Factorization for energy energy correlator in HIC

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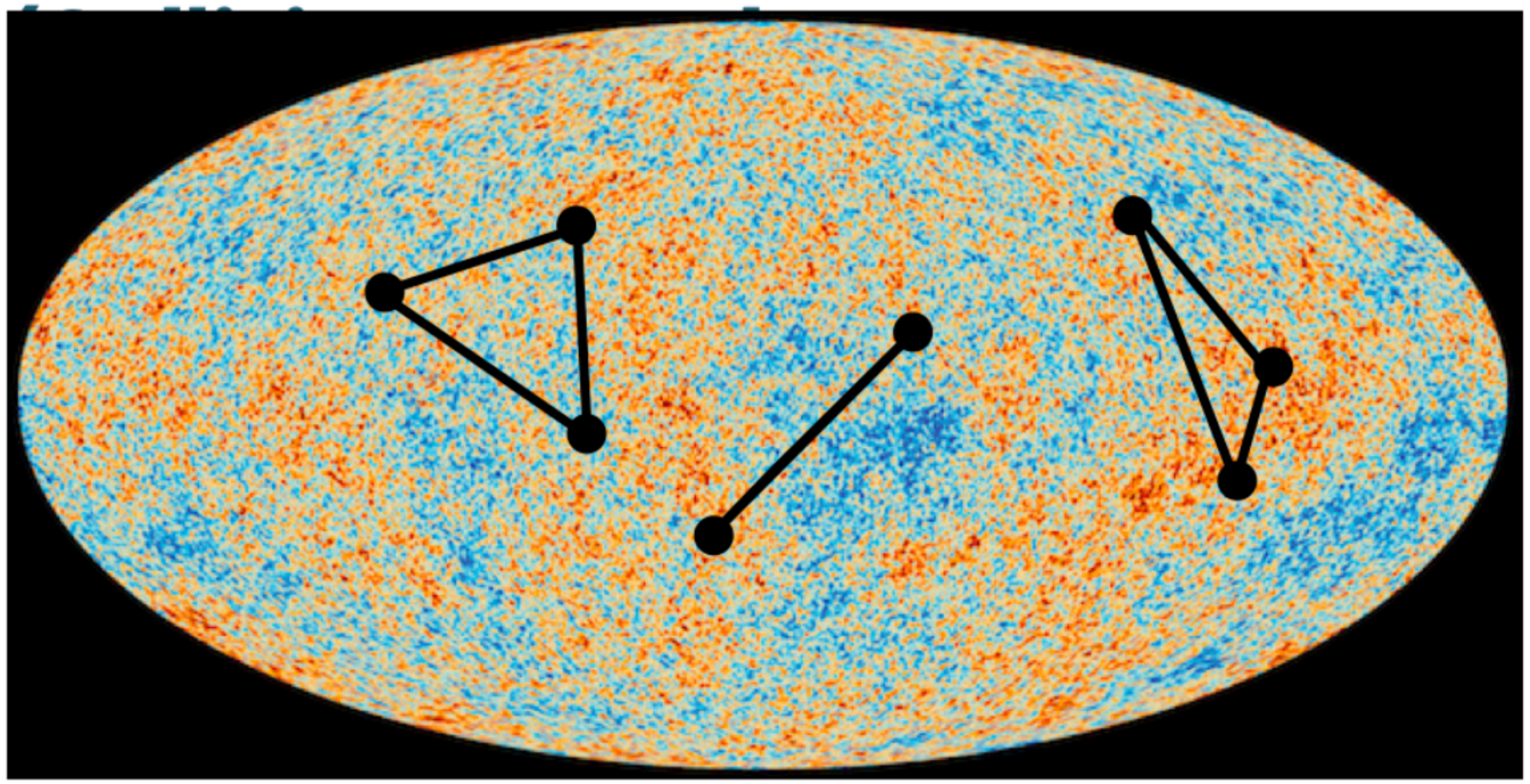
Based on : 2408.02753

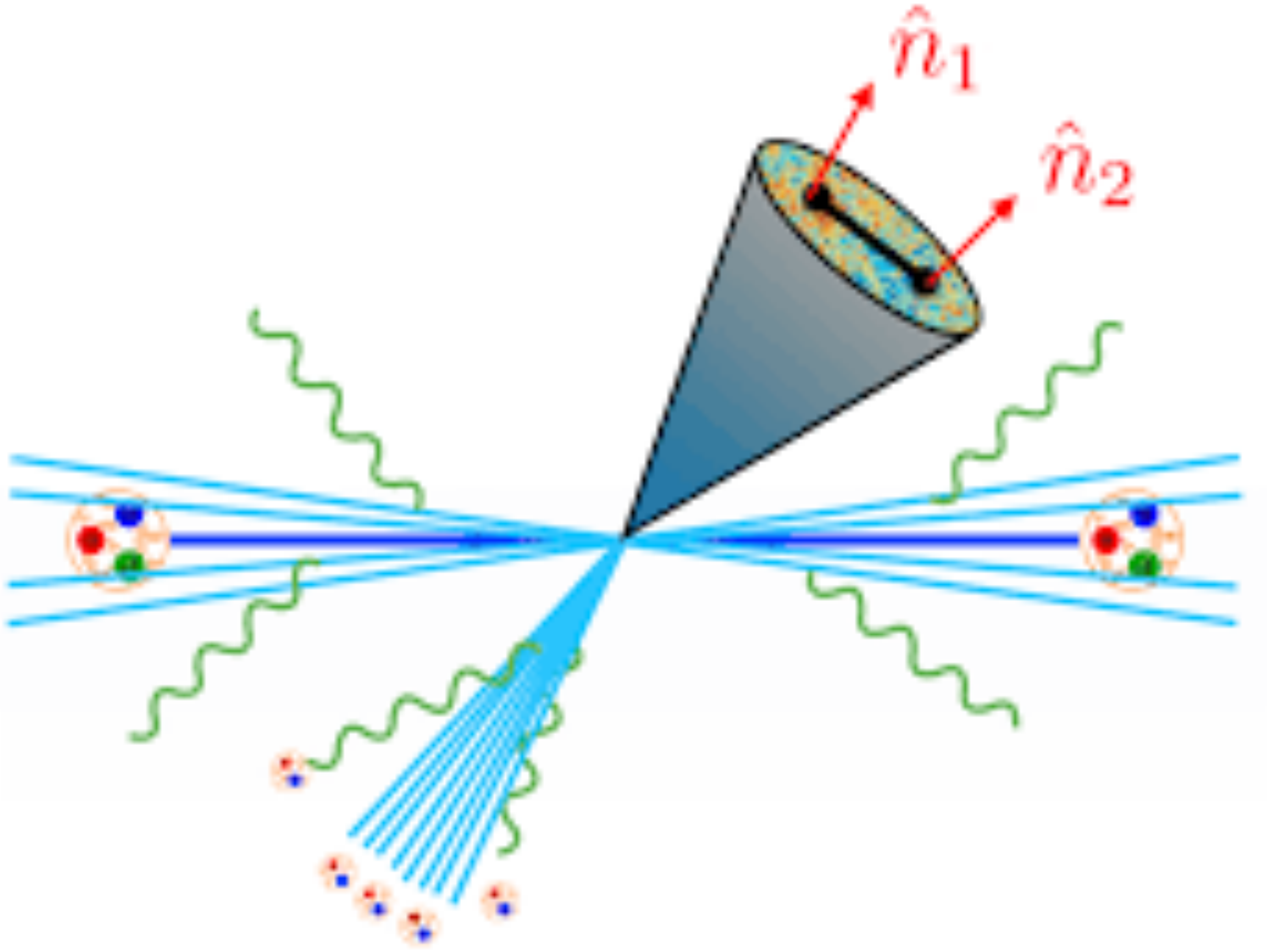
INT program Heavy ion collision in the EIC era

Seattle, August 7, 2024

Correlation functions : in general

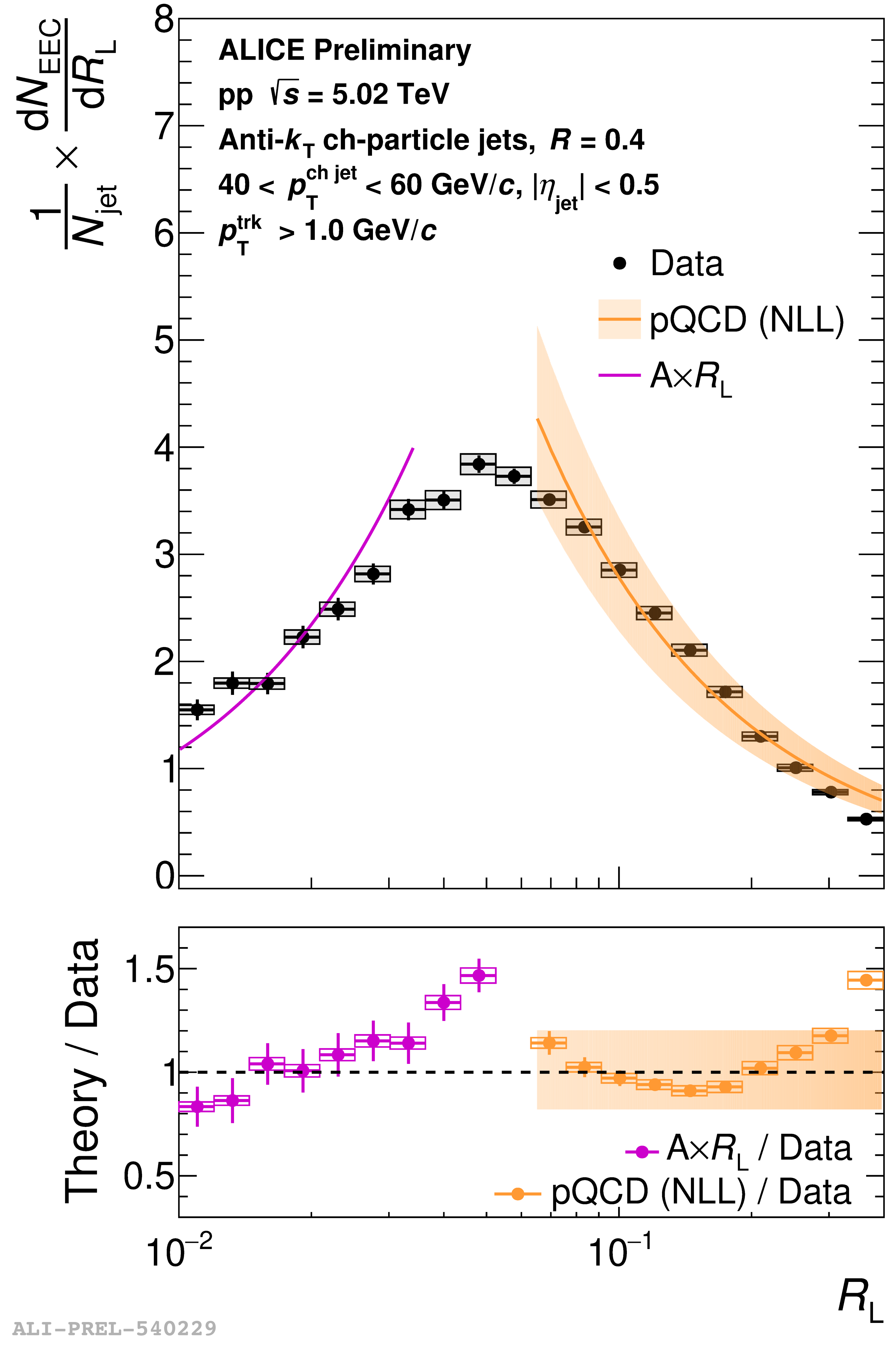
- Correlation functions are the fundamental objects that encode dynamics of underlying theory





- In cosmology, correlations in tempertaure fluctuations relevant to study the structure formation in early universe
- In particle physics, correlations in energy flow in final state jets/particles are useful to understand the dynamics of QCD

Energy correlators on jets



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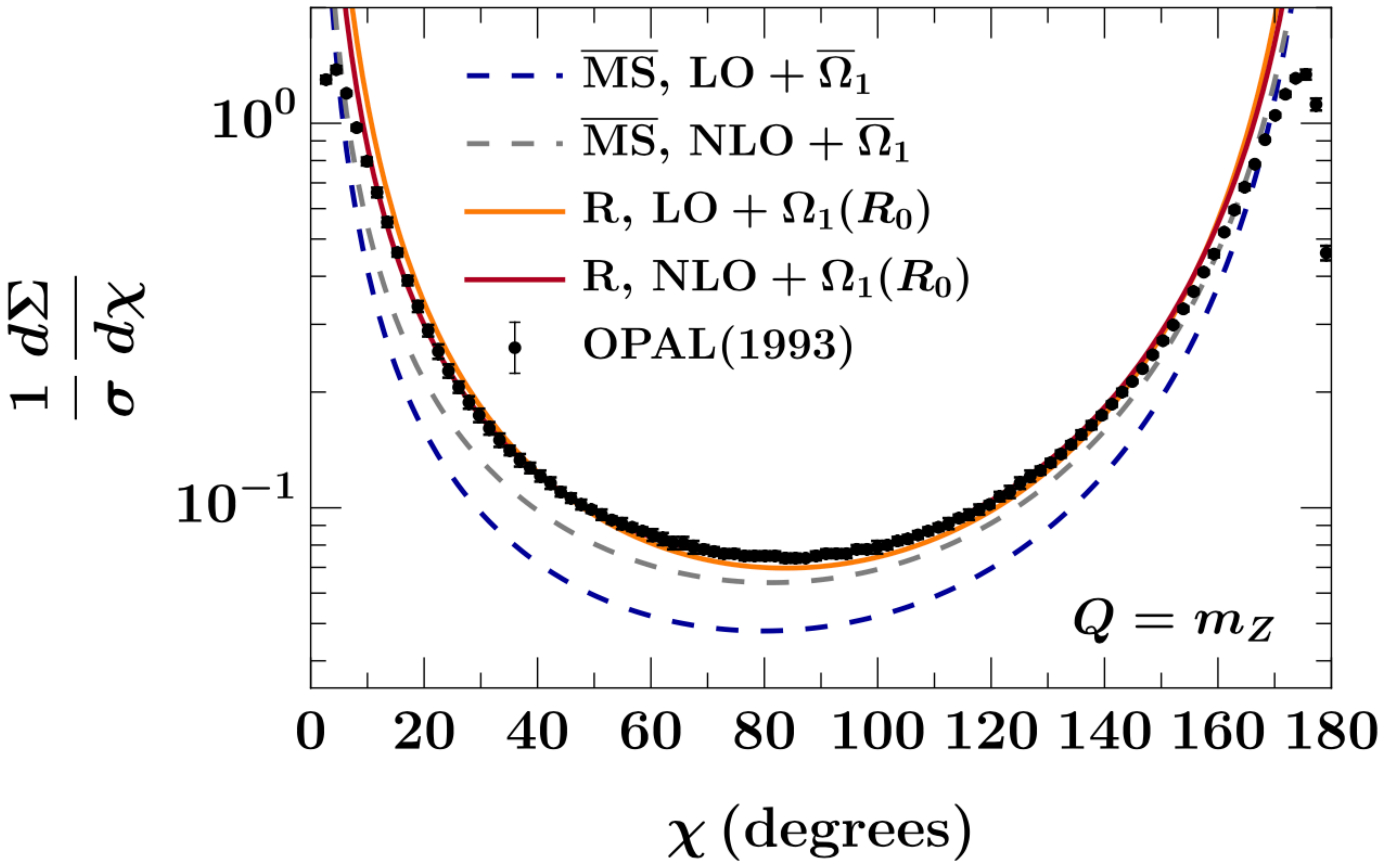
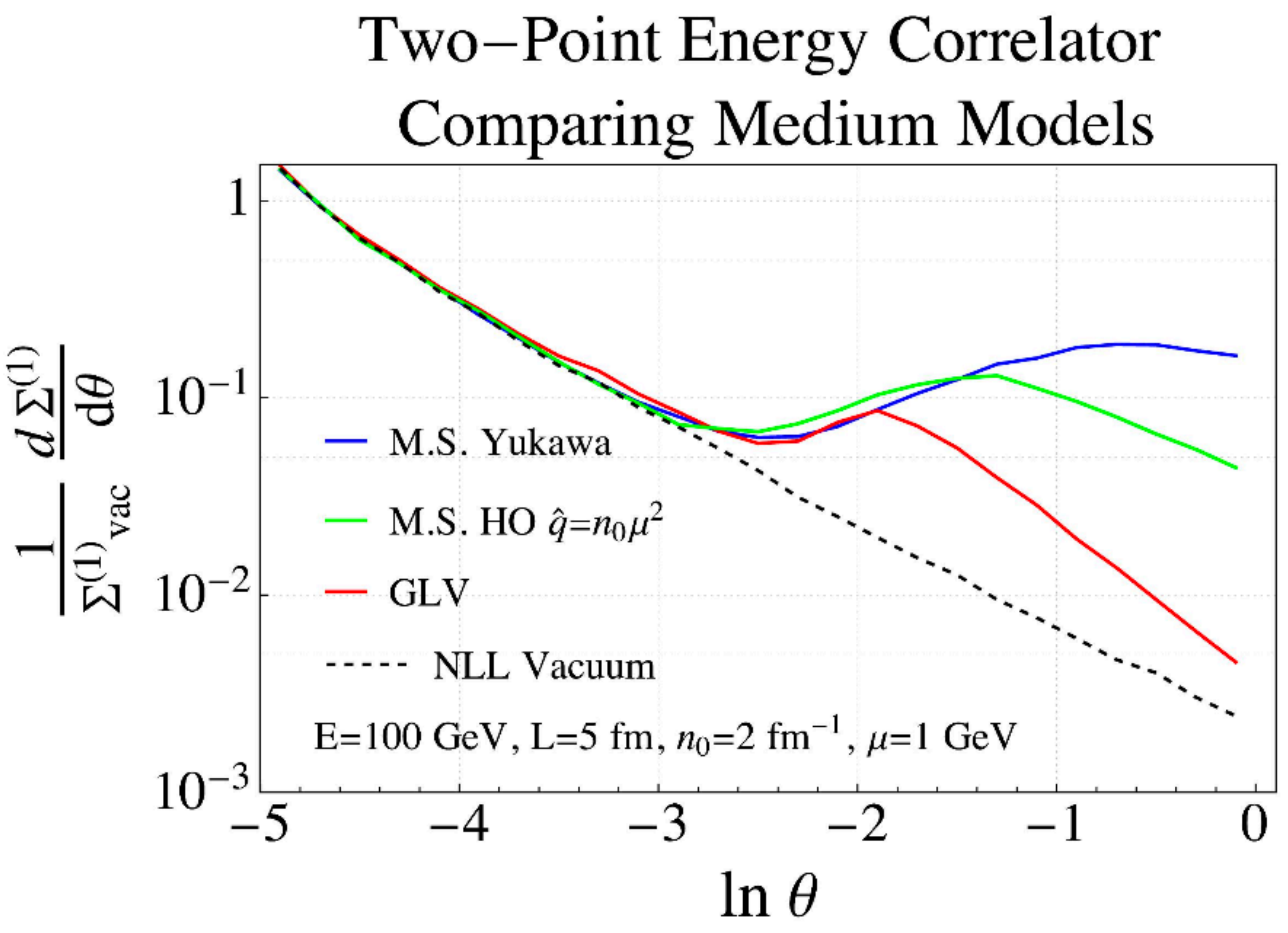
Two distinct scaling behaviours

Impressive agreement with data with leading non-perturbative effects



Schindler, Stewart, Sun '23

Some attempts for computing EEC in HIC



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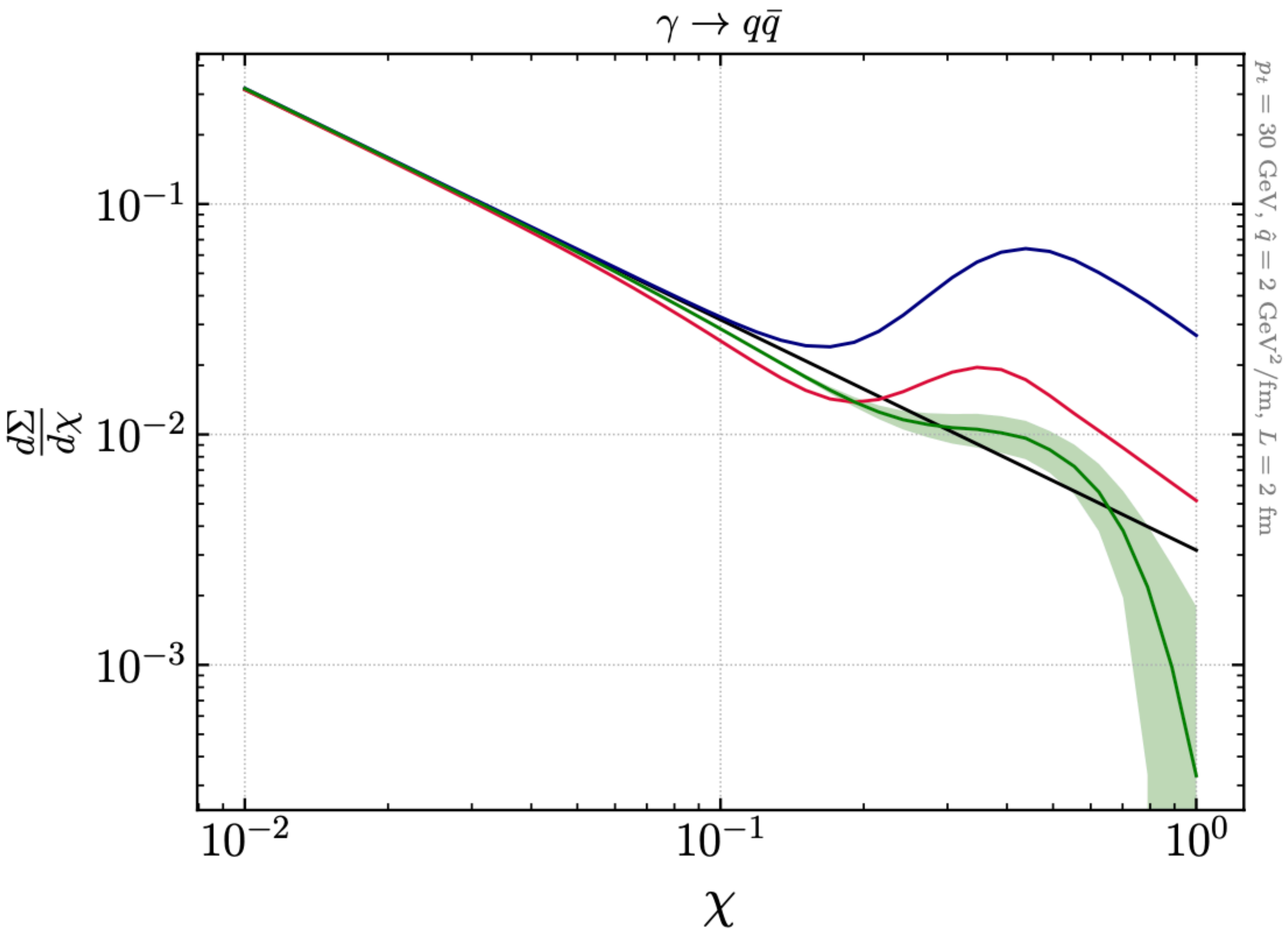

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- Recent first measurement for heavy ions by CMS indicates interesting modifications **CMS PAS HIN-23-004**
- I will focus on factorization for this observable in the EFT framework for HIC

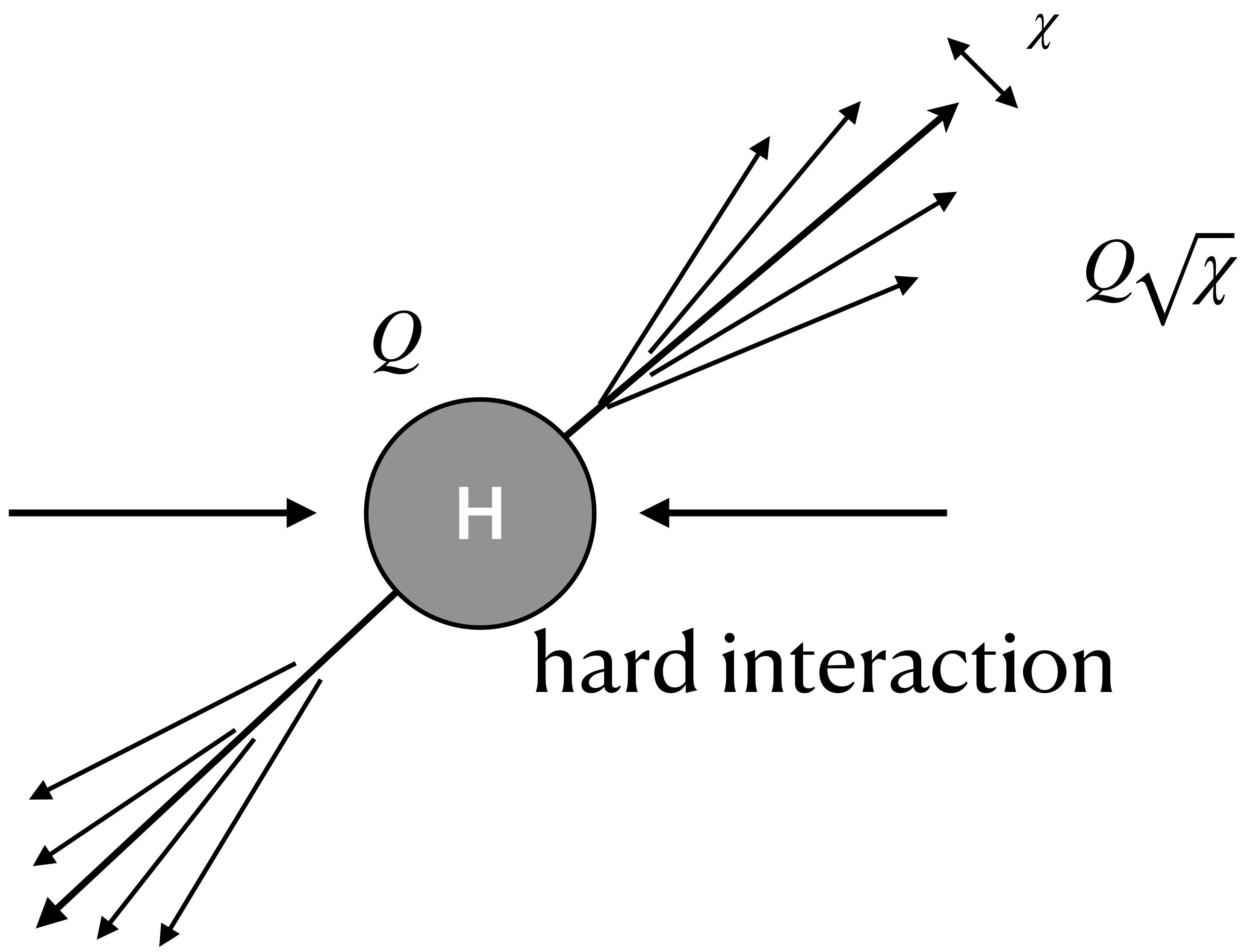
Recap : EEC in vacuum

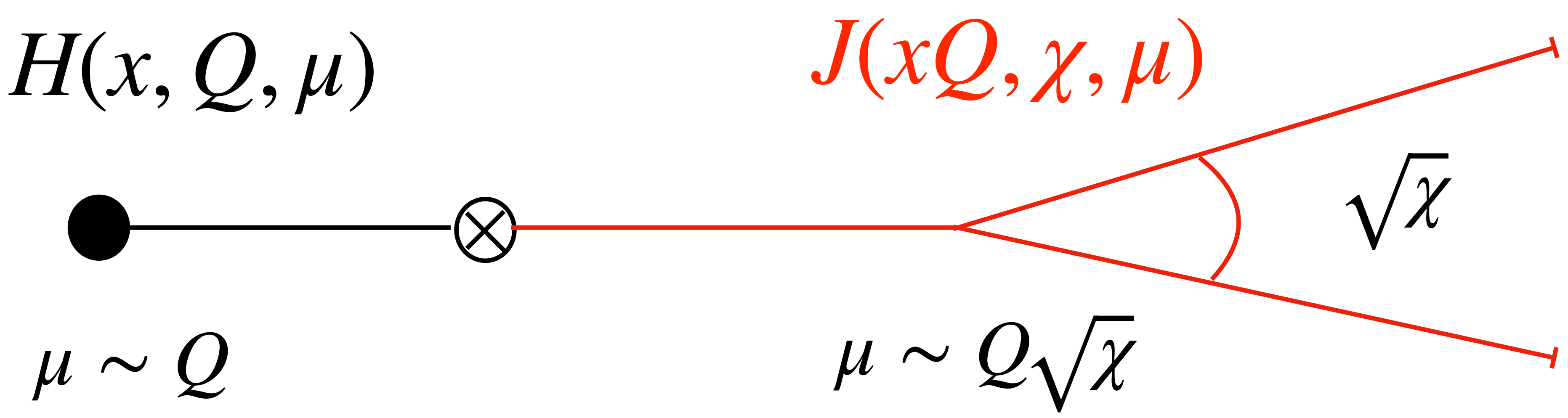
- Jets in vacuum: Only production and measurement scales

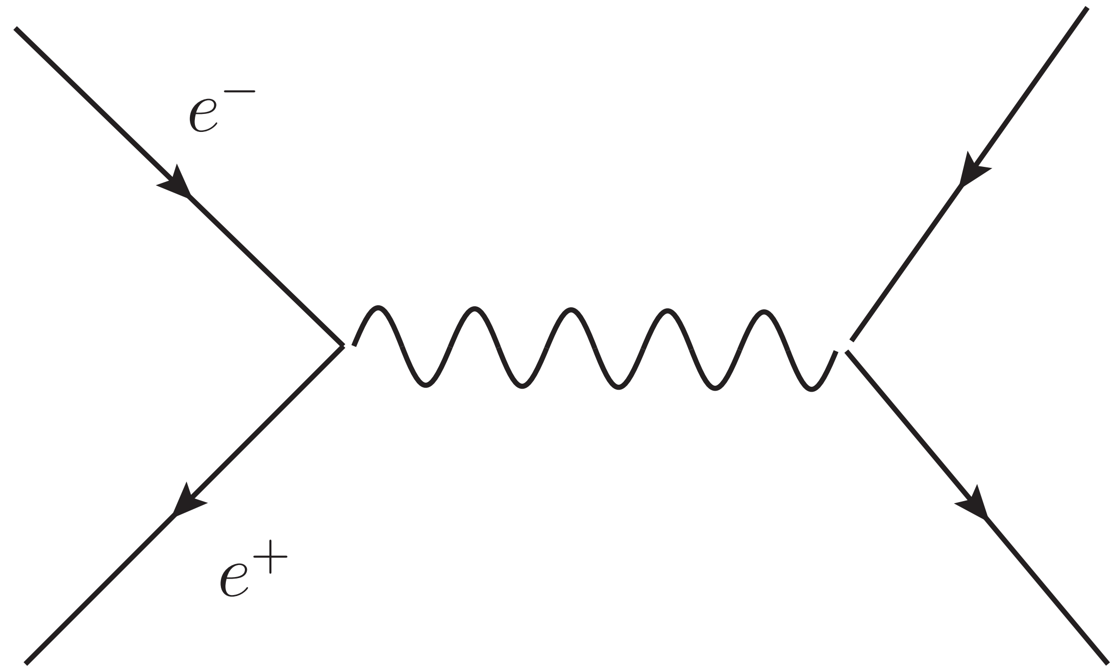
$$Q\sqrt{\chi} \ll Q \qquad \chi \ll 1$$

1905.01310

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\chi} = \sum_{i \in \{q,\bar{q},g\}} \int dx\, x^2 H_i(x, Q, \mu) J_i(xQ, \chi, \mu)$$







- Quark jet function

$$J_q(\omega, \chi, \mu) = \frac{1}{2N_c} \sum_{X_n} \mathrm{Tr}\left[\frac{\bar{n}}{2} \langle 0 | \chi_n(0) \mathscr{M} | X_n \rangle \langle X_n | \delta(\omega - \bar{n} \cdot \mathscr{P}) \delta^2(\mathbb{P}_\perp) \bar{\chi}_n(0) | 0 \rangle\right]$$

$$\chi_n = W_n \xi_n$$

$\mathscr{M} \rightarrow$ measurement operator

Wish list for jets in HIC

- Can we derive a similar factorization formula for any jet observable in HIC ?
- Can we separate out universal non-perturbative physics from the perturbative one ?
- Can we systematically improve computation/accuracy for jets in HIC ?
- Can we compute anomalous dimensions for jets in HIC ?
- Can we relax model dependence ?

We attempt to answer these questions taking energy correlator as jet observable

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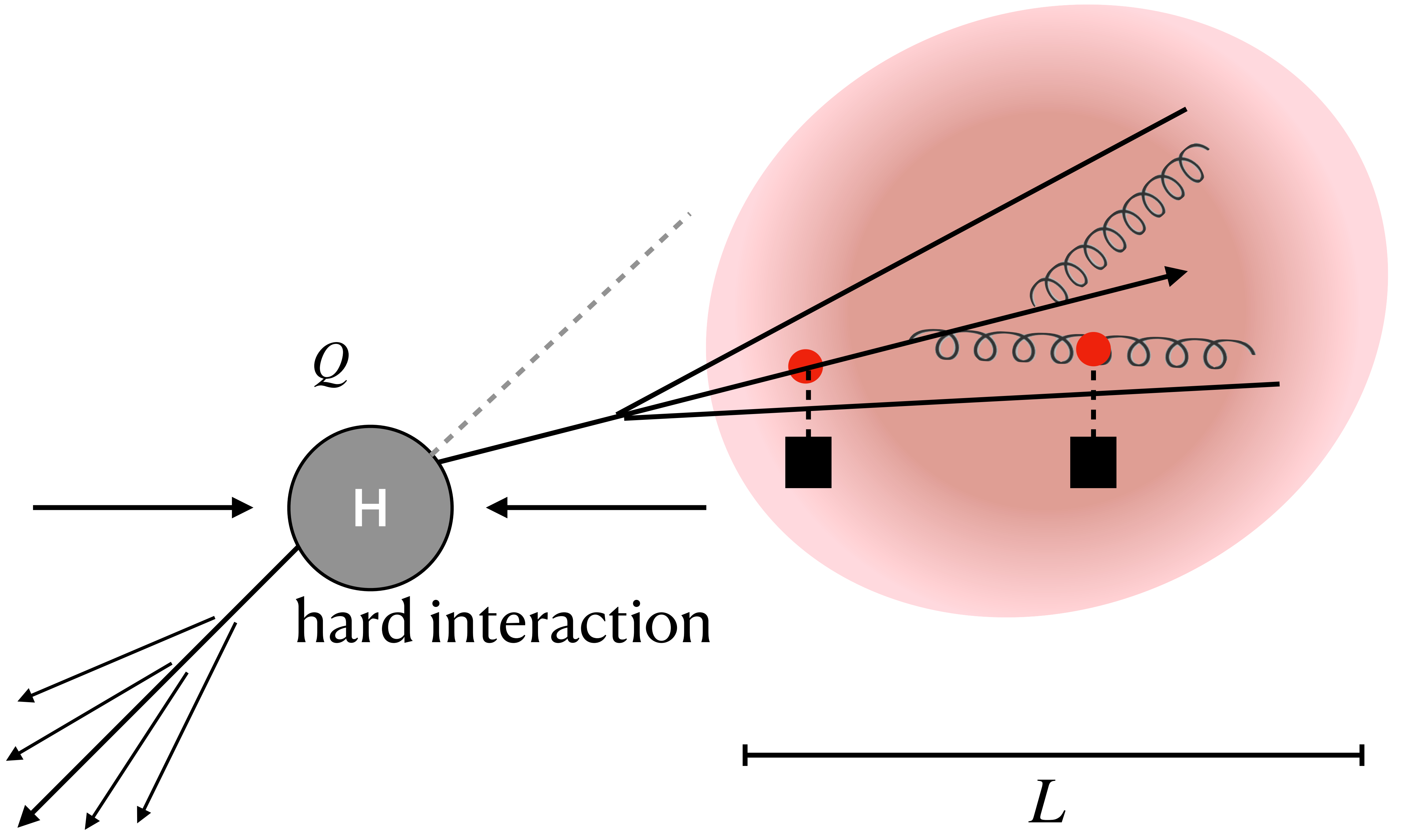
We attempt to answer some of these questions with energy correlators as jet observable

Step 0 : identify scales in presence of medium

- Jets in medium: Production, measurement, temperature and emergent scales

$Q\sqrt{\chi},\ T \sim m_D,\ L,\ t_f,\ \hat{q},\ \theta_c, \ldots$



$Q \gg T \sim m_D \gg \Lambda_{QCD}$

$T \sim m_D$ → medium temperature

t_f → formation time

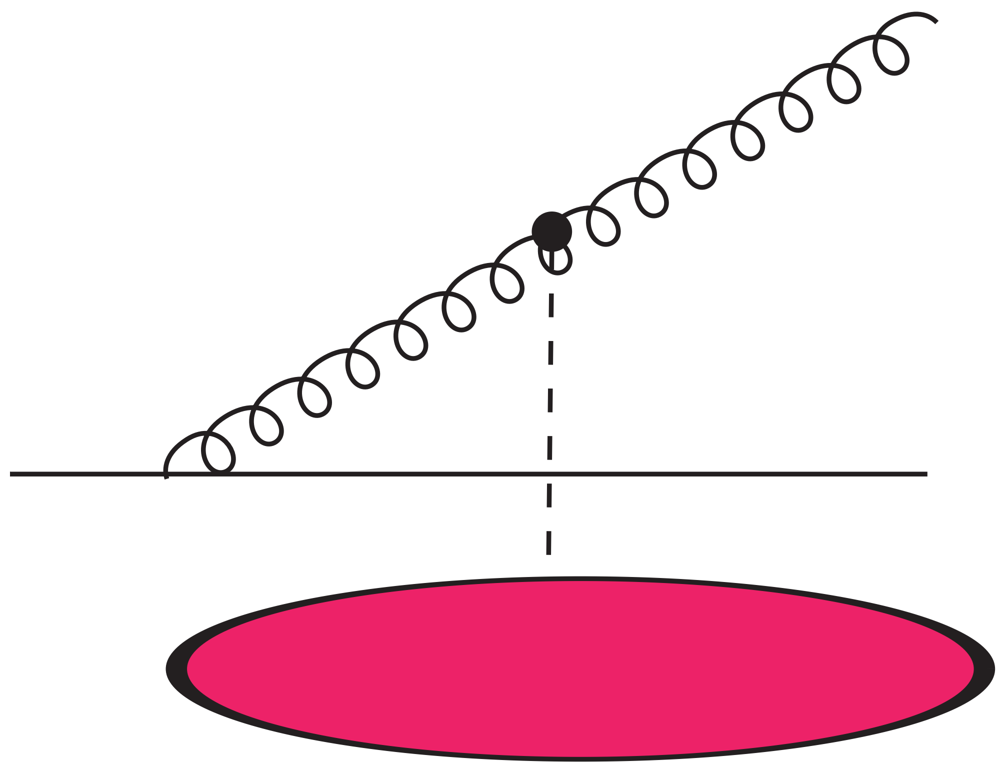
$\hat{q}$ → quenching parameter

$\theta_c \sim \frac{1}{\sqrt{\hat{q}L^3}}$ → critical angle

- So far hard interaction is same as in pp

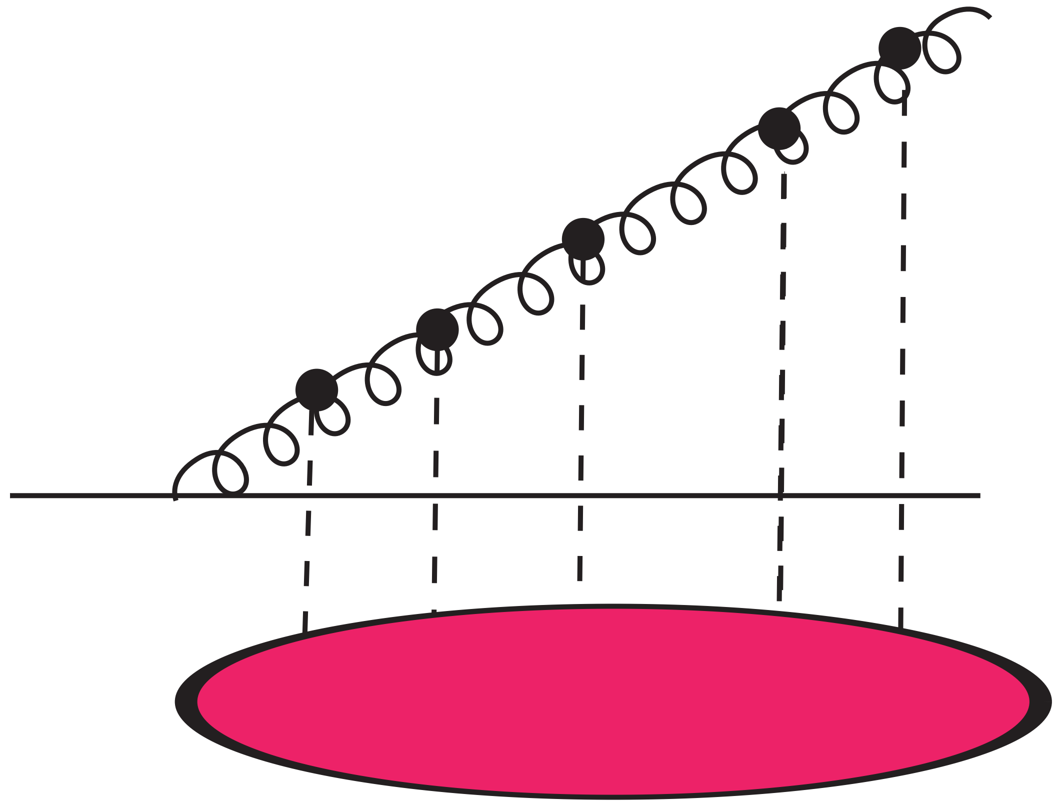
Jet-medium interaction and scale hierarchies

- While traversing the medium jet interact with the medium via elastic and inelastic processes



Single scattering

$k_\perp \sim Q_{\text{med}} \sim m_D$



Multiple scattering

$k_\perp \sim Q_{\text{med}} \sim \sqrt{\hat{q}L}$

$k_\perp$

$\downarrow$

Momentum imparted from medium

- Final jet and medium scales involved in the problem

$Q \rightarrow$ Hard scale

$Q\sqrt{\chi} \rightarrow$ Jet scale

$Q_{\text{med}} \rightarrow$ Characterize medium scale

Hierarchies

$Q \gg Q\sqrt{\chi} \sim Q_{\text{med}}$

$Q \gg Q\sqrt{\chi} \gg Q_{\text{med}}$

Case I: $Q \gg Q\sqrt{\chi} \sim Q_{\text{med}}$

- SCET, an EFT of QCD for soft and collinear radiation

$$\mathcal{L}^0_{\text{SCET}} = \sum_{n_i} \mathcal{L}^0_{n_i}(\xi_{n_i}, A_{n_i}) + \mathcal{L}_s(\psi_s, A_s) + \mathcal{L}_G(\xi_{n_i}, A_{n_i}, \psi_s, A_s)$$

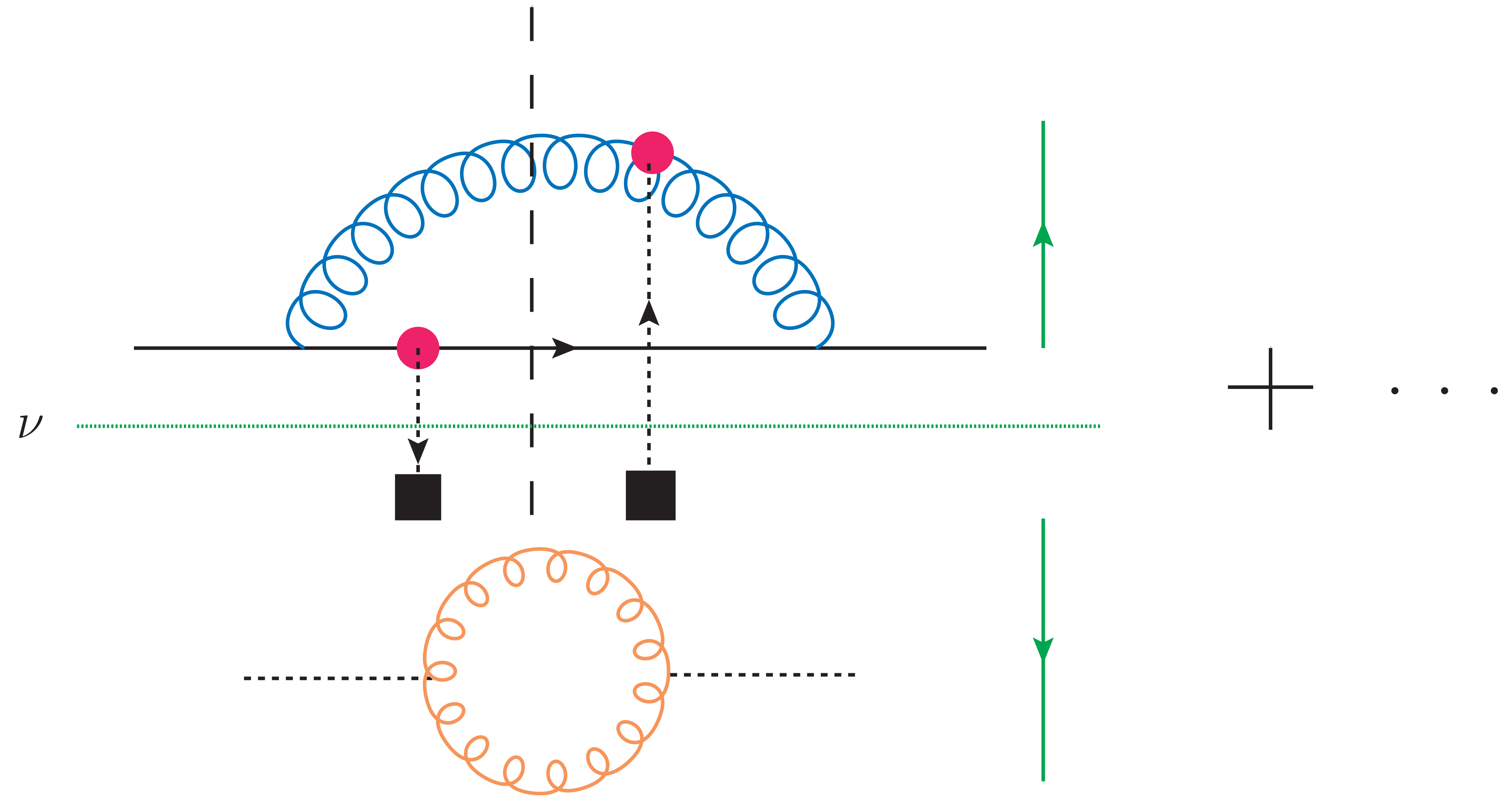
- Jets : collinear mode $\quad p_c \sim (p^-, p^+, p_\perp) \sim Q(1, \lambda^2, \lambda)$

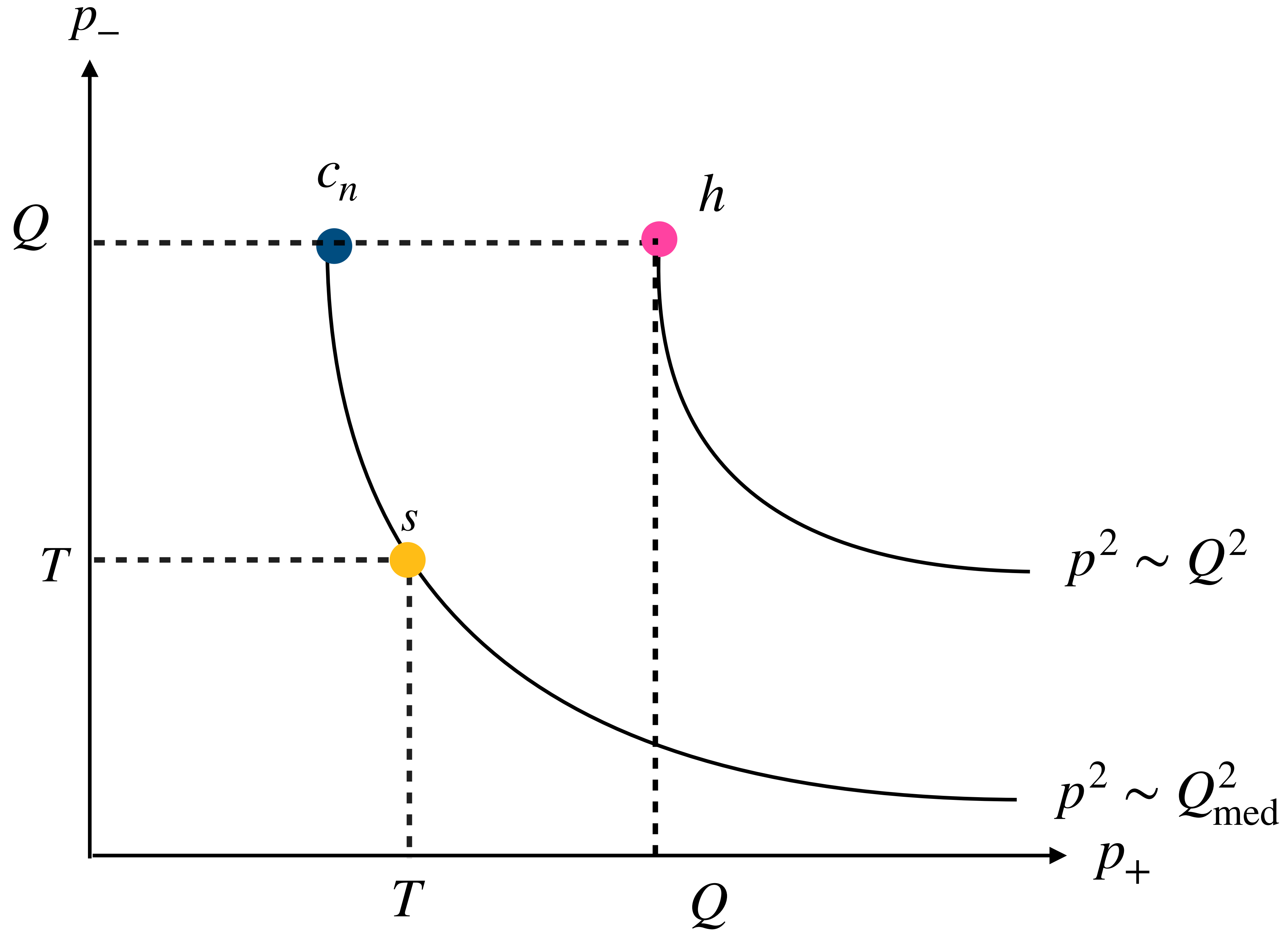
- Medium : soft mode $\quad p_s \sim Q(\lambda, \lambda, \lambda)$

- Exchange : Glauber modes $\quad p_G \sim Q(\lambda, \lambda^2, \lambda)$

  preserve offshellness of soft and collinear modes

- Collinear and soft modes are separated in rapidity





Lund plane representation

$k_\perp \to$ transverse momentum of radiated gluon

$\theta_c \to$ emergent resolution scale

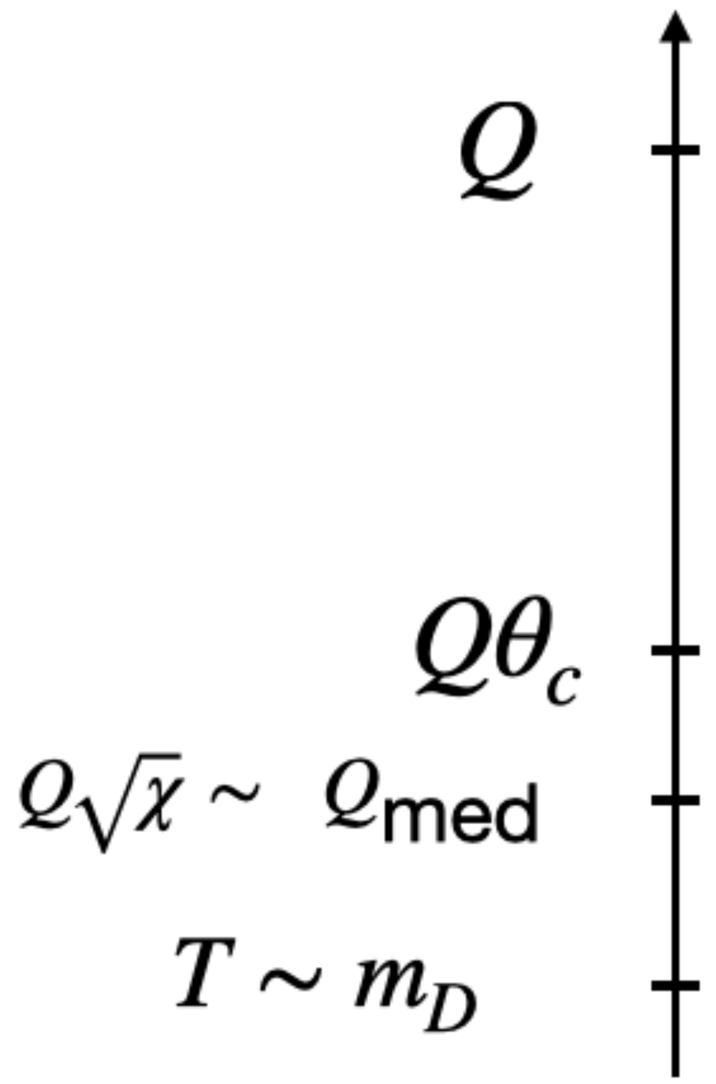
$t_f \to$ formation time of radiated gluon

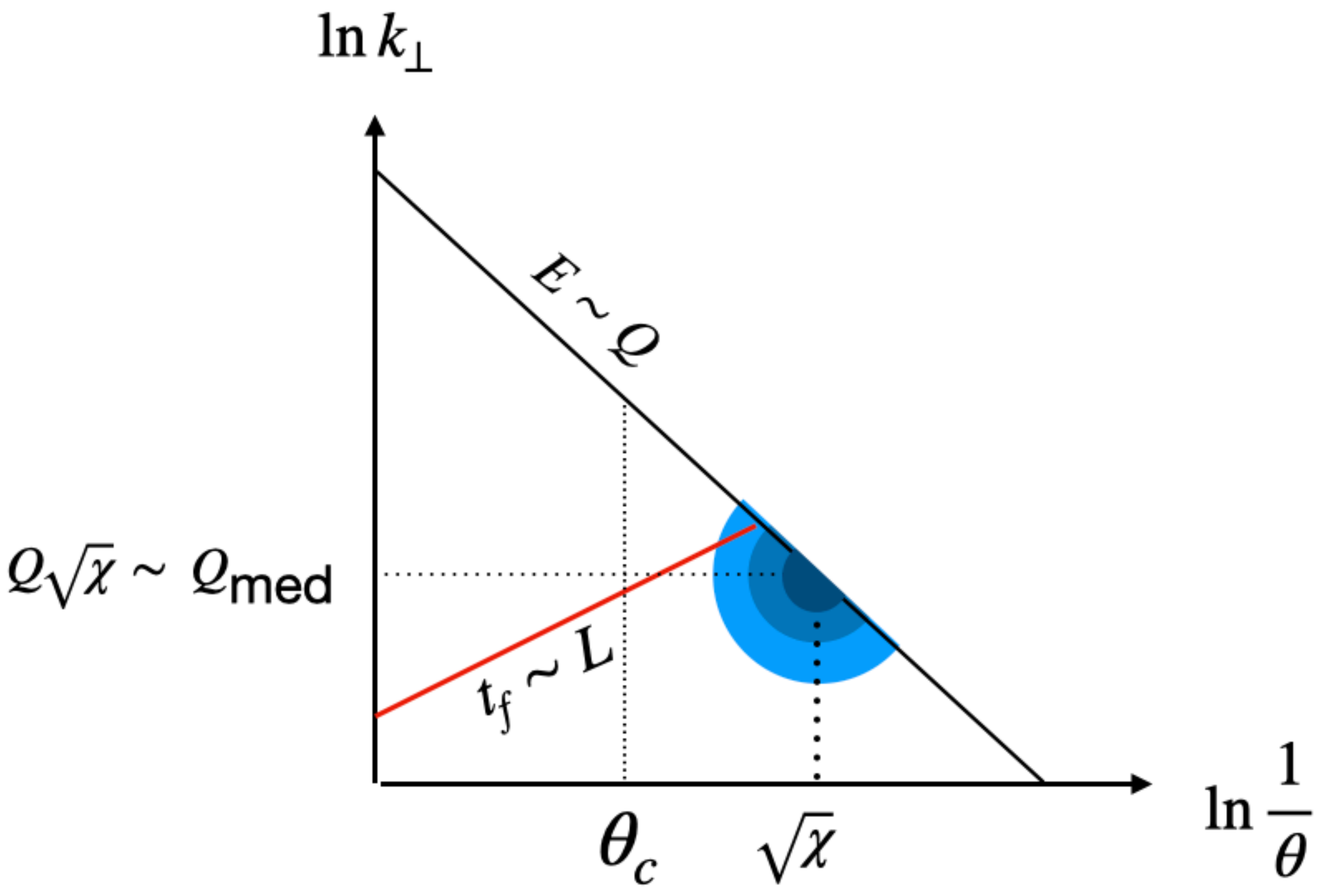
$E \sim Q \to$ hard emissions

$$Q \sim \mathcal{O}(10^2)\,\text{GeV}$$

$$\chi \sim \mathcal{O}(10^{-4} - 10^{-3})$$

$$t_f \sim \frac{Q}{Q^2_{\text{med}}} \sim \frac{1}{Q\chi}\,.$$





$Q = 200\,\text{GeV}$ $\quad \hat{q} = 1\,\text{GeV}^2\text{fm}^{-1}$ $\quad L = 5\,\text{fm}$ $\quad t_f = 8\,\text{fm}$

- Phase space is populated by unresolved emissions

Step I : time evolution of density matrix

- Factorized total initial density matrix

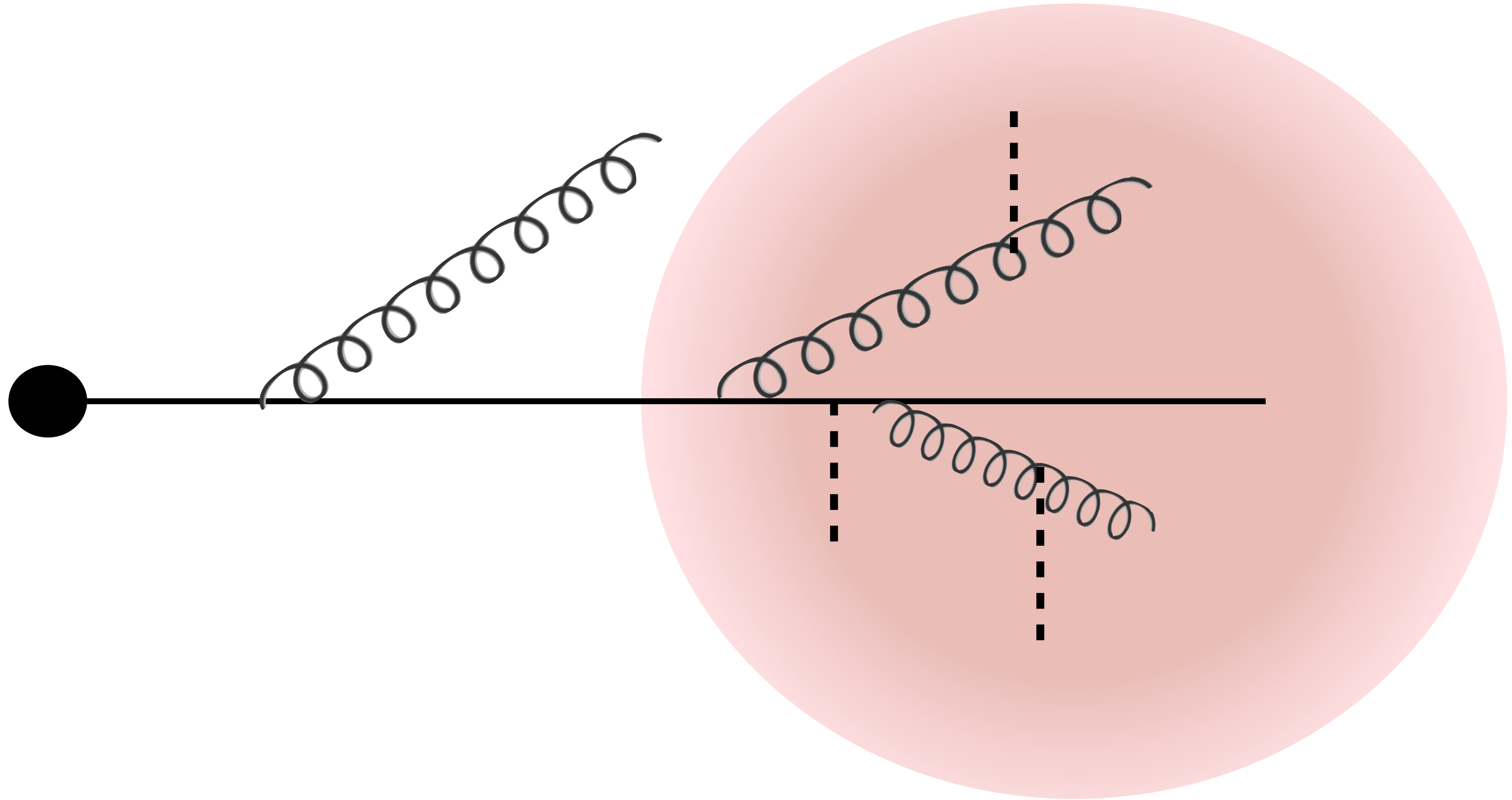
$$\rho(0) = |e^+e^-\rangle\langle e^+e^-| \otimes \rho_M(0)$$

- Time evolution is in system density matrix

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}$$

- Density matrix evolves with total Hamiltonian

$$H = H_n + H_s + H_G + C(Q)l^\mu J_\mu \equiv H_S + \underbrace{\mathcal{O}_H}$$



Vacuum and medium evolution

Hard interaction → required only once

$$\Sigma = \lim_{t\to\infty} \mathrm{Tr}[\rho(t)\mathcal{M}] = \underbrace{|C(Q)|^2} L_{\mu\nu} \lim_{t\to\infty} \int d^4x d^4y e^{iq\cdot(x-y)} \mathrm{Tr}[e^{-iH_S t} J^\mu(x)\rho(0)\underbrace{\mathcal{M}} J^\nu(y) e^{iH_S t}]$$

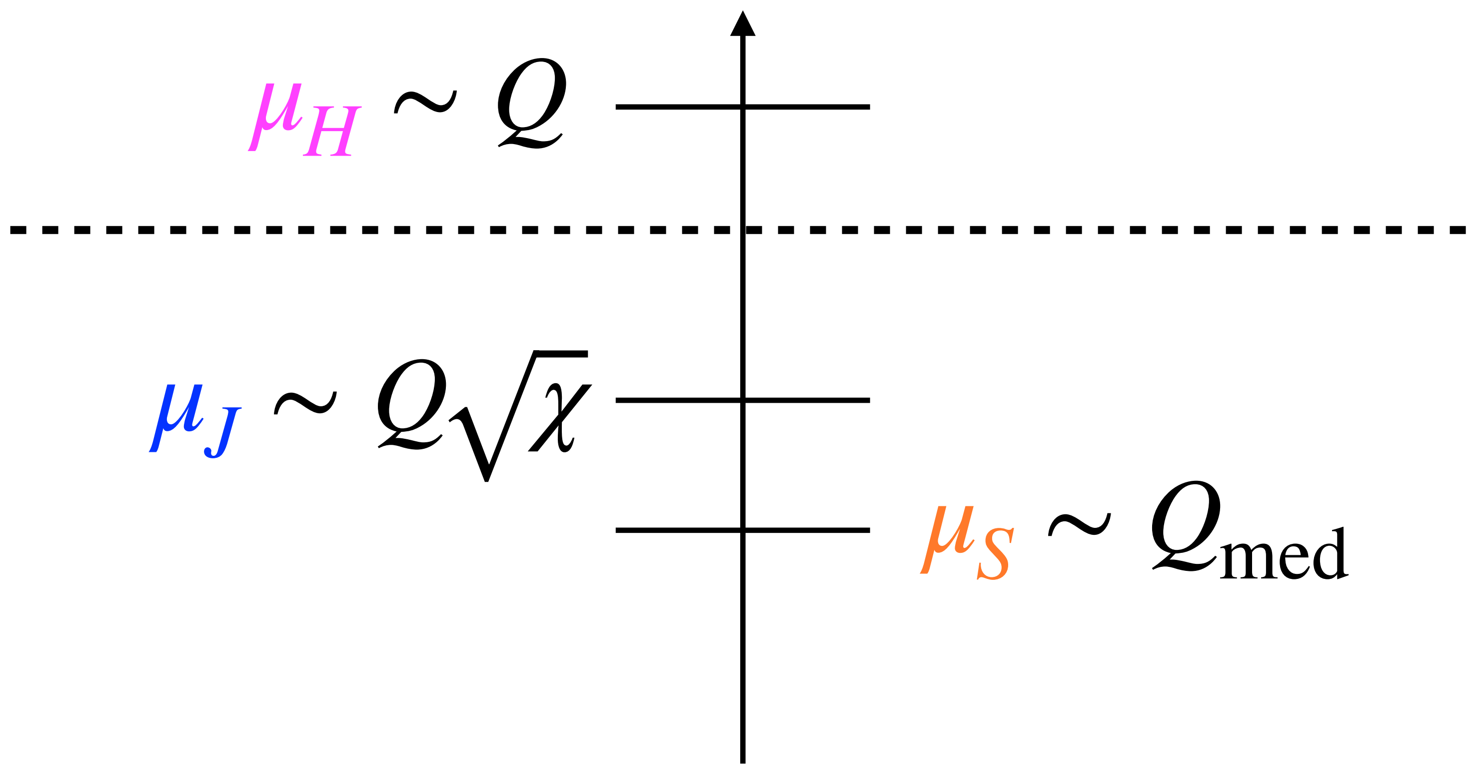
Hard matching Wilson coefficient

Measurement function

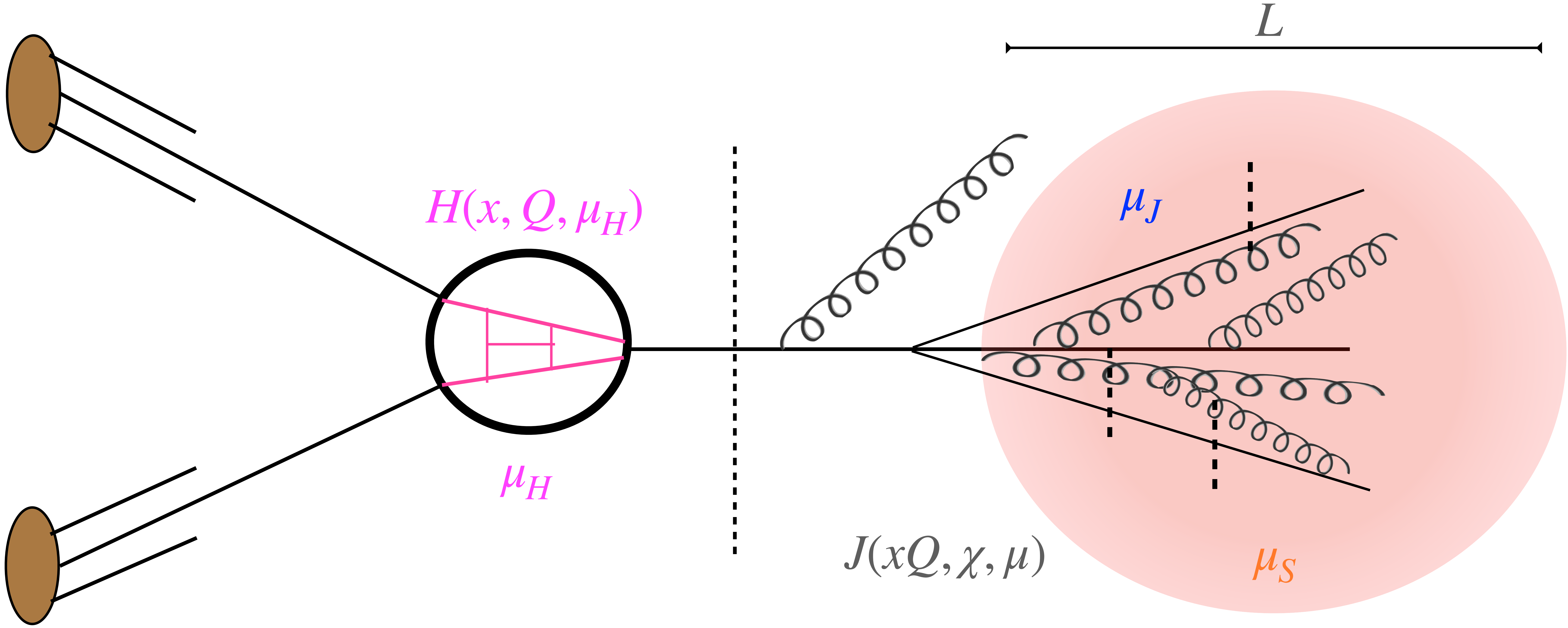
Step II : factorize jet dynamics from production

- OPE for factorizing hard scales

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\chi} = \sum_{i\in\{q,\bar{q},g\}} \int dx\, x^2 H_i(x,Q,\mu) J_i(xQ,\chi,\mu)$$



- At this stage $J(xQ,\chi,\mu)$ contains both vacuum and medium physics



Next separate jet dynamics μ_J including vacuum evolution from the medium μ_S

Step III : factorize the measurement function

- Jet function contains both medium and jet dynamics

$$J_q(\chi) = \frac{1}{2N_c}\sum_X \mathrm{Tr}\Big[\rho_M(0)\frac{\bar{n}}{2}e^{iH_{n+s}t}\bar{\mathbf{T}}\Big\{e^{-i\int_0^t dt_l H_{G,I}(t_l)}\chi_{n,I}(0)\Big\}\mathscr{M}\,|X\rangle\langle X|\,\mathbf{T}\Big\{\underbrace{e^{-i\int_0^t dt_r H_{G,I}(t_r)}}_{\text{Glauber insertion}}\delta^2(\mathbb{P}_\perp)\delta(\omega-\bar{n}\cdot\mathscr{P})\bar{\chi}_{n,I}(0)\Big\}e^{-iH_{n+s}t}\Big]$$

- Factorizing measurement function

$$\mathscr{M} = \hat{E}(\chi)\,|X\rangle = \frac{1}{Q}\sum_{i\in\{X_n,X_s\}}\Big(E_{i,n}\Theta(\chi-\theta_{n,i}) + E_{i,s}\Theta(\chi-\theta_{s,i})\Big)\,|X_n\rangle\,|X_s\rangle$$

Collinear mode

Soft mode

Glauber mode

- Soft contributions to the measurement are power suppressed
- Glaubers being off-shell modes do not contribute to the measurement
- Now we can separate vacuum and medium induced jet function in J_q

Leading order : vacuum jet function

- To recover the vacuum jet function we expand Glauber Hamiltonian

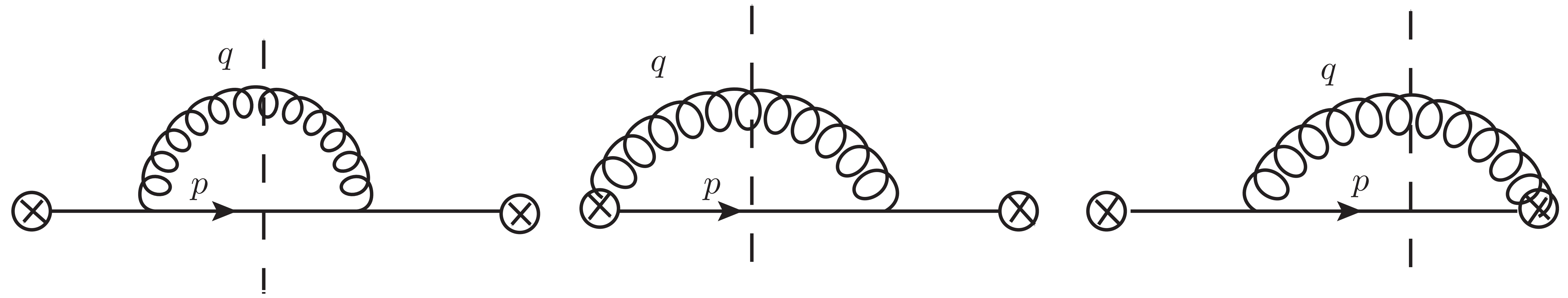
$$J_q(\omega,\chi,\mu) = \sum_{i=0}^{\infty} J_q^{(i)}(\omega,\chi,\mu)$$

$i = 0,$ vacuum
$i = 1,$ single scattering
$i \geq 2,$ multiple scattering

- Leading order $\qquad J_q(\omega,\chi,\mu) = J_q^{(0)}(\omega,\chi,\mu)$

- Soft function does not depend on the measurement and becomes identity $\qquad |X\rangle = |X_n\rangle|X_s\rangle$

$$J_q^{(0)}(\omega,\chi,\mu) = \frac{1}{2N_c}\sum_{X_n} \mathrm{Tr}\left[\frac{\bar{n}}{2}\langle 0|\chi_n(0)\mathscr{M}|X_n\rangle\langle X_n|\delta(\omega - \bar{n}\cdot\mathscr{P})\delta^2(\mathbb{P}_\perp)\bar{\chi}_n(0)|0\rangle\right]$$



$\omega = xQ$

$$J_q^{(0)}(\omega,\chi) = \delta(\chi) + \frac{\alpha_s C_F}{\pi}\left(-\frac{3}{2\epsilon}\delta(\chi) + \frac{3}{2}\left[\frac{1}{\chi}\right]_+ - \frac{3}{2}\delta(\chi)\ln\left(\frac{\mu^2}{\omega^2}\right) - \frac{19}{3}\delta(\chi) + \mathcal{O}(\epsilon)\right)$$

Log in cumulant

$$\frac{3}{2}\left[\frac{\mu^2}{\omega^2\chi}\right]_+$$

Single scattering : Glauber insertions on opposite side

- Real contribution with Glauber insertions

$$J_{q,o}(\chi, k_\perp; L) = \frac{1}{2N_c}\sum_X \int d^4x\,\Theta(L - x^-)\int d^4y\,\Theta(L - y^-)\mathrm{Tr}\Big[e^{iH_{n+s}t}\bar{\mathbf{T}}\Big\{H_{G,I}(x)\chi_{n,I}(0)\Big\}\rho_M(0)\frac{\bar{n}}{2}\mathbf{T}\Big\{H_{G,I}(y)\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathscr{P})\bar{\chi}_{n,I}(0)\Big\}e^{-iH_{n+s}t}\mathscr{M}\Big] + \mathcal{O}(H_G^4)$$

- Medium and jet interaction constraint to medium length L

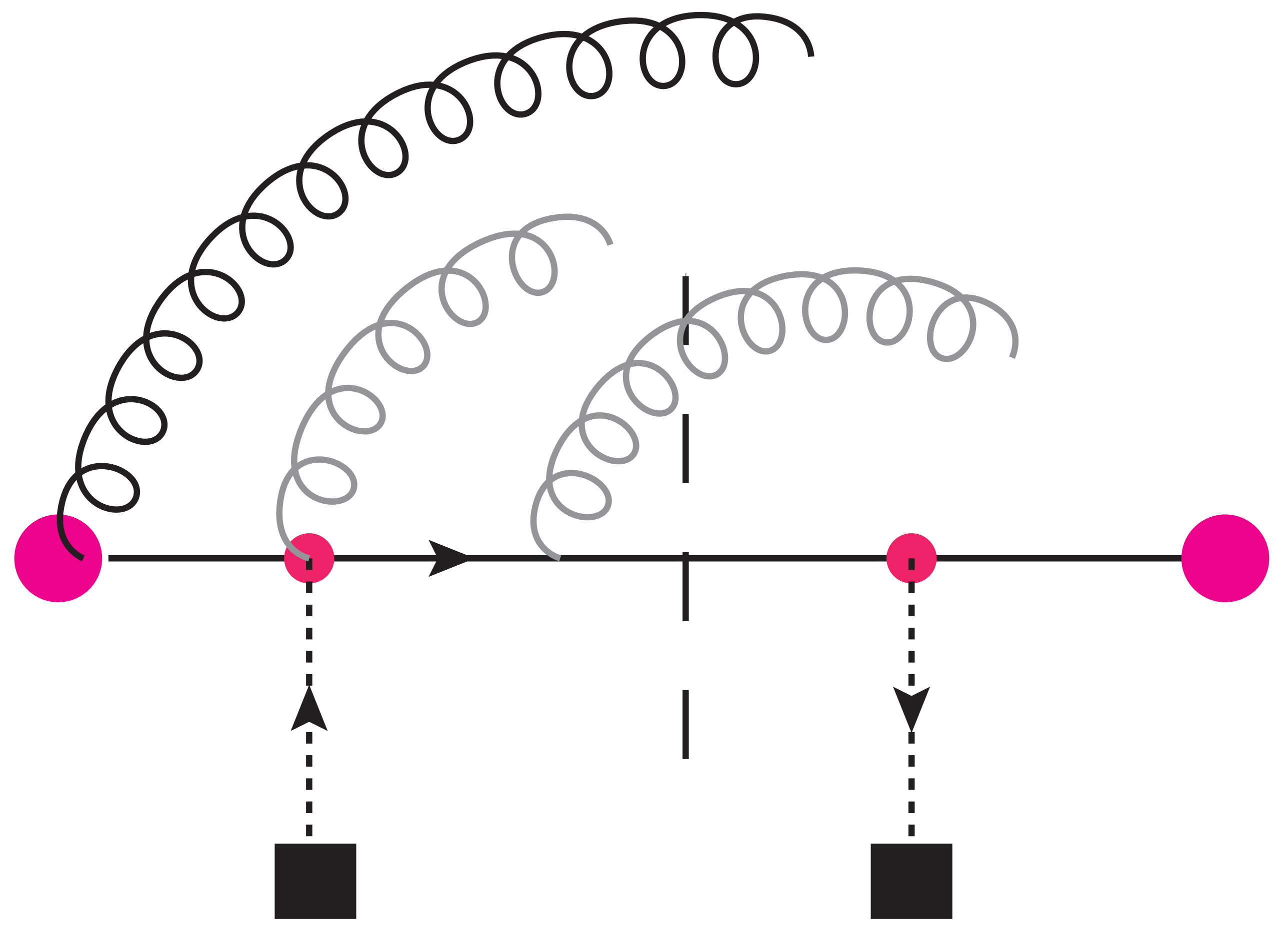
$$H_G(x) = \sum_{ij} C_{ij} O_{ns}^{ij}(x)$$

- $\mathbb{P}_\perp^2$ pulls out Glauber momentum

$$\mathcal{O}_{ns}^{qg} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{gA} \qquad \mathcal{O}_{ns}^{qq} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{qA}$$

- Order by order factorization for jet and medium functions

$$J_{q,o}(\omega, \chi; L) = L\int \frac{d^2k_\perp}{(2\pi)^2} J_{q,R}(\omega, \chi, k_\perp, \mu, \nu; L) \otimes \mathbf{B}(k_\perp, \mu, \nu)$$



$\mathbf{B}(k_\perp, \mu, \nu) \rightarrow$ medium function

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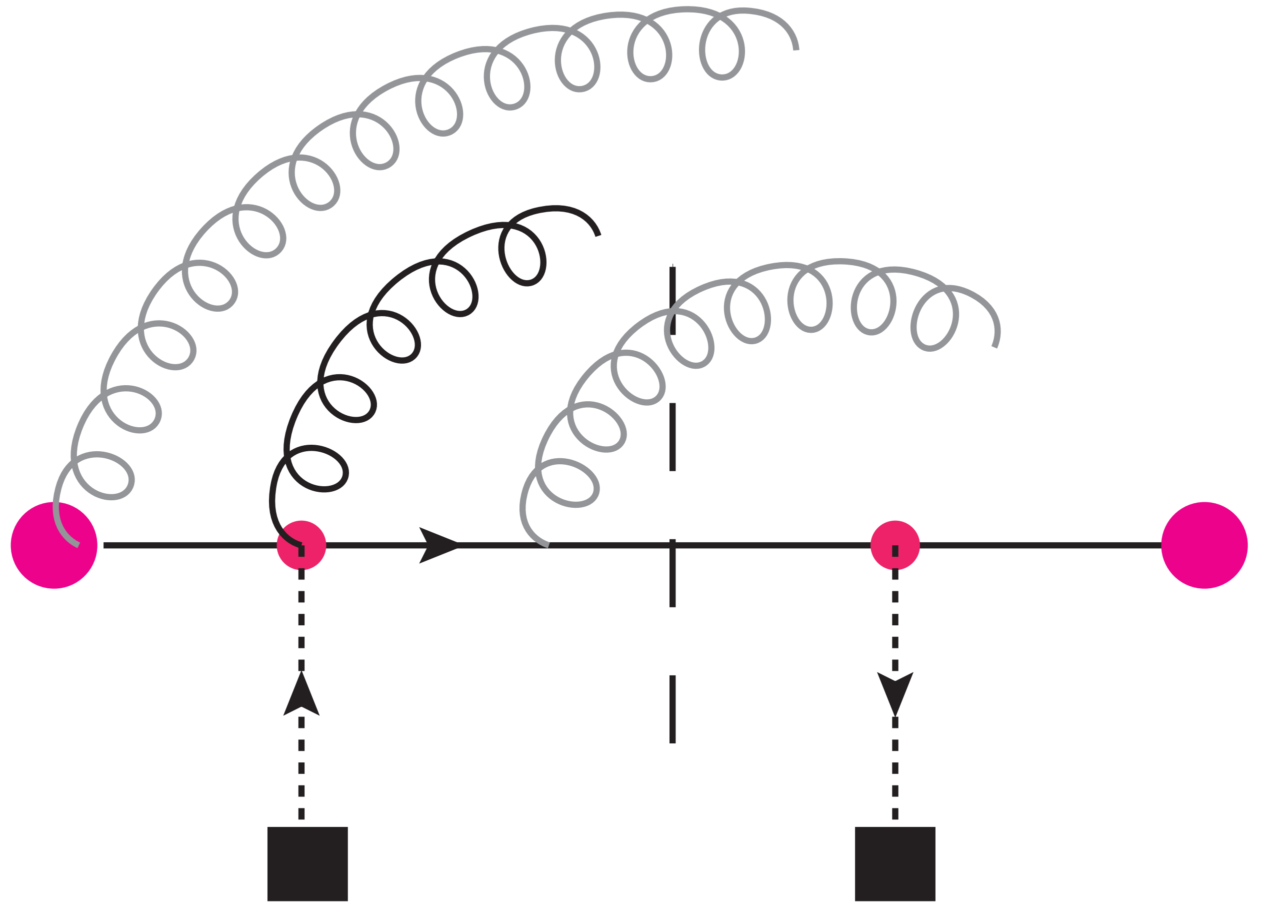

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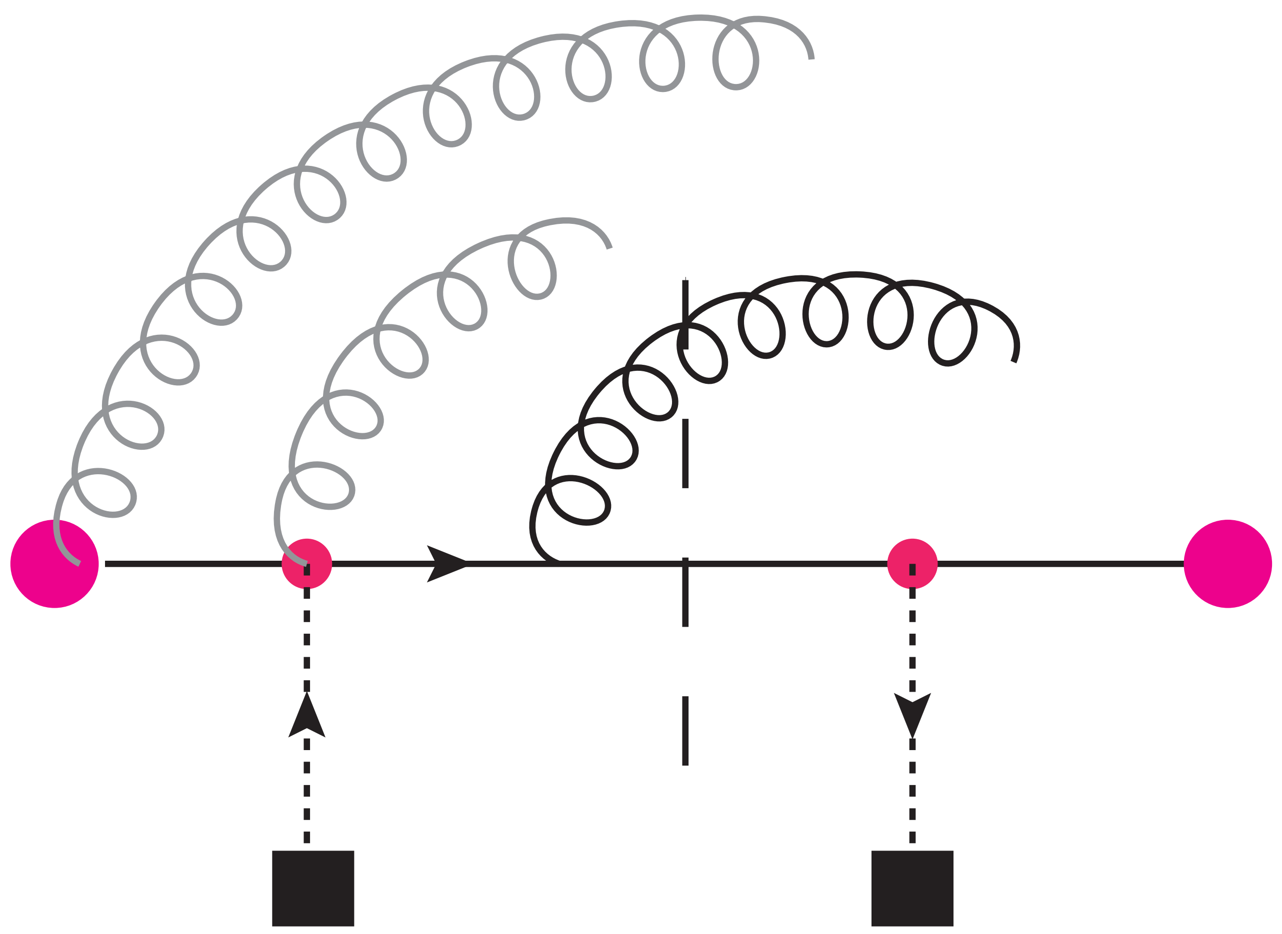

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Single scattering : Glauber insertions on opposite side

- sinc function leads to LPM terms

$$J_{q,R}(\chi, k_\perp; L) = \frac{e^{-i\frac{L}{2}(\mathbb{P}_+^A - \mathbb{P}_+^B)}}{2N_c}\text{sinc}\left[\frac{L}{2}(\mathbb{P}_+^A - \mathbb{P}_+^B)\right]\sum_X \mathrm{Tr}\left[\frac{\bar{n}}{2}\bar{\mathbf{T}}\left\{e^{-i\int dt H_n(t)}\left[\delta(\mathscr{P}^-)\delta^2(\mathbb{P}_\perp - k_\perp)O_n^{qB}(0)\right]\chi_n(0)\right\}\mathscr{M}\,|X\rangle\langle X|\right.$$
$$\left.\mathbf{T}\left\{e^{-i\int dt H_n(t)}\left[\delta(\mathscr{P}^-)\delta^2(\mathbb{P}_\perp + k_\perp)O_n^{qB}(0)\right]\left[\delta(\omega - \bar{n}\cdot\mathscr{P})\delta^2(\mathbb{P}^\perp)\bar{\chi}_n(0)\right]\right\}\right] + \mathcal{O}(H_G^4)$$

- Soft/Medium function explicitly factors out

$$\mathbf{B}_{AB}(x,y) = \mathrm{Tr}\left[\mathbf{T}\left\{e^{-i\int dt_l H_{s,I}(t_l)}\left(\frac{1}{\mathbb{P}_\perp^2}\mathcal{O}_s^A(x)\right)\right\}\rho_M(0)\bar{\mathbf{T}}\left\{e^{-i\int dt_r H_{s,I}(t_r)}\left(\frac{1}{\mathbb{P}_\perp^2}\mathcal{O}_s^B(y)\right)\right\}\right]$$

- $J_{q,R}(\omega, \chi, k_\perp; L)$ now depends on medium parameters
- $\mathbf{B}(x,y)$ does not depend on measurement
- $\mathbf{B}(x,y)$ depends only on medium parameters

SCET operators

$$\mathcal{O}_n^{qA} = \bar{\chi}_n T^A \frac{\bar{n}}{2}\chi_n$$
$$\mathcal{O}_s^{qA} = \bar{\chi}_s T^A \frac{n}{2}\chi_s$$
$$\mathcal{O}_s^{gA} = \frac{i}{2}f^{ACD}\mathscr{B}^C_{S\perp\mu}\frac{n}{2}\cdot(\mathscr{P} + \mathscr{P}^\dagger)\mathscr{B}^{D\mu}_{S\perp}$$

Single scattering : Glauber insertions on same side

- $\mathcal{O}(H_G^2)$ expansion at the same side

$$J_{q,s}(\chi, k_\perp; L) = \frac{1}{2N_c}\int d^4x\Theta(x^- - L)\int d^4y\Theta(y^- - L)\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\bar{\mathbf{T}}\Big\{e^{-i\int dtH_n(t)}\chi_n(0)\mathcal{M}|X\rangle\langle X|$$
$$\mathbf{T}\Big\{e^{-i\int dtH_n(t)}\{H_{G,I}(x)H_{G,I}(y)\}\Big\}\Big[\delta(\omega - \bar{n}\cdot\mathcal{P})\delta^2(\mathbb{P}^\perp)\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big] + \text{c.c} + \mathcal{O}(H_G^4)$$

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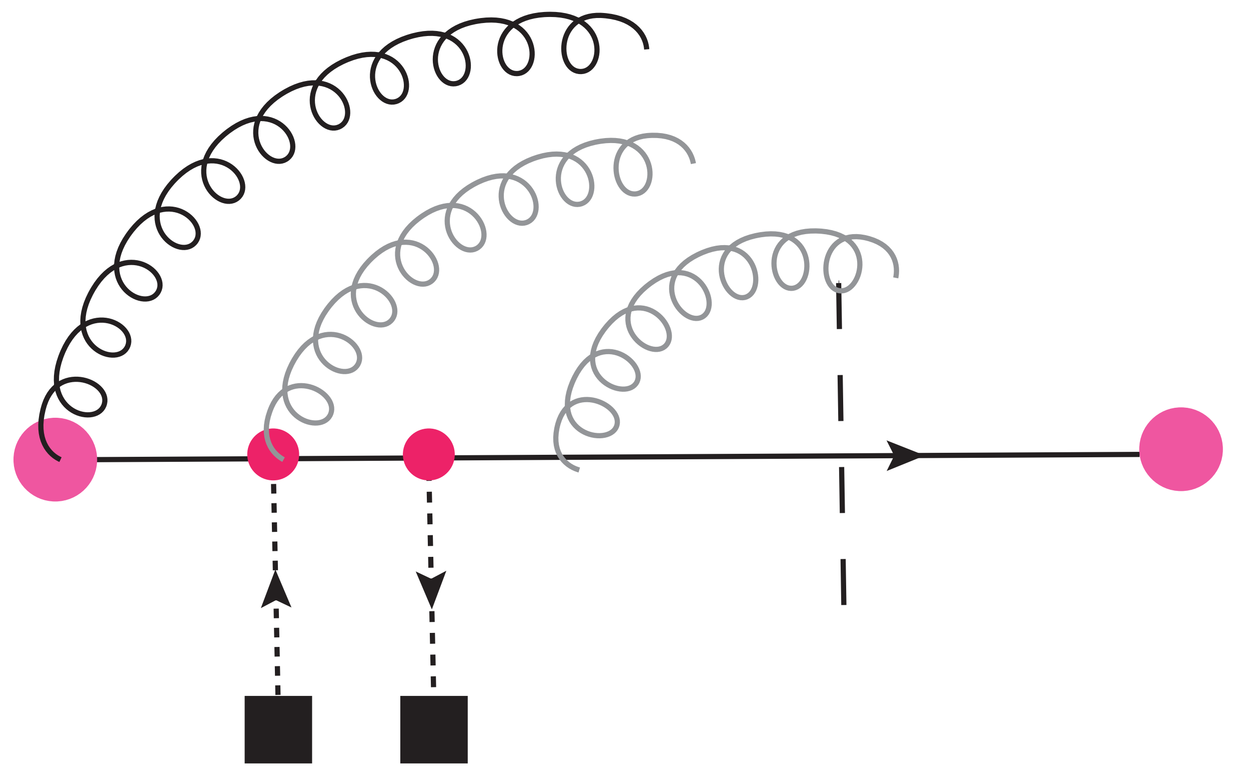
$$J_{q,s}(\omega,\chi;L) = L\int\frac{d^2k_\perp}{(2\pi)^2}J_{q,V}(\omega,\chi,k_\perp,\mu,\nu;L)\otimes\mathbf{B}(k_\perp,\mu,\nu)$$



$+ \quad c.c$

- Sinc function leads to LPM terms

$$J_{q,V}(\omega,\chi,k_\perp;L) = \frac{1}{2N_c}\frac{1}{2}e^{-i\frac{L}{2}(\mathbb{P}_+^A+\mathbb{P}_+^B)}\mathrm{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\bar{\mathbf{T}}\Big\{e^{-i\int dtH_{n,I}(t)}\chi_n(0)\Big\}\mathcal{M}|X\rangle\langle X|\mathbf{T}\Big\{e^{-i\int dtH_n(t)}\Big[\delta^2(\vec{\mathbb{P}}_\perp + \vec{k}_\perp)\delta(\mathcal{P}^-)O_n^A(0)\Big]$$
$$\times\Big[\delta^2(\vec{\mathbb{P}}_\perp - \vec{k}_\perp)\delta(\mathcal{P}^-)O_n^B(0)\Big]\Big[\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathcal{P})\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big]\delta^{AB} + c\,.\,c + \mathcal{O}(H_G^4)$$

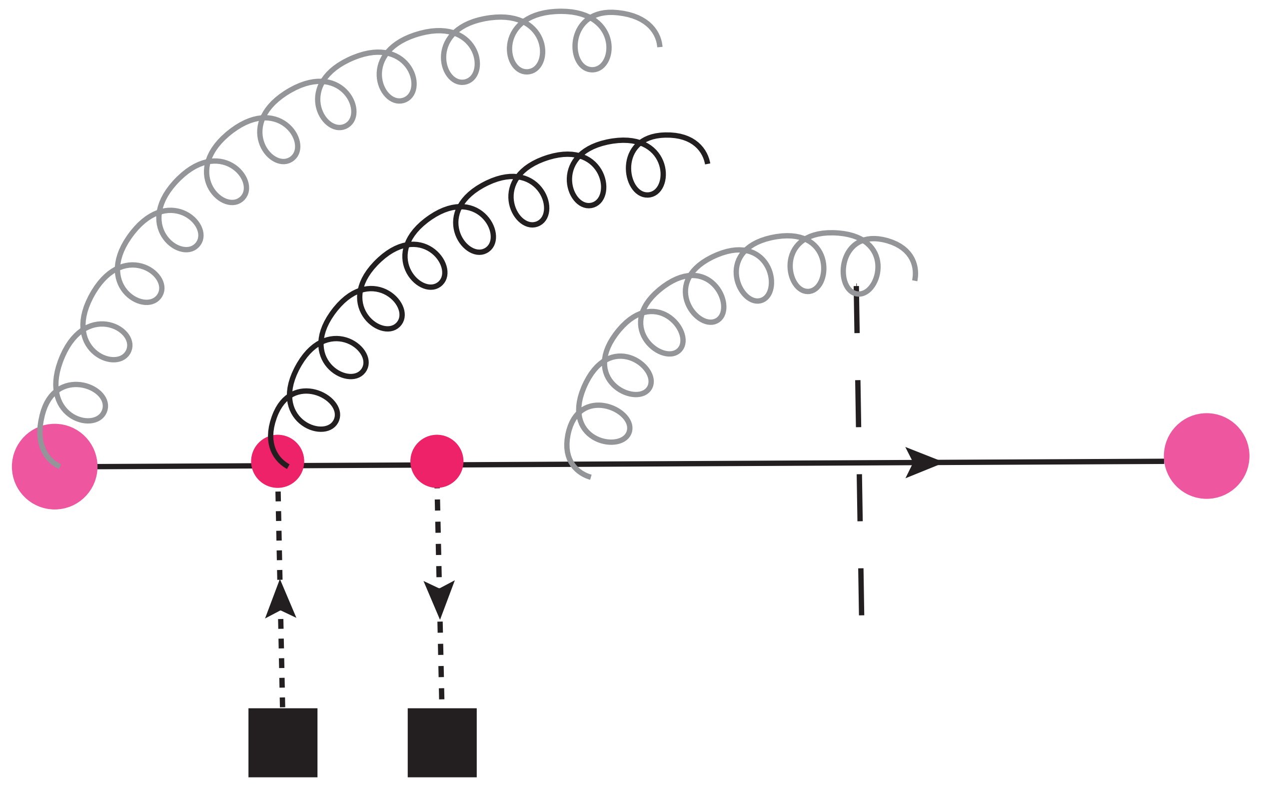
Single scattering : Glauber insertions on same side

- $\mathcal{O}(H_G^2)$ expansion at the same side

$$J_{q,s}(\chi, k_\perp; L) = \frac{1}{2N_c}\int d^4x\Theta(x^- - L)\int d^4y\Theta(y^- - L)\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\bar{\mathbf{T}}\Big\{e^{-i\int dtH_n(t)}\chi_n(0)\mathscr{M}|X\rangle\langle X|$$
$$\mathbf{T}\Big\{e^{-i\int dtH_n(t)}\{H_{G,I}(x)H_{G,I}(y)\}\Big\}\Big[\delta(\omega - \bar{n}\cdot\mathscr{P})\delta^2(\mathbb{P}^\perp)\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big] + \text{c.c} + \mathcal{O}(H_G^4)$$

- Soft/Medium function explicitly factors out

$$J_{q,s}(\omega,\chi;L) = L\int\frac{d^2k_\perp}{(2\pi)^2}J_{q,V}(\omega,\chi,k_\perp,\mu,\nu;L)\otimes\mathbf{B}(k_\perp,\mu,\nu)$$



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$$\times\Big[\delta^2(\vec{\mathbb{P}}_\perp - \vec{k}_\perp)\delta(\mathscr{P}^-)O_n^B(0)\Big]\Big[\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathscr{P})\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big]\delta^{AB} + c.c + \mathcal{O}(H_G^4)$$

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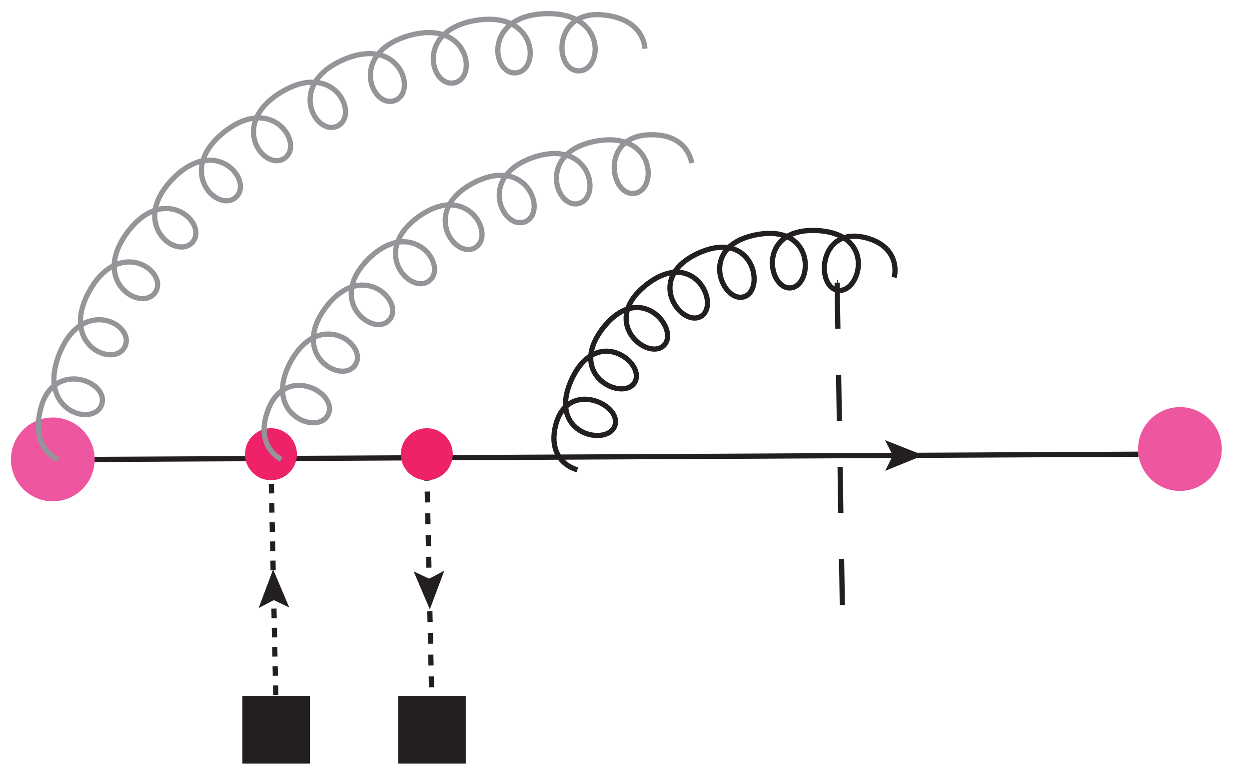
$$\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\{H_{G,I}(x)H_{G,I}(y)\}\Big\}\Big[\delta(\omega - \bar{n}\cdot\mathscr{P})\delta^2(\mathbb{P}^\perp)\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big] + \mathrm{c.c} + \mathcal{O}(H_G^4)$$

- Soft/Medium function explicitly factors out

$$J_{q,s}(\omega, \chi; L) = L\int \frac{d^2k_\perp}{(2\pi)^2}J_{q,V}(\omega, \chi, k_\perp, \mu, \nu; L)\otimes \mathbf{B}(k_\perp, \mu, \nu)$$



$+$ $c.c$

- Sinc function leads to LPM terms

$$J_{q,V}(\omega, \chi, k_\perp; L) = \frac{1}{2N_c}\frac{1}{2}e^{-i\frac{L}{2}(\mathbb{P}_+^A+\mathbb{P}_+^B)}\mathrm{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\bar{\mathbf{T}}\Big\{e^{-i\int dt H_{n,I}(t)}\chi_n(0)\Big\}\mathscr{M}|X\rangle\langle X|\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\Big[\delta^2(\vec{\mathbb{P}}_\perp + \vec{k}_\perp)\delta(\mathscr{P}^-)O_n^A(0)\Big]$$
$$\times\Big[\delta^2(\vec{\mathbb{P}}_\perp - \vec{k}_\perp)\delta(\mathscr{P}^-)O_n^B(0)\Big]\Big[\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathscr{P})\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big]\delta^{AB} + c.c + \mathcal{O}(H_G^4)$$

Medium induced jet function

- To get the total medium induced jet function add all real and virtual contributions

$$J_q^{(1)}(\omega,\chi;L) = L\int \frac{d^2k_\perp}{(2\pi)^2}\left[J_{q,R}(\omega,\chi,k_\perp,\mu,\nu;L) - J_{q,V}(\omega,\chi,k_\perp,\mu,\nu;L)\right]\mathbf{B}(k_\perp,\mu,\nu) + \mathcal{O}(H_G^4)$$

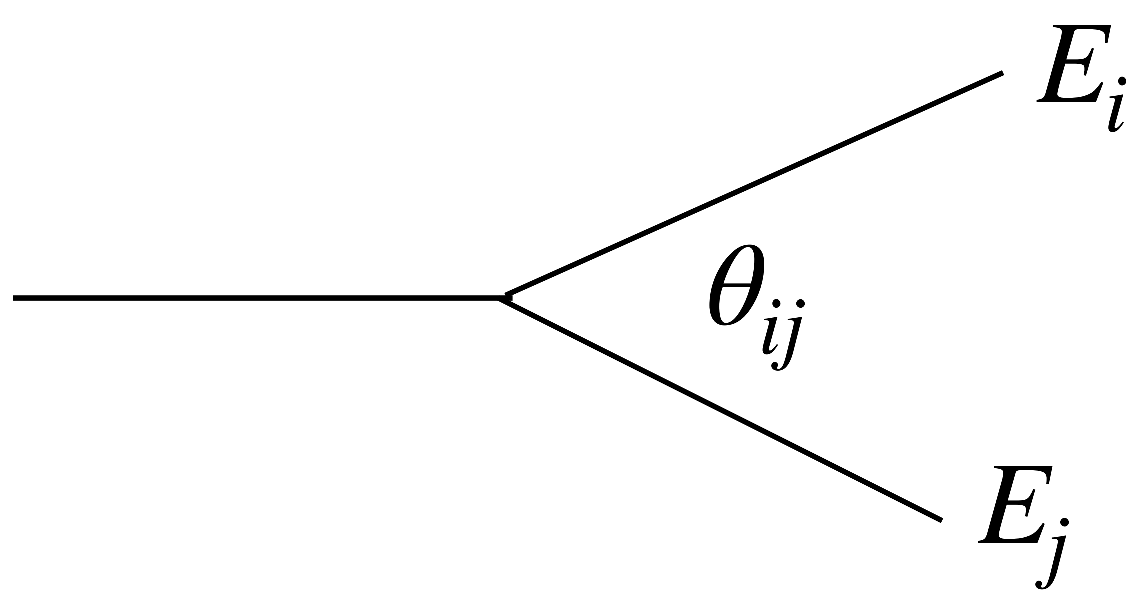
- At leading order, medium induced jet function enhance by L

- For total jet function add vacuum contribution

$$J_q(\omega,\chi) = J_q^0(\omega,\chi) + J_q^{(1)}(\omega,\chi;L)$$

- Measurement function

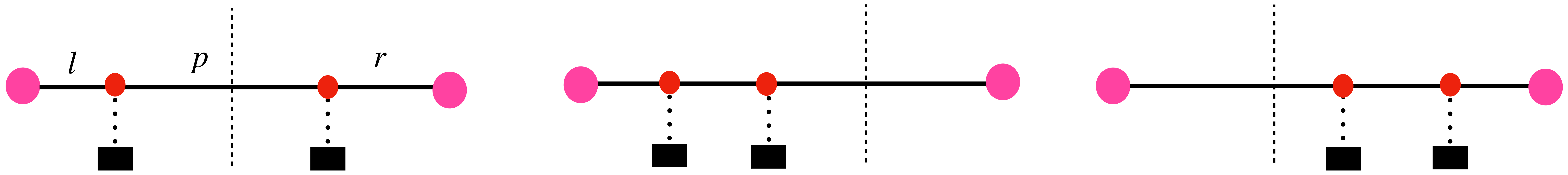
$$\mathscr{M} = \sum_{i,j\in X_n} \frac{1}{\omega^2}\left[E_i^2\delta(\chi) + E_j^2\delta(\chi) + 2E_iE_j\delta\left(\chi - \frac{\theta_{ij}^2}{4}\right)\right]$$



Next we explicitly evaluate these contributions

Compute medium induced jet function : LO

- At leading order jet parton gets kicks from the medium
- Both real and virtual contribution
- Measurement function is $\delta(\chi)$

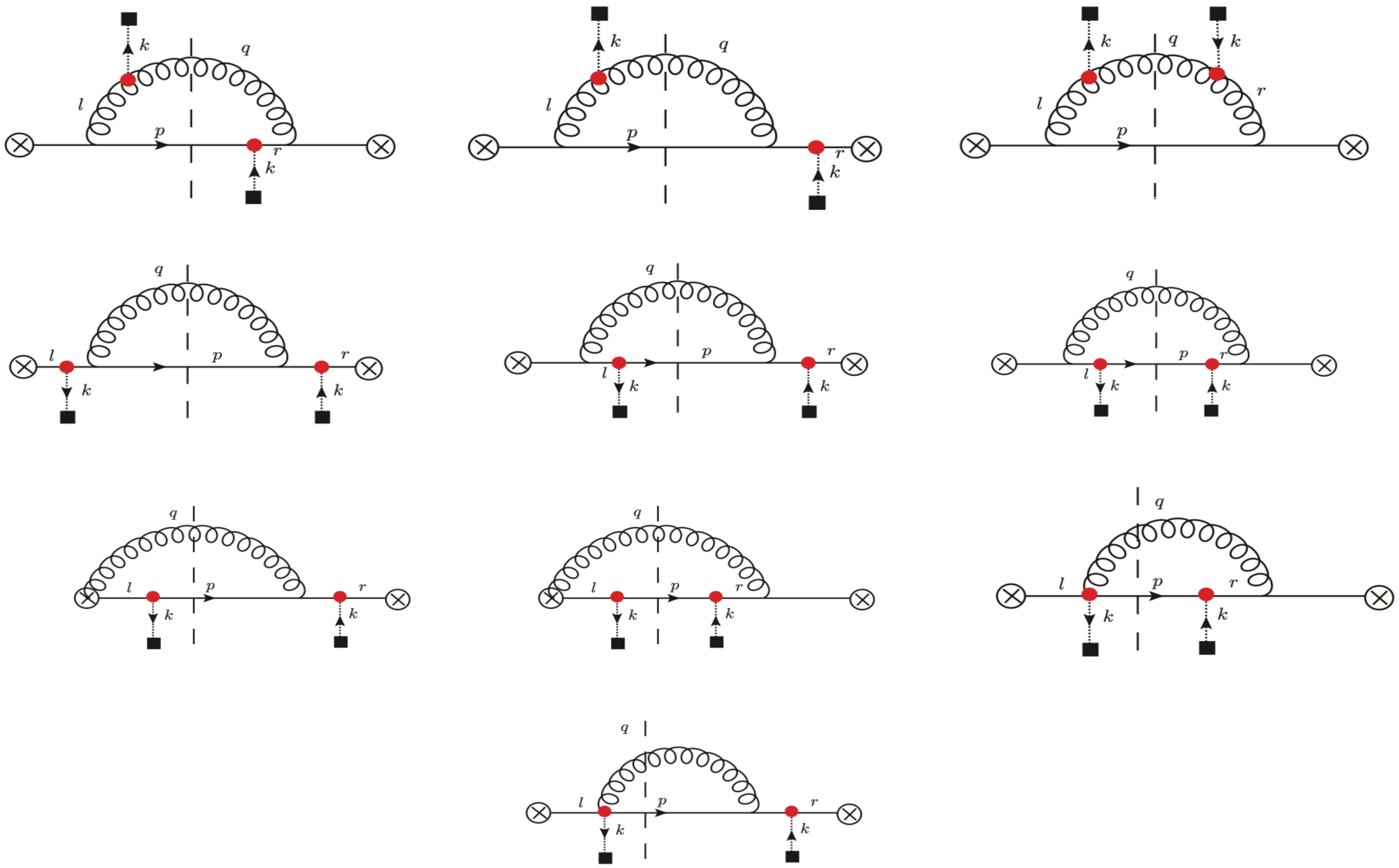


$$J_{R,\mathrm{LO}}(\omega,\chi) = -\,\delta_{AB}\int \frac{d^4p}{(2\pi)^4}\delta(p^2)\delta(p^- - \omega)\delta^2(\vec{p}_\perp - \vec{k}_\perp)\int\frac{dl^+}{2\pi}\int\frac{dr^+}{2\pi}\frac{\bar{n}\cdot l}{l^2+i\epsilon}\frac{\bar{n}\cdot r}{r^2-i\epsilon}\bar{n}\cdot p e^{-i(\frac{L}{2}(l^+-r^+))}\mathrm{Sinc}\left[\frac{L}{2}(l_+ - r_+)\right]\delta(\chi)\,.$$

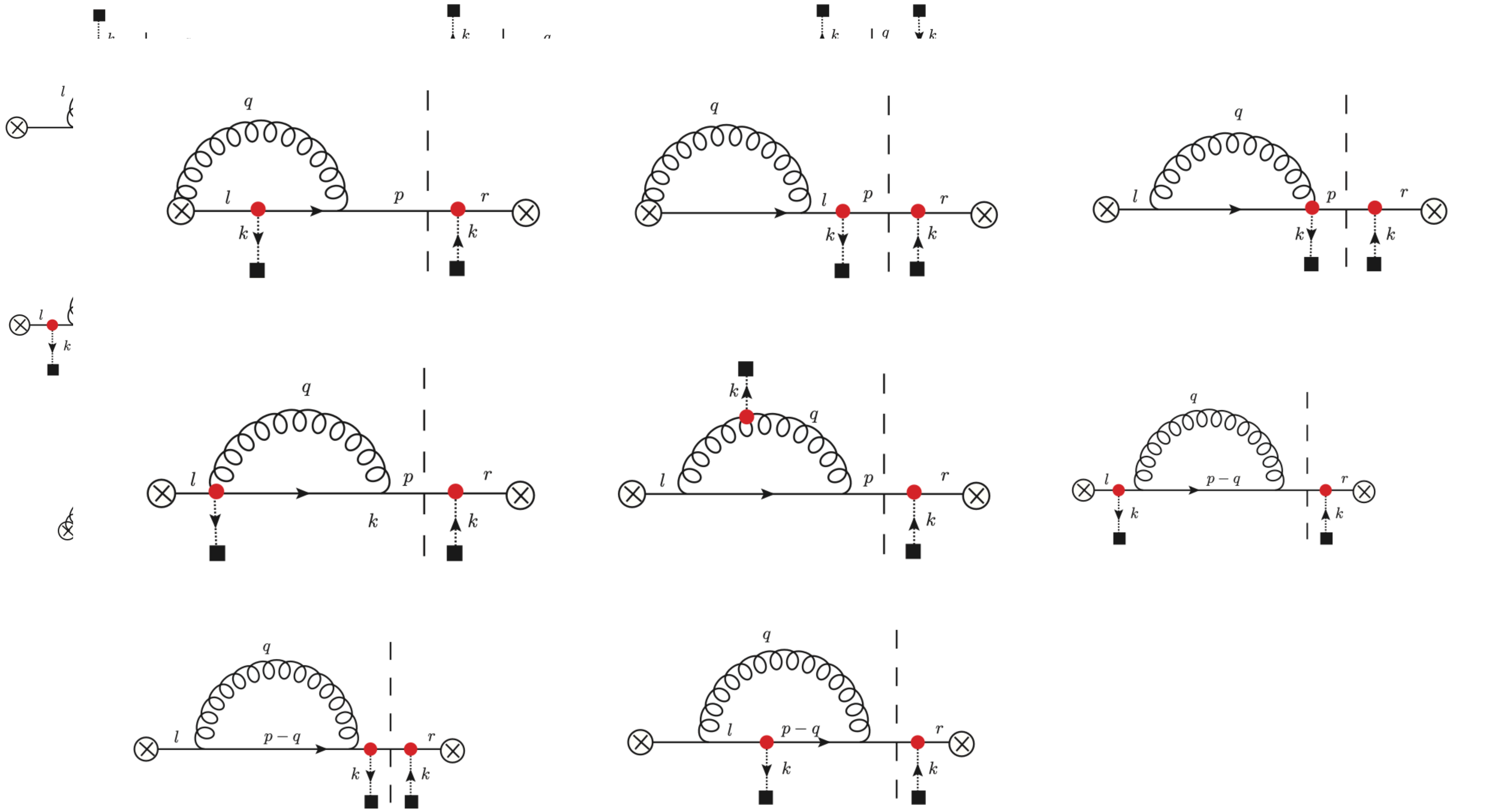
$$J_{R,\mathrm{LO}} = -\,4\delta_{AB}\delta(\chi) \qquad J_{V,\mathrm{LO}} = -\,4\delta_{AB}\delta(\chi)$$

- Total contribution vanishes

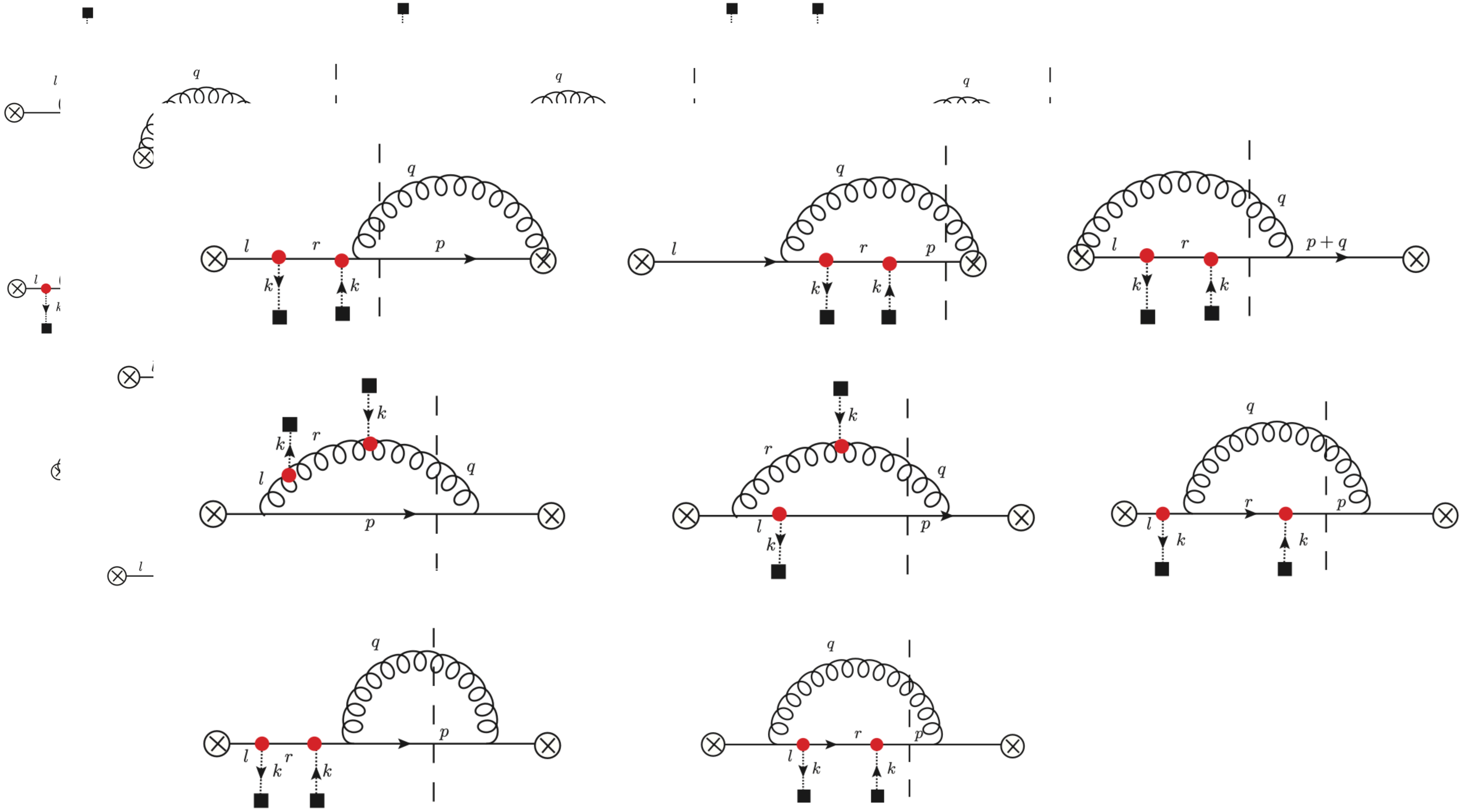
Compute medium induced jet function : NLO



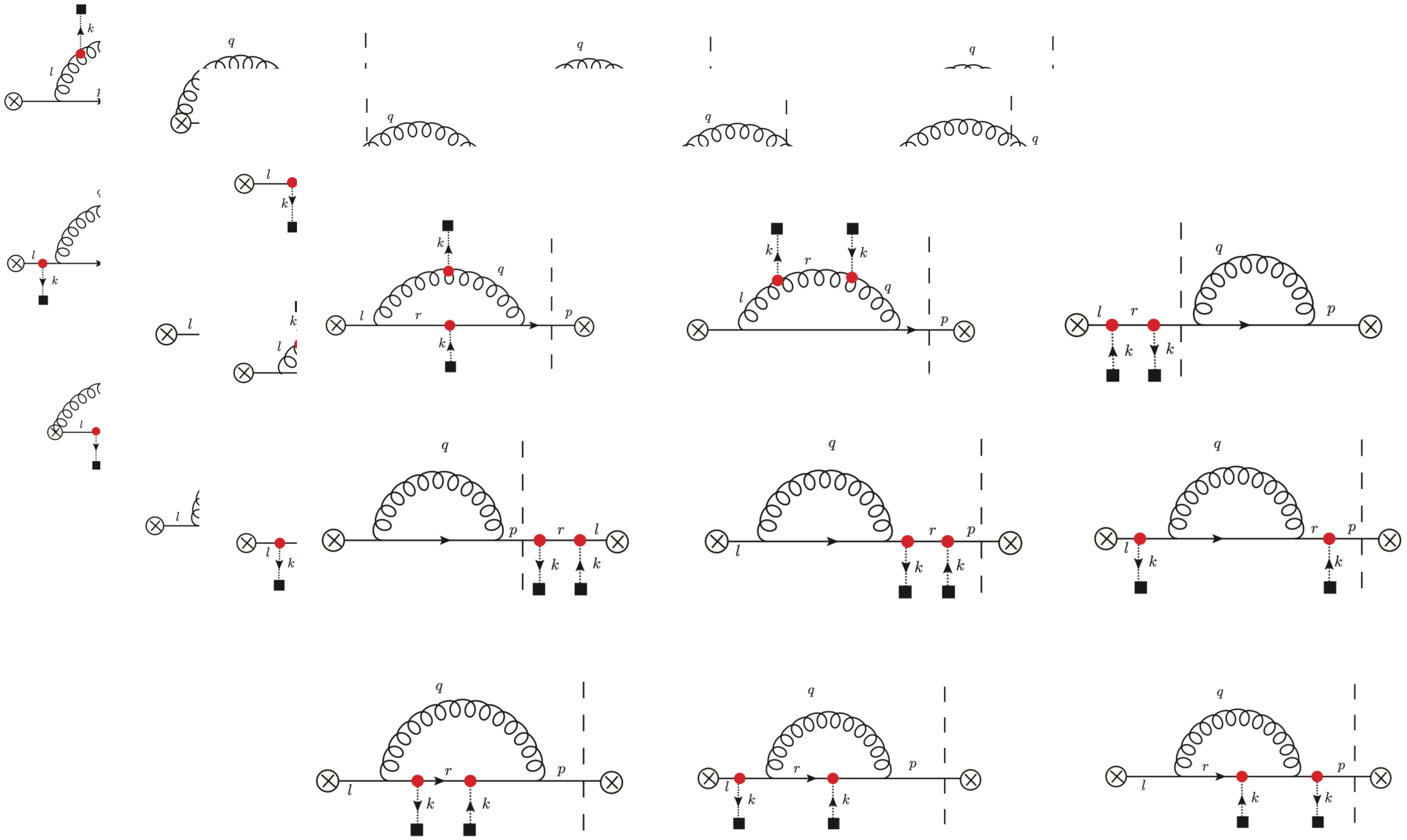
Compute medium induced jet function : NLO



Compute medium induced jet function : NLO



Compute medium induced jet function : NLO

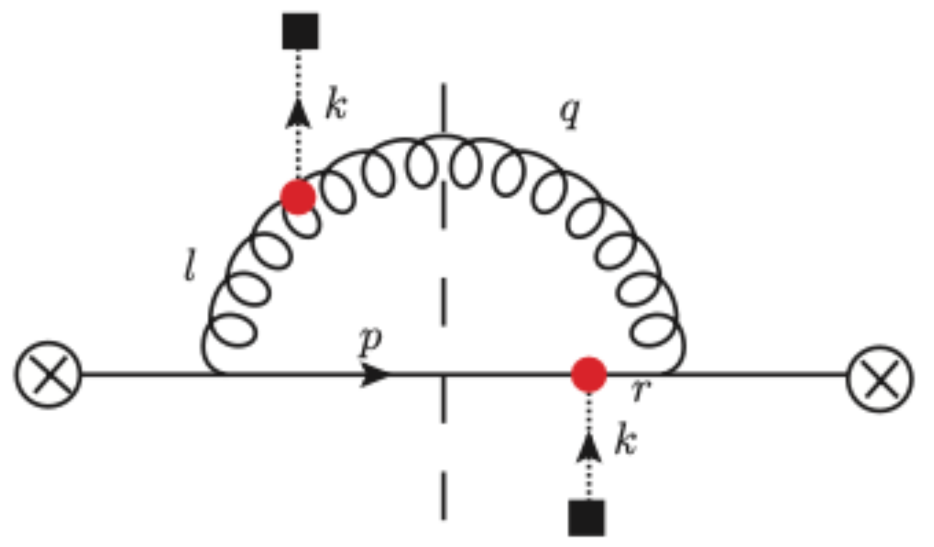


Medium induced jet function : NLO

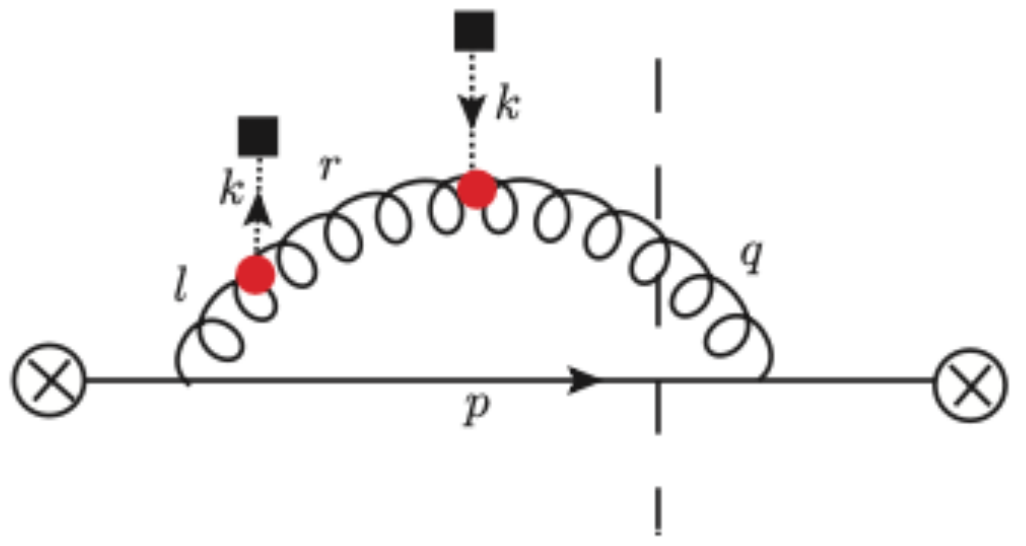
- Total transverse momentum of parton from hard vertex is zero so angle between final state parsons now depends Glauber momentum

$$\frac{\theta_{ij}^2}{4} = \frac{k_\perp^2 (q^-)^2 - 2q^- \vec{k}_\perp \cdot \vec{q}_\perp + q_\perp^2}{(q^-)^2(\omega - q^-)^2}$$

For opposite side insertions

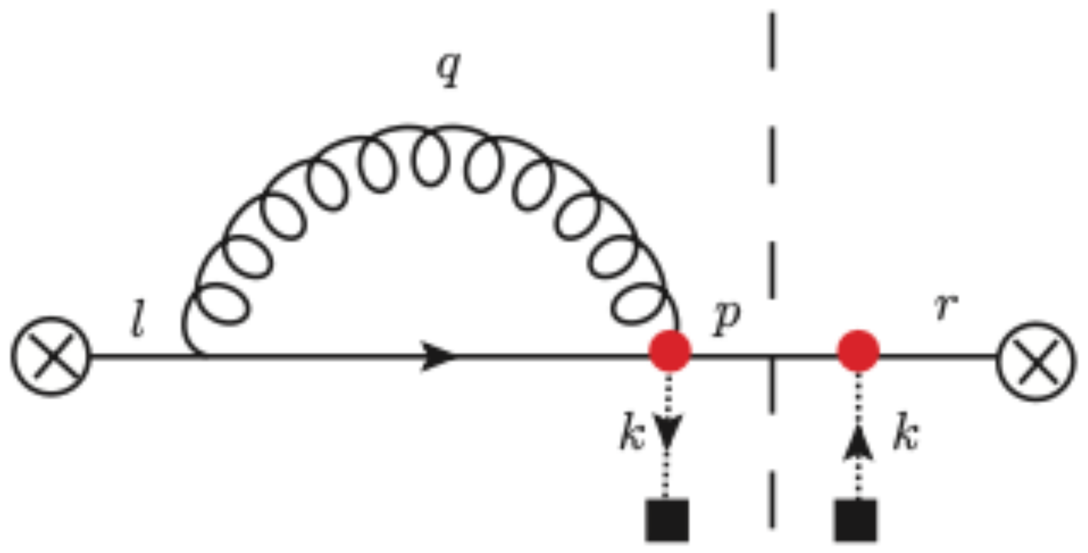


- Measurement function from virtual diagrams is $\delta(\chi)$



- Combine measurement from real and virtual diagrams

$$J_{1,\mathrm{NLO}}(\chi, \omega, k_\perp) = \frac{g^2 C_F}{(2\pi)^3} \int \frac{dz}{z} \int d^2 q_\perp \Big[\Big(|\mathscr{M}|^2_{RR} \{\delta(\chi - \theta_1^2) - \delta(\chi)\} - |\mathscr{M}|^2_{VR} \{\delta(\chi - \theta_2^2) - \delta(\chi)\} \Big) 2z(1-z) + (|\mathscr{M}|^2_{RR} + |\mathscr{M}|^2_{VR} - |\mathscr{M}|^2_{VR} - |\mathscr{M}|^2_{VV}) \delta(\chi) \Big]$$



$$\theta_1^2 = \frac{k_\perp^2 z^2 - 2z \vec{k}_\perp \cdot \vec{q}_\perp + q_\perp^2}{[z(1-z)\omega]^2} \qquad \theta_2^2 = \frac{q_\perp^2}{[z(1-z)\omega]^2}$$

Medium induced jet function : NLO

- NLO jet function for quark

$$
\begin{aligned}
J_{1,\mathrm{NLO}}(\chi,\omega,k_\perp) = \frac{\alpha_s C_F}{2\pi^2}\int dz z(1-z)\int d^2q_\perp\Bigg[\Bigg\{ -\frac{2N_c(\vec{q}_\perp\cdot\vec{\kappa}_\perp)}{q_\perp^2\kappa_\perp^2}f(z)\left(1-\frac{\sin\omega_1}{\omega_1}-\frac{\sin\omega_2}{\omega_2}+\frac{\sin(\omega_2-\omega_1)}{\omega_2-\omega_1}\right)-\frac{4N_c(1-z)^2}{\kappa_\perp^2 Q_\perp^2}\Bigg(\frac{\vec{q}_\perp\cdot\vec{\kappa}_\perp}{z} \\
+\frac{\kappa^2+\vec{q}_\perp\cdot\vec{\kappa}_\perp}{2(1-z)}+\frac{\vec{k}\cdot\vec{\kappa}_\perp}{2}+\frac{\kappa_\perp^2 z}{2(1-z)^2}\Bigg)\left(1-\frac{\sin\omega_1}{\omega_1}\right)+\frac{4N_c f(z)}{\kappa_\perp^2}\left(1-\frac{\sin\omega_1}{\omega_1}\right)+\frac{4C_F z}{q_\perp^2}\left(1-\frac{\sin\omega_1}{\omega_1}\right)+\frac{2}{3}\frac{z(1-z)^2}{q_\perp^2 Q_\perp^2} \\
\left(\frac{\vec{q}_\perp\cdot\vec{\kappa}_\perp}{(1-z)^2}+\frac{\vec{k}_\perp\cdot\vec{\kappa}_\perp}{1-z}\right)\left(1-\frac{\sin\omega_1}{\omega_1}\right)-2C_F\frac{z(1-z)^2}{Q_\perp^2}\left(k_\perp^2+\frac{\kappa_\perp^2}{(1-z)^2}+\frac{\vec{k}_\perp\cdot\vec{\kappa}_\perp}{(1-z)}\right)+\frac{4C_F(1-z)}{q_\perp^2 z}\left(1-\frac{\sin\omega_1}{\omega_1}\right) \\
-\frac{2}{3}\frac{(1-z)}{zQ_\perp^2}\frac{\sin\omega_1}{\omega_1}+\frac{2N_c(1-z)}{zq_\perp^2}\left(1-\frac{\sin\omega_1}{\omega_1}\right)+\frac{2N_c(1-z)}{zQ_\perp^2}\Bigg\}\left(\frac{(z(1-z)\omega)^2\delta(q_\perp-q_0)}{2|q_0-k_\perp z\cos\theta|}-\delta(\chi)\right) \\
-\left\{\frac{4C_F(1-z)}{q_\perp^2 z}-\frac{2N_c(\vec{q}_\perp\cdot\vec{\kappa}_\perp)}{q_\perp^2\kappa_\perp^2}f(z)\left(\frac{\sin(\omega_2-\omega_1)}{\omega_2-\omega_1}-\frac{\sin\omega_1}{\omega_1}\right)+\frac{2N_c}{q_\perp^2}f(z)\left(1-\frac{\sin\omega_1}{\omega_1}\right)\right\} \\
\left((z(1-z)\omega)^2\delta(q_\perp^2-[z(1-z)\omega]^2\chi)-\delta(\chi)\right)\Bigg]
\end{aligned}
$$

$$\omega_1=\frac{L\kappa_\perp^2}{z(1-z)\omega}\qquad \omega_2=\frac{Lq_\perp^2}{z(1-z)\omega}\qquad Q_\perp^2=\omega(\kappa_\perp^2 q^-+q_\perp^2p^-)-k_\perp^2p^-q^-\qquad \vec{\kappa}_\perp=\vec{q}_\perp-\vec{k}_\perp$$

Compute medium function

- Medium function can be obtained from spectral function

$$\mathbf{B}_{\mathrm{LO}}(k_\perp) = D^g_>(k) + D^q_>(k)$$

$D_>(k) \to$ Wightman correlator

- Wightman correlator can be obtained from medium spectral function

$$D_>(k) = (1 + f(k_0))\rho(k)$$

$\rho(k) \to$ Spectral function

- Wightman correlator can be obtained in imaginary time formalism

$$D_E^{AB}(K) = \int_0^\beta d\tau \int d^3x\, e^{iK\cdot X} \langle \frac{1}{\mathbb{P}_\perp^2} O_s^{g_n A}(X) \frac{1}{\mathbb{P}_\perp^2} O_s^{g_n B}(0) \rangle$$

SCET operators

$$\mathcal{O}_n^{qA} = \bar{\chi}_n T^A \frac{\bar{n}}{2} \chi_n$$

$$\mathcal{O}_s^{qA} = \bar{\chi}_s T^A \frac{n}{2} \chi_s$$

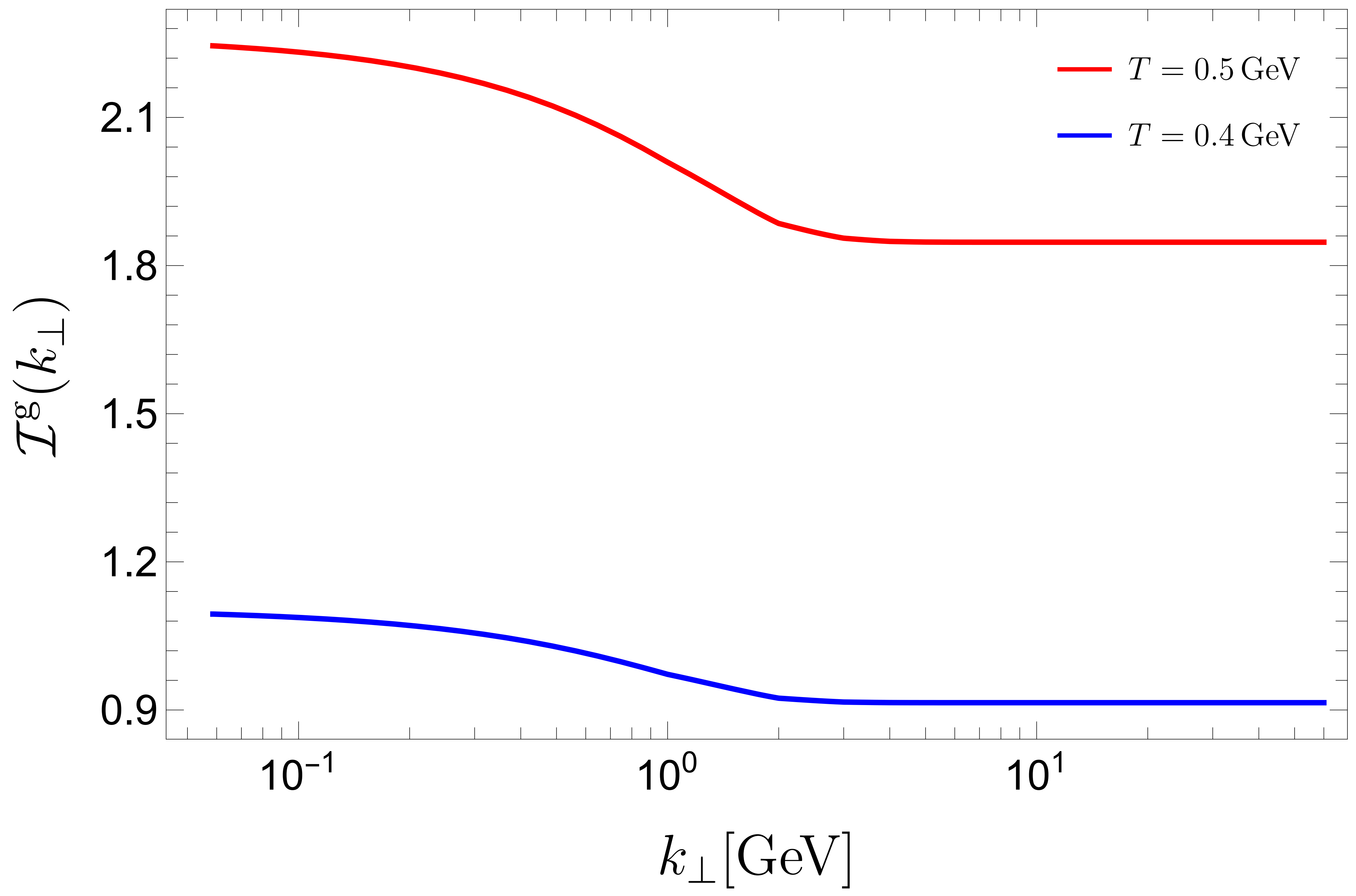
$$\mathcal{O}_s^{gA} = \frac{i}{2} f^{ACD} \mathcal{B}^C_{S\perp\mu} \frac{n}{2} \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}^{D\mu}_{S\perp}$$

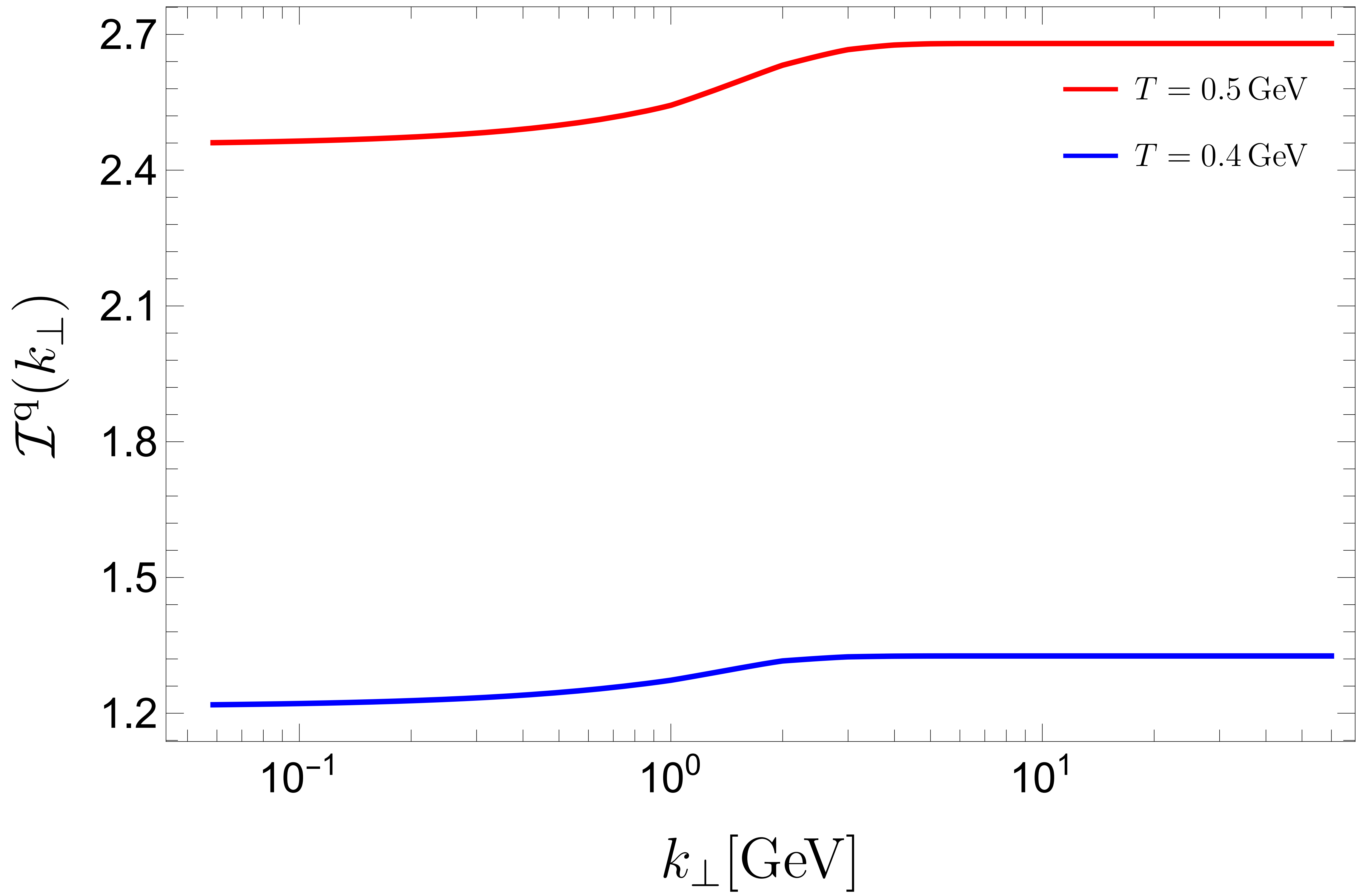
- Leading order medium function

$$\mathbf{B}_{\mathrm{LO}}(k_\perp) = (8\pi\alpha_s)^2 \left(\frac{2\pi N_c^2}{16 k_\perp^4} \mathcal{J}^g(k_\perp) + \frac{2\pi N_f}{k_\perp^4} \mathcal{J}^q(k_\perp) \right)$$

Result I : medium function

- Bose enhance and Pauli blocking for quark and gluons
- At leading order medium function has somewhat weak dependence on Glauber momentum

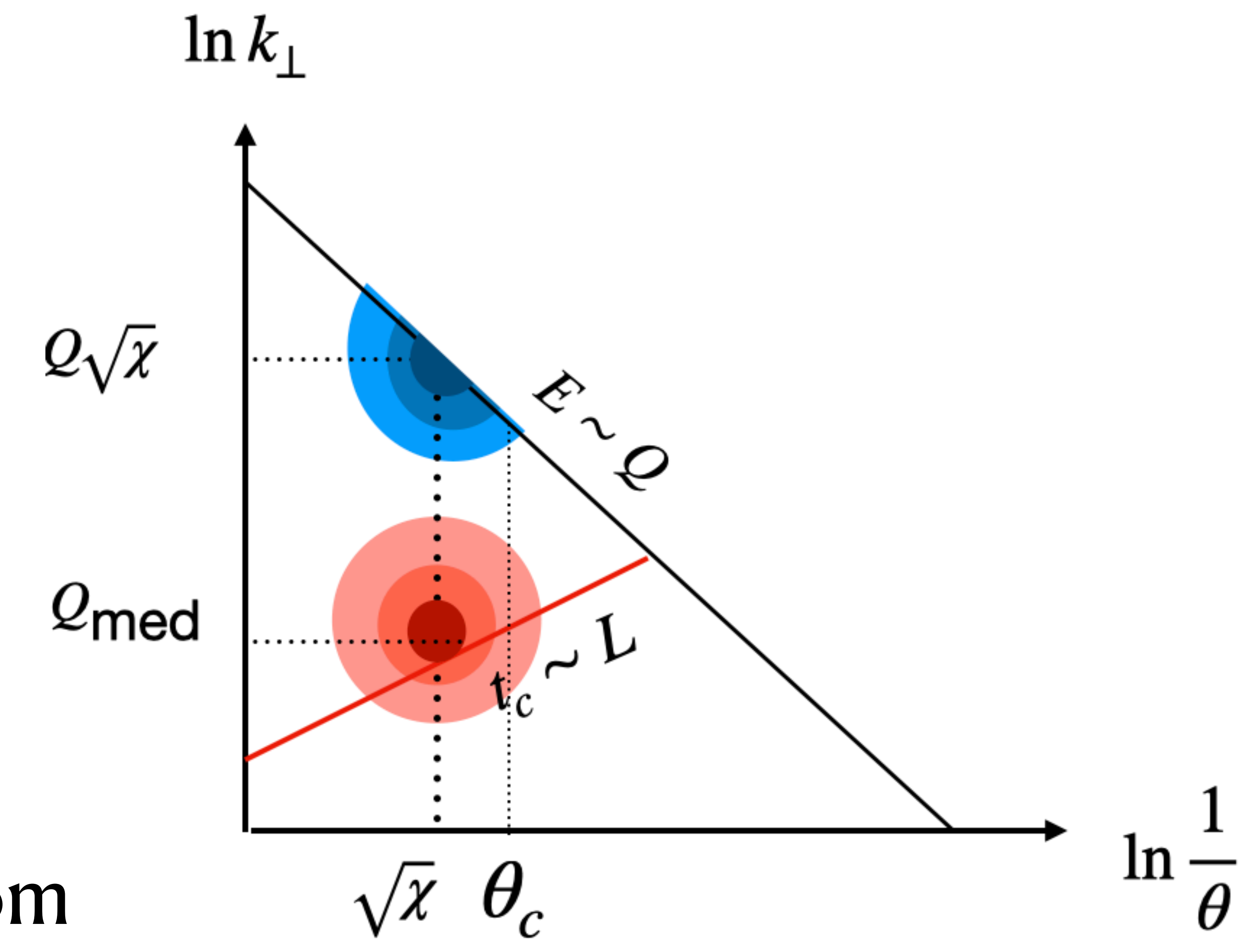




Case II : $Q \gg Q\sqrt{\chi} \gg Q_{\text{med}}$

- Refactorize the jet function

See Varun's talk, Monday 5 Aug

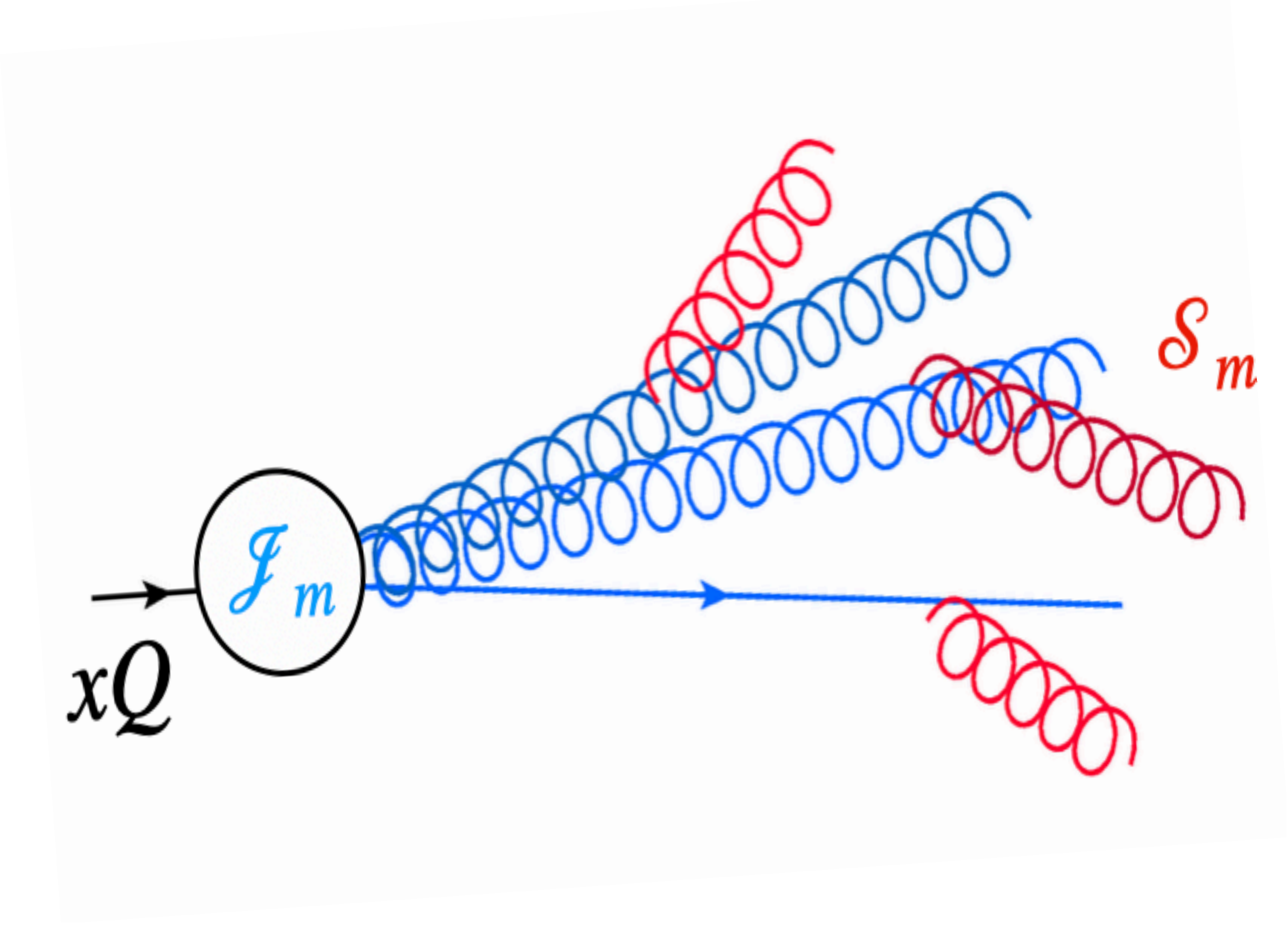


$$J_q = J_i^{(0)}(\omega,\chi,\mu) + \sum_{m=1}^{\infty}\sum_{j=1}^{m} \mathcal{J}^j_{i\to m}\underbrace{(\{\underline{m}\},\theta_c,\omega,\mu)}_{\text{Matching function}} \otimes_\theta \ \mathcal{S}_{m,j}\underbrace{(\{\underline{m}\},\chi,\mu)}_{\text{Collinear-soft function}}$$

- Matching function describes the production of m resolved hard partons from initial parton i

- Collinear soft function describes the production of medium induced radiation

- Perturbative matching coefficient $\mathcal{J}_{i\to m}$ starts at $\mathcal{O}(\alpha_s^{m-1})$



Compute jet function : NLO

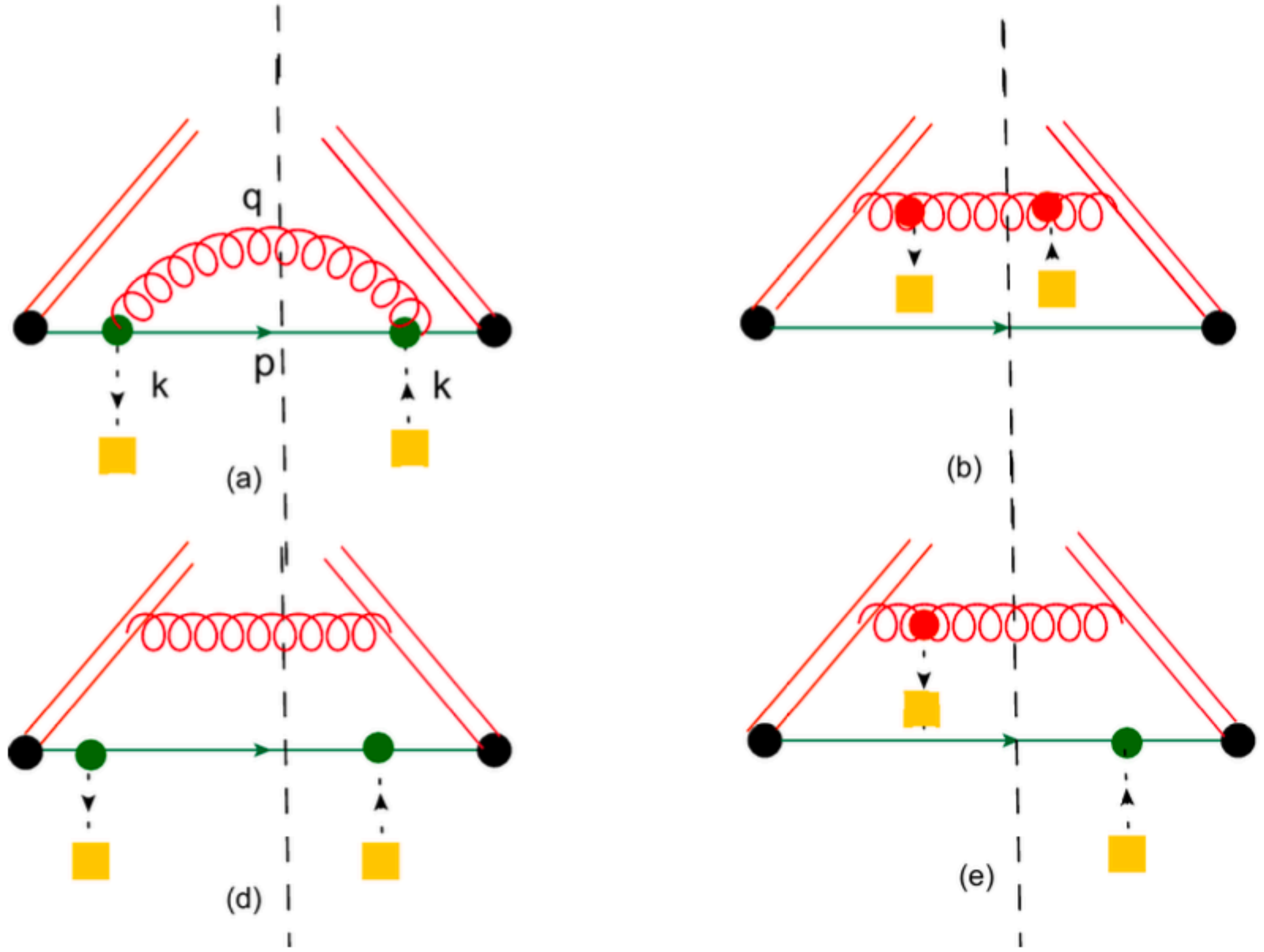
- Only one emission is resolved $\theta_c \sim R$

- First term for medium induced radiation

$$J_q^{(1)} = \mathcal{J}_{i\to 1}(\theta_c, \omega, \mu)\, \mathcal{S}_1(\chi, \mu) \qquad \mathcal{J}_{i\to 1}(\theta_c, \omega, \mu) = 1$$

- At leading order matching coefficient is one

- Medium function is contained inside collinear-soft function



$$\mathcal{S}_1^{(1)}(\chi) = L\int \frac{d^2k_\perp}{(2\pi)^3}\mathbf{S}_1^{(1)}(\chi, k_\perp; L)\mathbf{B}(k_\perp)$$

$$\mathbf{S}_1^{(1)} \sim \int \frac{dq^-}{(2\pi)^3}\int d^2q_\perp \frac{\vec{q}_\perp \cdot \vec{k}_\perp}{q_\perp^2 \kappa_\perp^2 q^-}\left(1 - \frac{q^-}{\kappa_\perp^2 L}\sin\left[\frac{L\kappa_\perp^2}{q^-}\right]\right)\mathcal{M}_1$$

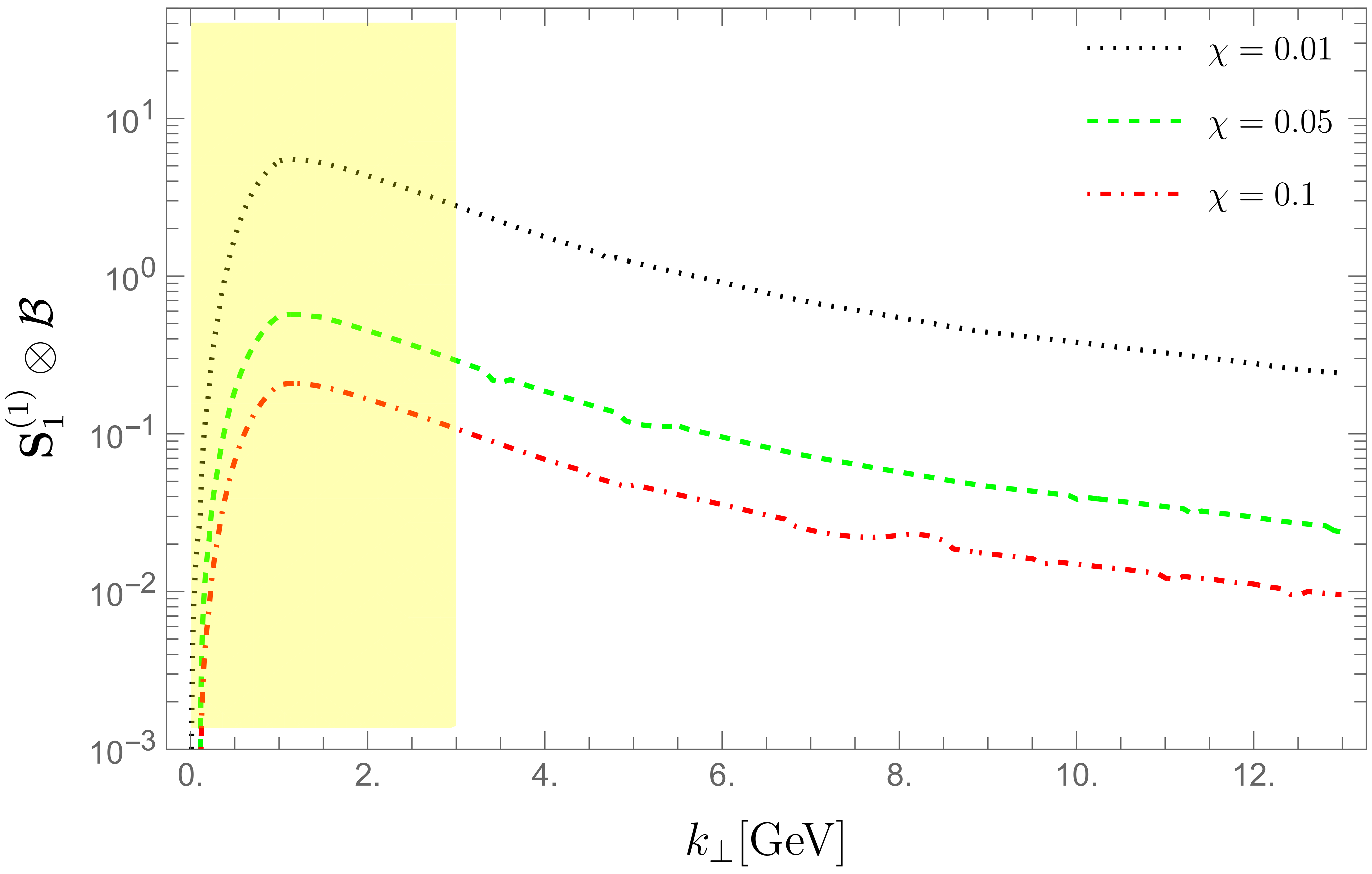
- Collinear-soft function function evolves with Hamiltonian

$$\mathcal{M}_1 = \delta\left(\chi - \frac{q_\perp^2}{z^2\omega^2}\right) - \delta(\chi)$$

$$\int dt H = \int dt\,\left(H_{cs} + H_s + H_{cs;s}\right) + \int ds \mathcal{O}_{cs;s}(sn)$$

Result II : Fixed order computation

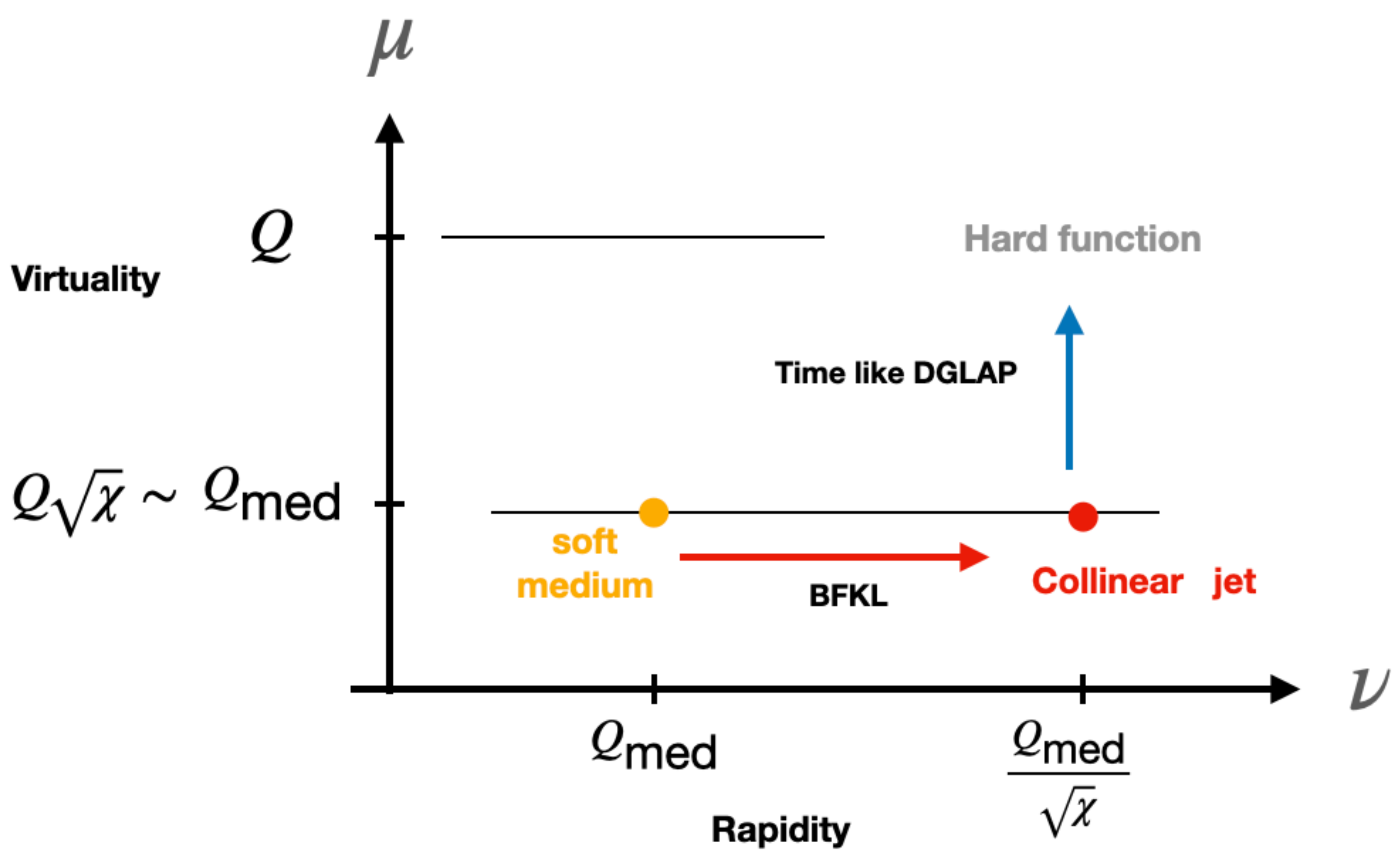
- For single scattering the dominant contribution appears to be in the non-perturbative regime

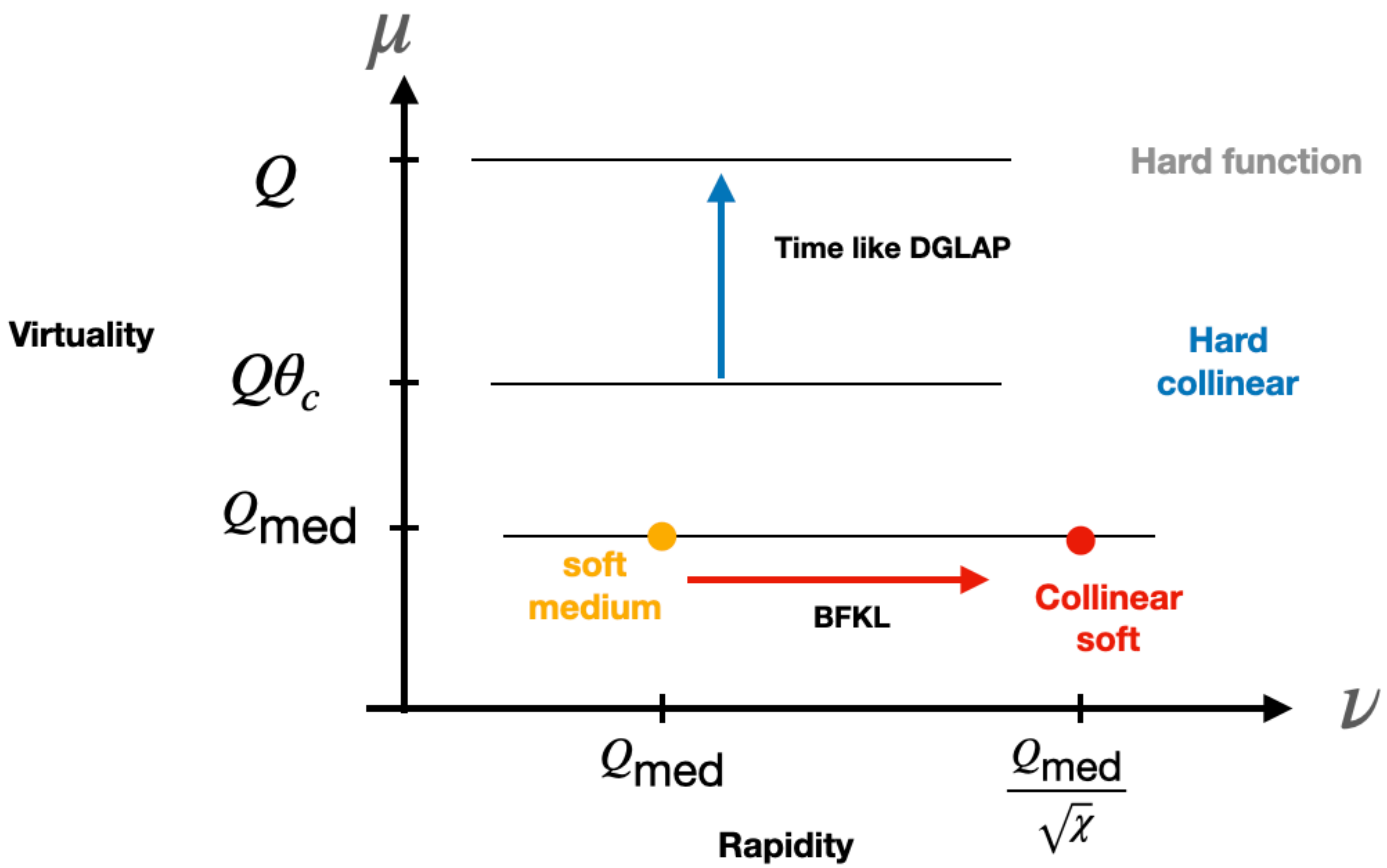


$m_D = 0.8$ GeV, $L = 5$ fm

Beyond fixed order : RG flow

- Medium function obeys BFKL, therefore, from RG consistency collinear-soft jet function also obeys BFKL evolution equation





- Natural scales for each functions are provided by the logs appearing in the functions

Solve BFKL evolution equation

- From RG consistency the jet function obeys BFKL evolution equation

$$\frac{d\sigma}{d\chi} \sim H \otimes (J^0(\chi) + \mathcal{J} \otimes \mathbf{S}_1^{(1)}(\chi) \otimes \mathbf{B})$$

$$\nu \frac{d\mathbf{S}_1^{(1)}(k_\perp, \nu)}{d\nu} = -\frac{\alpha_s(\mu)N_c}{\pi^2} \int d^2 l_\perp \left[\frac{\mathbf{S}_1^{(1)}(l_\perp, \nu)}{(\vec{l}_\perp - \vec{k}_\perp)^2} - \frac{k_\perp^2 \mathbf{S}_1^{(1)}(k_\perp, \nu)}{2l_\perp^2(\vec{l}_\perp - \vec{k}_\perp)^2} \right]$$

Scale for jet function

$$\nu_0 \sim \frac{Q_{\text{med}}}{\sqrt{\chi}}$$

- Fixed order NLO jet function sets the boundary condition

- Running from jet scale to medium scale

$$\mathbf{S}_{1,\mathrm{R}}^{(1)}(k_\perp, \mu, \nu_f) = \int d^2 l_\perp \mathbf{S}_1^{(1)}(l_\perp, \mu, \nu_0) \int \frac{d\nu}{2\pi} k_\perp^{-1+2i\nu} l_\perp^{-1-2i\nu} e^{in(\phi_k - \phi_l)} e^{-\frac{\alpha_s(\mu)N_c}{\pi}\chi(n,r)\log\frac{\nu_f}{\nu_0}}$$

Medium scale

$$\nu_f \sim Q_{\text{med}}$$

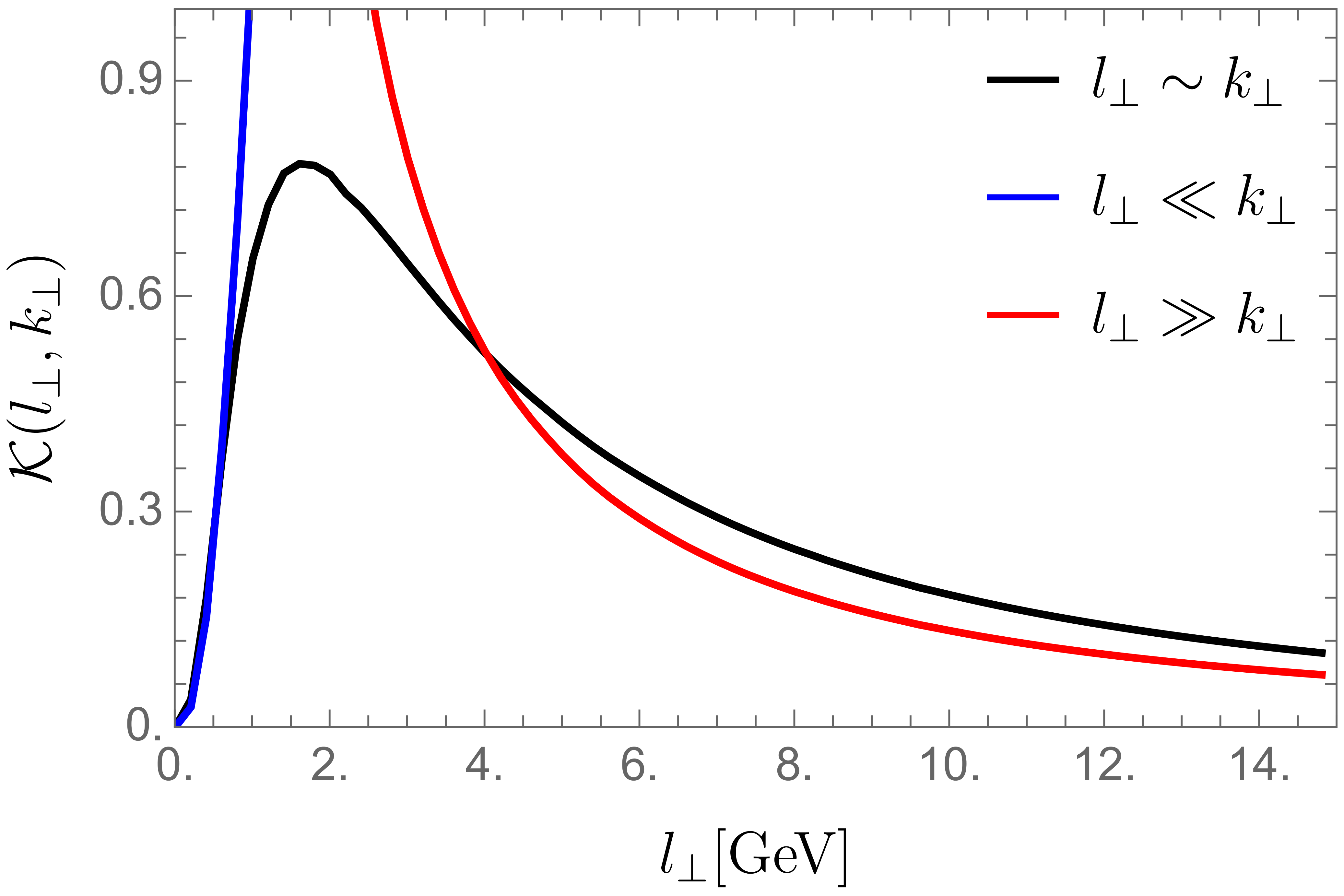
- Resums $(a_p - 1)\log(\nu_0/k_\perp)$

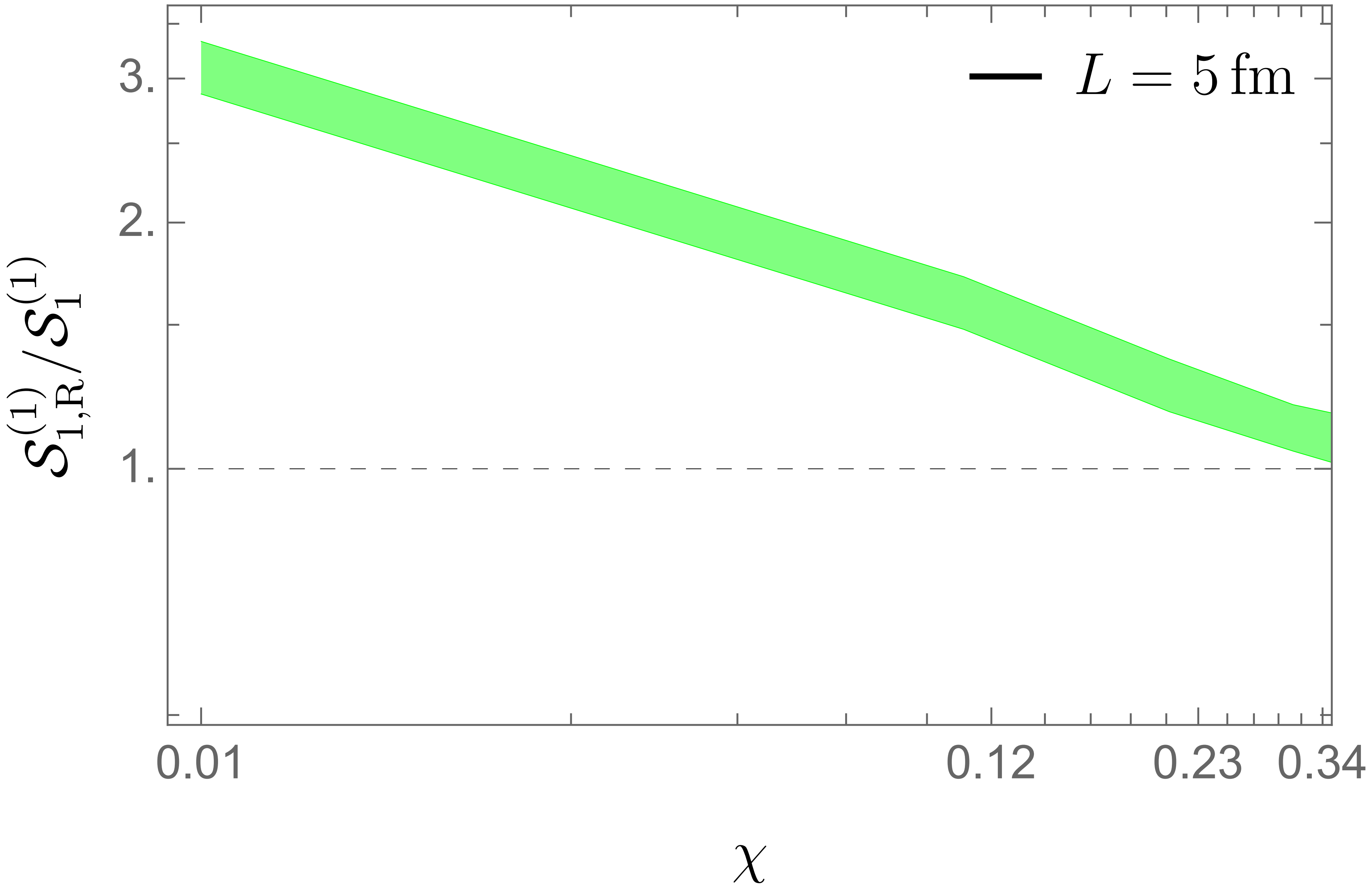
- Solution for $k_\perp \sim l_\perp$

$$\mathbf{S}_{1,R}^{(1)}(\chi, k_\perp; L) = \frac{1}{\pi k_\perp}\sqrt{\frac{\pi}{14\zeta(3)\bar{\alpha}Y}} e^{(a_p-1)Y} \int d^2 l_\perp \frac{\mathbf{S}_1^{(1)}(\chi, l_\perp; L)}{l_\perp} e^{-\frac{\log^2(k_\perp/l_\perp)}{14\zeta(3)\bar{\alpha}Y}}$$

Result III : BFKL resummation

- Resums $\sim \alpha_s \log \sqrt{\chi}$ terms which are relevant in small χ limit



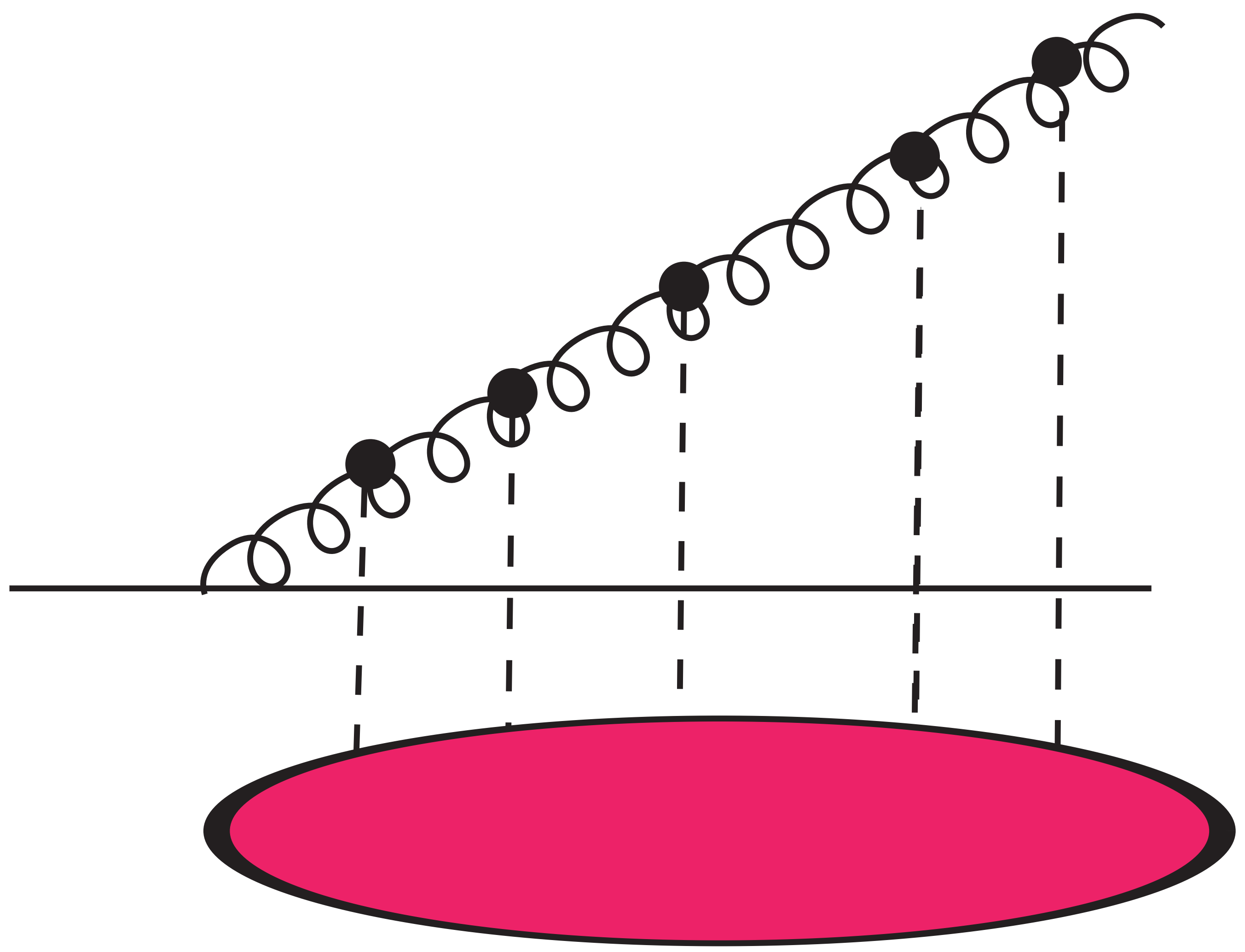


$$\alpha_s(\mu) = \frac{\alpha_s(m_Z)}{1 + \frac{\alpha_s(m_z)\beta_0}{2\pi}\log\left[\frac{\mu}{m_Z}\right]}$$

$\mu = Q_{\mathrm{med}} \sim \sqrt{\hat{q}L},\ m_Z = 90\,\mathrm{GeV}$ $\quad Q_{\mathrm{med}} \in (2-3)\,\mathrm{GeV}$

Towards an all order factorization

- For a dense medium multiple scatterings are important
- Need to sum over arbitrary number of Glauber interactions
- Expect $\hat{q}, \theta_c$ an emergent scale in the calculation
- ...



$$\frac{1}{\sigma^0}\frac{d\sigma}{d\chi} = \int dx\, x^2 H_i(\omega,\mu)\left\{ J_i^{(0)}(\omega,\chi,\mu) + \sum_{m=1}^{\infty}\sum_{j=1}^{m} \mathcal{J}^{j}_{i\to m}(\theta_c,\omega,\mu) \otimes_{\{\theta_1\ldots\theta_m\}} \left[\sum_{n=1}^{\infty}\frac{L^n}{n!}\left[\prod_{l=1}^{n}\int\frac{d^2k_{l\perp}}{(2\pi)^3}\mathbf{B}(k_{l\perp},\mu,\nu)\right]\mathbf{S}^{(n)}_{m,j}(\chi,k_{1\perp},\ldots k_{n\perp},L,\mu,\nu)\right]\right\}$$

Summary

- Factorisation can be achieved within SCET framework
- Factorization allows us to separate universal non-perturbative physics from perturbative one
- For single scattering, we reproduce GLV results
- Factorization allows us to go beyond leading order computations by resummation techniques
- EFT allows to systematically improve predictions by doing higher order computations for factorized functions
- For EEC, we resum $\log(\sqrt{\chi})$ by BFKL resummation relevant in small χ region

Thank you
for your attention