which is different from the standard TMD one $\zeta_f^{\mathrm{TMD}} = Q^2$

Threshold-TMDPDF

After solving RG evolution and CS evolution, we have TTMDPDF

$$\tilde{f}_{i/h}^{\mathrm{TTMD}}(N, b_T, Q) = \exp\left[-S_{\mathrm{pert}}(Q, \mu_{b_*}, \mu_F) - S_{\mathrm{NP}}^{f}\left(b_T, Q_0, \zeta_f^{\mathrm{TTMD}}\right)\right] \tilde{f}_{i/h}(N, \mu_F)$$

where the function form of the perturbative Sudakov factor and non-perturbative parts are the same as the usual TMD ones, but the CS scale is modified

TMD $\quad \zeta_f^{\mathrm{TMD}} = Q^2$

TTMD $\quad \zeta_f^{\mathrm{TTMD}} = Q^2/N^2 \sim Q^2(1-\hat{\tau})^2$

$\zeta_i = \mu_b^2 \sim q_T^2$

For large value of N, the Collins-Soper scale may go down the non-perturbative region, so we introduce a prescription to freeze the value of CS scale

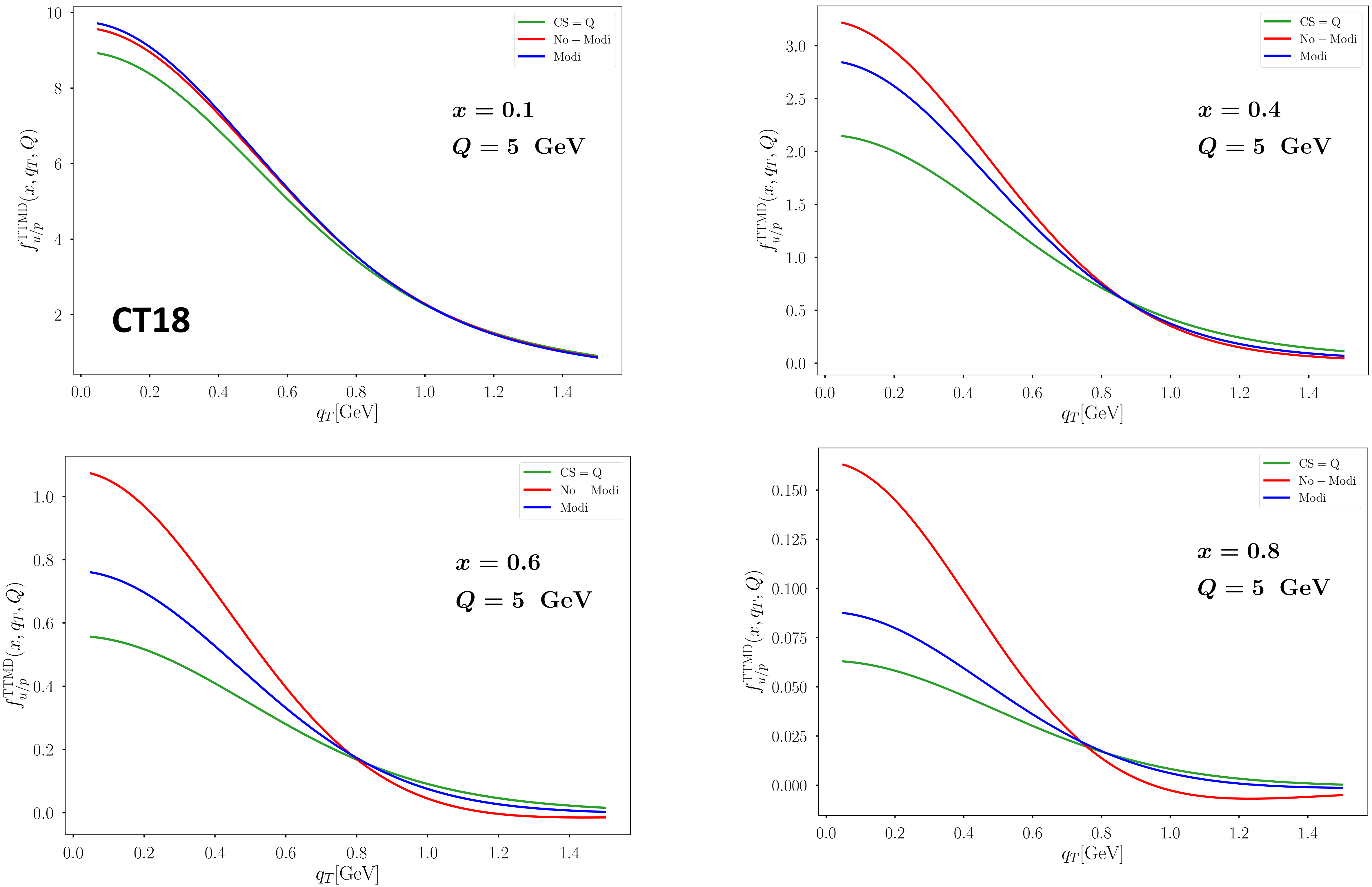
$$\zeta_* \equiv \zeta_*(\zeta_f^{\mathrm{TTMD}}, Q_0) = \left(\frac{Q}{\bar{N}}\right)^2 \left(1 + \frac{Q_0^2 \bar{N}^2}{Q^2}\right)$$

where Q_0 is given in the NP Sudakov

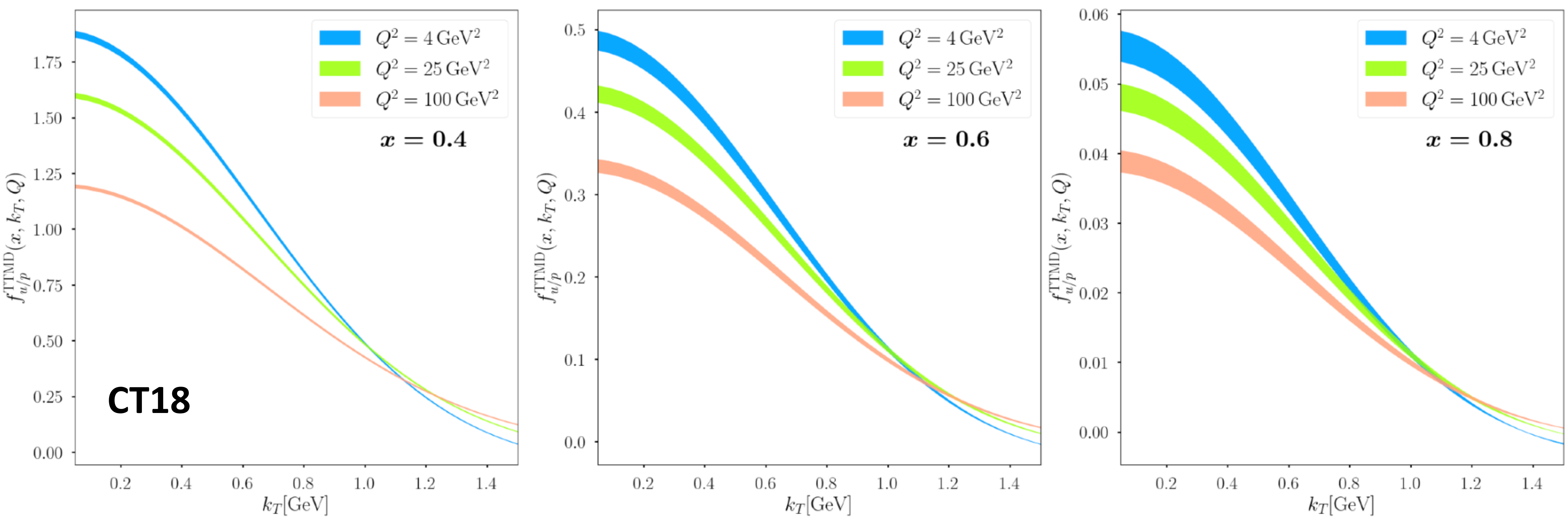
$$S_{\mathrm{NP}}\left(b_T, Q_0, \zeta_f\right) = g_1^f b_T^2 + \frac{g_2}{2} \ln\frac{\sqrt{\zeta_f}}{Q_0} \ln\frac{b_T}{b_*}$$

Numerical results for threshold-TMDPDF

TMD TTMD TTMD (ζ_*-prescription)



Numerical results for threshold-TMDPDFs

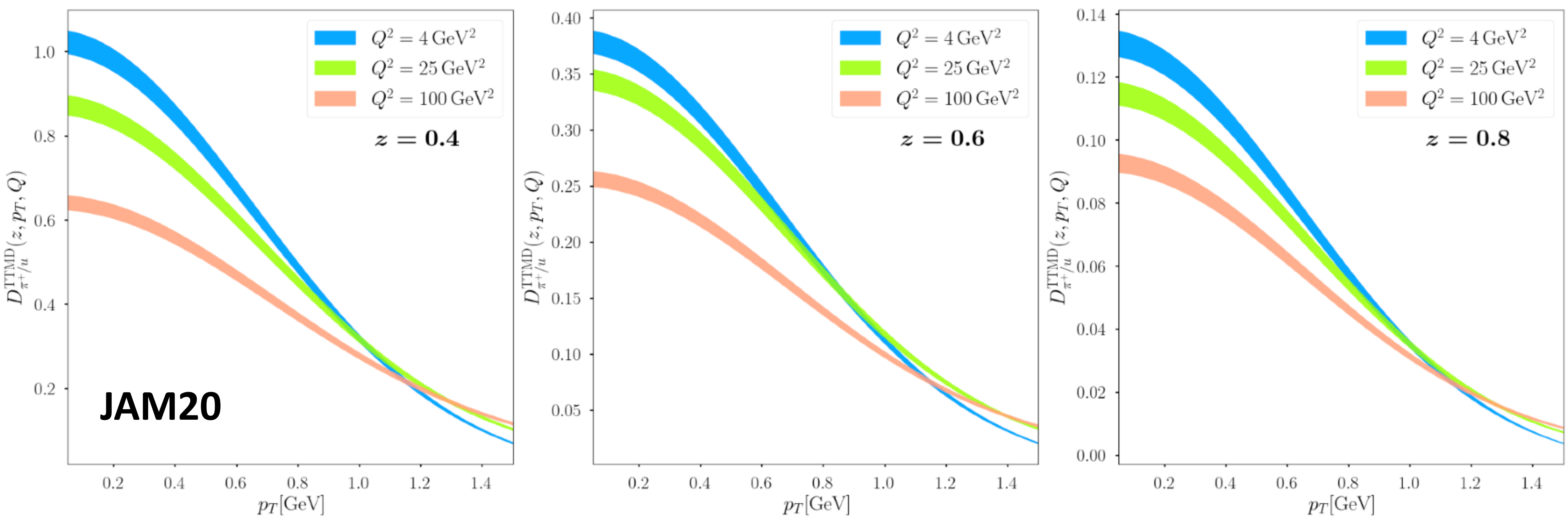


The uncertainty bands correspond to the 1-σ variation from CT18 PDFs using the Hessian method

Threshold-TMDFFs

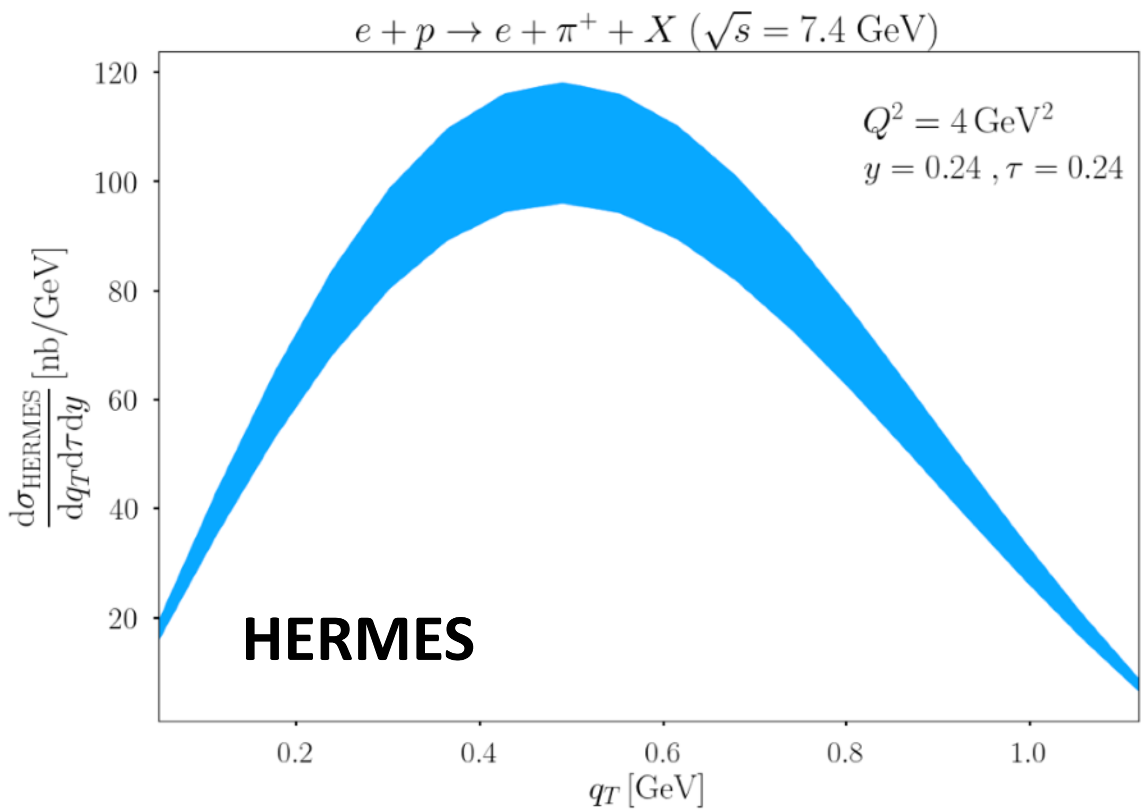
We show the universality of threshold-TMD functions among three standard processes, i.e. the Drell-Yan production in pp collisions, semi-inclusive deep-inelastic scattering and back-to-back two hadron production in e^+e^- collisions

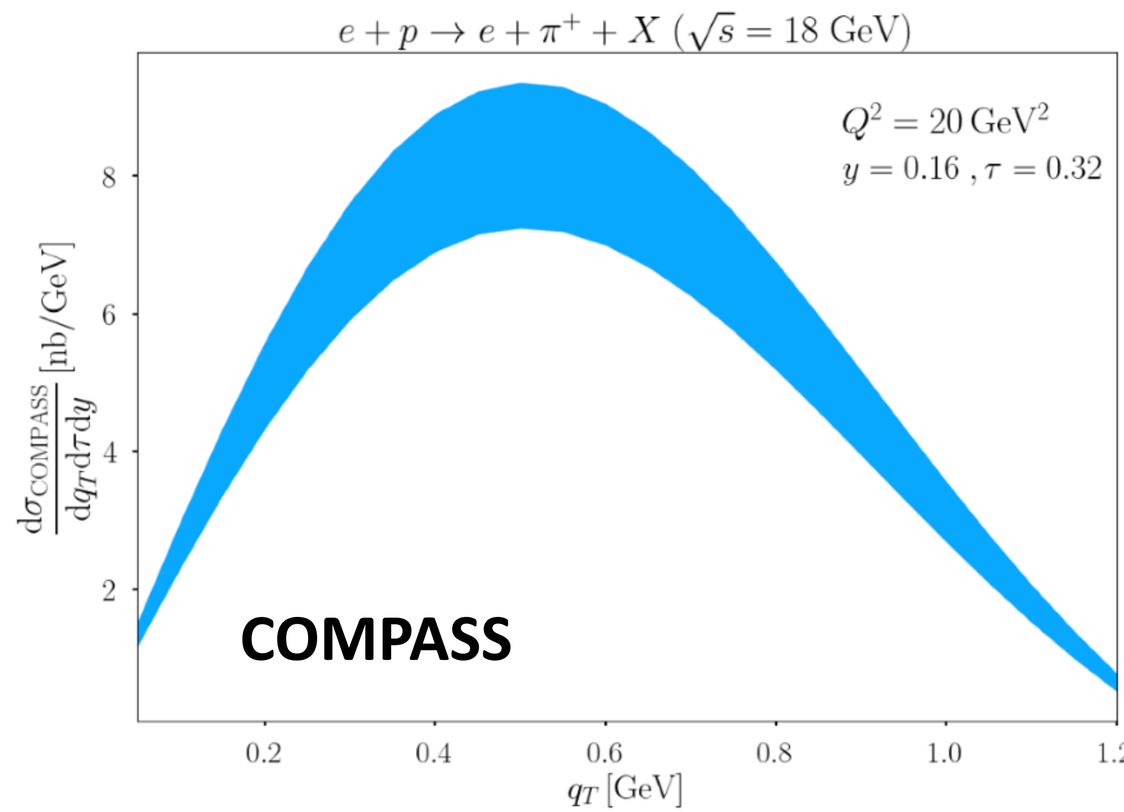
$$f_{q/p}^{\mathrm{TTMD}}(x, \boldsymbol{k}_T, Q)\big|_{\mathrm{SIDIS}} = f_{q/p}^{\mathrm{TTMD}}(x, \boldsymbol{k}_T, Q)\big|_{\mathrm{DY}},$$
$$D_{h/q}^{\mathrm{TTMD}}(z, \boldsymbol{p}_T, Q)\big|_{\mathrm{SIDIS}} = D_{h/q}^{\mathrm{TTMD}}(z, \boldsymbol{p}_T, Q)\big|_{e^+e^-}.$$

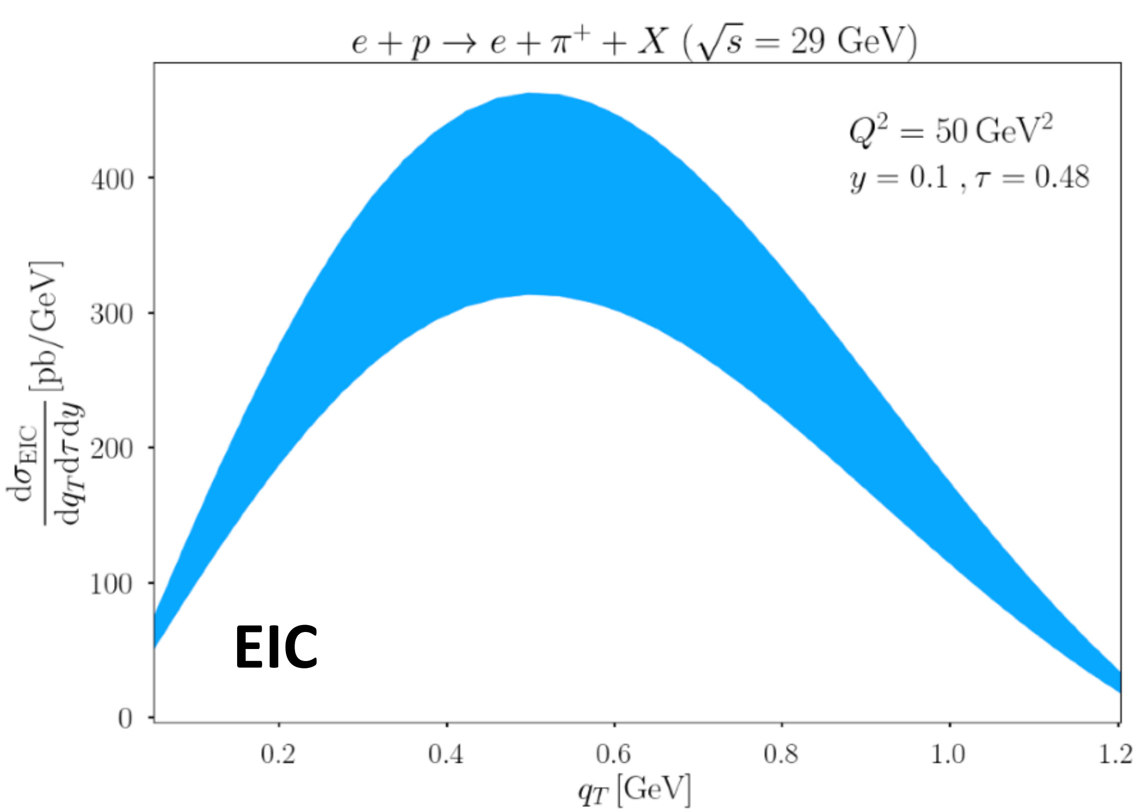


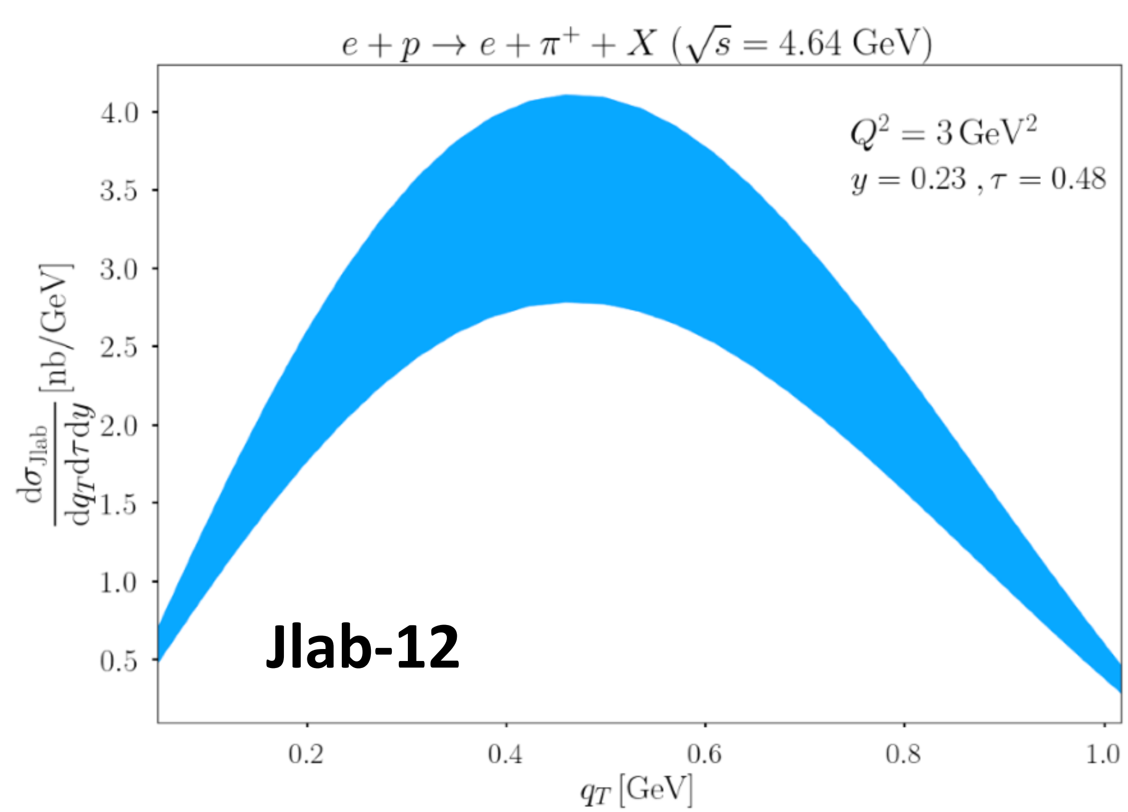
The uncertainty bands correspond to 1-σ variation of JAM FFs using the replica method.

Cross section for SIDIS









Summary and outlook

We provide a theoretical formalism for the threshold improved TMDs

In our analysis, we observe that to have a kinematic consistence result, one needs to modify Collins-Soper scale $\zeta_f^{\rm TTMD} = Q^2/N^2 \sim Q^2(1-\hat{\tau})^2$

We introduce a new ζ_* **-prescription to freeze the CS scale**

Our formalism will serve as a reliable theoretical input for extracting the TMD functions at large x value

Future experimental analysis and global fitting analysis will help in unveiling the three-dimensional picture of a hadron in the large x limit

The corresponding theoretical predictions on the spin asymmetry in the threshold limit will be explored in future work

Thank you