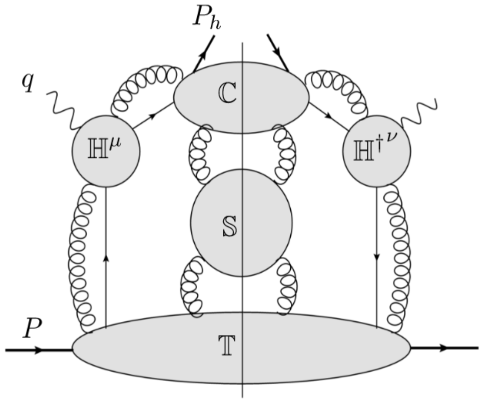
Andrea Simonelli

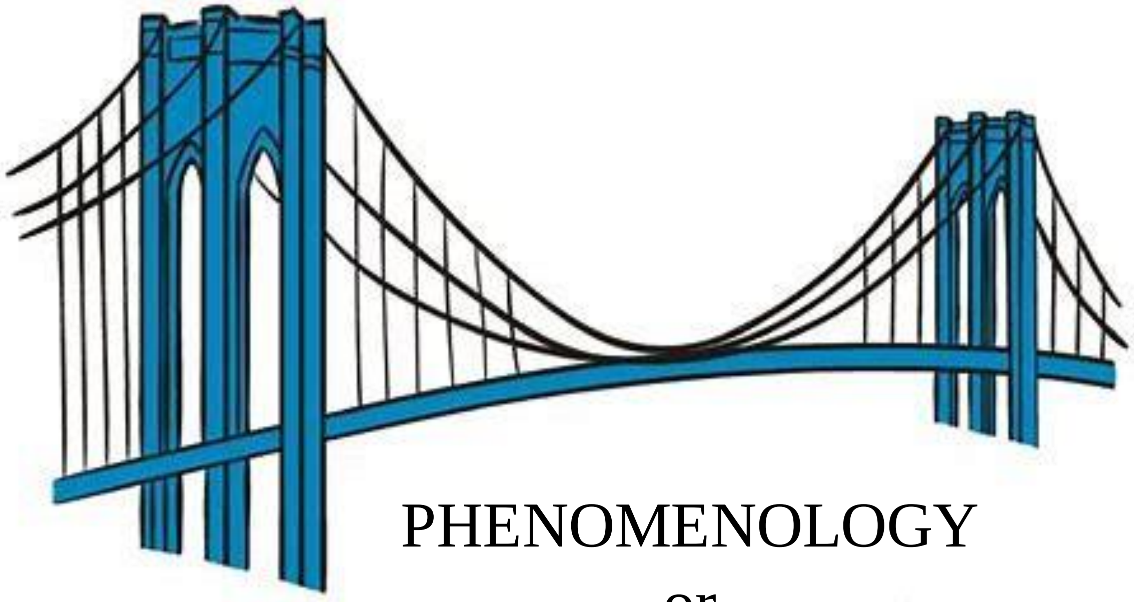
Bridging theory and experiment in TMD physics

OLD DOMINION UNIVERSITY

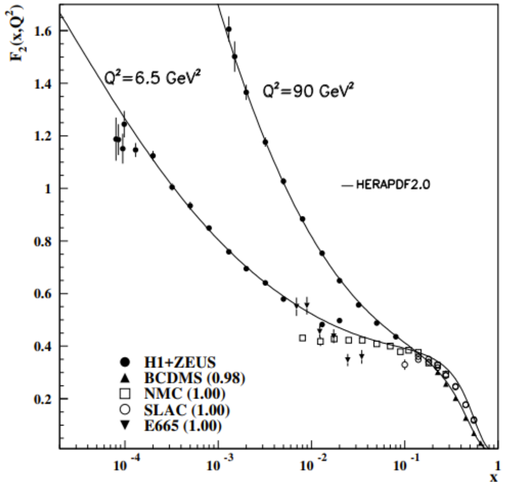
Jefferson Lab

THEORY
(QCD)





DATA



PHENOMENOLOGY
or
IMPLEMENTATION
SCHEMES

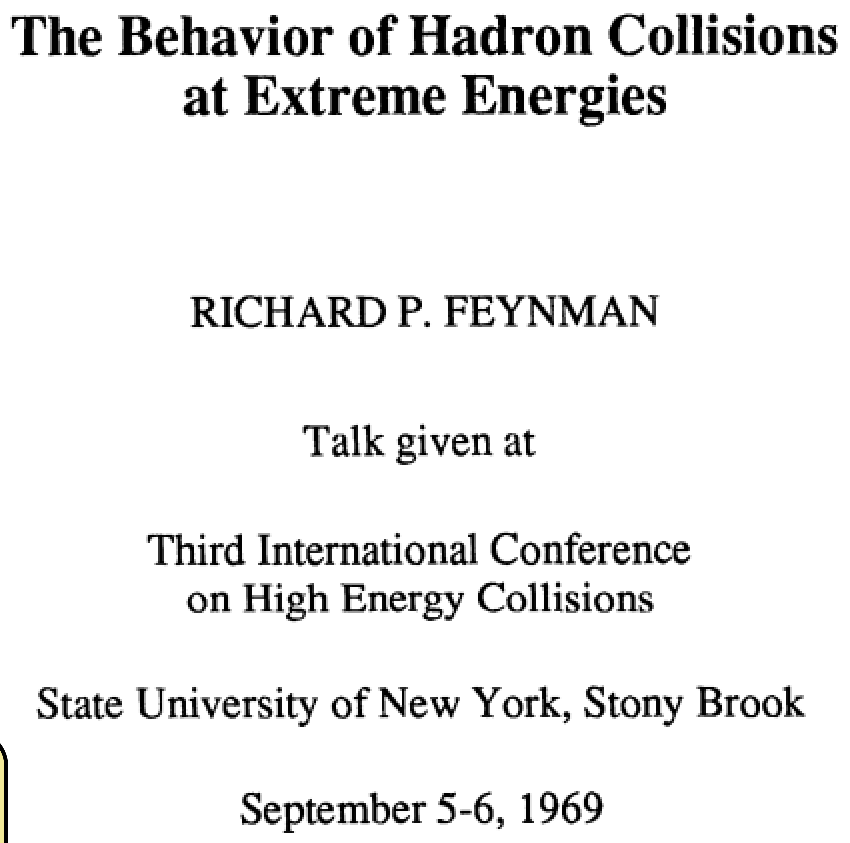
Crucial step to make sensible predictions

INTRODUCTION and MOTIVATIONS

or

How far we can go with the Parton Model picture

PARTON MODEL



The Behavior of Hadron Collisions at Extreme Energies

RICHARD P. FEYNMAN

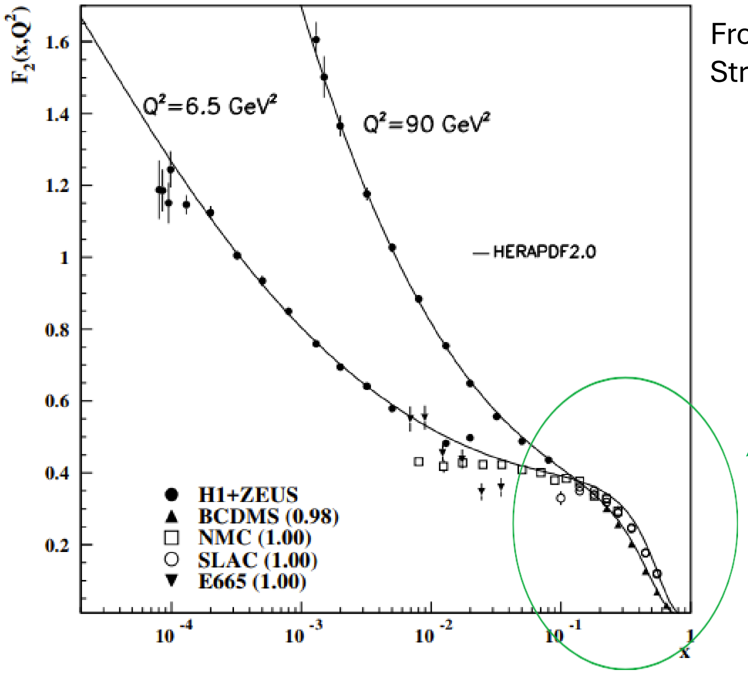
Talk given at

Third International Conference on High Energy Collisions

State University of New York, Stony Brook

September 5-6, 1969

Motivated by high energy lepton-nucleon scattering



From Particle Data Group 2022
Structure Functions

Bjorken scaling

Nucleons are composed by **elementary** consituents

Probability of hitting parton j

Probability of finding parton j inside the nucleon

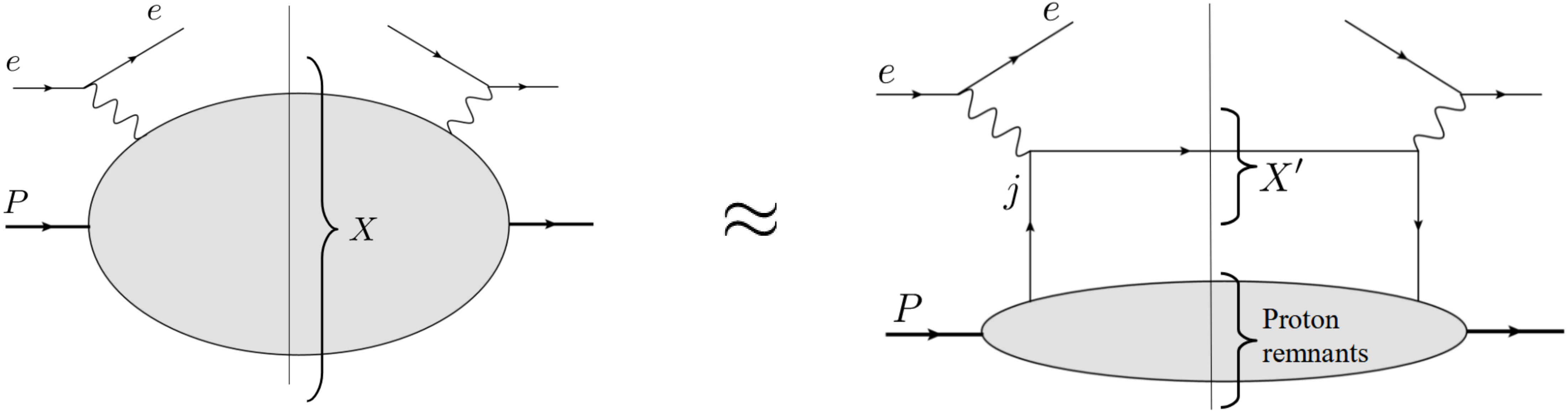
$$\sigma_{eP\to e\,X} = \sum_j \hat{\sigma}_{ej\to e\,X'} \otimes f_{j/P}$$

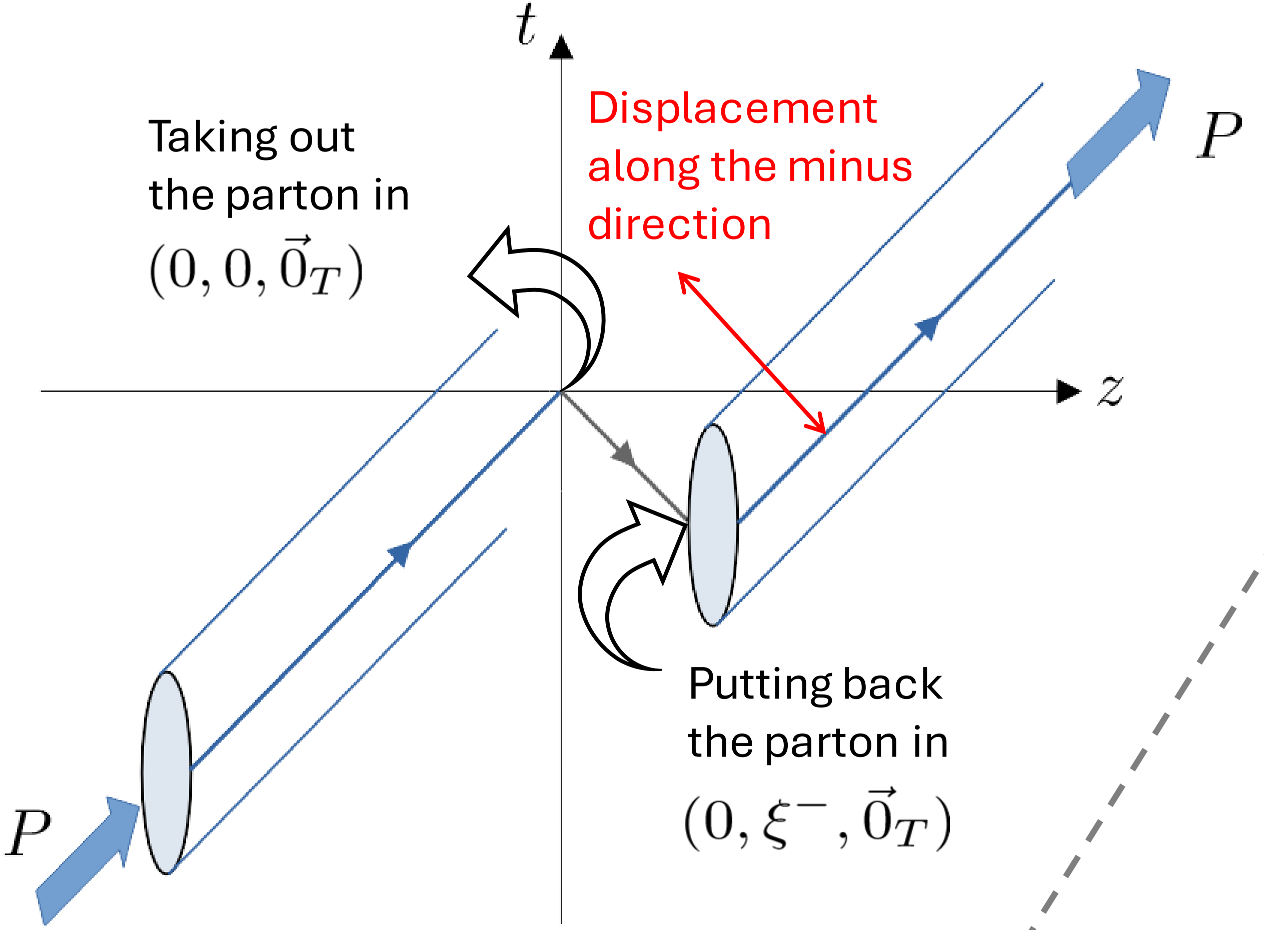
PARTON MODEL

Probability of hitting parton j

$$\sigma_{eP\to e\,X} = \sum_j \hat{\sigma}_{ej\to e\,X'} \otimes f_{j/P}$$

Probability of finding parton j inside the nucleon

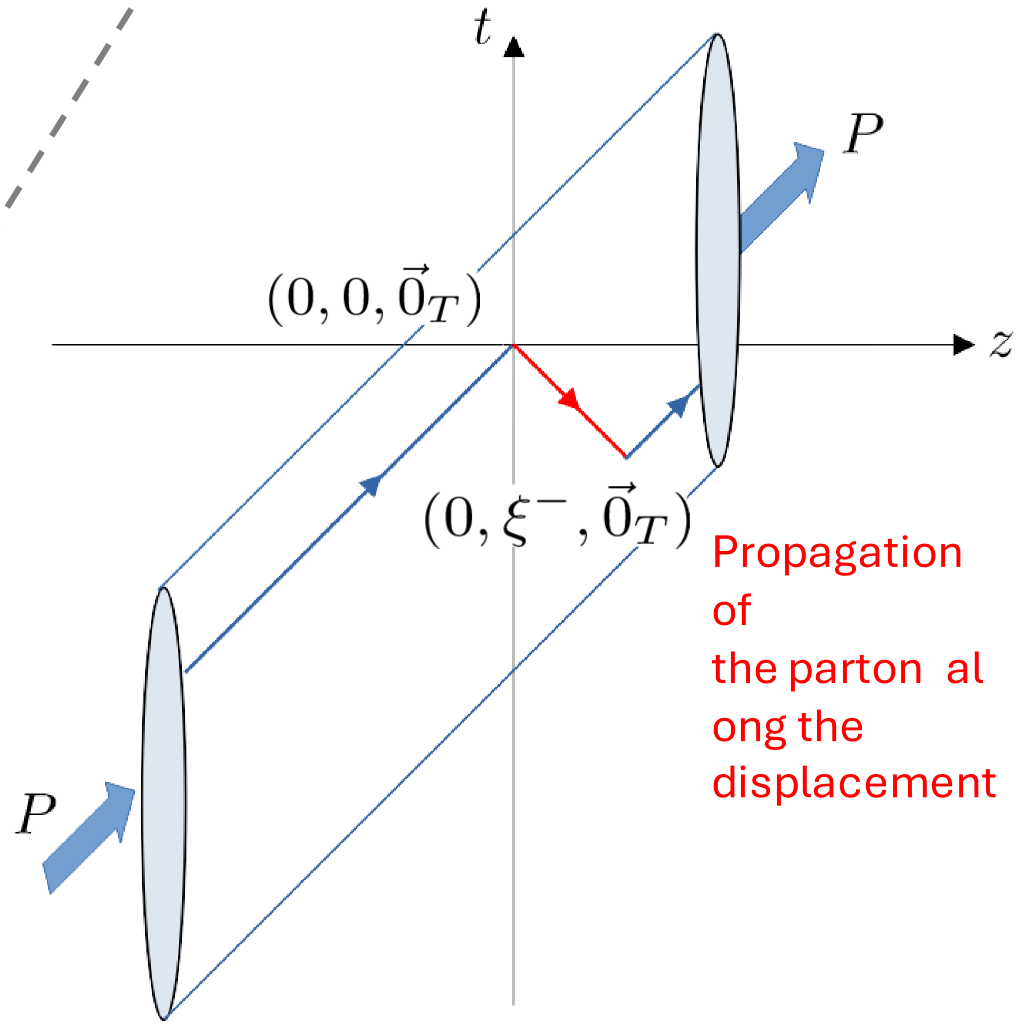




Taking out the parton in $(0, 0, \vec{0}_T)$

Displacement along the minus direction

Putting back the parton in $(0, \xi^-, \vec{0}_T)$



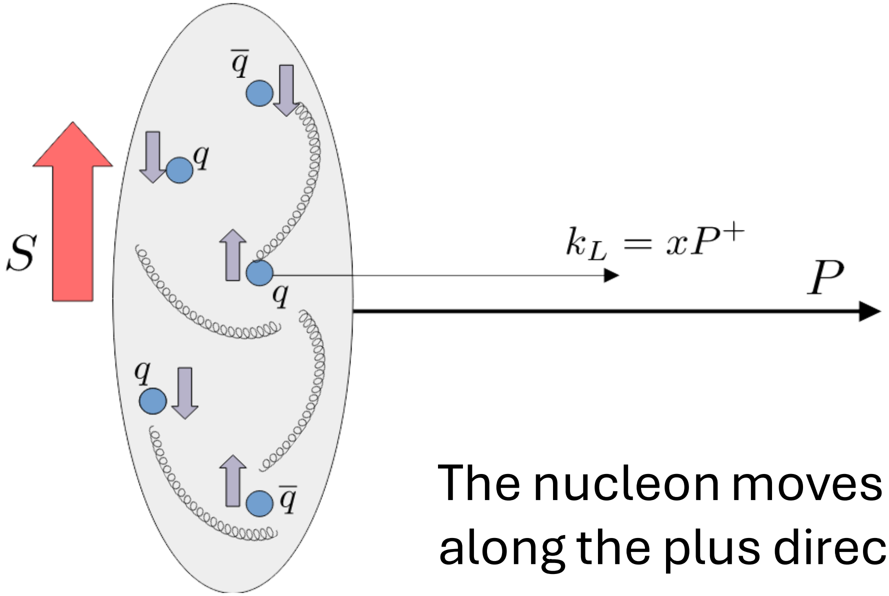
$(0, 0, \vec{0}_T)$

$(0, \xi^-, \vec{0}_T)$

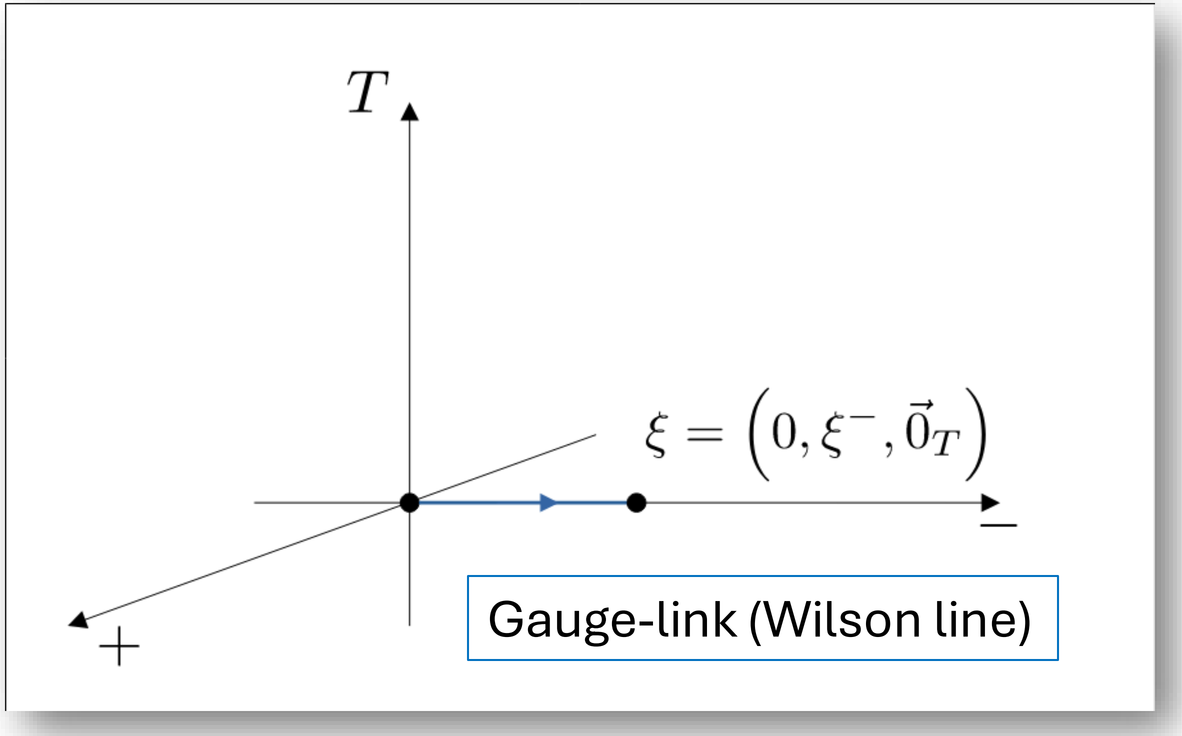
Propagation of the parton along the displacement

$$\Phi(x) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S|\overline{\psi}(\xi)\boxed{W_{0\to\xi}}\psi(0)|P, S\rangle$$

Gauge-invariant propagation of a quark inside a nucleon of momentum P and spin S



The nucleon moves fast along the plus direction



Gauge-link (Wilson line)

$$\Phi(x) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S|\overline{\psi}(\xi)\boxed{W_{0\to\xi}}\psi(0)|P, S\rangle$$

Gauge-invariant propagation of a quark inside a nucleon of momentum P and spin S

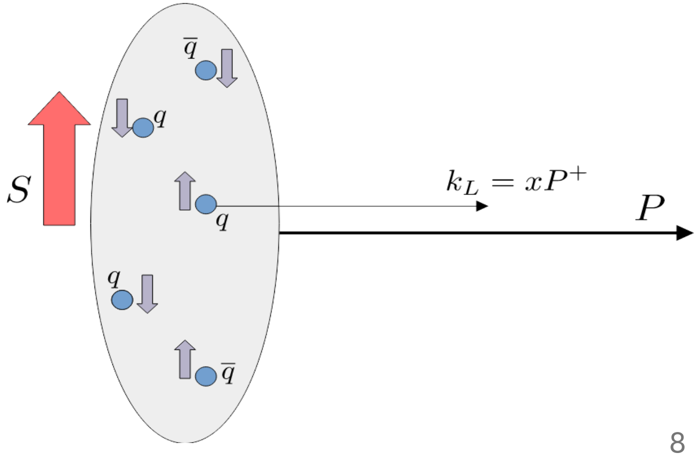
"Probability" of finding an **unpolarized quark** with momentum fraction x and transverse momentum k_T inside a nucleon of spin S

$$\Phi(x) = \boxed{f_1(x)}\gamma^- + S_L\, g_{1L}(x)\gamma^-\gamma^5 + i\, S_T^j\, h_{1T}(x)\gamma^-\gamma^j + \text{p.s.}$$

Quark Polarization

Nucleon Polarization

	U	L	T
U	f_1		
L		g_{1L}	
T			h_{1T}



$$\Phi(x) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S | \overline{\psi}(\xi) \boxed{W_{0\to\xi}} \psi(0) | P, S\rangle$$

Gauge-invariant propagation of a quark inside a nucleon of momentum P and spin S

"Probability" of finding a **longitudinally polarized quark** with momentum fraction x and transverse momentum k_T inside a nucleon of spin S

$$\Phi(x) = f_1(x)\gamma^- + S_L \boxed{g_{1L}(x)} \gamma^-\gamma^5 + i\, S_T^j\, h_{1T}(x)\gamma^-\gamma^j + \text{p.s.}$$

Quark Polarization

Nucleon Polarization

	U	L	T
U	f_1		
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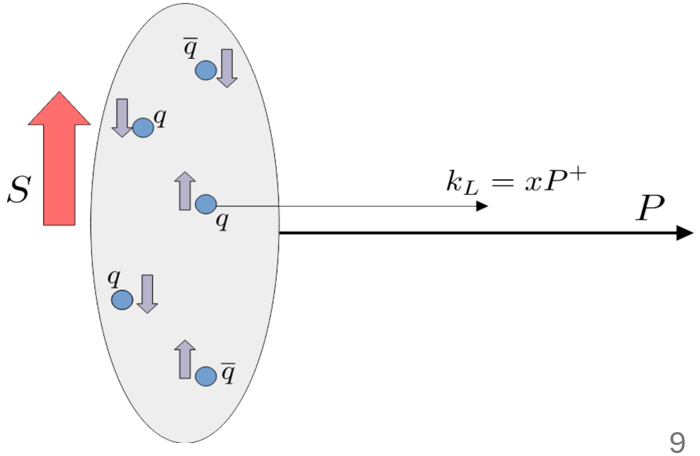


$$\Phi(x) = \int \frac{d\xi^-}{2\pi}\, e^{ixP^+\xi^-} \langle P, S|\overline{\psi}(\xi)\boxed{W_{0\to\xi}}\psi(0)|P, S\rangle$$

Gauge-invariant propagation of a quark inside a nucleon of momentum P and spin S

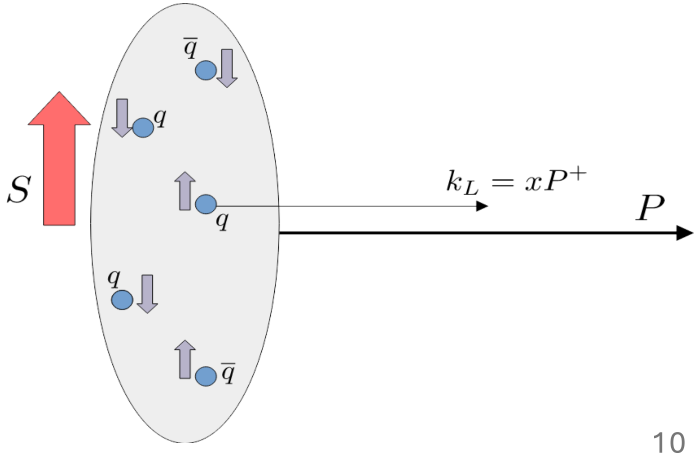
"Probability" of finding a **transversely polarized quark** with momentum fraction x and transverse momentum k_T inside a nucleon of spin S

$$\Phi(x) = f_1(x)\gamma^- + S_L\, g_{1L}(x)\gamma^-\gamma^5 + i\, S_T^j \boxed{h_{1T}(x)}\gamma^-\gamma^j + \text{p.s.}$$

Quark Polarization

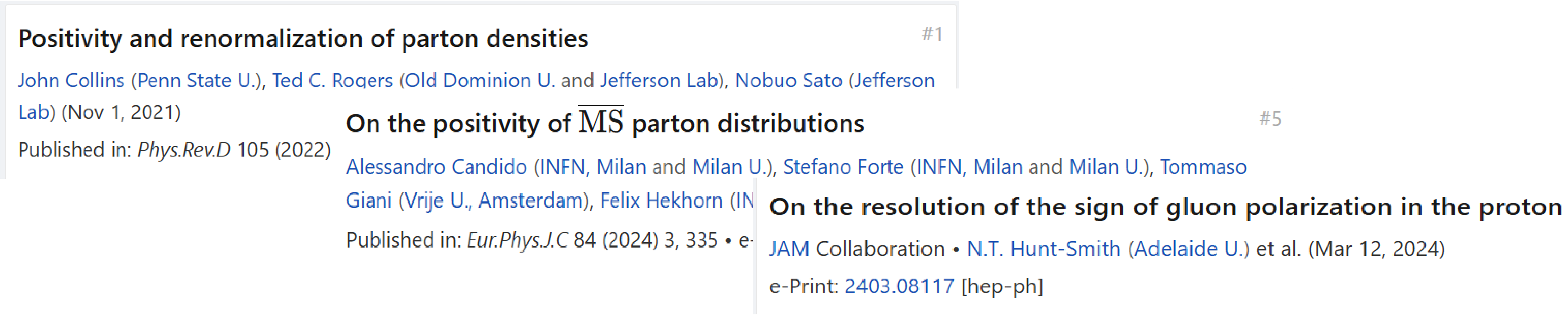
Nucleon Polarization

	U	L	T
U	f_1		
L		g_{1L}	
T			h_{1T}



Why "probabilities" and not probabilities?

POSITIVITY CONSTRAINT



Positivity and renormalization of parton densities #1

John Collins (Penn State U.), Ted C. Rogers (Old Dominion U. and Jefferson Lab), Nobuo Sato (Jefferson Lab) (Nov 1, 2021)

Published in: *Phys.Rev.D* 105 (2022)

On the positivity of $\overline{\mathrm{MS}}$ parton distributions #5

Alessandro Candido (INFN, Milan and Milan U.), Stefano Forte (INFN, Milan and Milan U.), Tommaso Giani (Vrije U., Amsterdam), Felix Hekhorn (IN

Published in: *Eur.Phys.J.C* 84 (2024) 3, 335 • e-

On the resolution of the sign of gluon polarization in the proton

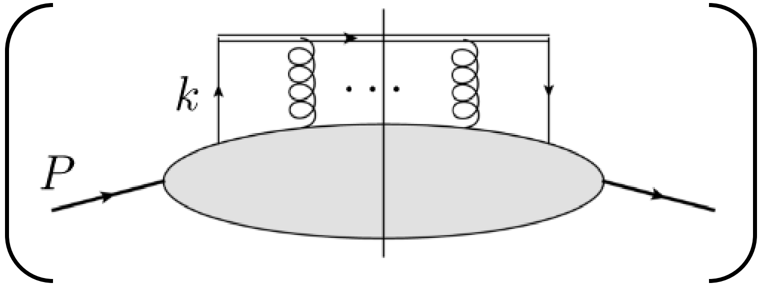
JAM Collaboration • N.T. Hunt-Smith (Adelaide U.) et al. (Mar 12, 2024)

e-Print: 2403.08117 [hep-ph]

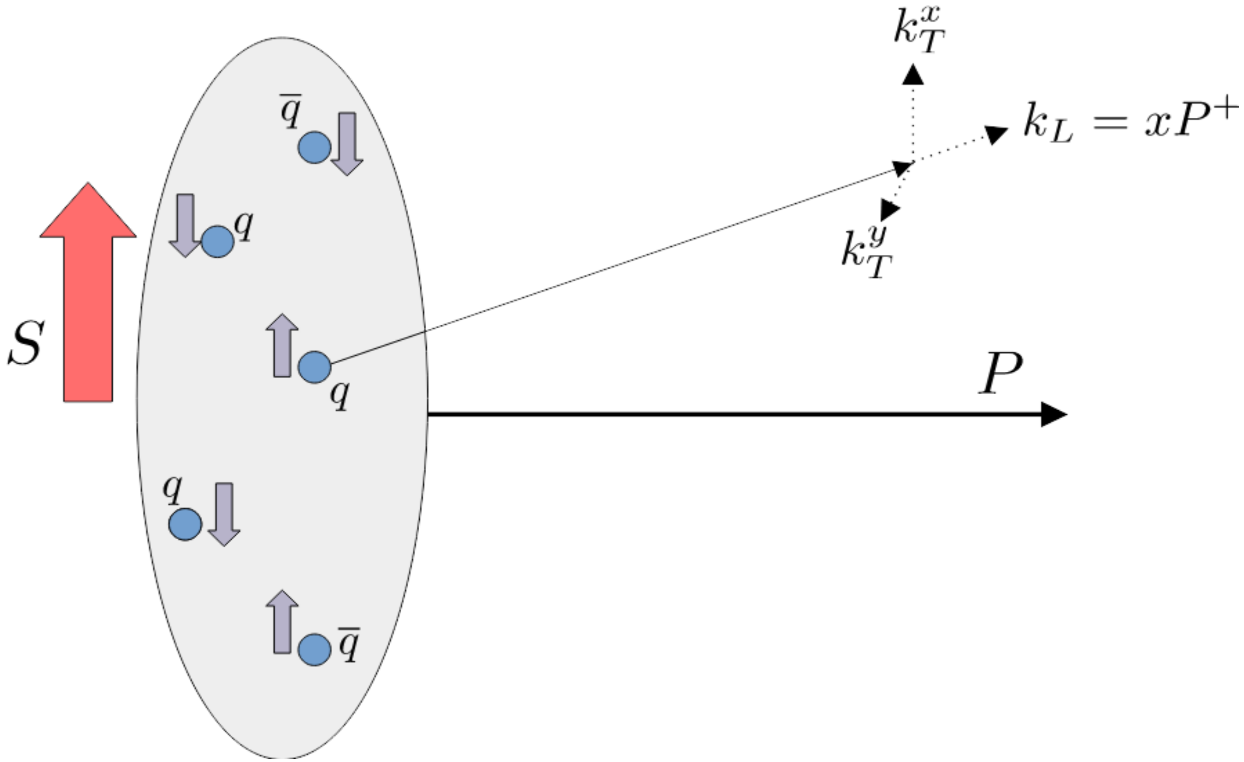
Positivity of PDFs is related to the UV renormalization scheme used to define them

$$f_1(x) = \left[\int \frac{d\xi^-}{2\pi}\, e^{ixP^+\xi^-} \langle P,S|\overline{\psi}(\xi) W^{\dagger}_{\infty\to\xi} \frac{\gamma^+}{2} W_{0\to\infty}\psi(0)|P,S\rangle \right]_{\text{UV ren.}}$$

$$= \int^{\boxed{\text{UV ren.}}} d^2\vec{k}_T \int dk^- \left(\quad \right)$$



Different treatment of large transverse momentum corresponds to different UV renormalization schemes



Generalization: **3D structure** of nucleon considering the *whole* intrinsic motion of partons

$$f_1(x) \overset{\text{naive}}{=} \int d^2\vec{k}_T\, f_1(x, k_T^2)$$

⇓

$$f_1(x) = \int^{\boxed{\text{UV ren.}}} d^2\vec{k}_T\, f_1(x, k_T^2)$$

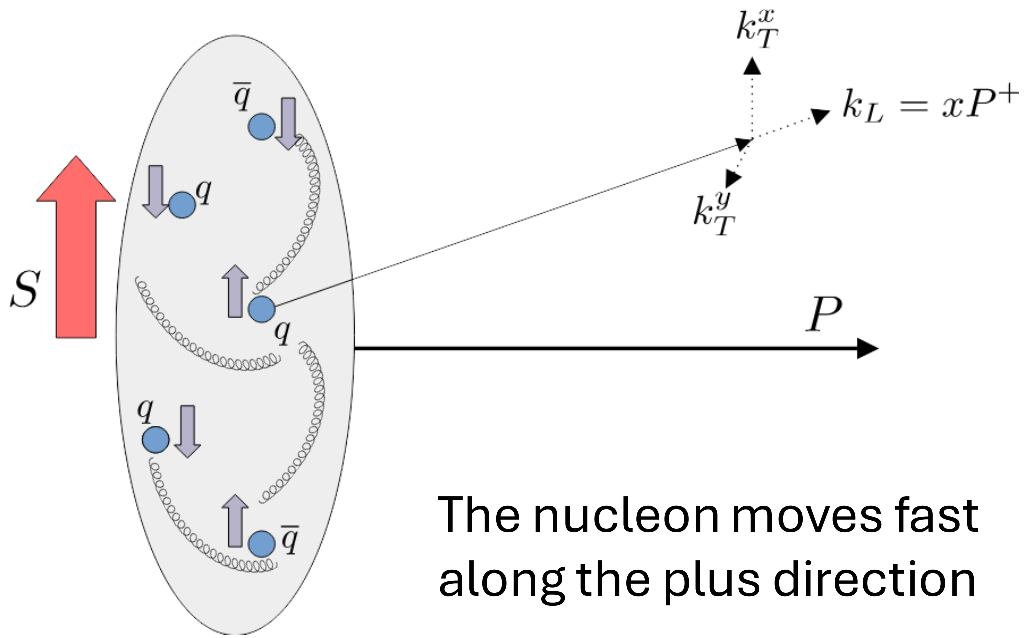
Quark Polarization

Nucleon Polarization

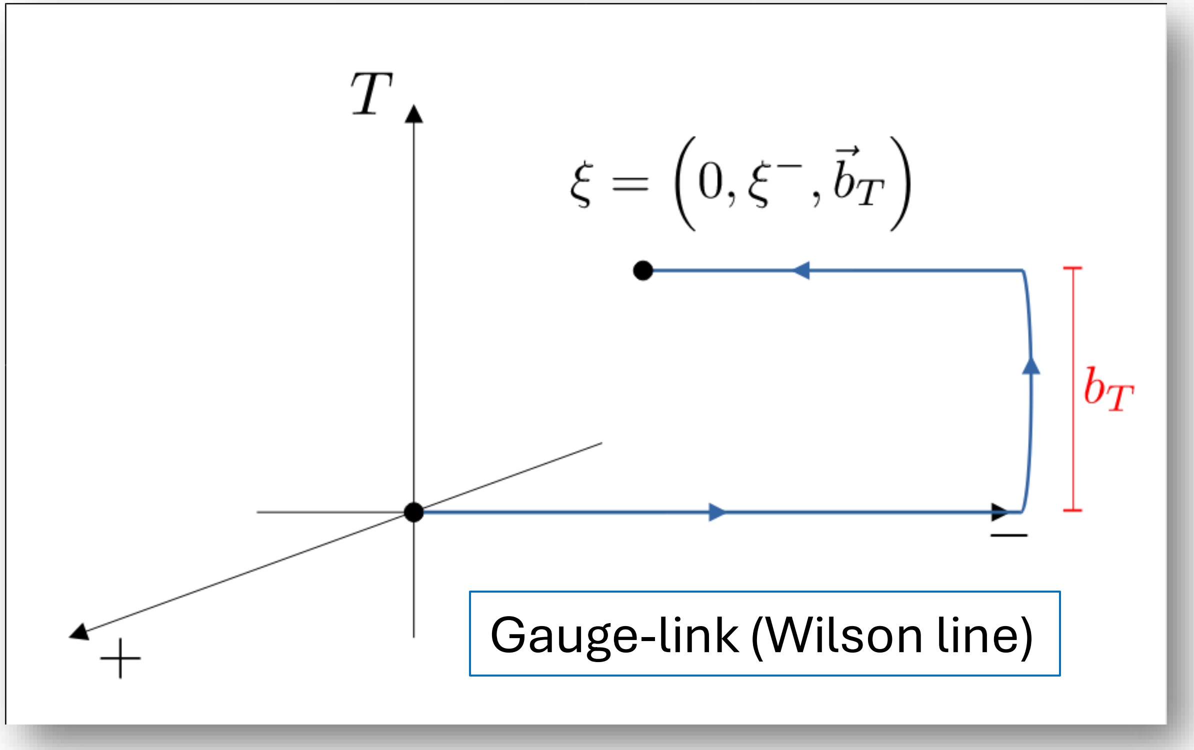
	U	L	T
U	$\boxed{f_1}$		$h_1^\perp$
L		$\boxed{g_{1L}}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boxed{h_{1T}}$, $h_{1T}^\perp$

$$\Phi(x,\vec{k}_T) = \int \frac{d\xi^-\, d^2\vec{b}_T}{(2\pi)^3}\, e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{b}_T} \langle P,S|\overline{\psi}(\xi)\boxed{W_{0\to\xi}}\psi(0)|P,S\rangle$$

Gauge-invariant propagation of a quark inside a nucleon of momentum P and spin S



The nucleon moves fast along the plus direction



Gauge-link (Wilson line)

TMD PARTON DENSITIES

"Probability" of finding an **unpolarized quark** with momentum fraction x and transverse momentum k_T inside a nucleon of spin S

$$\Phi^{[\gamma^+]}(x,\vec{k}_T) = f_1(x,k_T^2) - \frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x,k_T^2) + \text{p.s.}$$

Quark Polarization

Nucleon Polarization

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T},\, h_{1T}^{\perp}$

TMD PARTON DENSITIES

"Probability" of finding a **longitudinally polarized quark** with momentum fraction x and transverse momentum k_T inside a nucleon of spin S

$$\Phi^{[\gamma^+\gamma_5]}(x,\vec{k}_T) = S_L\, g_{1L}(x,k_T^2) - \frac{\vec{k}_T\cdot\vec{S}_T}{M} g_{1T}(x,k_T^2) + \text{p.s.}$$

Quark Polarization

Nucleon Polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T},\, h_{1T}^\perp$

TMD PARTON DENSITIES

"Probability" of finding a **transversely polarized quark** with momentum fraction x and transverse momentum k_T inside a nucleon of spin S

$$\Phi^{[i\sigma^{j+}\gamma^5]}(x,\vec{k}_T) = S_T^j\, h_{1T}(x,k_T^2) + S_L\,\frac{k_T^j}{M} h_{1L}^\perp(x,k_T^2)$$
$$-\frac{k_T^j\,\vec{k}_T\cdot\vec{S}_T + \frac{1}{2}k_T^2 S_T^j}{M^2} h_{1T}^\perp(x,k_T^2) - \frac{\epsilon^{jk}k_T^k}{M} h_1^\perp(x,k_T^2) + \text{p.s.}$$

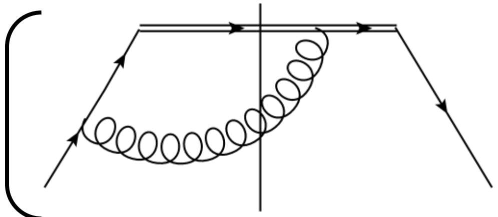
Quark Polarization

Nucleon Polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T},\ h_{1T}^\perp$

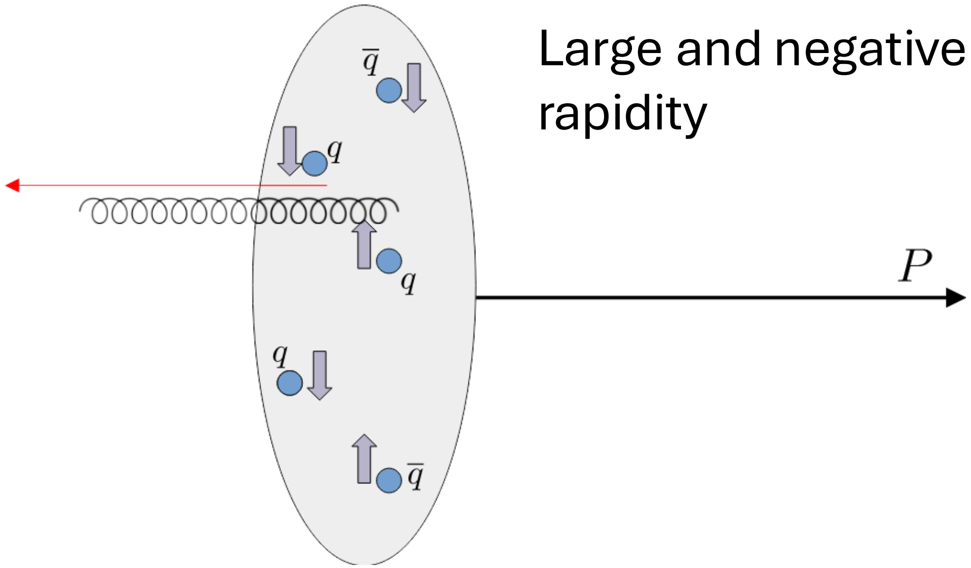
Nice, but it doesn't work:

Unregulated divergence when $k^+ = P^+$



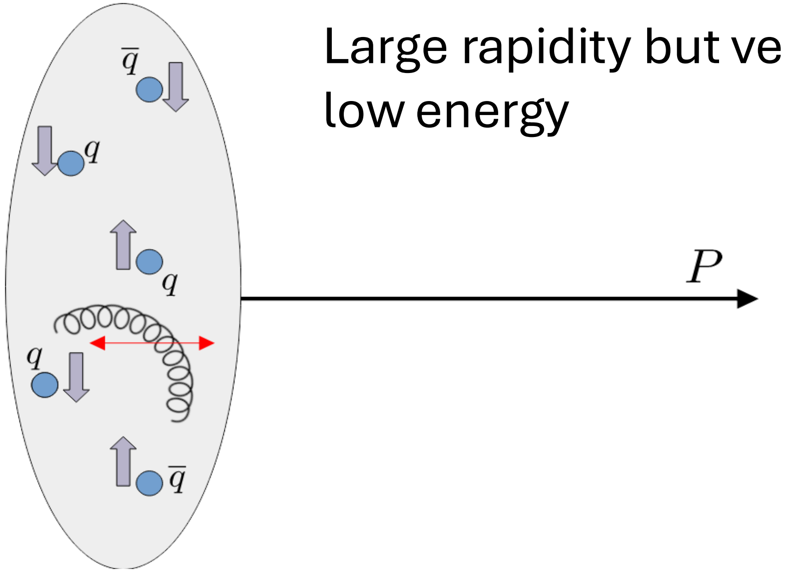
$$\int dk^- \frac{\mathrm{Tr}_C}{N}\frac{\mathrm{Tr}_D}{4}\left(\qquad \right) \sim a_S \frac{1}{k_T^2}\frac{1}{1-\xi}$$

Rapidity Divergences



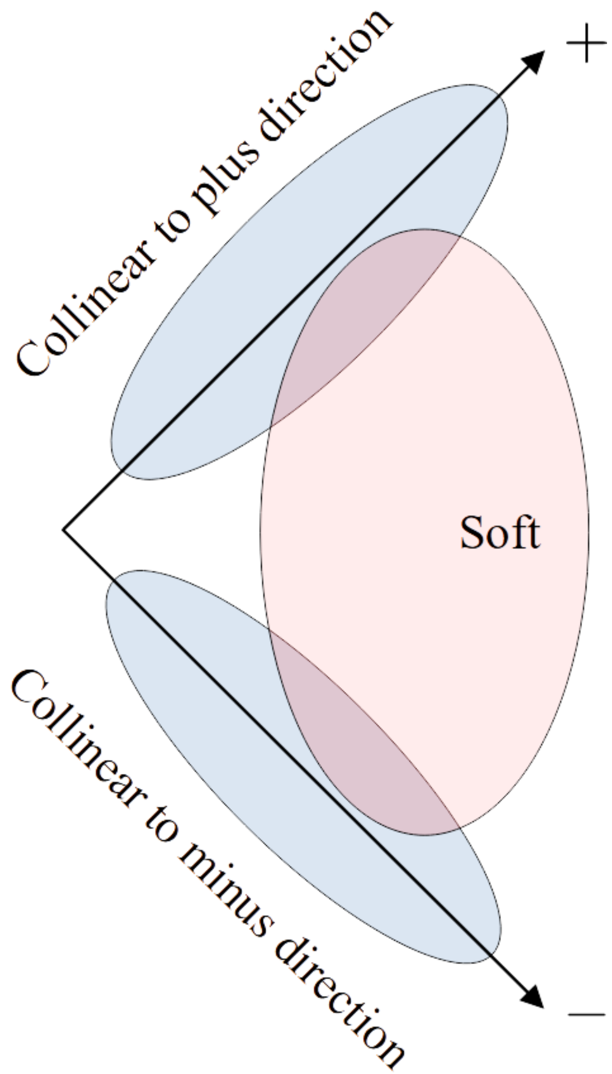
Large and negative rapidity

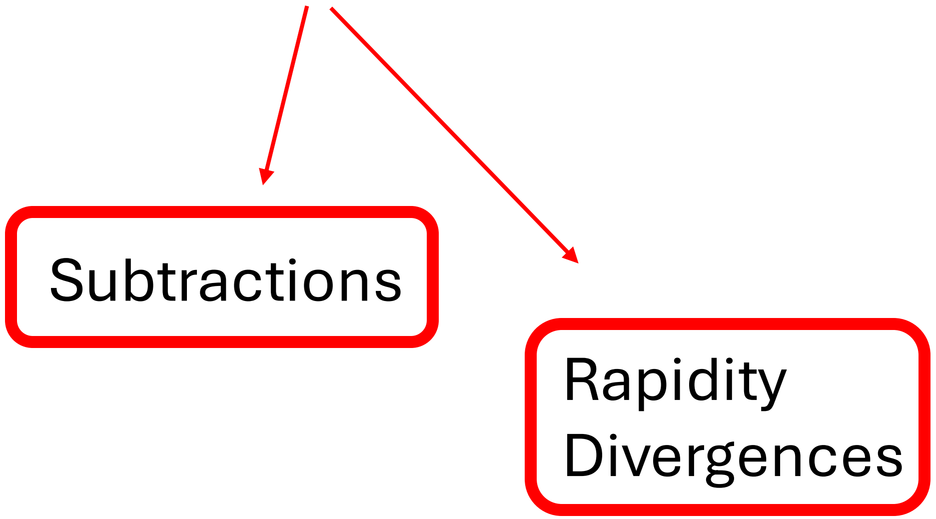
Soft-collinear contributions



Large rapidity but very low energy

The elephant in the room in TMD physics (and not only)



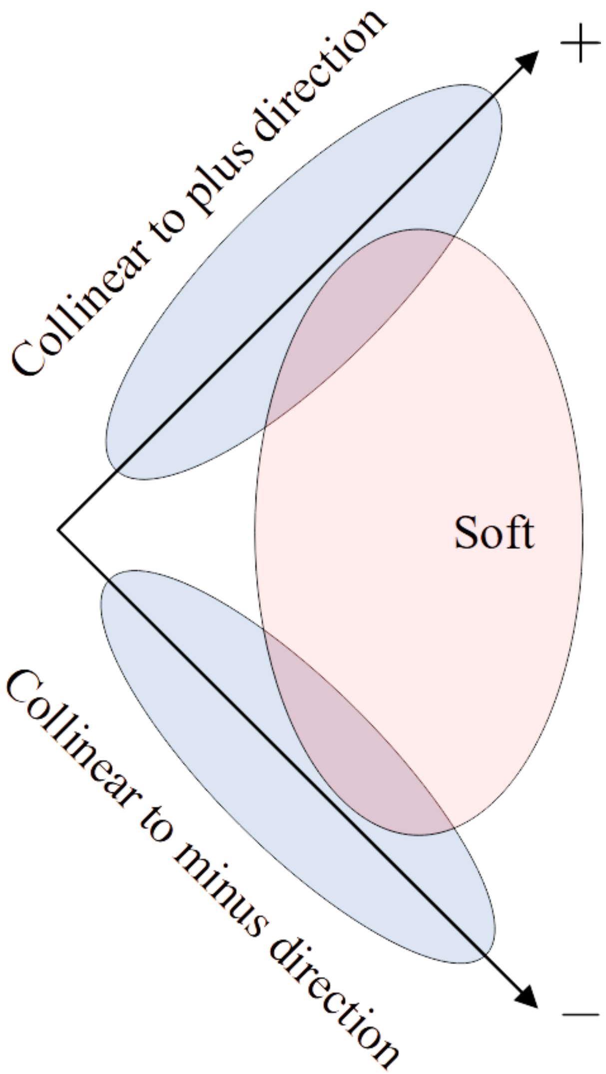


Entangling Soft Radiation

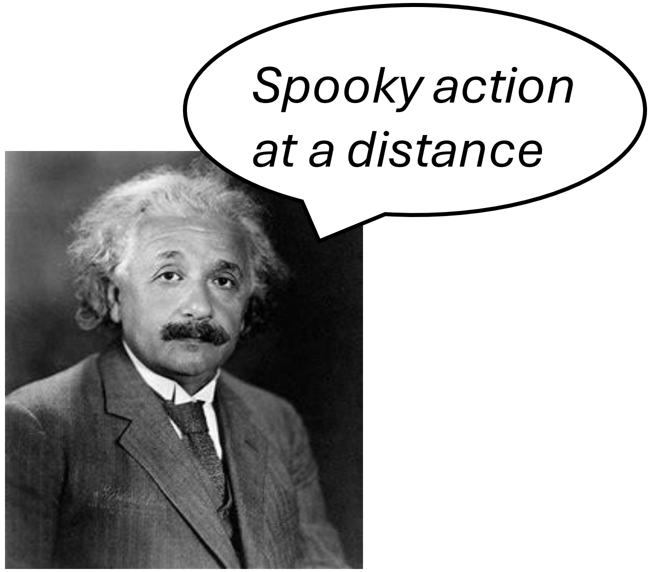
Subtractions

Rapidity Divergences

SOFT ENTANGLEMENT



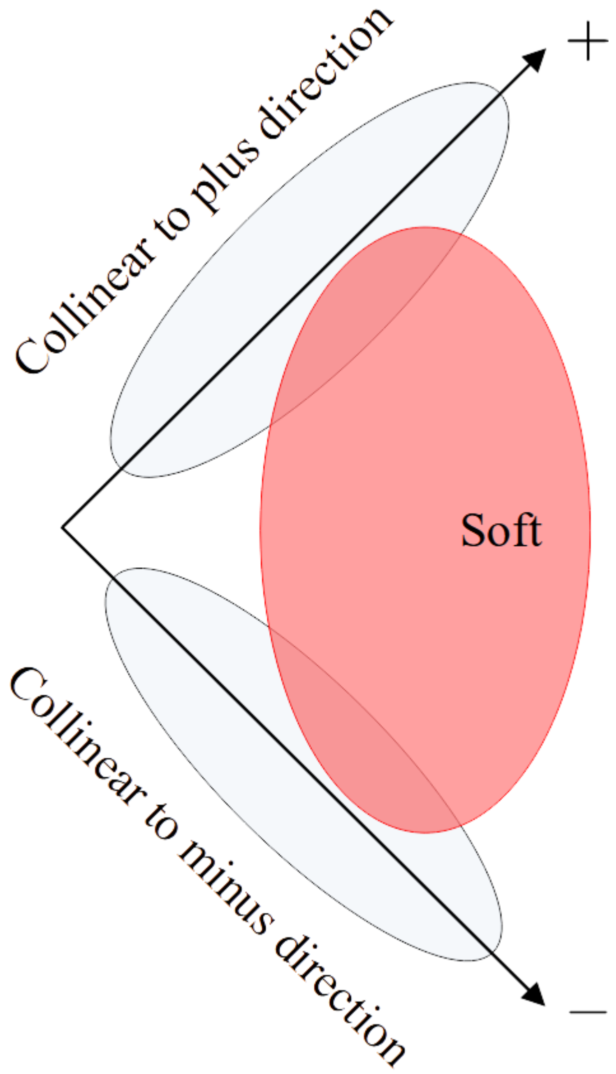
Two opposite light-cone directions ***entangled*** *by soft radiation*



Many examples:

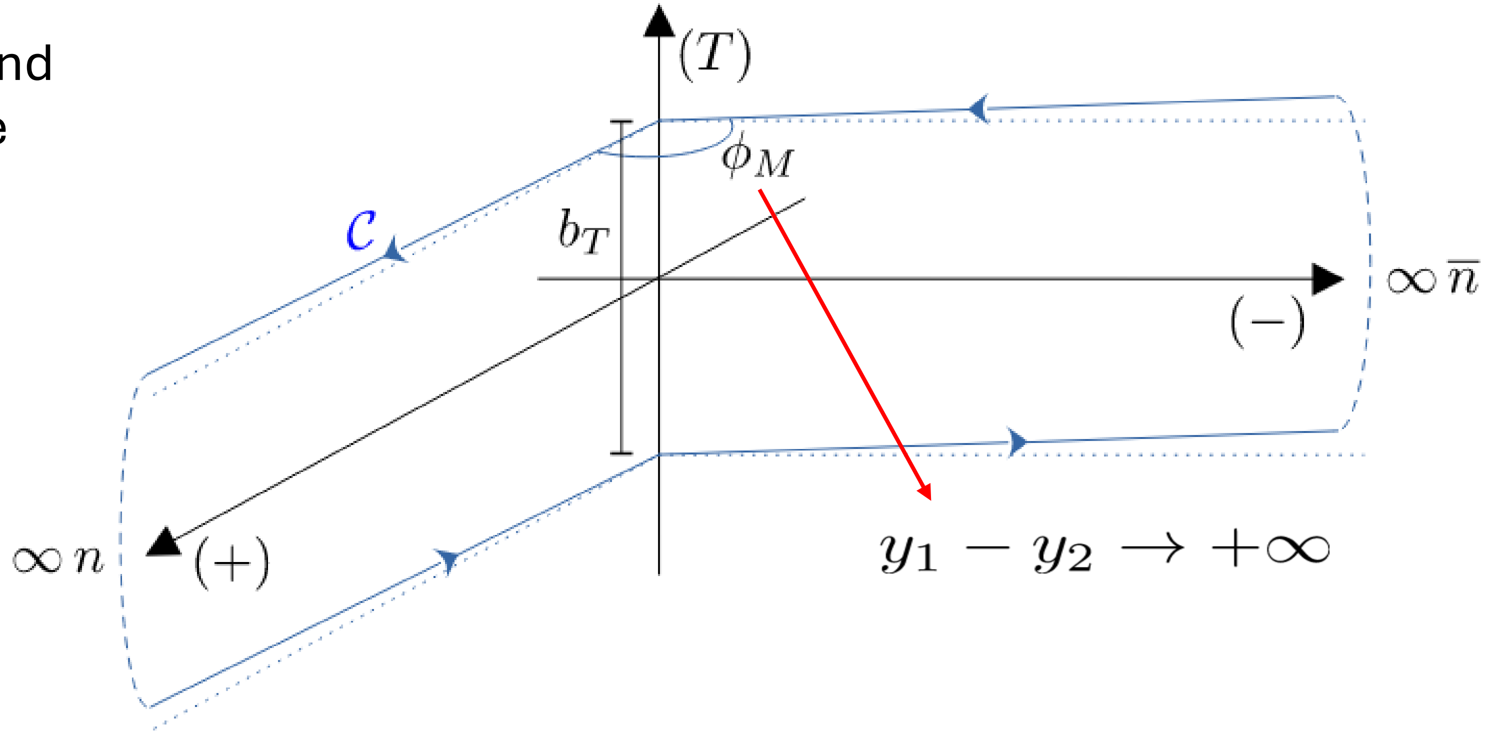
- Two back-to-back hadrons produced in e^+e^-annihilation
- Semi-Inclusive DIS at low $q_T = P_T/z$
- Drell-Yan with lepton pair almost back-to-back
- DIS at threshold
- Thrust distribution in the 2-jet limit
- Single hadron production from e^+e^- annihilation, reconstructing the thrust in the 2-jet limit

The world as seen by a soft particle:



Large and positive rapidity

Large and negative rapidity



$$S(b_T, \phi_M) = \langle 0 | W_{\mathcal{C}}(b_T, \phi_M) | 0 \rangle$$

A suitable *combination* of

$$\Phi(x,\vec{b}_T) = \int \frac{d\xi^-}{2\pi}\, e^{ixP^+\xi^-} \langle P,S|\overline{\psi}(\xi) W_{0\to\xi}\psi(0)|P,S\rangle$$

Collinear radiation

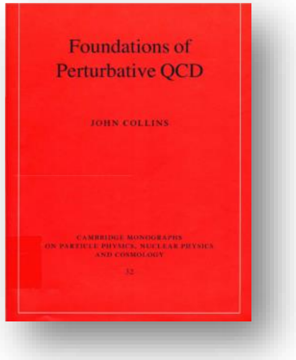
with

$$S(b_T,\phi_M) = \langle 0|W_C(b_T,\phi_M)|0\rangle$$

Soft and Soft-collinear radiation

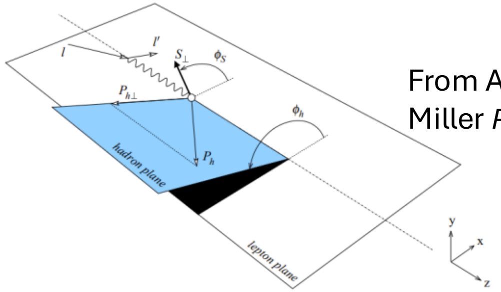
leads to a **well-defined** operator definition for the TMDs

Which combination?

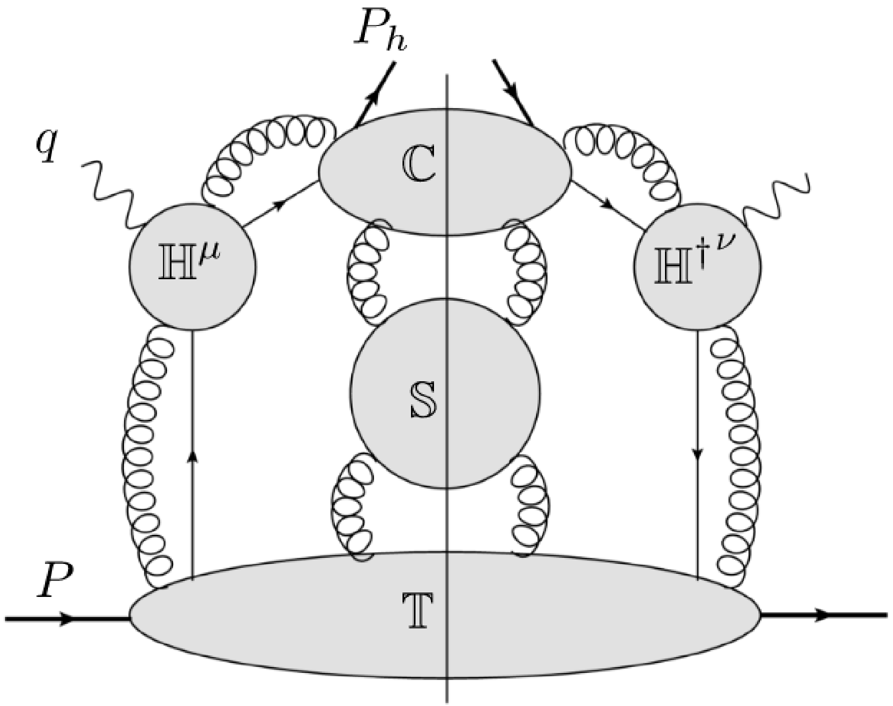


FACTORIZATION (in QCD) gives the answer

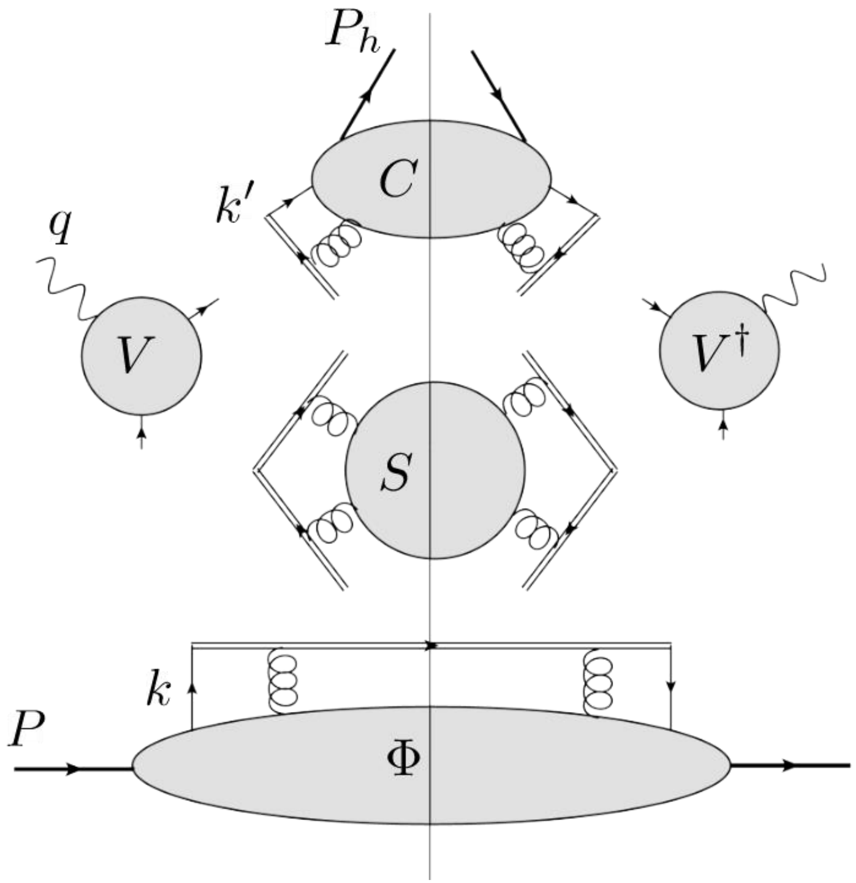
Consider SIDIS $e^- P \to h X$ in the low-q_T region:



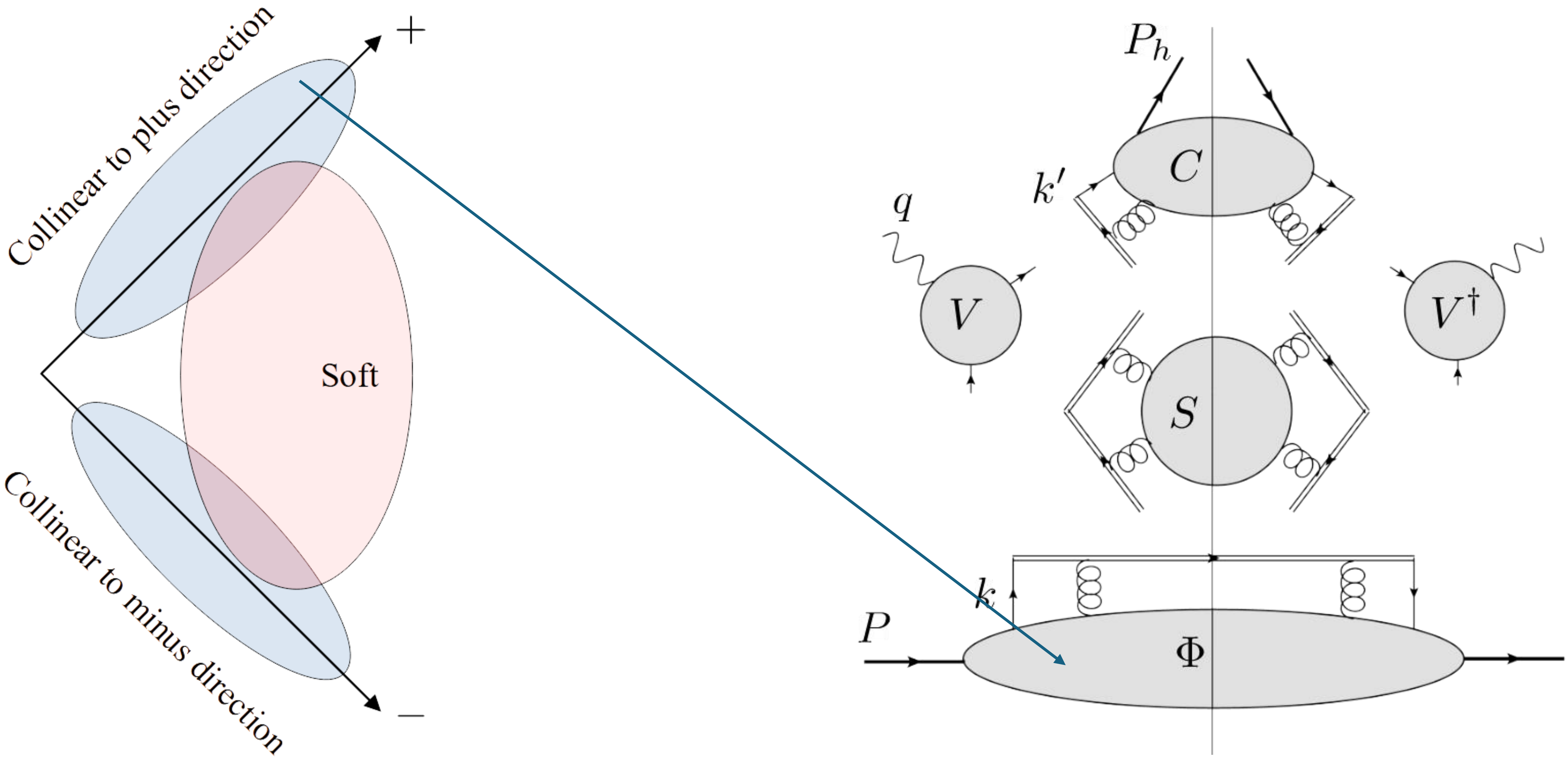
From A.Bacchetta, U.D'Alesio, M.Diehl, C. Miller *Phys.Rev.D* 70 (2004) 117504



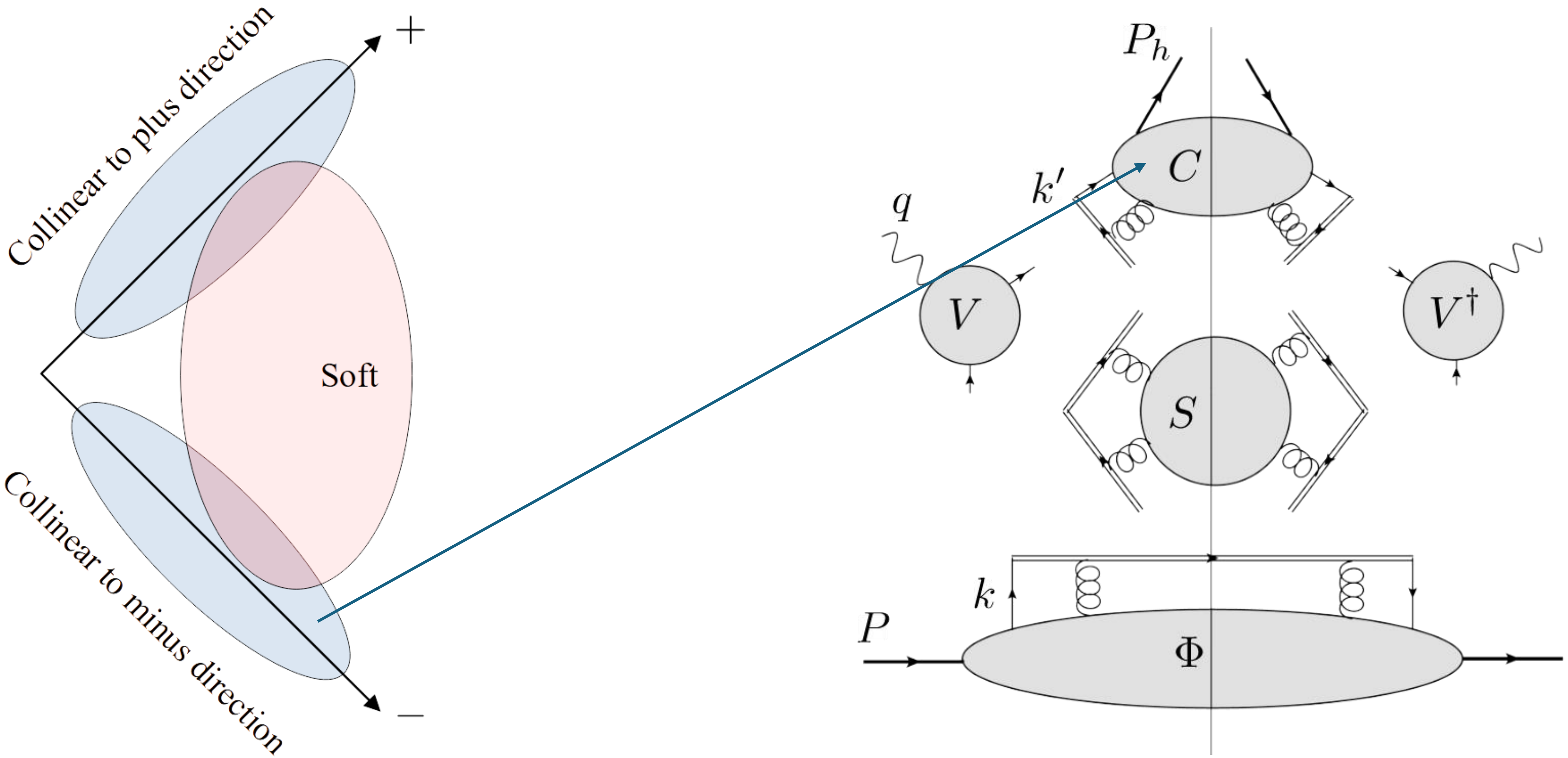
Factorization



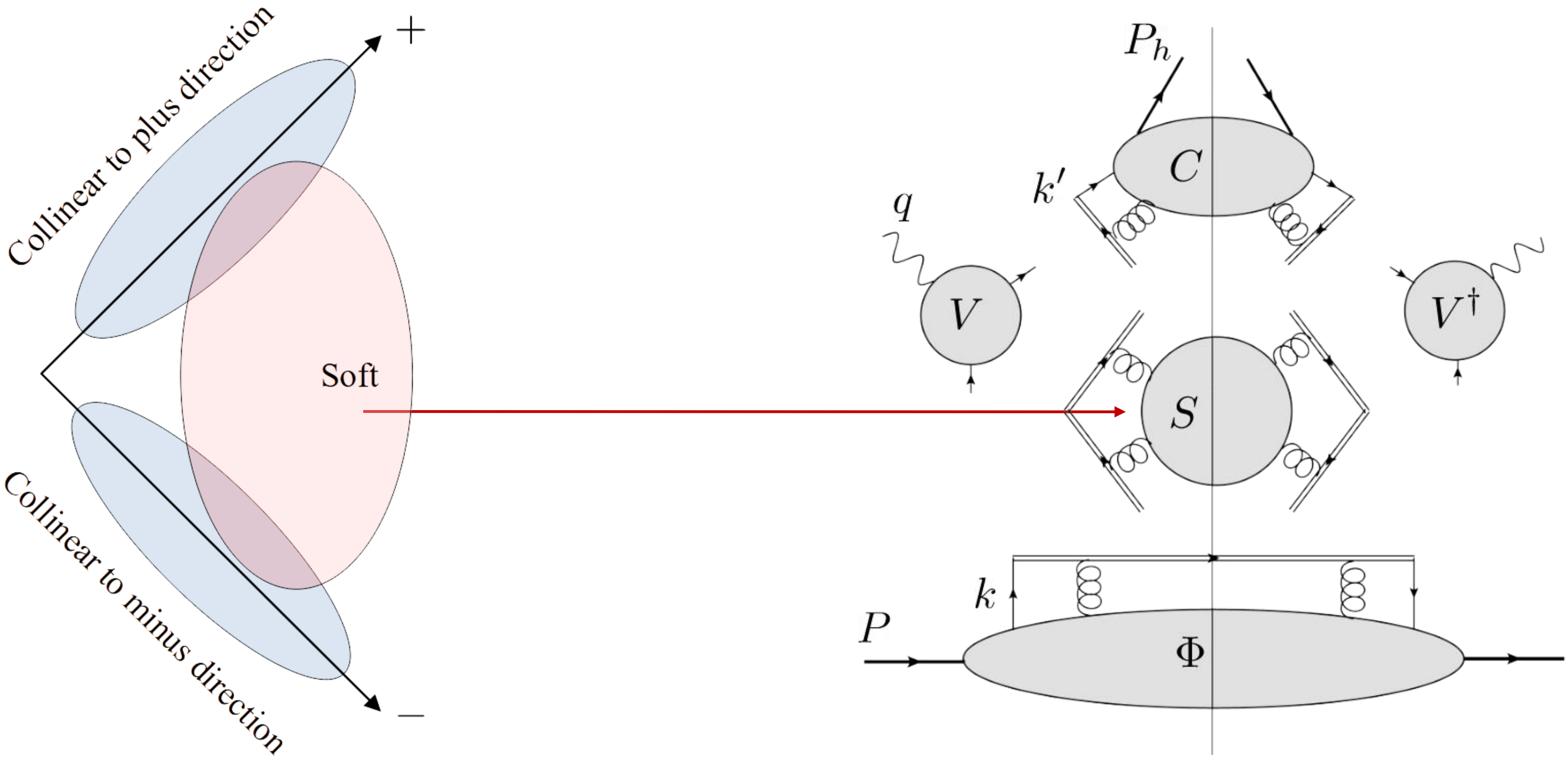
The factorization procedure is highly non-trivial!



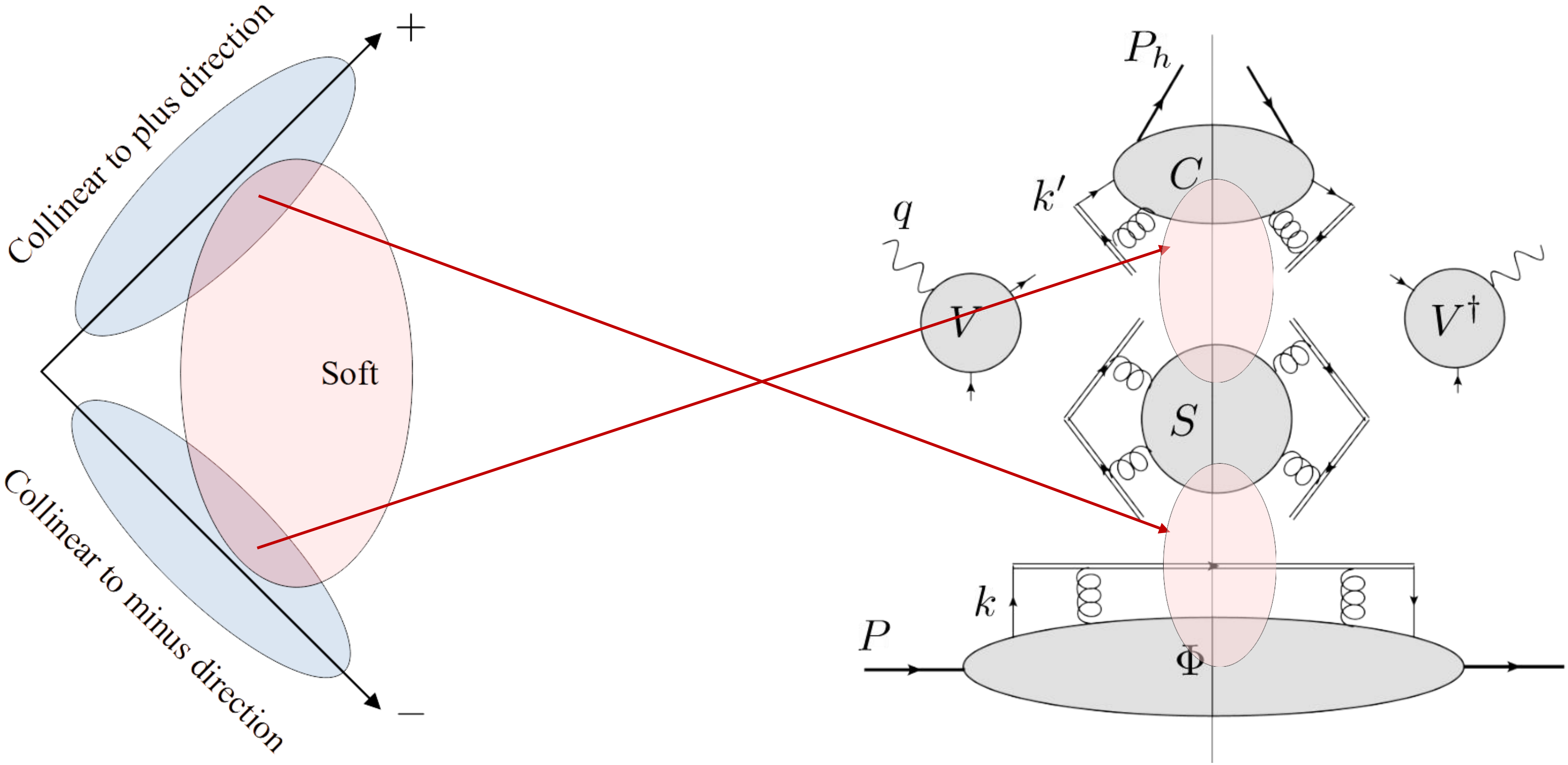
$$\Phi(x,\vec{b}_T) = \int \frac{d\xi^-}{2\pi}\, e^{ixP^+\xi^-} \langle P,S|\overline{\psi}(\xi) W_{0\to\xi}\psi(0)|P,S\rangle$$



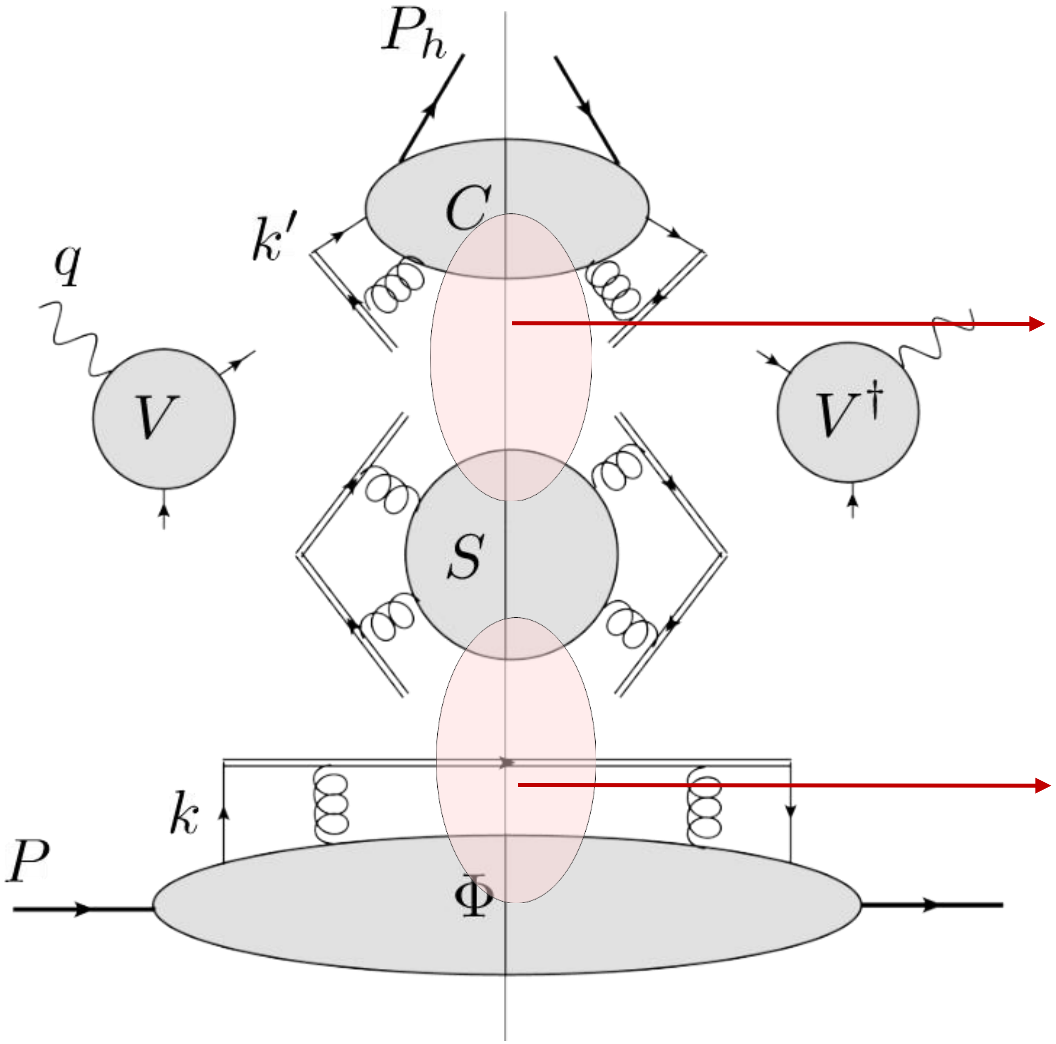
$$C(z,\vec{b}_T) = \sum_X \int \frac{d\xi^+}{2\pi}\, e^{i\frac{P_h^-}{z}\xi^+} \langle 0|W_{0\to\infty}\psi(0)|P_h,S_h;X\rangle\langle P_h,S_h;X|\overline{\psi}(\xi)W^\dagger_{\xi\to\infty}|0\rangle$$



$$S(b_T, y_1 - y_2) = \langle 0 | W_{\mathcal{C}}(b_T, y_1 - y_2) | 0 \rangle$$


Collinear to plus direction
+
Soft
Collinear to minus direction
−
P_h
C
k'
q
V
$V^\dagger$
S
k
P
Φ

Soft-collinear radiation feels the world as the soft radiation (same operator definition) **except for a different perception of collinear rapidities**:



$$S(b_T, +\infty - y_2) = \langle 0|W_{\mathcal{C}}(b_T, +\infty - y_2)|0\rangle$$

$$S(b_T, y_1 - (-\infty)) = \langle 0|W_{\mathcal{C}}(b_T, y_1 - (-\infty))|0\rangle$$

The **factorized** cross section is then:

$$d\sigma_{\text{SIDIS}} = H_{\text{SIDIS}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \boxed{\frac{\Phi^{[\Gamma]}(x, b_T)}{S(b_T, y_1 - (-\infty))}} S(b_T, y_1 - y_2) \boxed{\frac{C^{[\Gamma']}(z, b_T)}{S(b_T, +\infty - y_2)}} + \text{ p.s.}$$

Fully perturbative, process dependent hard part

$$H_{\text{SIDIS}} = 1 + a_S H^{[1]} + \mathcal{O}(a_S^2)$$

Soft-collinear subtractions and regularization of rapidity divergences

$$= H_{\text{SIDIS}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \Phi^{[\Gamma]}_{\text{sqrt.}}(x, b_T; y_n) C^{[\Gamma']}_{\text{sqrt.}}(z, b_T; y_n) + \text{ p.s.}$$

Smart rearrangement of soft factors

Soft troubles have been **absorbed** into the TMD definition

TMD DEFINITION

$$\Phi^{[\Gamma]}_{\text{sqrt.}}(x, b_T; \mu, y_n) = \Phi^{[\Gamma]}(x, b_T; \mu)\sqrt{\frac{S(b_T, \mu, y_n - (-\infty))}{S(b_T, \mu, +\infty - (-\infty))\, S(b_T, \mu, +\infty - y_n)}}$$

And analogous for final state part (plus – minus reversed role).

It cannot be obtained just speculating on the quark propagator inside a nucleon or through some parton model generalization.

The factorization theorem in full QCD is **crucial** to get here.

BRIDGING THEORY AND EXPERIMENTAL DATA

Implementing the QCD Operator Definition

What can we say about this complicate QCD operator?

- Adimensional (in b_T-space)

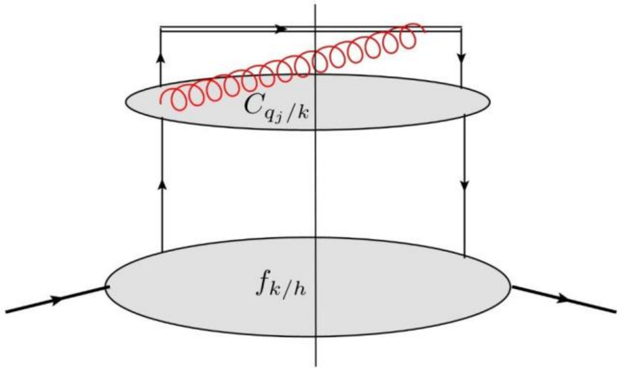
$$\Phi^{\mathrm{sqrt}}_{q_j/h}(x, b_T; \mu, y_n) = \Phi^{\mathrm{sqrt}}_{q_j/h}\left(x, a_S(\mu), \frac{\mu b_T}{c_1}, \frac{\overset{\sqrt{\zeta}}{\boxed{x\sqrt{2}P^+e^{-y_n}}}}{\mu}\right)$$

- Non-Perturbative at large distances $b_T \gg \frac{c_1}{\mu}$
- OPE at small distances $b_T \approx \frac{c_1}{\mu}$

Collinear PDFs

$$\Phi^{\mathrm{sqrt}}_{q_j/h} = \sum_k \int_x^1 \frac{d\rho}{\rho} \boxed{C_{q_j/k}\left(\rho, a_S(\mu), \log\left(\frac{\mu b_T}{c_1}\right), \log\left(\frac{\zeta}{\mu^2}\right)\right)} \boxed{f_{k/h}\left(\frac{x}{\rho}, \mu\right)} + \mathcal{O}(\mu b_T)^a$$

Wilson coefficients, fully perturbative*

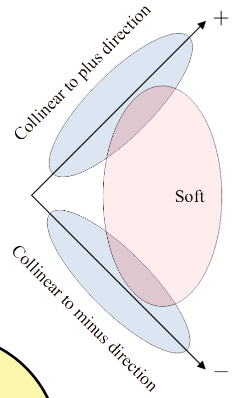


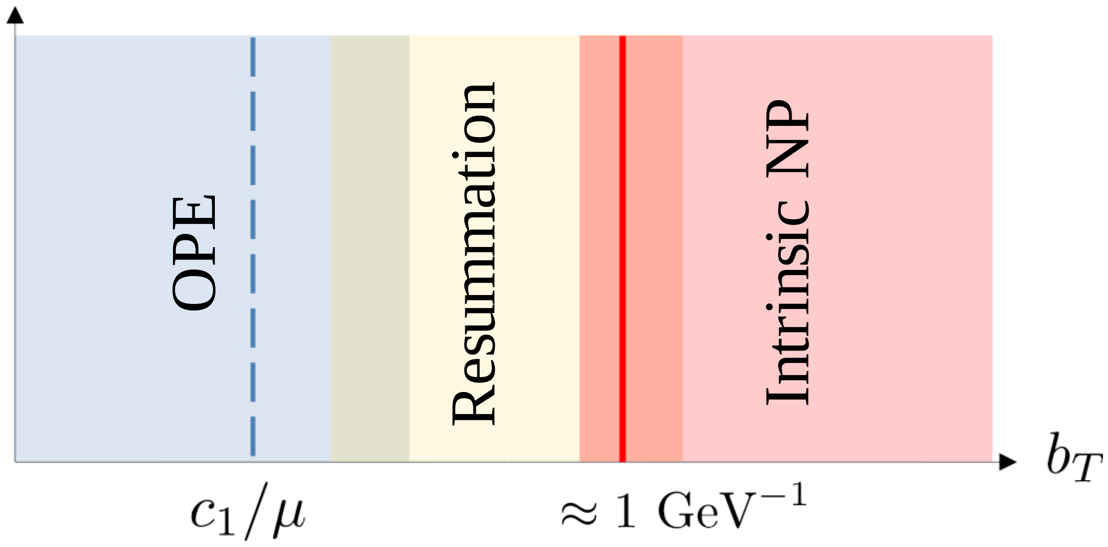
Most importantly, we know the Evolution Equations:

$$\begin{cases} \dfrac{\partial \log \Phi_{\mathrm{sqrt}}}{\partial \log \sqrt{\zeta}} = \boxed{K\left(a_S(\mu), \log\left(\dfrac{\mu b_T}{c_1}\right)\right)} \\ \dfrac{\partial \log \Phi_{\mathrm{sqrt}}}{\partial \log \mu} = \gamma_\Phi\left(a_S(\mu); \log\left(\dfrac{\zeta}{\mu^2}\right)\right) \end{cases}$$

With: $$\frac{dK}{d\log\mu} = -\gamma_K\left(a_S(\mu)\right)$$

Collins-Soper kernel, from Soft Entanglement





GOAL:
Implement the QCD operator definition such that:

1. OPE is satisfied at low bT
2. Transition to NP regime is smooth (natural)
3. pQCD is exploited as much as possible
4. The pheno (model) bias is minimized

General solution of Evolution Equations

$$\Phi_{q_j/h}(x,b_T;\mu,\zeta)=\boxed{\Phi_{q_j/h}(x,b_T;\mu_0,\zeta_0)}$$
$$\times\exp\left\{\frac{1}{2}\boxed{K(b_T;\mu_0)}\log\frac{\zeta}{\zeta_0}+\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\boxed{\gamma_\phi(a_S(\mu'))}-\frac{1}{2}\boxed{\gamma_K(a_S(\mu'))}\log\frac{\zeta}{\mu'^2}\right]\right\}$$

Reliable perturbative
expansion only at low b_T

Fully perturbative

The *reference scales* are totally arbitrary.
Nevertheless, their choice is crucial for the "**implementation scheme**".

NATURAL implementation scheme (PART 1)

$$\boxed{\mu_0 = c_1/b_T \equiv \mu_b, \quad \zeta_0 = \mu_b^2}$$

$$\begin{aligned}\Phi_{q_j/h}(x, b_T; \mu, \zeta) &= \Phi_{q_j/h}(x, b_T; \mu_b, \zeta_b) \\ &\times \exp\left\{\frac{1}{2}K(b_T;\mu_b)\log\frac{\zeta}{\mu_b^2} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_\phi(a_S(\mu')) - \frac{1}{2}\gamma_K(a_S(\mu'))\log\frac{\zeta}{\mu'^2}\right]\right\}\end{aligned}$$

$$a_S\left(c_1/b_T\right)$$

Problematic at large distances

$$\left\{\begin{aligned} &\Phi^{\text{sqrt}}_{q_j/h}(x, b_T; \mu_b, \mu_b^2) = f_{q_j/h}(x) + \\ &\qquad + a_S(\mu_b)\sum_k \int_x^1 \frac{d\rho}{\rho} C^{[1]}_{q_j/k}(\rho)\, f_{k/h}(x/\rho, \mu_b) + \ldots \\ &K(b_T, \mu_b) = \mathcal{O}\left(a_S^2(\mu_b)\right) \end{aligned}\right.$$

STANDARD implementation scheme

$$\boxed{\mu_0 = \frac{c_1}{b_T}\sqrt{1+\frac{b_T^2}{b_{\max}^2}} \equiv \mu_b^\star, \quad \zeta_0 = \mu_b^{\star 2}}$$

$$b_\star = \frac{b_T}{\sqrt{1+\frac{b_T^2}{b_{\max}^2}}}$$

$$\Phi_{q_j/h}(x, b_T; \mu, \zeta) = \Phi_{q_j/h}(x, b_T; \mu_b^\star, \mu_b^{\star 2})$$

$$\times \exp\left\{\frac{1}{2}K(b_T; \mu_b^\star)\log\frac{\zeta}{\mu_b^{\star 2}} + \int_{\mu_b^\star}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_\phi(a_S(\mu')) - \frac{1}{2}\gamma_K(a_S(\mu'))\log\frac{\zeta}{\mu'^2}\right]\right\}$$

Logs do not cancel. Problematic at large distances

$$K(b_T, \mu_b^\star) = -8a_S(\mu_b^\star)C_F\log\left(\sqrt{1+\frac{b_T^2}{b_{\max}^2}}\right) + \ldots$$

STANDARD implementation scheme

A way out (?): sum&subtract / multiply&divide

Purely **perturbative** (and **safe**)

$$K(b_T;\mu_b^\star) = \boxed{K(b_T^\star,\mu_b^\star)} - g_K(b_T,\dots)$$

$$\Phi_{q_j/h}(x,b_T;\mu_b^\star,\mu_b^{\star 2}) = \boxed{\Phi_{q_j/h}(x,b_T^\star;\mu_b^\star,\mu_b^{\star 2})}\,\Phi^{\mathrm{NP}}_{q_j/h}(b_T,\dots)$$
$$\times \exp\left\{-\frac{1}{2}g_K(b_T,\dots)\log\frac{\mu_b^{\star 2}}{\zeta_0}\right\}$$

$$\boxed{\begin{aligned}&\Phi^{\mathrm{sqrt}}_{q_j/h}(x,b_T^\star;\mu_b^\star,\mu_b^{\star 2}) = f_{q_j/h}(x) + \\ &\quad + a_S(\mu_b^\star)\sum_k\int_x^1\frac{d\rho}{\rho}C^{[1]}_{q_j/k}(\rho)\,f_{k/h}(x/\rho,\mu_b) + \mathcal{O}\left((b_T/b_{\max})^a;a_S^2(\mu_b^\star)\right) \\ &K(b_T^\star,\mu_b^\star) = \mathcal{O}\left(a_S^2(\mu_b^\star)\right)\end{aligned}}$$

STANDARD implementation scheme

Purely **non-perturbative** (**FIT**)

A way out (?): sum&subtract / multiply&divide

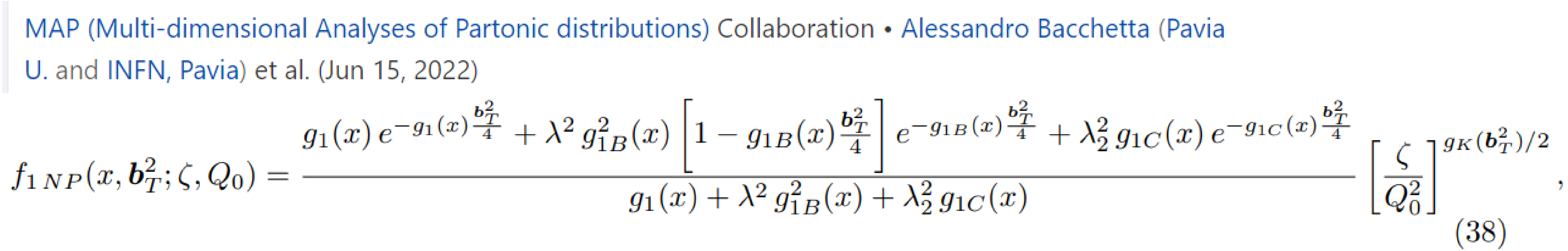
$$K(b_T;\mu_b^\star) = K(b_T^\star,\mu_b^\star) - \boxed{g_K(b_T,\dots)}$$

$$\Phi_{q_j/h}(x,b_T;\mu_b^\star,\mu_b^{\star 2}) = \Phi_{q_j/h}(x,b_T^\star;\mu_b^\star,\mu_b^{\star 2})\boxed{\Phi^{\mathrm{NP}}_{q_j/h}(b_T,\dots)}$$
$$\times \exp\left\{-\frac{1}{2}\boxed{g_K(b_T,\dots)}\log\frac{\mu_b^{\star 2}}{\zeta_0}\right\}$$

Nice. But does it *really* work?

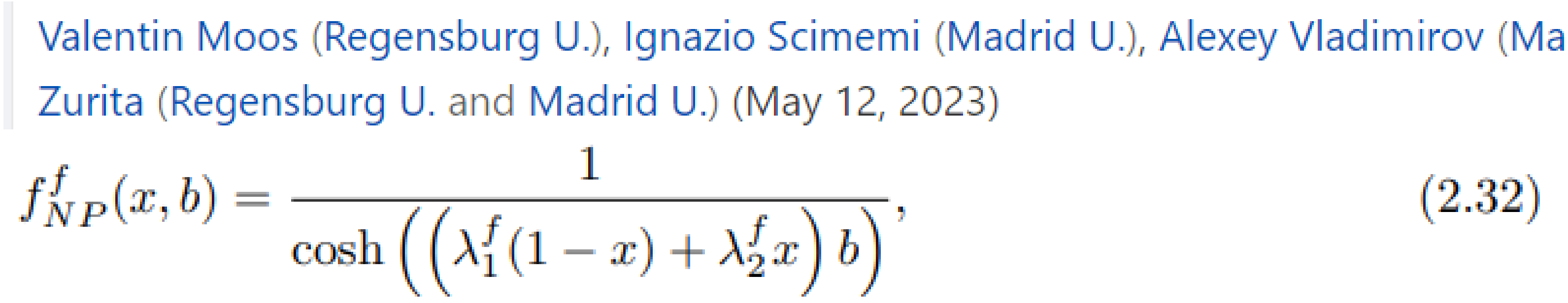
STANDARD implementation scheme

$$\boxed{\Phi^{\mathrm{NP}}_{q_j/h}(b_T,\dots)}$$



MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022)

$$f_{1\,NP}(x,\boldsymbol{b}_T^2;\zeta,Q_0) = \frac{g_1(x)\,e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\,g_{1B}^2(x)\left[1-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right]e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\,g_{1C}(x)\,e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x)+\lambda^2\,g_{1B}^2(x)+\lambda_2^2\,g_{1C}(x)}\left[\frac{\zeta}{Q_0^2}\right]^{g_K(\boldsymbol{b}_T^2)/2}, \qquad (38)$$



Valentin Moos (Regensburg U.), Ignazio Scimemi (Madrid U.), Alexey Vladimirov (Madrid U.), Pia Zurita (Regensburg U. and Madrid U.) (May 12, 2023)

$$f^f_{NP}(x,b)=\frac{1}{\cosh\left(\left(\lambda_1^f(1-x)+\lambda_2^f x\right)b\right)}, \qquad (2.32)$$

Both fit data very well!

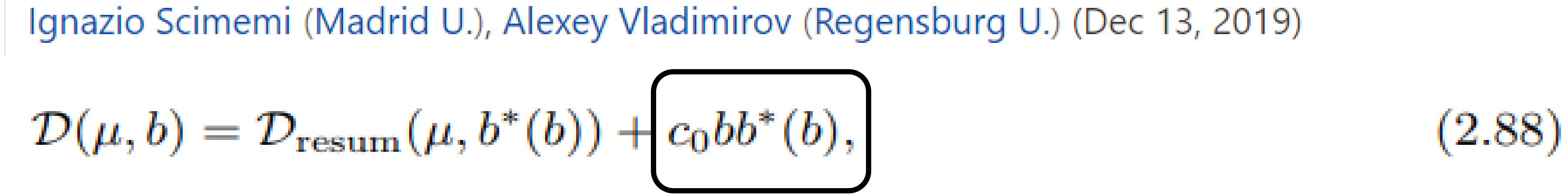
STANDARD implementation scheme

$$\boxed{g_K(b_T, \dots)}$$

MAP Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (May 22, 2024)

$$g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2} \, . \tag{30}$$

Both fit data very well!



Ignazio Scimemi (Madrid U.), Alexey Vladimirov (Regensburg U.) (Dec 13, 2019)

$$\mathcal{D}(\mu, b) = \mathcal{D}_{\text{resum}}(\mu, b^*(b)) + \boxed{c_0 b b^*(b),} \tag{2.88}$$

Nowadays, there is no agreement on NP content of TMDs

STANDARD implementation scheme

$$b_\star = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\rm max}^2}}}$$

The b* prescription is doing two different jobs:

1. Evolution (RG)

$$\frac{c_1}{b_T} \xrightarrow{\hspace{6em}} \frac{c_1}{b_{\rm max}}$$

Mild impact on pheno

2. Separation of Perturbative from Non-Perturbative regime

$$K(b_T;\mu_b^\star) = K(b_T^\star,\mu_b^\star) - g_K(b_T,\dots)$$

Strong impact on pheno

$$\Phi_{q_j/h}(x,b_T;\mu_b^\star,\mu_b^{\star 2}) = \Phi_{q_j/h}(x,b_T^\star;\mu_b^\star,\mu_b^{\star 2})\Phi^{\rm NP}_{q_j/h}(b_T,\dots)$$
$$\times \exp\left\{-\frac{1}{2}g_K(b_T,\dots)\log\frac{\mu_b^{\star 2}}{\zeta_0}\right\}$$

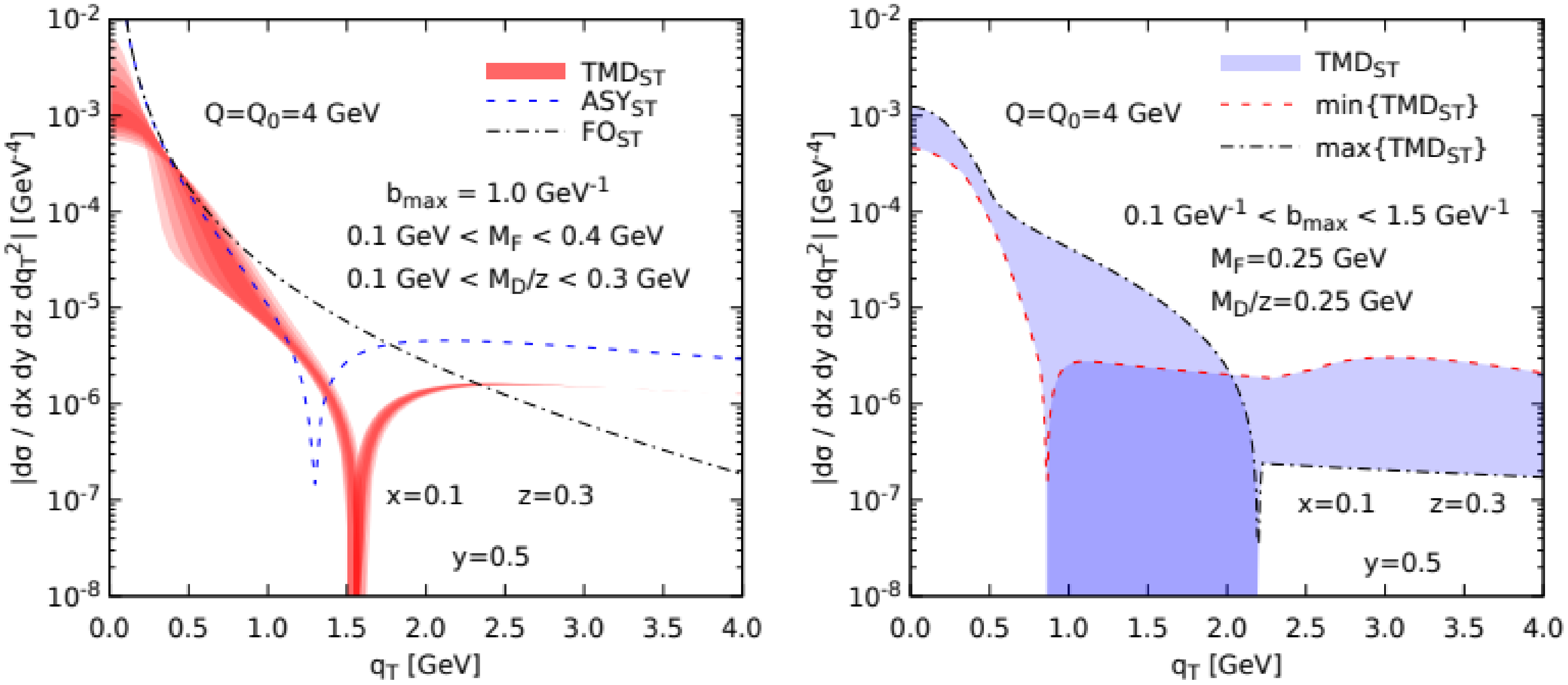
STANDARD implementation scheme

$$b_\star = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\max}^2}}}$$

Resolution to the problem of consistent large transverse momentum in TMDs #3

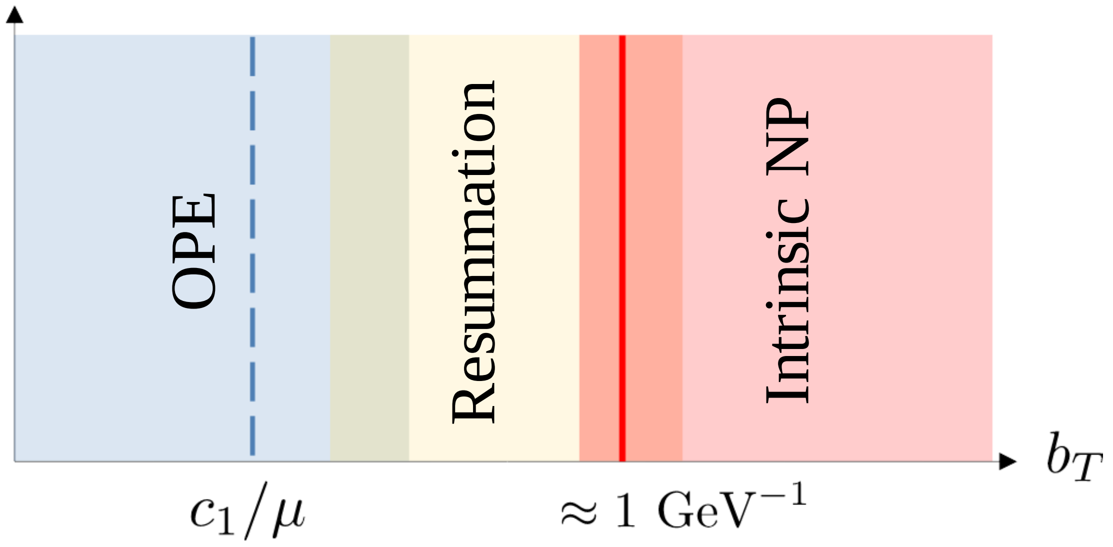
J.O. Gonzalez-Hernandez (Turin U. and INFN, Turin), T. Rainaldi (Old Dominion U. and Jefferson Lab),
T.C. Rogers (Old Dominion U. and Jefferson Lab) (Mar 8, 2023)

Published in: *Phys.Rev.D* 107 (2023) 9, 094029 • e-Print: 2303.04921 [hep-ph]



STANDARD implementation scheme

$$b_\star = \frac{b_T}{\sqrt{1+\frac{b_T^2}{b_{\rm max}^2}}}$$



GOAL:
Implement the QCD operator definition such that:

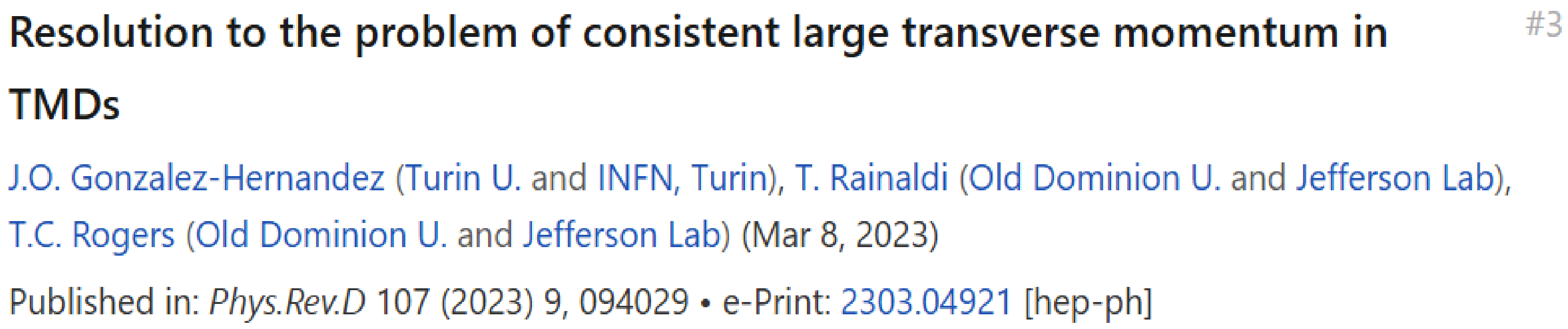
1. OPE is satisfied at low bT	✓	Not if $b_{\rm min}$ is used
2. Transition to NP regime is smooth (natural)	⚠	
3. pQCD is exploited as much as possible*	✓	Extractions at N3LL, N4LL…
4. The pheno (model) bias is minimized	✗	

HSO (Hadron Structure Oriented) implementation scheme

Combining nonperturbative transverse momentum dependence with TMD evolution #5

J.O. Gonzalez-Hernandez (Turin U.), T.C. Rogers (Old Dominion U. and Jefferson Lab), N. Sato (Jefferson Lab) (May 11, 2022)

Published in: *Phys.Rev.D* 106 (2022) 3, 034002 • e-Print: 2205.05750 [hep-ph]



Resolution to the problem of consistent large transverse momentum in TMDs #3

J.O. Gonzalez-Hernandez (Turin U. and INFN, Turin), T. Rainaldi (Old Dominion U. and Jefferson Lab), T.C. Rogers (Old Dominion U. and Jefferson Lab) (Mar 8, 2023)

Published in: *Phys.Rev.D* 107 (2023) 9, 094029 • e-Print: 2303.04921 [hep-ph]

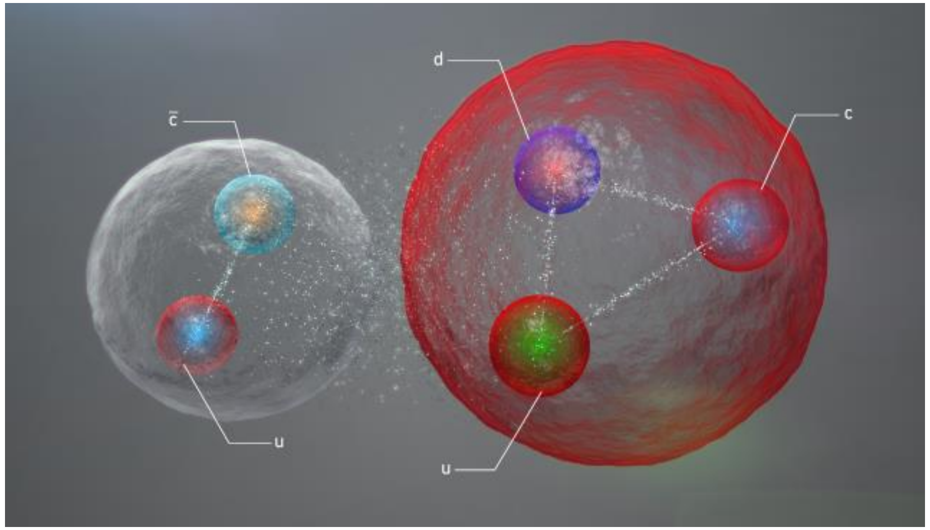
Phenomenology of TMD parton distributions in Drell-Yan and $With^0$ boson production in a hadron structure oriented approach #1

F. Aslan (Connecticut U. and Jefferson Lab), M. Boglione (Turin U. and INFN, Turin), J.O. Gonzalez-Hernandez (Turin U. and INFN, Turin), T. Rainaldi (Old Dominion U.), T.C. Rogers (Old Dominion U. and Jefferson Lab) et al. (Jan 25, 2024)

e-Print: 2401.14266 [hep-ph]

HSO (Hadron Structure Oriented) implementation scheme

Most of the information about hadron structure is at low energies



CERN website

Extract TMDs $Q_0 \approx \text{ few GeVs}$, then evolve and postdict high energy data

HSO (Hadron Structure Oriented) implementation scheme

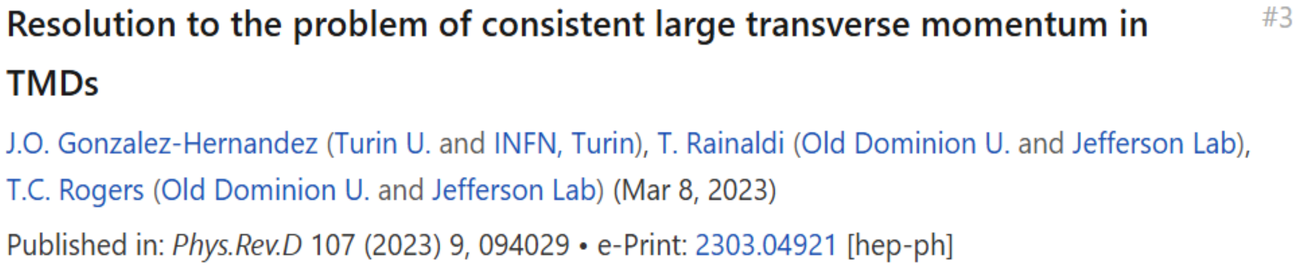
$$\boxed{\mu_0 = \frac{c_1}{b_T}\sqrt{1+\frac{b_T^2}{c_1^2/Q_0^2}} \equiv \overline{Q}_0(b_T), \quad \zeta_0 = Q_0^2}$$

*We used a different parametrization for the reference scale

$$\Phi_{q_j/h}(x, b_T; \mu, \zeta) = \Phi_{q_j/h}(x, b_T; \overline{Q}_0(b_T), Q_0^2)$$

$$\times \exp\left\{\frac{1}{2}K(b_T; \overline{Q}_0(b_T)) \log\frac{\zeta}{Q_0^2} + \int_{\overline{Q}_0(b_T)}^{\mu} \frac{d\mu'}{\mu'}\left[\gamma_\phi(a_S(\mu')) - \frac{1}{2}\gamma_K(a_S(\mu')) \log\frac{\zeta}{\mu'^2}\right]\right\}$$

Why the large logs are not a problem now?



Resolution to the problem of consistent large transverse momentum in TMDs #3

J.O. Gonzalez-Hernandez (Turin U. and INFN, Turin), T. Rainaldi (Old Dominion U. and Jefferson Lab), T.C. Rogers (Old Dominion U. and Jefferson Lab) (Mar 8, 2023)

Published in: *Phys.Rev.D* 107 (2023) 9, 094029 • e-Print: 2303.04921 [hep-ph]

HSO (Hadron Structure Oriented) implementation scheme

Because we parametrize (and fit) the large distance behavior:

$$K(b_T, Q_0) = \frac{a_S(Q_0) C_F}{2\pi} K_0(m_K b_T) + b_K e^{-m_K^2 b_T^2} + \boxed{D_K(Q_0, m_K, b_K)}$$

Fit parameters

Enforces the perturbative expansion (and the integral constraint)

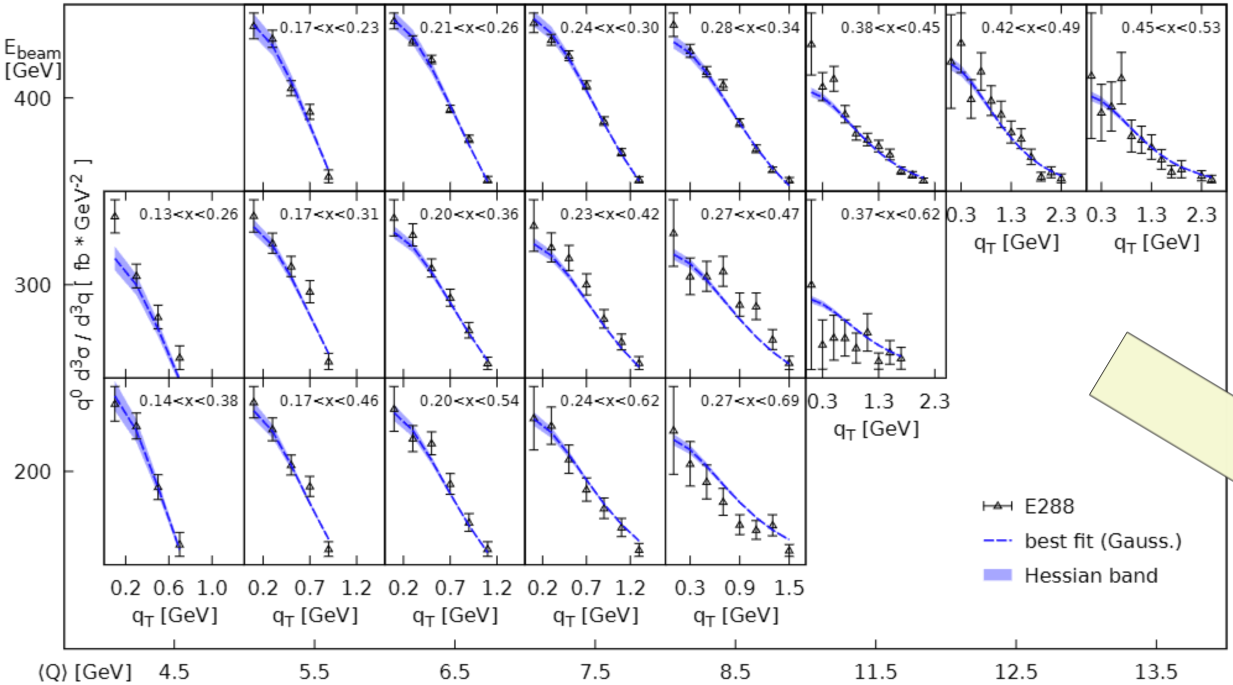
$$= -b_K + 8 a_S(Q_0) C_F \log \frac{m_K}{Q_0}$$

$$K(b_T \approx c_1/Q_0, Q_0) = -8 a_S(Q_0) C_F \log \frac{Q_0 b_T}{c_1} + \dots$$

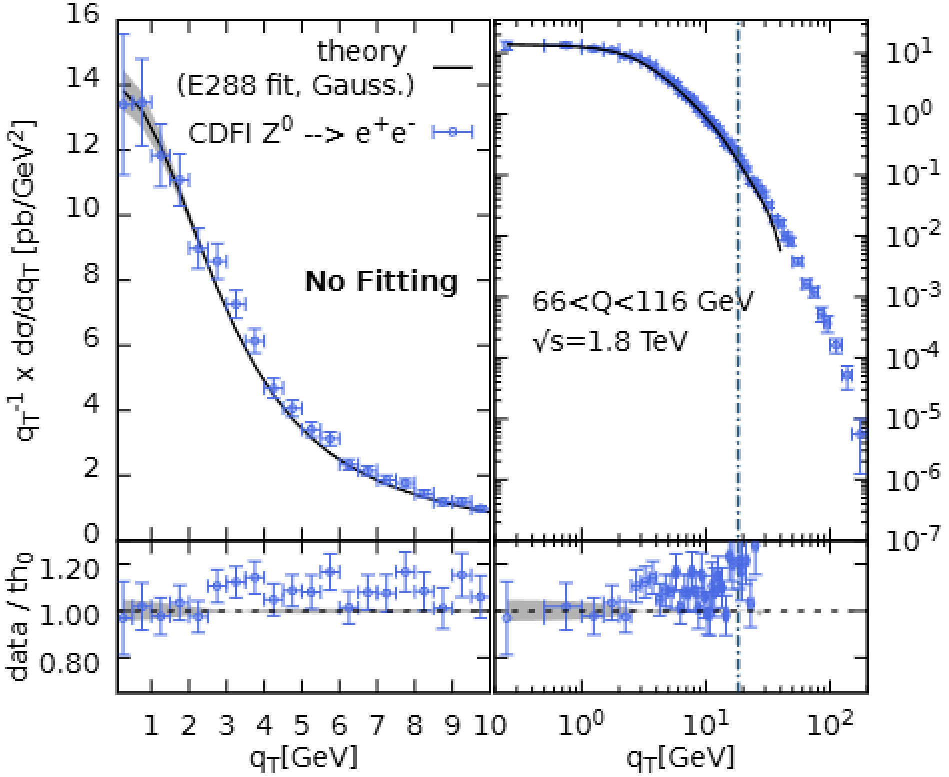
$$K(b_T \gg c_1/Q_0, Q_0) = D_K(Q_0, m_K, b_K) + \dots$$

An analogous treatment is adopted for the TMD at input scale

HSO (Hadron Structure Oriented) implementation scheme



High energy



Low energies

Phenomenology of TMD parton distributions in Drell-Yan and $With^0$ boson production in a hadron structure oriented approach #1

F. Aslan (Connecticut U. and Jefferson Lab), M. Boglione (Turin U. and INFN, Turin), J.O. Gonzalez-Hernandez (Turin U. and INFN, Turin), T. Rainaldi (Old Dominion U.), T.C. Rogers (Old Dominion U. and Jefferson Lab) et al. (Jan 25, 2024)

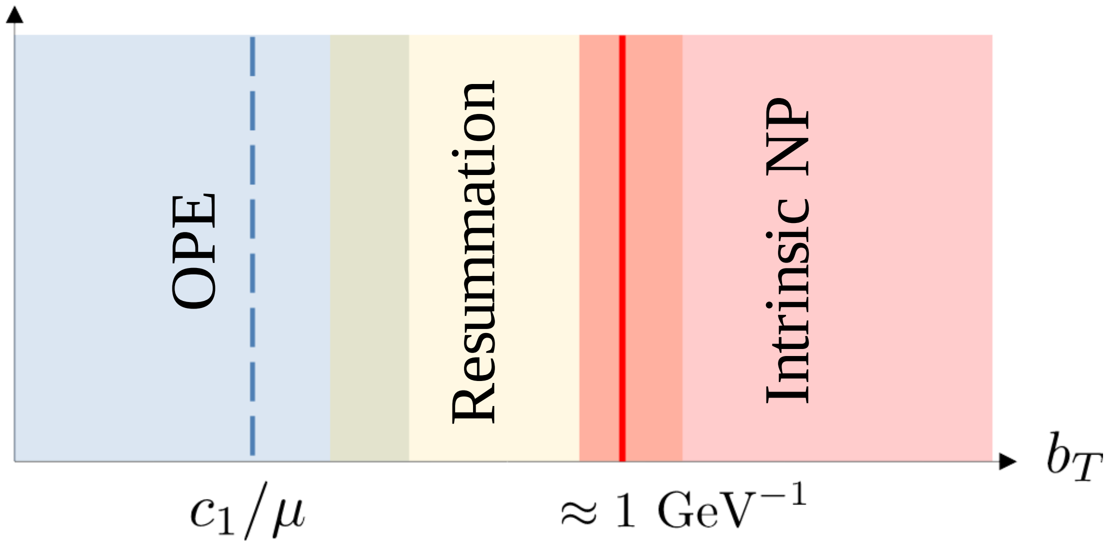
e-Print: 2401.14266 [hep-ph]

HSO (Hadron Structure Oriented) implementation scheme

Two jobs are now disentangled:

- Evolution $\frac{c_1}{b_T} \longrightarrow Q_0$
- Separation $m_K, m_F, b_K \ldots$

pheno



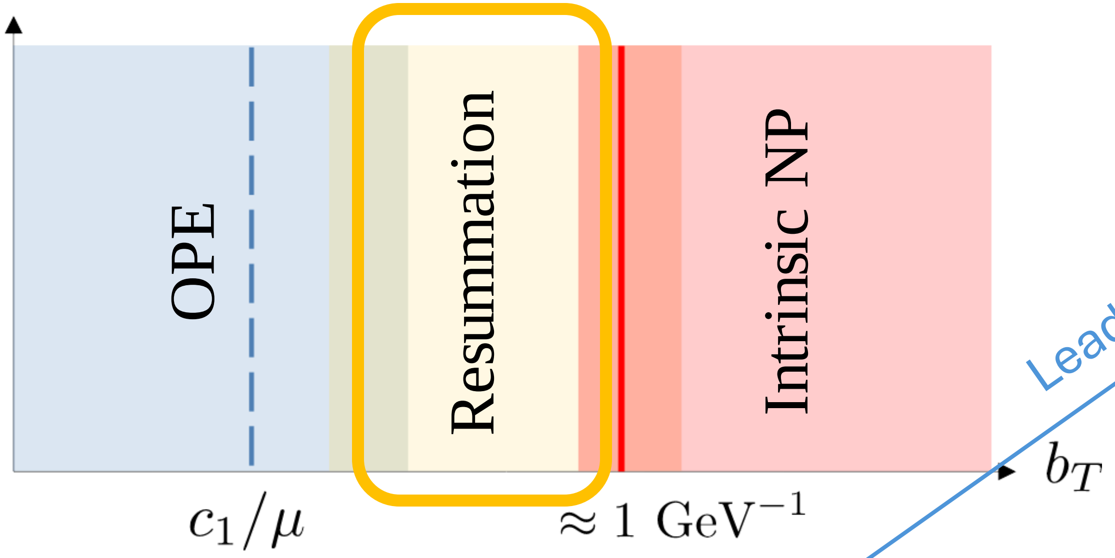
GOAL:
Implement the QCD operator definition such that:

1. OPE is satisfied at low bT ✓
2. Transition to NP regime is smooth (natural) ✓
3. pQCD is exploited as much as possible* ✓ ⚠ Our extraction at NLO
4. The pheno (model) bias is minimized ✓ ⚠

Where can we improve the scheme:

- Transition (intermediate) region, i.e. *where the logs gets large*

Catani, Frixione, Trentadue etc… late 80s-90s



Resumming **towers** of logs:

$$\boxed{a_S^n L^{2n}}$$

$$a_S^n L^{2n-1}$$

$$a_S^n L^{2n-2}$$

Leading Log (LL)

$$O = O|_{\text{ref. scale}}\, e^{L\boxed{g_1(\lambda)}+g_2(\lambda)+\frac{1}{L}g_3(\lambda)+\ldots}$$

$$L = \log\frac{Q}{Q_{\text{ref.}}}$$

$$\lambda \propto a_S(Q)L$$

Where can we improve the scheme:

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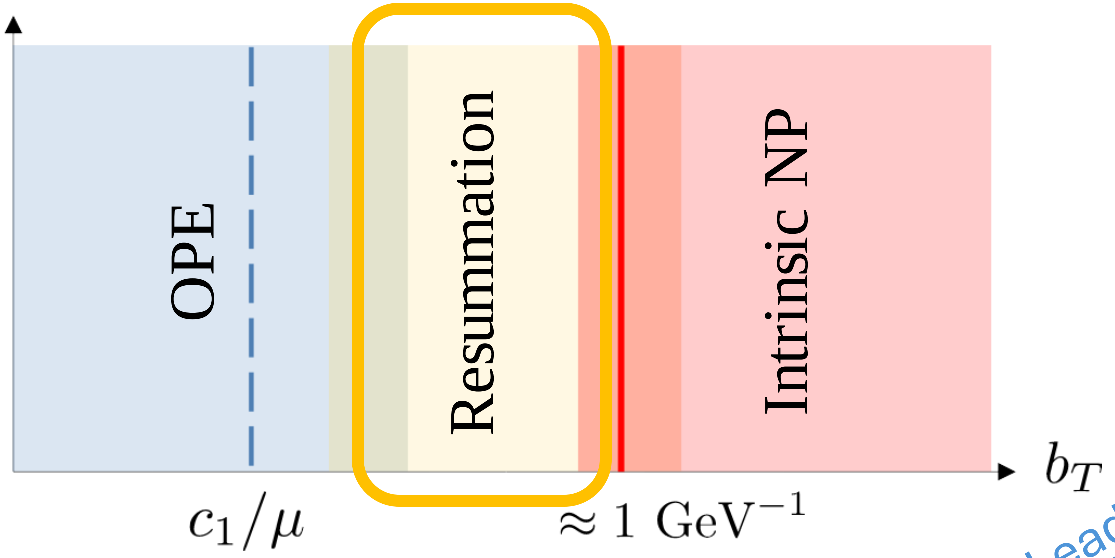
Catani, Frixione, Trentadue etc… late 80s-90s



Resumming **towers** of logs:

$$a_S^n L^{2n}$$

$$\boxed{a_S^n L^{2n-1}}$$

$$a_S^n L^{2n-2}$$

Next-Leading Log (NLL)

$$O = O|_{\text{ref. scale}}\, e^{Lg_1(\lambda)+\boxed{g_2(\lambda)}+\frac{1}{L}g_3(\lambda)+\dots}$$

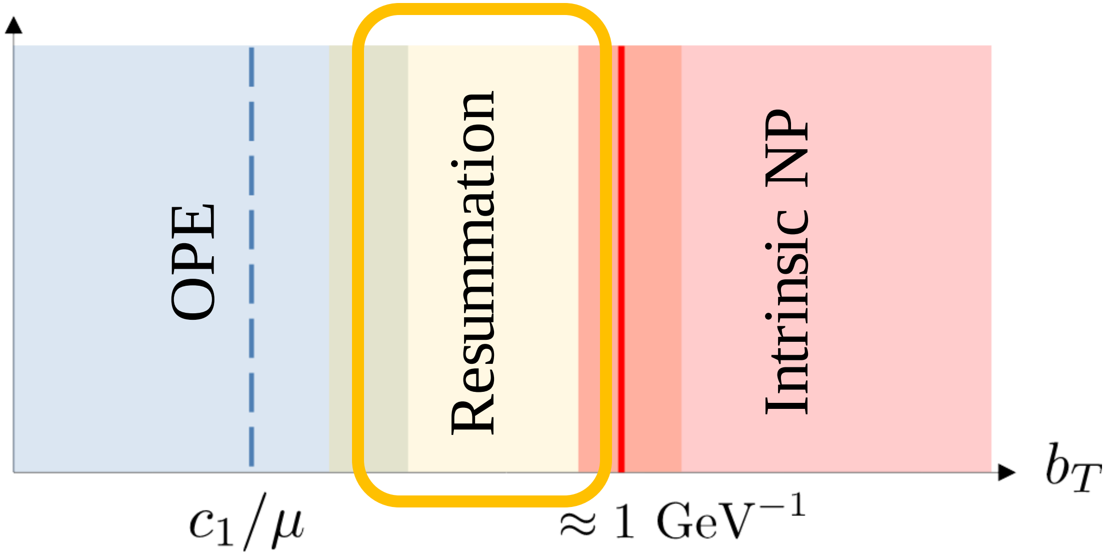
$$L = \log\frac{Q}{Q_{\text{ref.}}}$$

$$\lambda \propto a_S(Q)L$$

Where can we improve the scheme:

- Transition (intermediate) region, i.e. *where the logs gets large*

Catani, Frixione, Trentadue etc… late 80s-90s



Resumming **towers** of logs:

$$a_S^n L^{2n}$$

$$a_S^n L^{2n-1}$$

$$\boxed{a_S^n L^{2n-2}}$$

Next-next-Leading Log (N2LL)

$$O = O|_{\text{ref. scale}}\, e^{L g_1(\lambda) + g_2(\lambda) + \frac{1}{L}\boxed{g_3(\lambda)} + \ldots}$$

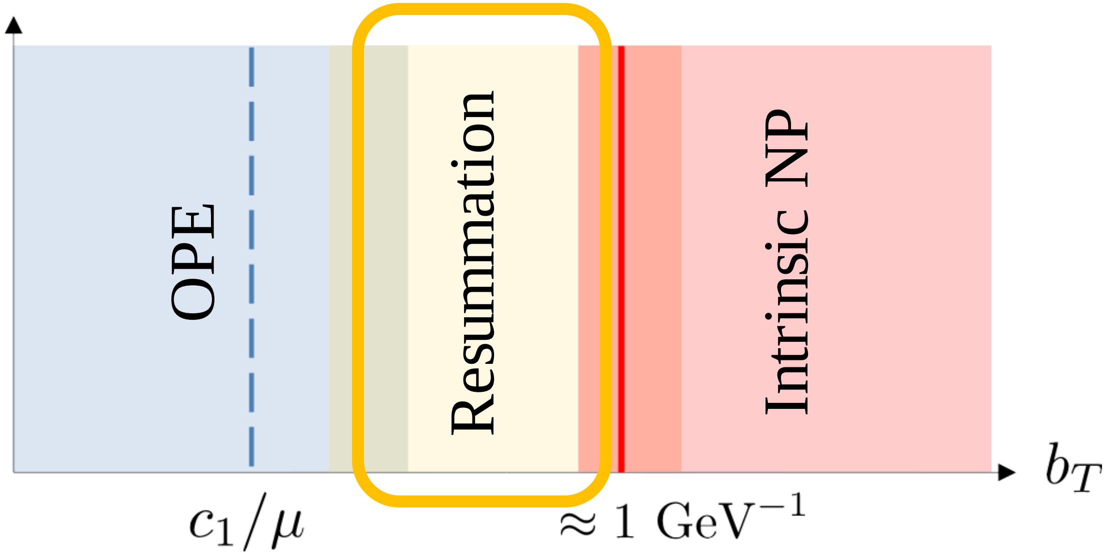
$$L = \log \frac{Q}{Q_{\text{ref.}}}$$

$$\lambda \propto a_S(Q) L$$

Where can we improve the scheme:

- Transition (intermediate) region, i.e. *where the logs gets large*

Catani, Frixione, Trentadue etc… late 80s-90s



Can't be the final answer:

$$\lambda \lesssim 1$$

Resumming **towers** of logs:

$$a_S^n L^{2n}$$

$$a_S^n L^{2n-1}$$

$$\boxed{a_S^n L^{2n-2}}$$

Next-next-Leading Log (N2LL)

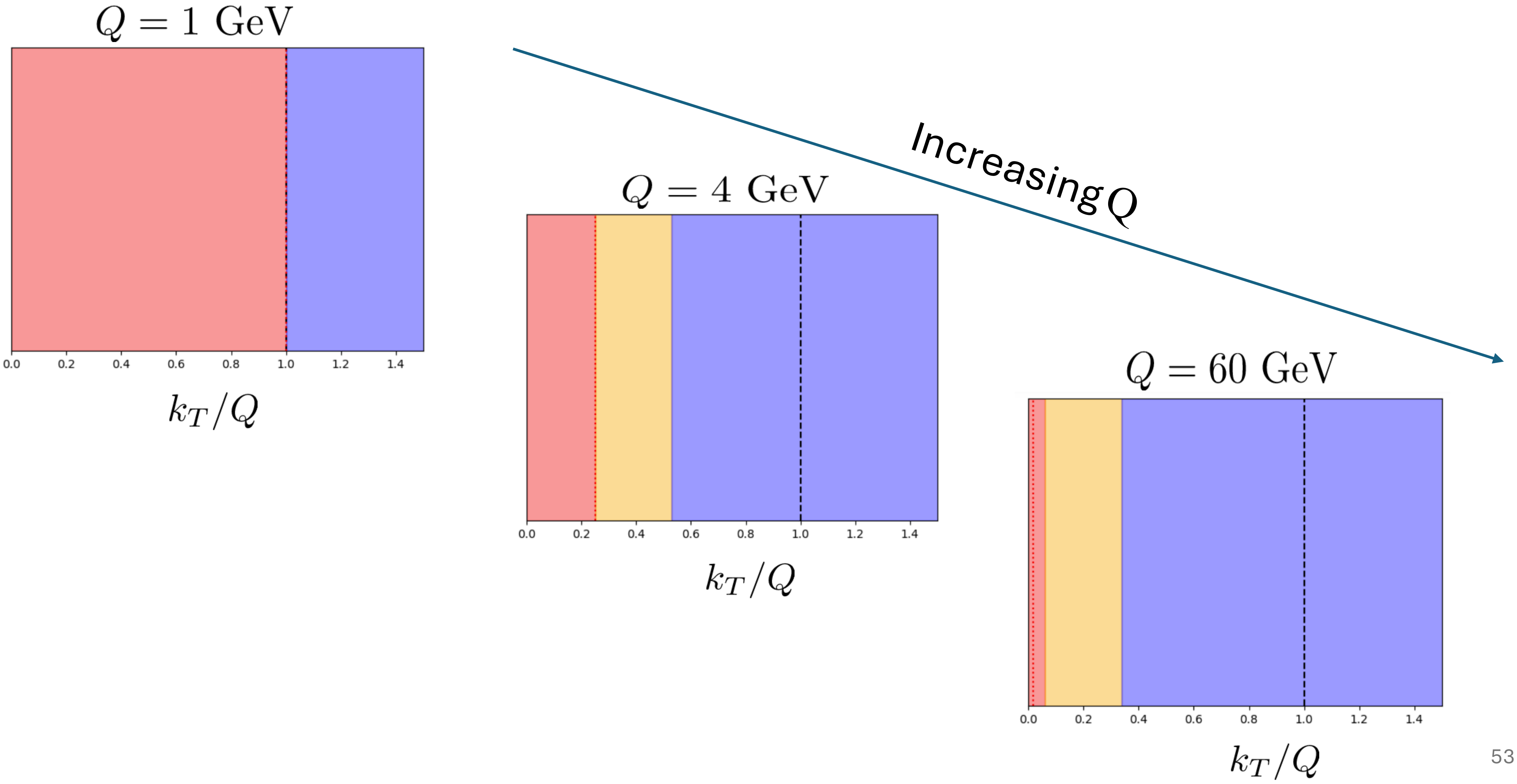
$$O = O\big|_{\mathrm{ref.\ scale}}\, e^{L g_1(\lambda)+g_2(\lambda)+\frac{1}{L}\boxed{g_3(\lambda)}+\dots}$$

$$L = \log\frac{Q}{Q_{\mathrm{ref.}}}$$

$$\lambda \propto a_S(Q) L$$

Where can we improve the scheme:

- Transition (intermediate) region, i.e. *where the logs gets large*



$Q = 1$ GeV

$Q = 4$ GeV

$Q = 60$ GeV

Increasing Q

Where can we improve the scheme:

- Collinear PDFs are (usually) *just* inputs

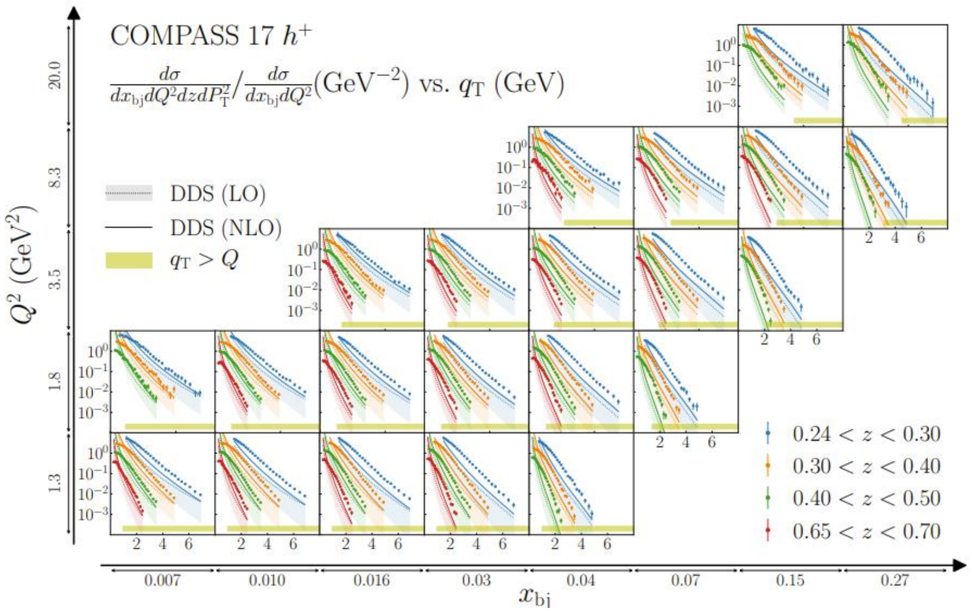
The efforts of TMD community are concentrated here

$$\Phi_{q_j/h} = \sum_k C_{q_j/k} \otimes \boxed{f_{k/h}} \times e^{\cdots} \times \boxed{(\text{NP content})}$$

From LHAPDF for instance, ultimately from global analyses of collinear (**integrated**) data

The problem of the tails

Gonzalez, Rogers, Sato, Wang,
Phys.Rev.D 98 (2018) 11, 114005



> *"We are basically trying to describe the integrand from the integral"*
>
> T. Rogers

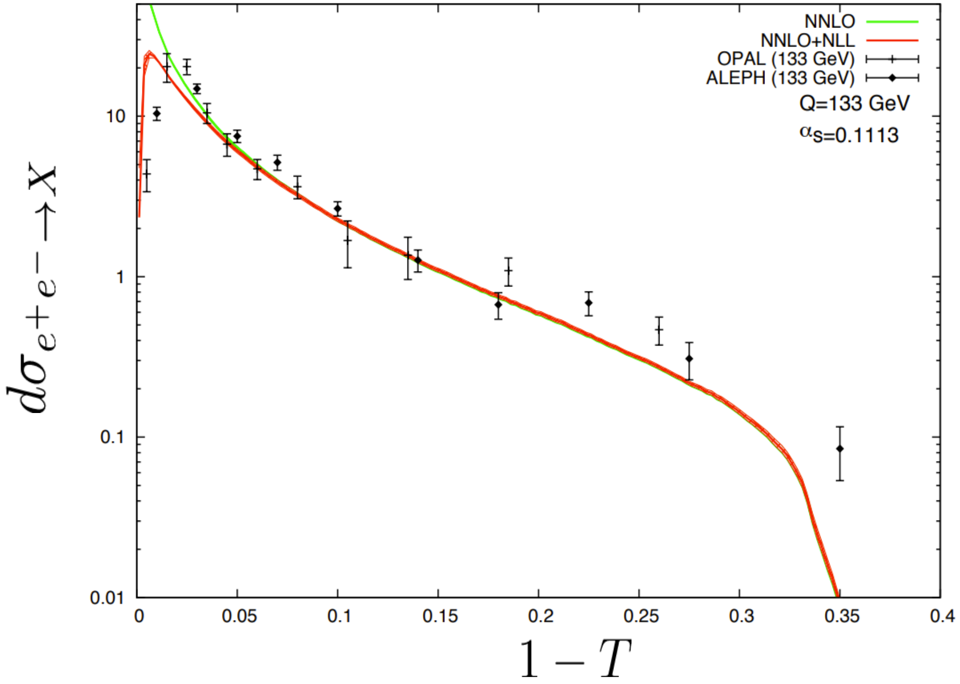
Where can we improve the scheme:

○ We struggle to implement TMDs in $\mathrm{b_T}$-space, but data are in $\mathrm{k_T}$ space

- Inverse Fourier Transform are an expensive and crucial part of TMD pheno analyses
- Inverse Fourier Transform necessarily requires to introduce a pheno bias (e.g. prescription to avoid the Landau pole)

$$b_\star = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\mathrm{max}}^2}}}$$

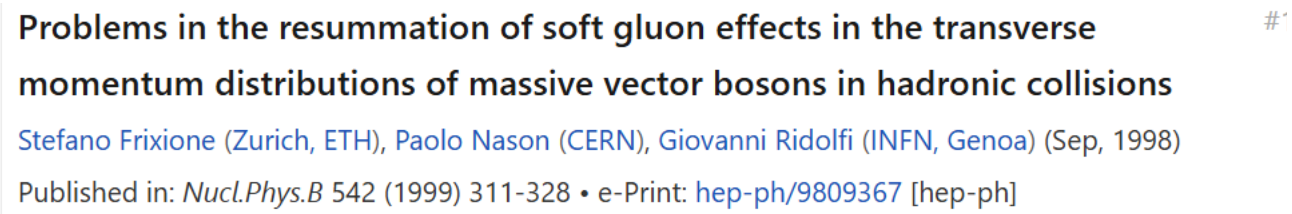
Why don't we try to replicate what people do for event-shape observables?



Where can we improve the scheme:

- We struggle to implement TMDs in $\mathrm{b_T}$-space, but data are in $\mathrm{k_T}$ space

The problem is that it is very difficult to obtain analytic expressions in $\mathrm{k_T}$ space and also being consistent with Evolution (log-counting)

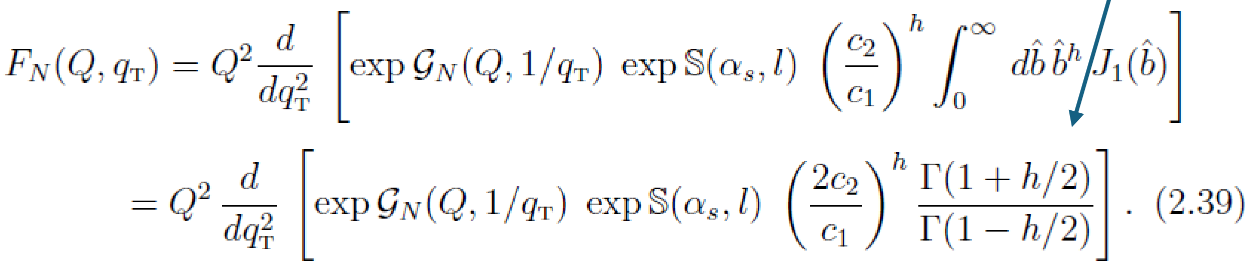


Problems in the resummation of soft gluon effects in the transverse momentum distributions of massive vector bosons in hadronic collisions

Stefano Frixione (Zurich, ETH), Paolo Nason (CERN), Giovanni Ridolfi (INFN, Genoa) (Sep, 1998)

Published in: *Nucl.Phys.B* 542 (1999) 311-328 • e-Print: hep-ph/9809367 [hep-ph]

Non-analytic expression!



$$F_N(Q,q_\mathrm{T}) = Q^2 \frac{d}{dq_\mathrm{T}^2}\left[\exp \mathcal{G}_N(Q,1/q_\mathrm{T})\ \exp \mathbb{S}(\alpha_s,l)\ \left(\frac{c_2}{c_1}\right)^h \int_0^\infty d\hat{b}\,\hat{b}^h J_1(\hat{b})\right]$$

$$= Q^2 \frac{d}{dq_\mathrm{T}^2}\left[\exp \mathcal{G}_N(Q,1/q_\mathrm{T})\ \exp \mathbb{S}(\alpha_s,l)\ \left(\frac{2c_2}{c_1}\right)^h \frac{\Gamma(1+h/2)}{\Gamma(1-h/2)}\right]. \quad (2.39)$$

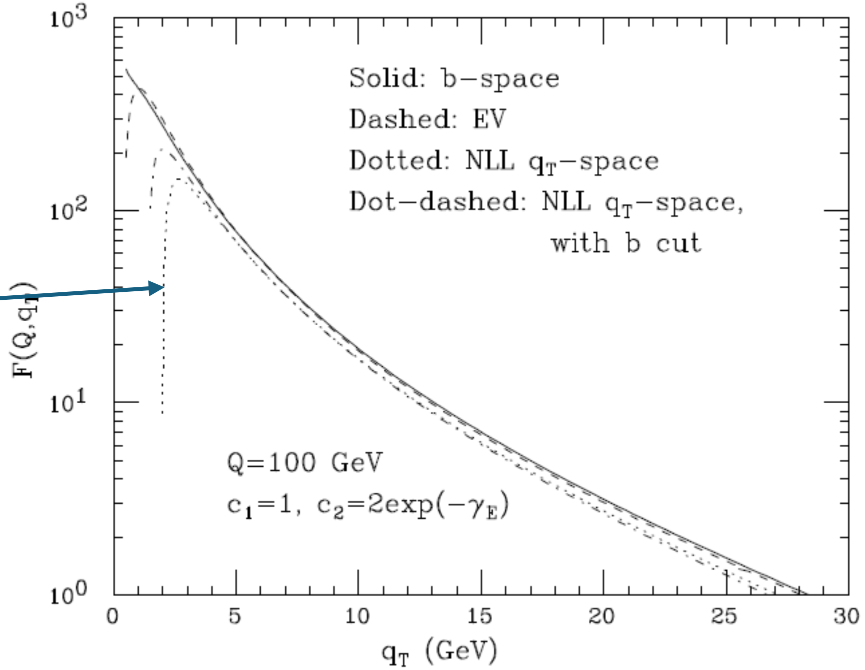
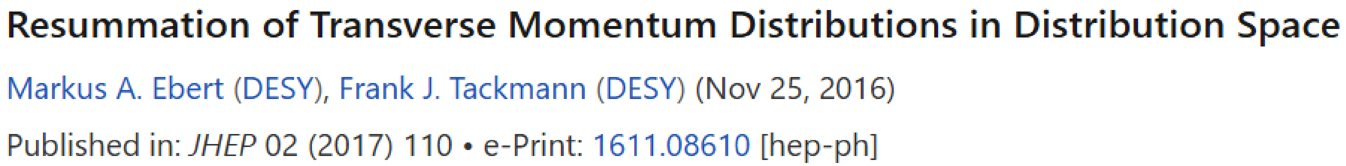


Figure 1: *Comparison of the numerical results of the various approaches for the calculation of the form factor.*

Where can we improve the scheme:

- We struggle to implement TMDs in b_T-space, but data are in k_T space

More recently

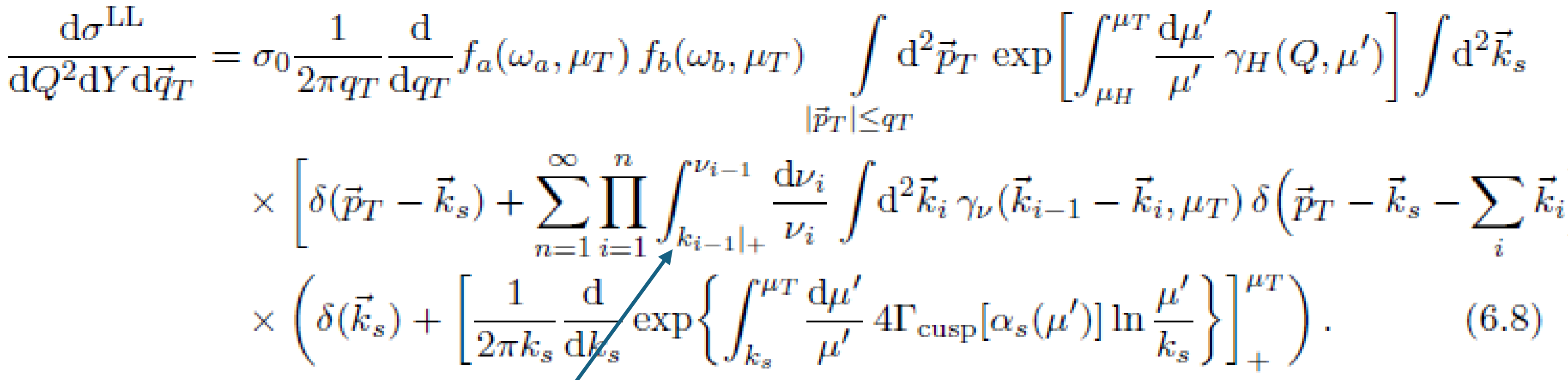


Resummation of Transverse Momentum Distributions in Distribution Space

Markus A. Ebert (DESY), Frank J. Tackmann (DESY) (Nov 25, 2016)

Published in: *JHEP* 02 (2017) 110 • e-Print: 1611.08610 [hep-ph]

Solution of Evolution directly in k_T-space



$$\frac{\mathrm{d}\sigma^{\mathrm{LL}}}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}\vec{q}_T} = \sigma_0 \frac{1}{2\pi q_T}\frac{\mathrm{d}}{\mathrm{d}q_T} f_a(\omega_a,\mu_T)\, f_b(\omega_b,\mu_T) \int\limits_{|\vec{p}_T|\le q_T} \mathrm{d}^2\vec{p}_T\, \exp\Big[\int_{\mu_H}^{\mu_T}\frac{\mathrm{d}\mu'}{\mu'}\,\gamma_H(Q,\mu')\Big]\int \mathrm{d}^2\vec{k}_s$$
$$\times \Big[\delta(\vec{p}_T-\vec{k}_s) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_{|k_{i-1}|_+}^{\nu_{i-1}}\frac{\mathrm{d}\nu_i}{\nu_i}\int \mathrm{d}^2\vec{k}_i\,\gamma_\nu(\vec{k}_{i-1}-\vec{k}_i,\mu_T)\,\delta\Big(\vec{p}_T-\vec{k}_s-\sum_i \vec{k}_i\Big)\Big]$$
$$\times \bigg(\delta(\vec{k}_s) + \Big[\frac{1}{2\pi k_s}\frac{\mathrm{d}}{\mathrm{d}k_s}\exp\Big\{\int_{k_s}^{\mu_T}\frac{\mathrm{d}\mu'}{\mu'}\,4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu')]\ln\frac{\mu'}{k_s}\Big\}\Big]_+^{\mu_T}\bigg). \qquad (6.8)$$

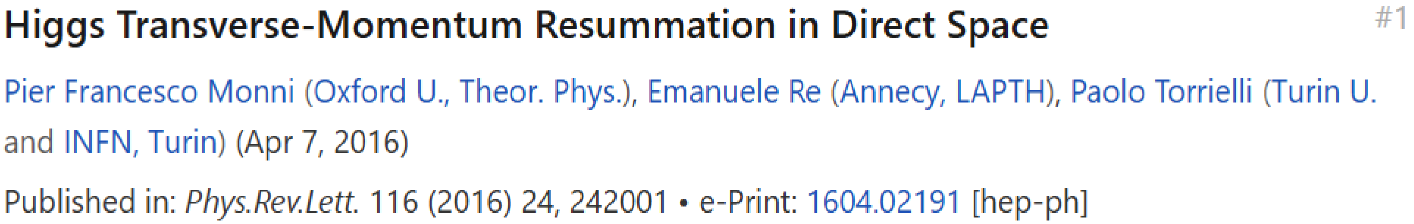
Infinite convolutions due to rapidity evolution

Where can we improve the scheme:

○ We struggle to implement TMDs in $\mathrm{b_T}$-space, but data are in $\mathrm{k_T}$ space

Also:

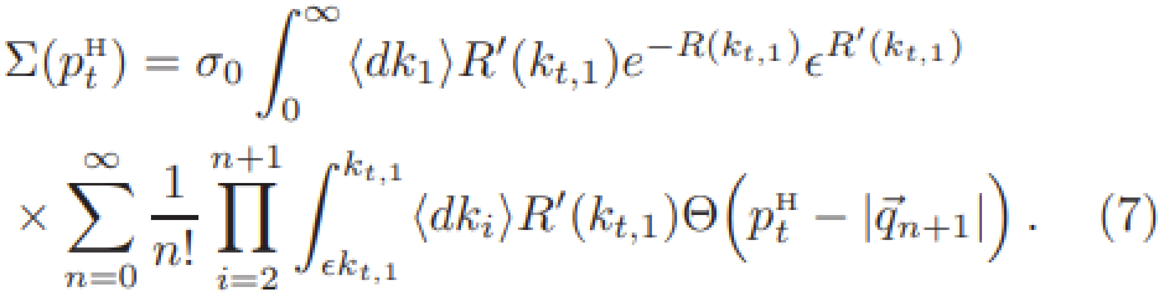
- Coherent branching algorithm



Higgs Transverse-Momentum Resummation in Direct Space #1

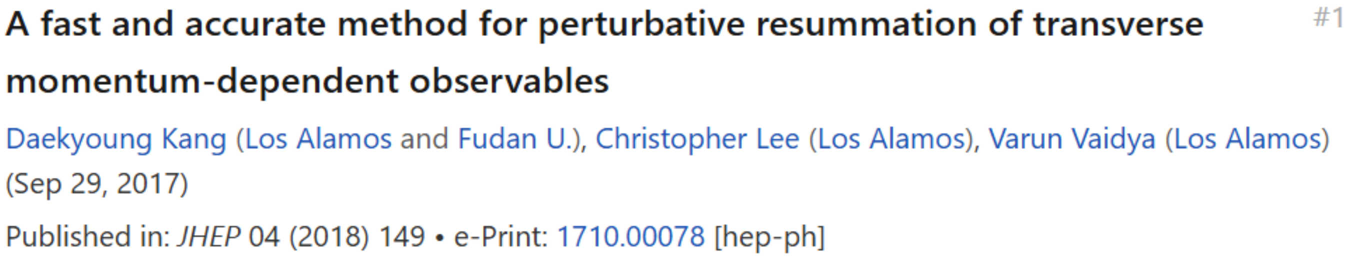
Pier Francesco Monni (Oxford U., Theor. Phys.), Emanuele Re (Annecy, LAPTH), Paolo Torrielli (Turin U. and INFN, Turin) (Apr 7, 2016)

Published in: *Phys.Rev.Lett.* 116 (2016) 24, 242001 • e-Print: 1604.02191 [hep-ph]



$$\Sigma(p_t^{\mathrm{H}}) = \sigma_0 \int_0^{\infty} \langle dk_1\rangle R'(k_{t,1}) e^{-R(k_{t,1})} \epsilon^{R'(k_{t,1})} \times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon k_{t,1}}^{k_{t,1}} \langle dk_i\rangle R'(k_{t,1}) \Theta\left(p_t^{\mathrm{H}} - |\vec{q}_{n+1}|\right) . \quad (7)$$

- Semi-analytic resummation



A fast and accurate method for perturbative resummation of transverse momentum-dependent observables #1

Daekyoung Kang (Los Alamos and Fudan U.), Christopher Lee (Los Alamos), Varun Vaidya (Los Alamos) (Sep 29, 2017)

Published in: *JHEP* 04 (2018) 149 • e-Print: 1710.00078 [hep-ph]

NATURAL implementation scheme (PART 2)

$a_S\left(c_1/b_T\right)$

Problematic at large distances

$$\boxed{\mu_0 = c_1/b_T \equiv \mu_b, \quad \zeta_0 = \mu_b^2}$$

Let's trade this problem for the analogous issue in k_T-space

$$\log \frac{\mu b_T}{c_1} = \boxed{\log \frac{\mu}{k_T}} + \boxed{\log \frac{k_T b_T}{c_1}}$$

Log-counting in transverse momentum space L

$\mathcal{L}$ Taylor expansion variable

NATURAL implementation scheme (PART 2)

$$\boxed{\lambda = 2\beta_0 a_S(\mu) L}$$

$$\Phi^{\mathrm{LL}}_{q_j/h}(x, k_T; \mu, \mu^2) = -\frac{1}{2\pi k_T^2} h_1(\lambda) e^{L g_1(\lambda)} f_{q_j/h}(x, k_T)$$

$$\Phi^{\mathrm{NLL}}_{q_j/h}(x, k_T, \mu, \mu^2) = -\frac{1}{2\pi k_T^2} e^{L g_1(\lambda) + g_2(\lambda)}$$
$$\times \left\{ \left[\left(h_1(\lambda) + \frac{1}{L}\vartheta_1(\lambda) \right) \psi(\lambda, L) + \frac{1}{L}\vartheta_2(\lambda) + \psi'(\lambda, L) \right] f_{q_j/k}(x, k_T) + \frac{d}{d \log k_T} f_{q_j/h}(x, k_T) \right\}$$

- ✓ Full analytic
- ✓ Full perturbative → Can't be the final answer: $\lambda \lesssim 1$

NATURAL implementation scheme (PART 2)

$$\boxed{\lambda = 2\beta_0 a_S(\mu) L}$$

$$\Phi^{\mathrm{LL}}_{q_j/h}(x, k_T; \mu, \mu^2) = -\frac{1}{2\pi k_T^2} h_1(\lambda) e^{L g_1(\lambda)} f_{q_j/h}(x, k_T)$$

$$\Phi^{\mathrm{NLL}}_{q_j/h}(x, k_T, \mu, \mu^2) = -\frac{1}{2\pi k_T^2} e^{L g_1(\lambda) + g_2(\lambda)}$$

$$\times \left\{ \left[\left(h_1(\lambda) + \frac{1}{L} \vartheta_1(\lambda) \right) \psi(\lambda, L) + \frac{1}{L} \vartheta_2(\lambda) + \psi'(\lambda, L) \right] f_{q_j/k}(x, k_T) + \frac{d}{d \log k_T} f_{q_j/h}(x, k_T) \right\}$$

$$\frac{1}{(k_T^2)^{1+1/2 g_1(\lambda)}} \Rightarrow \frac{1}{(k_T^2 + m^2)^{1+1/2 g_1(\lambda)}}$$

- ✓ Full analytic
- ✓ Full perturbative
- ✓ Natural extension to NP

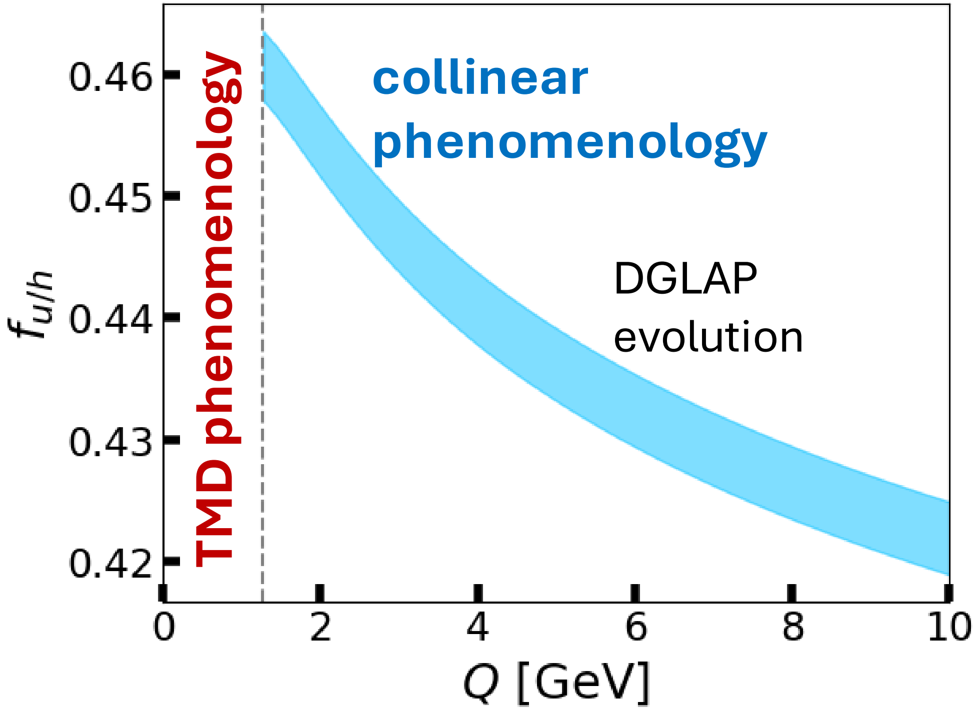
Power-law seems the functional form suggested by pQCD

NATURAL implementation scheme (PART 2)

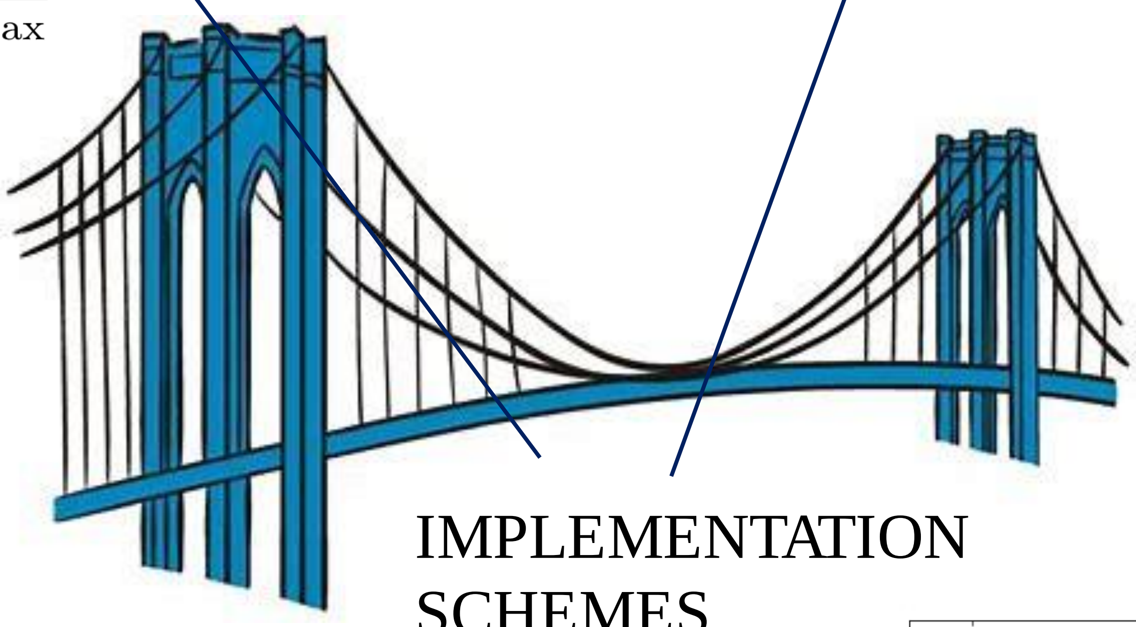
- Clean test of how far we can get with the sole perturbative QCD (no pheno biases **at all**)
- Clean(er) test of QCD factorization
- Exploration of collinear PDFs in the Deep InfraRed

$$\Phi_{q_j/h} \hookleftarrow f_{q_j/h}(x, \boxed{k_T})$$

BONUS: Natural flavor dependence inside the TMD Non-Perturbative content



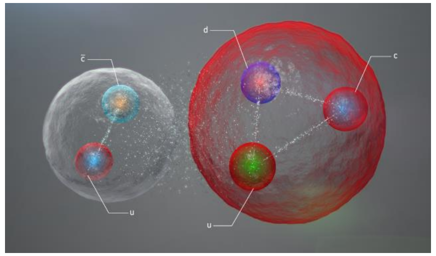
CONCLUSIONS



STANDARD

$$b_\star = \frac{b_T}{\sqrt{1+\frac{b_T^2}{b_{\max}^2}}}$$

HSO



THEORY

DATA

IMPLEMENTATION SCHEMES

NATURAL

GOAL:

1. OPE is satisfied at low bT
2. Transition to NP regime is smooth (natural)
3. pQCD is exploited as much as possible
4. The pheno (model) bias is minimized