Some Recent Results on n-$\bar{n}$ Transitions

Robert Shrock

C. N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy,
Stony Brook University

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Outline

- Motivations
- General formalism and current experimental limits
- An illustrative model in which n-$\bar{n}$ transitions are dominant manifestation of baryon number violation (BNV)
- A corresponding model arising from a left-right symmetric theory
- n-$\bar{n}$ transitions in neutron stars
- Conclusions

This work has involved recent collaborations with S. Girmohanta, I. Goldman, R. Mohapatra, and S. Nussinov; and earlier collaboration with S. Rao (1982-84): main recent references:

- S. Girmohanta and RS, "Baryon-Number-Violating Nucleon and Dinucleon Decays in a Model with Large Extra Dimensions", Phys. Rev. D 101, 015017 (2020) [1911.05102].
- S. Girmohanta and R. Shrock, "Baryon-Number-Violating Processes in a Left-Right Symmetric Model with Large Extra Dimensions", Phys. Rev. D 101, 095012 (2020) [2003.14185].
- I. Goldman, R. N. Mohapatra, S. Nussinov, and R. Shrock, Effects of Neutron-Antineutron Transitions in Neutron Stars, Phys. Rev. Lett. in press, [2408.14555].

See also S. Nussinov and R. Shrock, Phys. Rev. Lett. 88, 171601 (2002); S. Girmohanta, R. N. Mohapatra, and R. Shrock, Phys. Rev. D 103, 015021 (2021) [arXiv:2011.01237]; Phys. Rev. D 104, 115021 (2021) [2109.02670].

Recent reviews include

A. Addazi et al., New High-Sensitivity Searches for Neutrons Converting into Antineutrons and/or Sterile Neutrons at the European Spallation Source, J. Phys. G 48, 070501 (2021) [2006.04907].

K. Babu, J. Barrow, Z. Berezhiani, L. Broussard et al., $|\Delta B| = 2$: State of the Field, and Looking Forward, [2010.0299].

J. Barrow, L. Broussard, J. Cline, P. Bhupal Dev, M. Drewes, et al., Theories and Experiments for Testable Baryogenesis: a Snowmass White Paper, [2203.07059].

Motivations

Producing the observed baryon asymmetry in the universe requires interactions that violate baryon number, B (as well as CP violation and deviation from thermal equilibrium) (Sakharov, 1967).

Suggestion of n-$\bar{n}$ transitions as a mechanism involved in generating baryon asymmetry in the universe (Kuzmin, 1970).

Standard Model (SM) conserves B perturbatively. SU(2)$_L$ instantons produce nonperturbative violation of B and lepton number, L, while conserving $B-L$ ('t Hooft, 1976). These SU(2)$_L$ instantons have a negligibly small effect at temperatures $T \ll v_{EW}$, but are important for $T \gtrsim v_{EW}$ (Kuzmin, Rubakov, Shaposhnikov, 1985).

Since (anti)quarks and (anti)leptons are placed in same representations in grand unified theories (GUT's), the violation of B and L is natural in these theories. Besides proton decay, n - $\bar{n}$ transitions can occur and may be the dominant manifestation of baryon number violation (Glashow, 1980; Mohapatra and Marshak, 1980).

Some other early work: Chang+Chang, Kuo+Love, Cowsik+Nussinov, Rao+RS,...

A continuing question about B is whether it is just a global symmetry or whether it is gauged. In the SM, B is a global symmetry, while in the left-right symmetric (LRS) theory with gauge group (Mohapatra, Marshak, Senjanović, 1975...)

$$G_{LRS} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$$

$B - L$ is gauged. Electric charge in SM: $Q_{em} = T_{3L} + (Y/2)$; in LRS theory,

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

Further embedding of $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{B-L}$ in $\mathrm{SU}(4)$ (Pati-Salam): gauge group $\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, and as $\mathrm{SO}(6) \otimes \mathrm{SO}(4)$ in SO(10) GUT.

Lepton number L is a global symmetry in the original SM. Neutrino masses and lepton mixing are confirmed physics beyond the SM; the most natural mechanism to explain light neutrino masses is the seesaw mechanism, which involves a combination of Dirac mass terms $\bar{\nu}_{iL} M_{ij}^{(D)} \nu_{j,R} + h.c.$ and Majorana mass terms $\nu_{i,R}^T C M_{ij}^{(R)} \nu_{j,R} + h.c.$; the Majorana terms break L, as $\Delta L = 2$ operators.

The occurrence of $\Delta L = 2$ operators, possibly at a low-scale, in neutrino mass models gives further motivation to explore the possibility that there might also be $\Delta B = 2$

operators at scales well below a GUT scale. This is particularly natural in models with a gauged $U(1)_{B-L}$, containing Higgs with $|B-L|=2$, whose vacuum expectation values (VEVs) thus lead to both $|\Delta L|=2$ and $|\Delta B|=2$ processes.

These are good motivations for new experimental searches for n - $\bar{n}$ transitions and associated $\Delta B=-2$ dinucleon decays as well as proton and bound neutron decay, as manifestations of baryon number violation (BNV).

Plan for future n-$\bar{n}$ search exp. at European Spallation Source, ESS Searches for n-$\bar{n}$ transitions and associated $\Delta B=-2$ dinucleon decays in deep underground detectors, most recently, Super-K and SNO; future searches at Hyper-K and DUNE.

Also interest in n-n' transitions with n' in possible mirror universe [Lee, Yang (1956), Kobzarev, Okun, Pomeranchuk (1966), Foot, Lew, Volkas (1991), Berezhiani, Mohapatra, Dolgov (1996)...); connection with beam-bottle τ_n issue; connection with dark matter; searches for n-n' transitions, $n \to n' \to n$ regeneration (Serebrov, Berezhiani, Kamyshkov, Broussard, Barrow, Milstead, Young...); astrophysical constraints on dark matter and n-n': Goldman, Mohapatra, Nussinov, Zhang, Baym et al., McKeen, Nelson, Reddy, Zhou, Pospelov, Raj, Berezhiani et al. Gardner et al., Thompson et al..).

Here we focus on n-$\bar{n}$ transitions.

General Formalism for n - $\bar{n}$ Transitions

n - $\bar{n}$ Transitions in Field-Free Vacuum:

CPT: $\langle n|H_{eff}|n\rangle = \langle\bar{n}|H_{eff}|\bar{n}\rangle = m_n - i\lambda_n/2$, where H_{eff} denotes relevant effective Hamiltonian and $\lambda_n^{-1} = \tau_n = 0.88\times 10^3$ sec. H_{eff} may also mediate $n \leftrightarrow \bar{n}$ transitions: $\langle\bar{n}|H_{eff}|n\rangle \equiv \delta m$. Consider the matrix in $(n,\bar{n})$ basis:

$$\mathcal{M} = \begin{pmatrix} m_n - i\lambda_n/2 & \delta m \\ \delta m & m_n - i\lambda_n/2 \end{pmatrix}$$

Diagonalizing $\mathcal{M}$ yields mass eigenstates

$$|n_\pm\rangle = \frac{1}{\sqrt{2}}(|n\rangle \pm |\bar{n}\rangle)$$

with mass eigenvalues $m_\pm = (m_n \pm \delta m) - i\lambda_n/2$.

So if start with pure $|n\rangle$ state at $t=0$, then there is a finite probability P for it to be an $|\bar{n}\rangle$ at $t \neq 0$. Denote $\tau_{n\bar{n}} = 1/|\delta m|$. Then

$$P(n(t)=\bar{n}) = |\langle\bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-\lambda_n t}$$

More generally, in the $(n, \bar{n})$ basis, write

$$\mathcal{M} = \begin{pmatrix} M_{11} & \delta m \\ \delta m & M_{22} \end{pmatrix}$$

Diagonalization yields mass eigenstates $|n_1\rangle$ and $|n_2\rangle$:

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}$$

where

$$\tan(2\theta) = \frac{2\delta m}{\Delta M}$$

and $\Delta M = M_{11} - M_{22}$. The energy eigenvalues are

$$E_{1,2} = \frac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(\Delta M)^2 + 4(\delta m)^2} \right]$$

Let $\Delta E = E_1 - E_2 = \sqrt{(\Delta M)^2 + 4(\delta m)^2}$; transition probability:

$$P(n(t) \to \bar{n}) = |\langle \bar{n} | n(t) \rangle|^2 = \sin^2(2\theta)\ \sin^2[(\Delta E)t/2]\, e^{-\lambda_n t}$$

$$= \left[\frac{(\delta m)^2}{(\Delta M/2)^2 + (\delta m)^2} \right] \sin^2 \left[\sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t \right] e^{-\lambda_n t}$$

In realistic free-neutron experiment, $|\Delta M| \gg |\delta m|$, due to ambient magnetic field, but the exp. achieves sensitivity to δm by arranging that $[(1/2)\Delta E]t \ll 1$, i.e.,

$$[(\Delta M/2)^2 + (\delta m)^2]^{1/2}\, t \ll 1\ ,$$

then by Taylor-expanding the sine squared, the quantity $(\Delta M/2)^2 + (\delta m)^2$ cancels, so in this case

$$P(n(t) \to \bar{n}) \simeq [(\delta m)t]^2\, e^{-\lambda_n t} = (t/\tau_{n\bar{n}})^2\, e^{-\lambda_n t}$$

n - $\bar{n}$ Transitions in a Magnetic Field $\vec{B}$:

Even with magnetic shielding, the neutrons in a free-neutron exp. searching for $n - \bar{n}$ transitions are subject to a nonzero external magnetic field $\vec{B}$ due to the earth. The n and $\bar{n}$ interact with $\vec{B}$ via magnetic moment $\vec{\mu} = \mu\,\vec{\sigma}$, $\mu_n = -\mu_{\bar{n}} = \kappa\mu_N$, where $\kappa = -1.91$, $\mu_N = e/(2m_N) = 3.15 \times 10^{-14}$ MeV/Tesla, so

$$\mathcal{M} = \begin{pmatrix} m_n - \vec{\mu}_n \cdot \vec{B} - i\lambda_n/2 & \delta m \\ \delta m & m_n + \vec{\mu}_n \cdot \vec{B} - i\lambda_n/2 \end{pmatrix}$$

So $\Delta M = M_{11} - M_{22} = -2\vec{\mu}_n \cdot \vec{B}$ and diagonalization yields mass eigenstates $|n_1\rangle$, $|n_2\rangle$, with energy eigenvalues

$$E_{1,2} = m_n \pm \sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} - i\lambda_n/2$$

ILL experiment reduced $|\vec{B}| = B$ to $\sim 10^{-4}\ \mathrm{G} = 10^{-8}$ T, so

$$|\mu_n|B = (6.03 \times 10^{-22}\ \mathrm{MeV}) \left(\frac{B}{10^{-8}\ \mathrm{T}}\right)$$

Now $|\delta m| \lesssim 10^{-30}$ from Super-K, so $|\delta m| \ll |\mu_n|B$, and

$$\Delta E = 2\sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} \simeq 2|\vec{\mu}_n \cdot \vec{B}|$$

Experimentally, arrange that n's propagate a time t such that $[(1/2)\Delta E]t \ll 1$, i.e,

$$|\vec{\mu}_n \cdot \vec{B}|t = 0.92 \left(\frac{B}{10^{-8}\ \mathrm{T}}\right)\left(\frac{t}{1\ \mathrm{sec}}\right) \ll 1 \quad \text{and } t \ll \tau_n$$

Then the exp. is sensitive to δm

$$P(n(t) \to \bar{n}) \simeq [(\delta m)\, t]^2 = (t/\tau_{n\bar{n}})^2$$

Denoting the total number of neutrons measured as N_n, the resultant total number of $\bar{n}$'s produced in an exp. is

$$N_{\bar{n}} = P(n(t) \to \bar{n})\ N_n$$

Here, $N_n = \phi T_{run}$ where $\phi =$ is the neutron flux and $T_{run} =$ the exp. running time.

The sensitivity of exp. depends in part on the product

$$N_n \left(\frac{t}{\tau_{n\bar{n}}}\right)^2 = \phi T_{run} \left(\frac{t}{\tau_{n\bar{n}}}\right)^2$$

so, with adequate magnetic shielding, want to maximize t, subject to condition $|\vec{\mu}_n \cdot \vec{B}|t \ll 1$

Most sensitive reactor n - $\bar{n}$ exp. done with ILL High Flux Reactor (HFR) at Grenoble (Baldo-Ceolin, Fidecaro,.., 1985-1994; M. Baldo-Ceolin et al., Z. Phys. C63, 409 (1994))) with neutrons cooled to liquid D_2 temp., kinetic energy $E \simeq 2 \times 10^{-3}$ eV, typical velocity $v \simeq 700$ m/s, $L \simeq 80$ m, $t \simeq 0.1$ sec., $\phi \sim 1.25 \times 10^{11}$ n/s, so $\phi t^2 = 1.5 \times 10^9 \ \mathrm{n \cdot s}$; set limit

$$\tau_{n\bar{n}} \geq 0.86 \times 10^8 \ \mathrm{sec} \quad (90\ \%\ CL)$$

i.e., $|\delta m| = \hbar/\tau_{n\bar{n}} = (6.58 \times 10^{-22} \ \mathrm{MeV \cdot s})/\tau_{n\bar{n}} \leq 0.77 \times 10^{-29}$ MeV.

In general, $|\delta m| = (0.658 \times 10^{-29} \ \mathrm{MeV})(10^8 \ \mathrm{s}/\tau_{n\bar{n}})$.

n - $\bar{n}$ Transitions in Matter: For n - $\bar{n}$ transitions involving a neutron bound in a nucleus, consider

$$\mathcal{M} = \begin{pmatrix} m_{n,eff.} & \delta m \\ \delta m & m_{\bar{n},eff.} \end{pmatrix}$$

$$m_{n,eff} = m_n + V_n \ , \quad m_{\bar{n},eff.} = m_n + V_{\bar{n}}$$

where the nuclear potential V_n is real, $V_n = V_{nR}$, but $V_{\bar{n}}$ has an imaginary part representing the $\bar{n}N$ annihilation: $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$ with $V_{nR}, \ V_{\bar{n}R}, \ V_{\bar{n}I} \sim O(100)$ MeV (Dover, Gal, Richard; Friedman; recently work by Barrow, Golubeva, Richard.. Oosterhof, de Vries, van Kolck et al.; Haidenbauer, Meissner; Syritsyn, Wagman, et al.; talks by van Kolck (EFT), Wagman (LQCD)).

Mixing is thus strongly suppressed; $\tan(2\theta)$ is determined by

$$\frac{2\delta m}{|m_{n,eff.} - m_{\bar{n},eff.}|} = \frac{2\delta m}{\sqrt{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}} \ll 1$$

Using the reactor exp. bound on $|\delta m|$, this gives $|\theta| \lesssim 10^{-31}$. This suppression in mixing is compensated by the large number of nucleons in a nucleon decay detector, $\sim 10^{33}$ n's in Super-K.

Eigenvalues:

$$m_{1,2} = \frac{1}{2}\left[m_{n,eff.} + m_{\bar{n},eff.} \pm \sqrt{(m_{n,eff.} - m_{\bar{n},eff.})^2 + 4(\delta m)^2}\;\right]$$

Expanding m_1 for the mostly n mass eigenstate $|n_1\rangle \simeq |n\rangle$,

$$m_1 \simeq m_n + V_n - i\frac{(\delta m)^2\, V_{\bar{n}I}}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

Imaginary part leads to matter instability, mainly via $\bar{n}n,\ \bar{n}p \to \pi$'s, with rate

$$\Gamma_{m.i.} = \frac{1}{\tau_{m.i.}} = \frac{2(\delta m)^2|V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

So $\tau_{m.i.} \propto (\delta m)^{-2} = \tau_{n\bar{n}}^2$.

Writing $\tau_{m.i.} = R\,\tau_{n\bar{n}}^2$, one has $R \sim O(100)$ MeV, dependent on nucleus.

With $\hbar = 6.6 \times 10^{-22}$ MeV-sec, equiv. $R \sim 10^{23}$ sec^{-1}.

Searches for matter instability due to n-$\bar{n}$ transitions with large nucleon decay detectors are complementary to searches with free neutrons at reactors or spallation sources. Searches for matter instability due to n-$\bar{n}$ transitions were performed most recently by Soudan, Super-K, and SNO experiments.

Generic signature is a multipion final state resulting from the annihilation of the $\bar{n}$ with a neighboring neutron or proton.

A lower bound on $\tau_{m.i.}$ yields a lower bound on $\tau_{n\bar{n}}$ via $\tau_{n\bar{n}} = (\tau_{m.i.}/R)^{1/2}$. Current best bound is from Super-Kamiokande, $\tau_{m.i.} > 3.6 \times 10^{32}$ yrs, giving

$\tau_{n\bar{n}} > 4.7 \times 10^8$ sec (90 % CL) - Linyan Wan talk

[K. Abe,.. L. Wan,.. et al., Phys. Rev. D 103, 012008 (2021)] and thus

$$|\delta m| = \frac{1}{\tau_{n\bar{n}}} < 1.4 \times 10^{-30}\ \mathrm{MeV}$$

The future $n - \bar{n}$ search experiment at ESS should significantly improve this limit or see a signal (Womersley talk).

n - $\bar{n}$ Transitions in an Extra-Dimensional Model

We discuss a model in which proton decay can easily be suppressed well below experimental limits while n-$\bar{n}$ transitions can occur at level comparable to existing limits (work with Nussinov and Girmohanta, op. cit.)

Consider a model with a $d = 4 + n$ dimensional spacetime, with n extra spatial dimensions. Denote usual spacetime coords. as x_ν, $\nu = 0, 1, 2, 3$ and consider n extra compact coordinates, y_λ with $0 \le y_\lambda \le L$, i.e., size of extra dimension(s) is L. Each SM fermion f has the form

$$\Psi_f(x, y) = \psi_f(x)\chi_f(y)$$

with strong localization at a point y_f in the extra dimensions, with a Gaussian profile of half-width $\sigma \equiv 1/\mu \ll L$:

$$\chi_f(y) = Ae^{-\mu^2\|y-y_f\|^2} = Ae^{-\|\eta-\eta_f\|^2}$$

where $\|y_f\| = (\sum_{\lambda=1}^{n} y_{f,\lambda}^2)^{1/2}$, A is a normalization constant, and we define a convenient dimensionless variable $\eta_f = \mu y_f$.

Such models are of interest because they can provide a mechanism for a generational hierarchy in fermion masses and quark mixing.

We use a low-energy effective field theory (EFT) approach with an ultraviolet cutoff M_*, where $M_* > \mu$ for self-consistency. Only the lowest mode in the Kaluza-Klein (KK) mode decompositions of each Ψ field will be needed here; effects of higher modes are considered in our papers.

Starting from the Lagrangian in the d-dimensional spacetime, one obtains the resultant low-energy EFT in 4D by integrating over the extra n dimension(s). For canonical normalization of the 4D fermion kinetic term,

$$A = \left(\frac{2}{\pi}\right)^{n/4} \mu^{n/2}$$

The localization is achieved by coupling to auxiliary “localizer” scalar fields with kink form for $n = 1$, and similarly for higher n (Arkani-Hamed + Schmaltz; Mirabelli+Schmaltz; Grossman+Perez); Higgs fields are taken flat in extra dims. As in these refs., a full UV completion is not specified.

Define $\Lambda_L \equiv 1/L$; take $\Lambda_L \sim 10^2$ TeV, $\sigma \equiv 1/\mu \sim L/30$; this gives adequate separation of fermions while fitting in interval $[0, L]$, consistent with precision electroweak data, collider bounds, flavor-changing neutral current constraints. Corresponding compactification length: $L = 2 \times 10^{-19}$ cm.

With $\Lambda_L = 10^2$ TeV, this yields $\mu \sim 3 \times 10^3$ TeV.

This extra-dimensional model (ED) is quite different from ED models with low quantum gravity scales (Arkani-Hamed, Dimopoulos, Dvali; Dienes, Dudas, Gherghetta), as is clear from the fact that, e.g., for $n = 2$ and quantum gravity scale of 30 TeV, the ADD-DDG models have a compactification size $\sim 3 \times 10^{-4}$ cm., much larger than the scale $L \simeq 2 \times 10^{-19}$ cm in the ED model that we use.

Given the localization of fermion wavefunctions on scale $\sigma \ll L$, in the integration over the extra dimensions, can extend $\int_0^L \longrightarrow \int_{-\infty}^{\infty}$ to good approximation.

Integrals over extra dimensions have the general form (with $\int d^n\eta = \int_{-\infty}^{\infty} d^n\eta$)

$$\int d^n\eta \, \exp\Big[-\sum_{i=1}^{m} a_i \|\eta - \eta_{f_i}\|^2\Big] = \left[\frac{\pi}{\sum_{i=1}^m a_i}\right]^{n/2} \exp\left[\frac{-\sum_{j,k=1;\ j<k}^m a_j a_k \|\eta_{f_j} - \eta_{f_k}\|^2}{\sum_{s=1}^m a_s}\right]$$

For example, for $m = 3$,

$$\int d^n\eta \, \exp\Big[-\Big(a_1\|\eta - \eta_{f_1}\|^2 + a_2\|\eta - \eta_{f_2}\|^2 + a_3\|\eta - \eta_{f_3}\|^2\Big)\Big] =$$

$$= \left[\frac{\pi}{a_1 + a_2 + a_3}\right]^{n/2} \exp\left[\frac{-\Big(a_1 a_2\|\eta_{f_1} - \eta_{f_2}\|^2 + a_2 a_3\|\eta_{f_2} - \eta_{f_3}\|^2 + a_3 a_1\|\eta_{f_3} - \eta_{f_1}\|^2\Big)}{a_1 + a_2 + a_3}\right]$$

If only one fermion involved in integrand, then no exponential suppression:

$$\int d^n\eta \, \exp\Big[-a_1\|\eta - \eta_{f_1}\|^2\Big] = \left[\frac{\pi}{a_1}\right]^{n/2}$$

A Yukawa interaction in the d-dimensional space with coefficients of order unity and moderate separation of localized fermion wavefunction centers yields a strong hierarchy in the low-energy 4D Yukawa interaction,

$$\int d^n y \, \bar{\chi}(y_{f_L}) \chi(y_{f_R}) \sim \int d^n \eta \, e^{-\|\eta - \eta_{f_L}\|^2} e^{-\|\eta - \eta_{f_R}\|^2} \sim e^{-(1/2)\|\eta_{f_L} - \eta_{f_R}\|^2}$$

Resultant fermion masses m_f:

$$m_f \simeq h^{(f)} \frac{v}{\sqrt{2}} \exp\Big[-\frac{1}{2} \|\eta_{f_L} - \eta_{f_R}\|^2 \Big] \, ,$$

where $v/\sqrt{2}$ is SM Higgs VEV. With $h^{(f)} \simeq 1$, produce fermion generational hierarchy via different separation distances $\|\eta_{f_L} - \eta_{f_R}\|$ for different generations.

Leading nucleon decay operators are of the form $qqq\ell$. Hence, one can suppress nucleon decay well below experimental limits by arranging that the wavefunction centers of the u and d quarks are separated far from those of the leptons.

Key point: this does not suppress n - $\bar{n}$ transitions because the n-$\bar{n}$ transition operators do not involve leptons.

For example, one nucleon decay operator is (with $\ell = e,\ \mu$)

$$\mathcal{O}_1^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha\ T} C d_R^\beta][u_R^{\gamma\ T} C \ell_R]$$

where $\alpha,\ \beta,\ \gamma$ are SU(3)$_c$ color indices.

The product of y-dependent fermion wavefunctions in this operator is

$$A^4 \exp\Big[-\Big\{2\|\eta - \eta_{u_R}\|^2 + \|\eta - \eta_{d_R}\|^2 + \|\eta - \eta_{\ell_R}\|^2\Big\}\Big]$$

The integral over y yields

$$I_1^{(Nd)} = b_4 \ \exp\Big[-\frac{1}{4}\Big\{2\|\eta_{u_R} - \eta_{d_R}\|^2 + 2\|\eta_{u_R} - \eta_{\ell_R}\|^2 + \|\eta_{d_R} - \eta_{\ell_R}\|^2\Big\}\Big]$$

where $b_4 = (\mu/\sqrt{\pi})^n$.

One can guarantee that this is sufficiently small by taking the distances between wavefunction centers $\|\eta_{u_R} - \eta_{\ell_R}\|$ and/or $\|\eta_{d_R} - \eta_{\ell_R}\|^2$ sufficiently large.

Similarly for other nucleon decay operators.

At the quark level $n \to \bar{n}$ is $(udd) \to (u^c d^c d^c)$. This is mediated by 6-quark operators $\mathcal{O}_r^{(n\bar{n})} \sim uddudd$.

In $d = 4$ dims., effective Lagrangian for the $n - \bar{n}$ transition is

$$\mathcal{L}_{eff}^{(n\bar{n})}(x) = \sum_r c_r^{(n\bar{n})} \, \mathcal{O}_r^{(n\bar{n})}(x) + h.c.$$

Correspondingly, in $d = 4 + n$ dimensions,

$$\mathcal{L}_{eff,4+n}^{(n\bar{n})}(x, y) = \sum_r \kappa_r^{(n\bar{n})} O_r^{(n\bar{n})}(x, y) + h.c.$$

where the $\mathcal{O}_r^{(n\bar{n})}(x)$ and $O_r^{(n\bar{n})}(x, y)$ are 6-quark operators in $d = 4$ and $d = 4 + n$ dims.

In d-dimensional spacetime the dimension of a fermion field ψ in mass units is $\dim(\psi) = (d - 1)/2$, so $\dim(\mathcal{O}_r^{(n\bar{n})}) = 6d_\psi = 3(d - 1)$ and

$$\dim(\kappa_r) = d - \dim(\mathcal{O}_r^{(n\bar{n})}) = 3 - 2d = 3 - 2(4 + n) = -(5 + 2n)$$

So the coefficients κ_r have the form

$$\kappa_r^{(n\bar{n})} = \frac{\bar{\kappa}_r^{(n\bar{n})}}{M_{n\bar{n}}^{5+2n}}$$

where $\bar{\kappa}_r^{(n\bar{n})}$ are dimensionless and $M_{n\bar{n}}$ is the effective mass characterizing the physics responsible for the n-$\bar{n}$ transition. We can set $\bar{\kappa}_r^{(n\bar{n})} = 1$ for the dominant $O_r^{(n\bar{n})}$ in defining $M_{n\bar{n}}$.

Integration of fermion wavefunctions in the $O_r^{(n\bar{n})}(x, y)$ over y yield the coeffs. $c_r^{(n\bar{n})}$ in terms of $\kappa_r^{(n\bar{n})}$

Operators $\mathcal{O}_r^{(n\bar{n})}$ must be color singlets and, for $M_{n\bar{n}}$ larger than the electroweak symmetry breaking scale, also $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$-singlets. Relevant operators in SM EFT ($C = i\gamma^2\gamma^0$; $C^T = -C$)

$$\mathcal{O}_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_2^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

$$\mathcal{O}_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}\epsilon_{km}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][d_R^{\rho T} C d_R^{\sigma}]$$

where $Q_L = \binom{u}{d}_L$, $i, j..$ are SU(2)$_L$ indices, and color tensors are

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$$

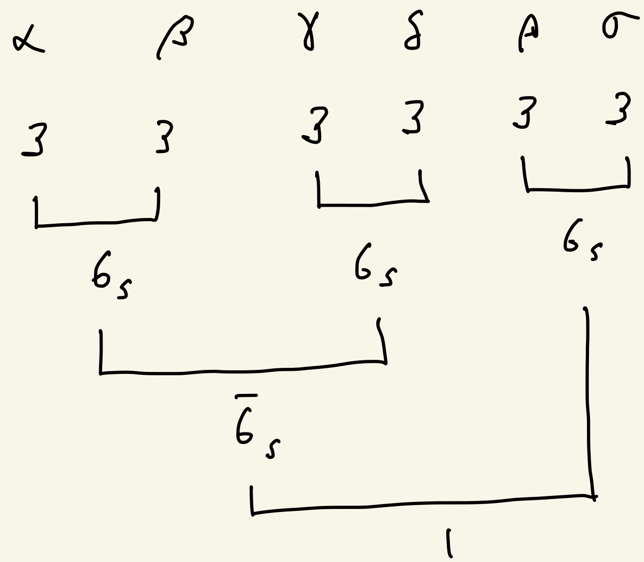
$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$ is symmetric in the indices $(\alpha\beta)$, $(\gamma\delta)$, $(\rho\sigma)$.

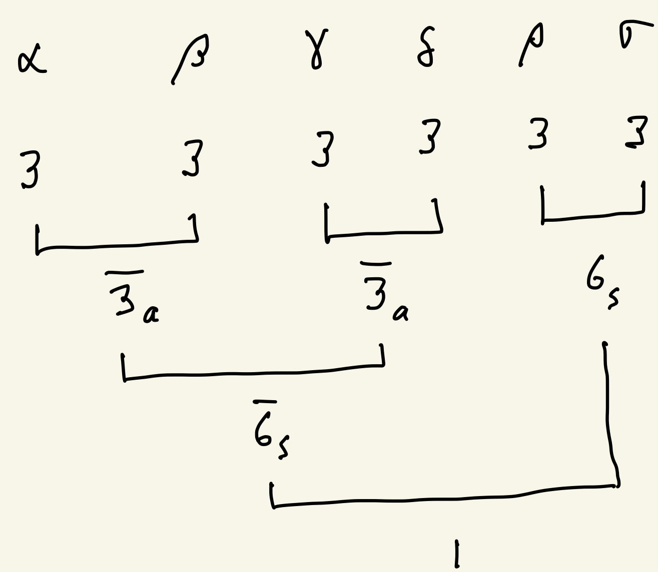
$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$ is antisymmetric in $(\alpha\beta)$ and $(\gamma\delta)$ and symmetric in $(\rho\sigma)$.

$SU(3)_c$ color contractions

$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$



$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$



The integrals of these operators over y comprise three classes: operators $O_1^{(n\bar{n})}$ and $O_2^{(n\bar{n})}$ yield the integral

$$I_{r12}^{(n\bar{n})} = b_6 \, \exp\left[-\frac{4}{3}\|\eta_{u_R} - \eta_{d_R}\|^2 \right]$$

$O_3^{(n\bar{n})}$ yields the integral

$$I_{r3}^{(n\bar{n})} = b_6 \, \exp\left[-\frac{1}{6}\Big\{ 2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 6\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2 \Big\} \right]$$

$O_4^{(n\bar{n})}$ yields the integral

$$I_{r4}^{(n\bar{n})} = b_6 \, \exp\left[-\frac{4}{3}\|\eta_{Q_L} - \eta_{d_R}\|^2 \right]$$

where $b_6 = (2 \cdot 3^{-1/2} \, \pi^{-1} \mu^2)^n$.

The coeffs. $c_r^{(n\bar{n})} = \bar{\kappa}_r^{(n\bar{n})}/(M_{n\bar{n}})^5$ times these $I_r^{(n\bar{n})}$ integrals.

Consider, e.g., case $n = 2$: one can fit data on quark masses, mixing with

$$\|\eta_{Q_L} - \eta_{u_R}\| = 4.75, \quad \|\eta_{Q_L} - \eta_{d_R}\| \simeq 4.60$$

$$\|\eta_{u_R} - \eta_{d_R}\| \simeq 7$$

We find that the $|c_r^{(n\bar{n})}|$ for $r = 1, 2, 3$ are $\ll |c_4^{(n\bar{n})}|$, and hence we focus on $c_4^{(n\bar{n})}$:

To leading order (neglecting small CKM mixings), $\|\eta_{Q_L} - \eta_{d_R}\|$ is determined by m_d via relation (with Higgs vev $v = 246$ GeV)

$$m_d = h_d \frac{v}{\sqrt{2}}$$

with

$$h_d = h_{d,0} \exp[-(1/2)\|\eta_{Q_L} - \eta_{d_R}\|^2]$$

where $h_{d,0}$ is the Yukawa coupling in $(4 + n)$-dims. so that

$$\exp\left[-(1/2)\|\eta_{Q_L} - \eta_{d_R}\|^2\right] = \frac{2^{1/2} m_d}{h_{d,0} v}$$

With $h_{d,0} \sim 1$

$$\delta m \simeq c_4^{(n\bar{n})} \langle \bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle \simeq \left(\frac{4\mu^4}{3\pi^2 M_{n\bar{n}}^9}\right) \left(\frac{2^{1/2} m_d}{v}\right)^{8/3} \langle \bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle$$

Requiring that $\tau_{n\bar{n}} = 1/|\delta m|$ agree with the lower limit from Super-K, $\tau_{n\bar{n}} > 4.7 \times 10^8$ sec. yields the lower bound on the mass scale of $n - \bar{n}$ transitions:

$$M_{n\bar{n}} > (51 \text{ TeV})\left(\frac{\tau_{n\bar{n}}}{4.7 \times 10^8 \text{ sec}}\right)^{1/9} \left(\frac{\mu}{3 \times 10^3 \text{ TeV}}\right)^{4/9} \left(\frac{|\langle \bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle|}{\Lambda_{QCD}^6}\right)^{1/9}$$

where $\Lambda_{QCD} = 0.25$ GeV. This bound is not very sensitive to the precise size of $\langle \bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle$ because of the 1/9 power in the exponent.

$\mathcal{O}_4^{(n\bar{n})} = -Q_3$ in notation of lattice QCD calculation (Rinaldi, Syritsyn, Wagman, Buchoff, Schroeder, Wasem, 2019), with LQCD matrix element $|\langle \bar{n}|Q_3|n\rangle| \simeq 5 \times 10^{-4}$ GeV$^6 = 2\Lambda_{QCD}^6$; substituting this yields factor $2^{1/9} = 1.08$ so lower bound is (1.08)51 TeV = 55 TeV.

Hence, for relevant values of $M_{n\bar{n}}$ in this model, $n - \bar{n}$ transitions could occur at levels that are close to the current limit.

This model also illustrates how baryon number violation can occur via $n - \bar{n}$ transitions with strongly suppressed proton decay.

With SM fermion wavefunction centers chosen to suppress BNV nucleon decays adequately in this model, an interesting question is what are the predictions for other $\Delta B = -1$ nucleon and $\Delta B = -2$ dinucleon decays, including

- (i) the $\Delta L = -3$ nucleon decays $p \to \ell^+ \bar{\nu}\bar{\nu}'$ and $n \to \bar{\nu}\bar{\nu}'\bar{\nu}''$
- (ii) the $\Delta L = 1$ nucleon decays $p \to \ell^+ \nu\nu'$ and $n \to \bar{\nu}\nu'\nu''$
- (iii) the $\Delta L = -2$ dinucleon decays $pp \to (e^+e^+, \mu^+\mu^+,\ e^+\mu^+,\ e^+\tau^+,\ \text{or}\ \mu^+\tau^+)$, $np \to \ell^+\bar{\nu}$, and $nn \to \bar{\nu}\bar{\nu}'$, where $\ell^+ = e^+,\ \mu^+$, or τ^+;
- (iv) the $\Delta L = 2$ dineutron decays $nn \to \nu\nu'$.

The decays of type (i) and (ii) are mediated by 6-fermion operators, while the decays of type (iii) and (iv) are mediated by 8-fermion operators. In S. Girmohanta and RS, PRD 101, 015017 (2020) we show that the predictions of the model are in accord with experimental constraints.

n-$\bar{n}$ Transitions in an Extra-Dimensional Model with G_{LRS} Gauge Group

We have also studied n-$\bar{n}$ transitions in an extra-dimensional model with the gauge group $G_{LRS} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$

in Girmohanta + RS, PRD 101, 095012 (2020).

This model provides a useful contrast to the previous study because in the SM the n-$\bar{n}$ transitions do not break the SM gauge symmetry, while in the LRS model, they occur via the breaking of the U(1)$_{B-L}$ gauge symmetry.

Recall field content of LRS model (Mohapatra, Pati, Senjanović, 1975...) for fermions (first gen.):

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L : \ (3,2,1)_{1/3,L} \ , \quad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R : \ (3,1,2)_{1/3,R}$$

$$L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L : \ (1,2,1)_{-1,L} \ , \quad L_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R : \ (1,1,2)_{-1,R} \ ,$$

Higgs sector:

$$\Phi : (1,2,2)_0 : \quad \Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \ .$$

$\Delta_L : (1,3,1)_2, \quad \Delta_R : (1,1,3)_2$

$$\Delta_{L,R} = \begin{pmatrix} \Delta_{L,R}^+/\sqrt{2} & \Delta_{L,R}^{++} \\ \Delta_{L,R}^0 & -\Delta_{L,R}^+/\sqrt{2} \end{pmatrix} \ ,$$

Minimization of Higgs potential yields VEVs

$$\langle \Phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 e^{i\theta_\Phi} \end{pmatrix} \ ,$$

$$\langle \Delta_L \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_\Delta} & 0 \end{pmatrix}$$

$$\langle \Delta_R \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \ .$$

At highest scale, v_R breaks $\mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \longrightarrow \mathrm{U}(1)_Y$ with $|\Delta(B-L)| = 2$. This naturally yields $n - \bar{n}$ transitions and connects them with the Majorana neutrino mass generation. So in this model,

$$M_{n\bar{n}} = v_R$$

At electroweak level, $\kappa,\ \kappa'$ break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_{em}$. Take $v_L \ll \kappa,\ \kappa'$ to preserve $\rho = 1$ where $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$.

As in the SM EFT, nucleon decay can be suppressed well below experimental limits by separating the wavefunction centers of the quarks from those of the leptons.

Since the adjoint rep. of SU(2) is the rank-2 symmetric tensor, can write Δ_L as $(\Delta_L)^{ij}$ and Δ_R as $(\Delta_R)^{i'j'}$, where i, j are SU(2)$_L$ indices and i', j' are SU(2)$_R$ indices.

$O_r^{(n\bar{n})}$ operators:

$$O_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}\,(\epsilon_{i'k'}\epsilon_{j'm'} + \epsilon_{j'k'}\epsilon_{i'm'})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$

$$\times\ [Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_2^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{i'j'}\epsilon_{k'm'}\,(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$

$$\times\ [Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{k'm'}\,(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\,\epsilon_{ij}\epsilon_{km}\,(\epsilon_{p'r'}\epsilon_{q's'}+\epsilon_{q'r'}\epsilon_{p's'})\,[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

$$O_5^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}(\epsilon_{ik}\epsilon_{jm} + \epsilon_{jk}\epsilon_{im})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})\times$$

$$\times\ [Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}]\,(\Delta_R^\dagger)^{r's'}$$

After symmetry breaking of U(1)$_{B-L}$, replace Δ_R by VEV, v_R.

In the same way as before, we obtain the low-energy 4D EFT by integrating the operator products over the n extra dimensions.

Because $O_1^{(n\bar{n})}$ and $O_2^{(n\bar{n})}$ involve only one kind of fermion field (namely, Q_R), we find that for these two operators the integral over y does not yield any exponential (Gaussian) suppression factor. Coeffs. $\bar{\kappa}_r^{(n\bar{n})}$ can naturally be $\sim$ O(1) in the model for these operators.

This is in contrast to the SM EFT, where the integrals of all $n-\bar{n}$ operators involved exponential suppression factors.

Because of this, the constraint that this model should agree with the experimental lower limit on $\tau_{n\bar{n}}$ imposes a more stringent lower bound on the scale $M_{n\bar{n}}$ in this model than in the SM EFT analysis:

$$M_{n\bar{n}} \gtrsim \max\Bigg[(1\times 10^3 \text{ TeV})\Big(\frac{\tau_{n\bar{n}}}{4.7\times 10^8 \text{ sec}}\Big)^{1/9} \times \Big(\frac{\mu}{3\times 10^3 \text{ TeV}}\Big)^{4/9}\Bigg(\frac{|\bar{\kappa}_r^{(n\bar{n})}\, \langle \bar{n}|\mathcal{O}_r^{(n\bar{n})}|n\rangle|}{\Lambda_{QCD}^6}\Bigg)^{1/9}\Bigg], \quad r=1,2$$

Other models can predict n-$\bar{n}$ transitions near to current limits (Mohapatra + Marshak, 1980; Rao + RS, 1984; Babu + Mohapatra, 2001; Babu, Bhupal Dev, Mohapatra... 2006-present (post-sphaleron baryogenesis); Wise et al. (2013); Barrow et al. (2022)... others... In several of these models, nucleon decay is absent or suppressed so that n-$\bar{n}$ oscillations are main manifestation of BNV.

Among $\Delta B = -2$ processes, in the context of a given EFT (not dependent on an extra-dimensional model framework), one can relate limits on $\Delta B = -2$ dinucleon decays dilepton final states, as discussed in:

S. Girmohanta and RS, Phys. Lett. B 803, 135296 (2020) [arXiv:1910.08356]

S. Nussinov and RS, Phys. Rev. D 102, 035003 (2020) [arXiv:2005.12493]

Similar technique applied to $\Delta B = -1$ nucleon decays to multi-lepton final states:

S. Girmohanta and RS, Phys. Rev. D 100, 115025 (2019) [arXiv:1910.08106]

n-$\bar{n}$ Transitions in Neutron Stars

It is of interest to investigate effects of n-$\bar{n}$ transitions in a neutron star (NS). An early estimate by Buchella, Gualdi, and Orlandini, Nuovo Cim. 100, 809 (1987) found this to be negligible. In 2405.08591, using the same framework without any assumptions for dark matter, n', etc., Fu, Ge, Guo, and Wang claimed a larger effect by many orders of magnitude, $M_{n\bar{n}} \gg M_{GUT}$. If correct, this claim would remove the justification for further terrestrial n-$\bar{n}$ search experiments.

This motivated a reanalysis in the same framework and we have done this in Goldman, Mohapatra, Nussinov, and RS, Phys. Rev. Lett. in press, 2408.14555. We find the effect to be negligible and strengthen the upper bound on NS heating obtained by Buchella et al. by $\sim 10^{-5}$.

Recall some basic properties of neutron stars. These originate as remnants of supernova. The Fermi energy of degenerate electrons becomes sufficiently high that the reaction $e + p \to \nu_e + n$ takes place, producing a compact object consisting mainly of a degenerate Fermi sea of neutrons.

The stability of the neutron star arises from a combination of neutron degeneracy pressure and the hard-core repulsion of the neutrons.

Typical neutron star mass is $M_{NS} \sim 1.4 M_\odot$ (where $M_\odot = 2.0 \times 10^{33}$ g is the solar mass), and NS masses have been observed up to $\sim 2.1 M_\odot$.

Radius: $R_{NS} \sim O(10)$ km; typical value $R_{NS} \sim 12$ km.

Mass density increases from outer region to core; typical average value $\rho \sim 5 \times 10^{14}$ g/cm^3, and correspondingly, ave. neutron number density ρ_n is somewhat larger than the saturation nuclear value $\rho_{n,nuc} = 0.16$ fm^{-3}. Number of neutrons in NS $\sim 10^{57}$.

Owing to the contraction from stellar radii to ~ 10 km, neutron stars have large rotation rates with periods $P \sim 0.05 - 10$ sec and large magnetic fields $B = |\vec{B}| \sim 10^{12}$ Gauss or more. Many have been observed as pulsars.

Compendium of NS, e.g., Potekhin et al., MNRAS 496, 5052 (2020).
After initially cooling mainly by neutrino emission, subsequent cooling is via photon emission, approximately described by thermal blackbody relation with power

$$L = 4\pi R_{NS}^2 \sigma_{SB} T^4$$

where T refers to surface temp. and $\sigma_{SB} = \pi^2 k_B^4/(60\hbar^3 c^2) = 5.67 \times 10^{-5}$ erg/(sec-cm^2-K^4) is Stefan-Boltzmann const.

The n-$\bar{n}$ transition is suppressed as in nuclei. The associated $\bar{n}$ annihilation or equivalently $\Delta B = -2$ dinucleon decay releases $E \sim 2m_n$ energy. The annihilation yields mainly pions with average multiplicity ~ 5, as in normal matter.

These pions will undergo strong reactions with adjacent neutrons on a time scale $\sim 10^{-23}$ s, including $\pi^+ n \longrightarrow \pi^0 p$. The π^0s produced directly from the $\bar{n}$ annihilation and via this charge-exchange reaction will then decay via $\pi^0 \longrightarrow \gamma\gamma$ on a time scale $\tau_{\pi^0} = 0.85 \times 10^{-16}$ sec.

Other effects slightly increase or decrease the energy release in photon luminosity. The effective neutron mass is somewhat decreased in the NS, but this will largely cancel. Another decrease is due to energy loss from antineutrinos from $\pi^- \longrightarrow \mu^- \bar{\nu}_\mu$. A slight increase is due to the energy released as high-lying neutrons in the Fermi sea move down to occupy the holes left by the $\Delta B = -2$ annihilation.

The matter instability due to $n - \bar{n}$ transitions and consequent annihilation is characterized by the matter decay rate $\Gamma_m = 1/\tau_m$. We have

$$N_n(t) = N_n(0)e^{-t/\tau_m}$$

so

$$\frac{dN_n}{dt} = -\frac{N_n(0)}{\tau_m}e^{-t/\tau_m} \simeq -\frac{N_n(0)}{\tau_m}$$

where $N_n(0)$ denotes the initial number of neutrons in the neutron star and we have used the fact that $\tau_m \gg t_U$, where $t_U = 1.38 \times 10^{10}$ yrs to approximate $e^{-t/\tau_m} \simeq 1$ in dN_n/dt.

Hence, the number of neutrons that transform to $\bar{n}$, denoted $N_{n\to\bar{n}}$, divided by the initial number of neutrons, $N_n(0)$, is

$$\frac{N_{n\to\bar{n}}(t)}{N_n(0)} = \frac{1}{N_n(0)}\left|\frac{dN_n}{dt}\right|t = \left(\frac{t}{\tau_m}\right) < 2.8 \times 10^{-29}\left(\frac{t}{10^4 \text{ yr}}\right)$$

(Typical ages of observed NS range from $\sim 10^3$ yrs to $\sim 10^6$ yrs.)

The energy deposition rate resulting from the n-$\bar{n}$ transitions followed by annihilation is then

$$\frac{dU}{dt} = \left|\frac{dN_n}{dt}\right|(2m_n) = \left(\frac{N_n(0)}{\tau_m}\right)(2m_n) = \frac{2M_{NS}}{\tau_m}$$

Note that with $N_n(0) \simeq M_{NS}/m_n$, the dependence on m_n divides out in this expression for dU/dt. Numerically,

$$\frac{dU}{dt} = (4.4 \times 10^{14}\ \mathrm{erg/s})\left(\frac{M_{NS}}{1.4M_\odot}\right)\left(\frac{3.6 \times 10^{32}\ \mathrm{yr}}{\tau_m}\right)$$

This can also be expressed in terms of the fundamental quantity $\tau_{n\bar{n}}$, as

$$\frac{dU}{dt} = (4.4 \times 10^{14}\ \mathrm{erg/s})\left(\frac{M_{NS}}{1.4M_\odot}\right)\left(\frac{4.7 \times 10^{8}\ \mathrm{s}}{\tau_{n\bar{n}}}\right)^2$$

This is an upper bound on this energy deposition rate in our analysis, since values we used for τ_m and $\tau_{n\bar{n}}$ are experimental lower bounds.

To see if this energy deposition is significant, we choose an old neutron star that has undergone a long period of cooling, since the fractional effect on the surface temperature T_s is largest for the lowest T_s. Typical values for moderately old NS are $T_s \sim 5 \times 10^5$ K to 10^6 K.

The corresponding thermal radiative luminosity is

$$L_{NS} = 4\pi R_{NS}^2 \sigma_{SB} T_s^4 = (7.1 \times 10^{32}\ \mathrm{erg/s}) \Big(\frac{R_{NS}}{10\ \mathrm{km}}\Big)^2 \Big(\frac{T_s}{10^6\ \mathrm{K}}\Big)^4 \quad (1)$$

The fractional change due to the n-$\bar{n}$ transitions is thus

$$\frac{(dU/dt)}{L_{NS}} \lesssim 10^{-18}$$

which is negligibly small.

This is in agreement with the earlier study by Buchella et al. and improves the bound on the effect by $\sim 10^5$, mainly due to increase in lower limit on $\tau_{n\bar{n}}$.

In addition to calculating the effect of the $\Delta B = -2$ annihilations on (i) NS luminosity, we also calculate the effect on (ii) the rotation of a NS and (iii) the periods of binary (NS) pulsars.

Effect on Rotation period: Let us denote the NS rotation period as P, the angular frequency of rotation as $\omega = 2\pi/P$, and the moment of inertia as I. We have $I \simeq (2/5)M_{NS}R_{NS}^2$. For a typical neutron star of mass $M_{NS} \sim 1.4M_\odot$ and radius $R_{NS} \simeq 10$ km, $I \simeq 10^{45}$ g cm^2.

Because of the emission of electromagnetic radiation, ω decreases (spin-down process), and hence $E_{\rm rot} = (1/2)I\omega^2$ decreases.

Hence, $dE_{\rm rot}/dt = I\omega\dot{\omega} = -(2\pi)^2 I(\dot{P})/P^3$, From the observed values of P and $\dot{P}$, one calculates a time $t_c = (1/2)P/\dot{P}$ that is approximately characteristic of the age of the pulsar.

For example, consider the Vela pulsar, with $P = 0.0893$ sec and $t_c = 1.13 \times 10^4$ yr, and hence $\dot{P} = P/(2t_c) = 1.2 \times 10^{-13}$. Consequently, $-\dot{E}_{\rm rot} \simeq 7 \times 10^{36}$ erg/s.

The energy deposition rate dU/dt from n-$\bar{n}$ is $\lesssim 10^{-22}$ of this spin-down energy loss rate and therefore has a negligible effect on it. We reach the same conclusion analyzing other pulsars with a range of values of P and $\dot{P}$.

Finally, we estimate the effect of n-$\bar{n}$ transitions on the orbital period P_b of binary pulsars. We use the Jeans relation $\dot{P}_b/P_b = -2\dot{M}/M$, where P_b denotes the orbital period, $M = M_1 + M_2$ is total binary system mass.

Let us consider, for example, the well-studied Taylor-Hulse binary pulsar system, PSR B1913+16 (= PSR J1915+1606), which was used as a test of general relativity (GR).

For this system, $P_b = 0.322997$ days, $M = 2.83 M_\odot$, and $\dot{P}_{b,\mathrm{int}} = (-2.393 \pm 0.004) \times 10^{-12}$ for the intrinsic (int) $\dot{P}_b$ (Weisberg, Nice, Taylor (2010); Weisberg, Huang, 2016).

This agrees with the GR prediction $\dot{P}_{b,\mathrm{GR}} = (-2.40263 \pm 0.00005) \times 10^{-12}$:

$$\frac{\dot{P}_{b,\mathrm{int}}}{\dot{P}_{b,\mathrm{GR}}} = 0.9983 \pm 0.0016$$

So residual (res) $\dot{P}_{b,\mathrm{res}} \equiv \dot{P}_{b,\mathrm{int}} - \dot{P}_{b,\mathrm{GR}} = (4.6 \pm 4.0) \times 10^{-15}$ and hence

$$\frac{\dot{P}_{b,\mathrm{res}}}{P_b} = (5.2 \pm 4.5) \times 10^{-12}\ \mathrm{yr}^{-1}$$

Substituting $M = 2.83 M_\odot$, denoting this mass/energy loss as $\dot{M}_{n-\bar{n}} = -dU/dt$, and using Jeans relation, the change due to the $\Delta B = -2$ annihilation is

$$\frac{\dot{P}_{b,n\bar{n}}}{P_b} = -2\frac{\dot{M}_{n-\bar{n}}}{M} = (1.1 \times 10^{-32}\ \mathrm{yr}^{-1})\left(\frac{4.7 \times 10^{8}\ \mathrm{s}}{\tau_{n\bar{n}}}\right)^2$$

Again, this is an upper limit, since the value used for $\tau_{n\bar{n}}$ is the experimental lower limit.

Thus, the increase in $\dot{P}_b/P_b$ due to possible n-$\bar{n}$ transitions is about 10^{20} times smaller than the observed residual $\dot{P}_{b,\mathrm{res}}/P_b$ for the Taylor-Hulse binary pulsar. We find similar results for other binary pulsars.

This shows that possible n-$\bar{n}$ transitions have a negligible effect on the period of binary pulsars, just as they have a negligible effect on the pulsar luminosities and spin-down rates.

As with the Buchella et al. study, we do not assume any particular model of dark matter, since there is no consensus on a preferred model and there are many DM models (recent DM review: 2406.01705 by Cirelli, Strumia, and Zupan).

In ongoing work we are studying the effects of n-$\bar{n}$ transitions in NS in the presence of various types of dark matter (DM).

These DM models are constrained by NS properties even in the absence of n-$\bar{n}$ transitions. For example, in mirror DM models, constraints on n-n' mixing in NS have been derived by many groups (Goldman, Mohapatra, Nussinov, Zhang; Baym et al.; McKeen, Nelson, Reddy, Zhou, Pospelov, Raj; Motta et al.; Berezhiani et al.; Gardner et al.; Thompson et al...).

Conclusions

- General theoretical expectation that baryon number is violated, and this is borne out in many BSM scenarios.

- n-$\bar{n}$ transitions are an interesting possible manifestation of baryon number violation, of $|\Delta B| = 2$ type, complementary to proton decay, motivating further experimental searches.

- We have discussed two models that show how new physics beyond the SM can produce n-$\bar{n}$ transitions at rates comparable with current limits. These models also show that n-$\bar{n}$ transitions can be the main manifestation of baryon number violation, since proton decay is strongly suppressed.

- Analysis of effects of n-$\bar{n}$ transitions in neutron stars.

Thank you