

Inferring chiral three-nucleon forces from third-generation gravitational-wave detectors

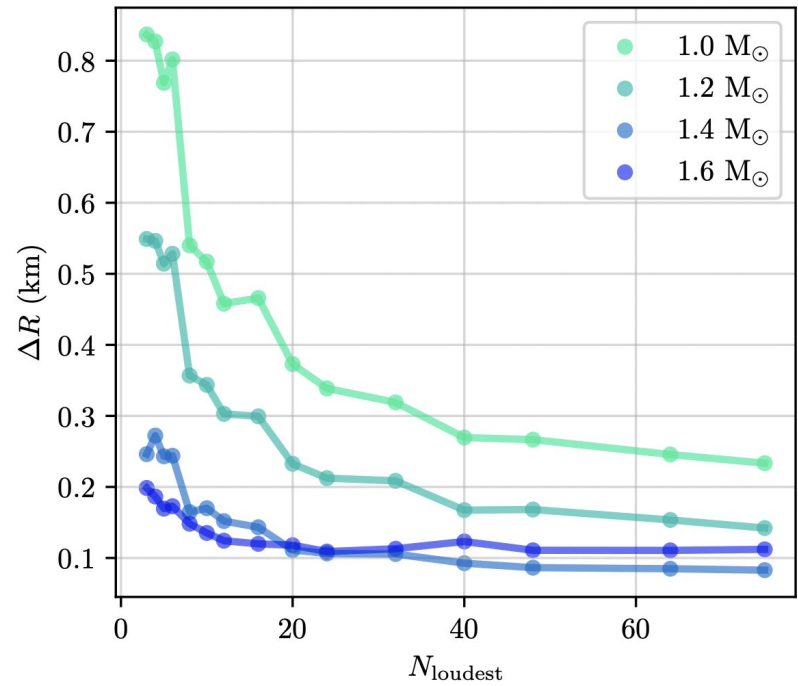
Rahul Somasundaram
Los Alamos National Laboratory and Syracuse University

Collaborators: Isak Svensson, Philippe Landry, Ingo Tews, Duncan Brown, Achim Schwenk, Soumi De, Andrew Deneris, Yannick Dietz, Brendan Reed, Pablo Giuliani, Cassandra Armstrong

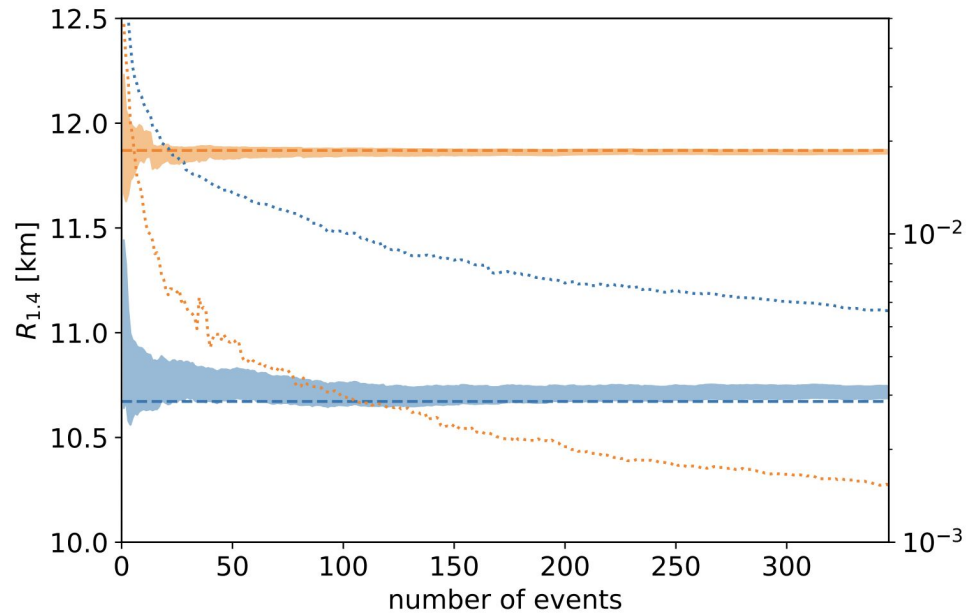
08/30/2024, INT Workshop: EOS Measurements with Next-Generation GW Detectors



High precision measurements of the EOS from next-generation detectors



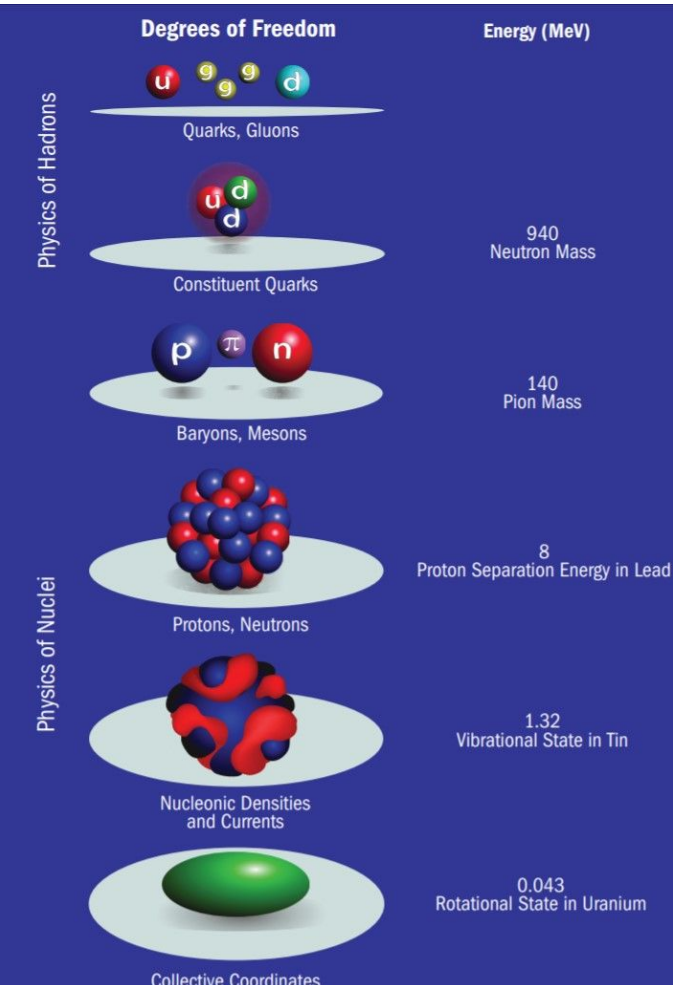
Walker et al., arXiv:2401.02604



Finstad et al., Astrophys.J. 955 (2023) 1, 45

But can we go beyond the EOS? What can we learn about the underlying nuclear interactions?

Chiral Effective Field theory



	NN	3N	4N
LO $\mathcal{O}(Q^0/\Lambda^0)$	1990 [151,152] 2 	—	—
NLO $\mathcal{O}(Q^2/\Lambda^2)$	1992 [164,165] 7 	1992,1994 [166-169] —	—
N ² LO $\mathcal{O}(Q^3/\Lambda^3)$	1992 [164,165] 0 	1994 [167,170] 2 	—
N ³ LO $\mathcal{O}(Q^4/\Lambda^4)$	2000–2002 [179-182] 12 	2008–2011 [183-185] 0 	2006 [186] 0
N ⁴ LO $\mathcal{O}(Q^5/\Lambda^5)$	2015 [188,189] 0 	2011– [190-192] ? 	?

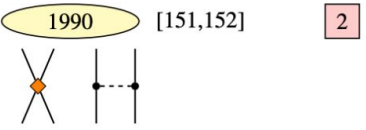
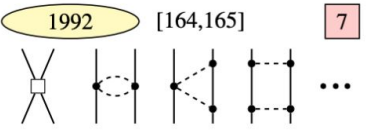
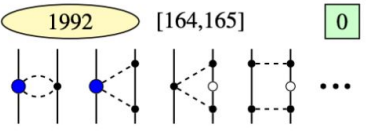
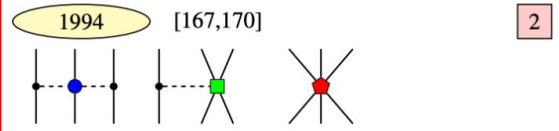
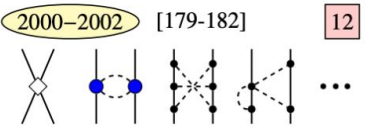
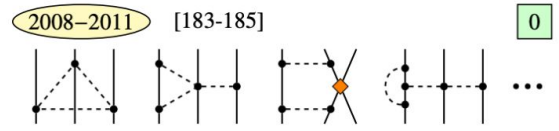
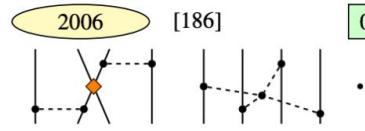
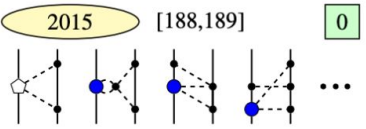
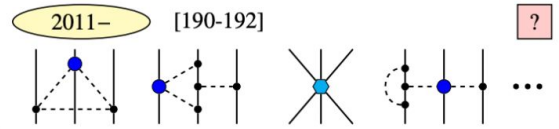

Hebeler, Phys.Rept. 890 (2021) 1-116

Chiral Effective Field theory

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Fit to NN scattering

Chiral Effective Field theory

	NN	3N	4N
LO $\mathcal{O}(Q^0/\Lambda^0)$	<p>1990 [151,152] 2</p> 	<p>—</p>	<p>—</p>
NLO $\mathcal{O}(Q^2/\Lambda^2)$	<p>1992 [164,165] 7</p> 	<p>1992,1994 [166-169]</p> <p>—</p> <p>Fit to $A \sim 3$ nuclei</p>	<p>—</p>
N ² LO $\mathcal{O}(Q^3/\Lambda^3)$	<p>1992 [164,165] 0</p> 	<p>1994 [167,170] 2</p> 	<p>—</p>
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Chiral Effective Field theory

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Fit to ???
Astrophysical NS observations?

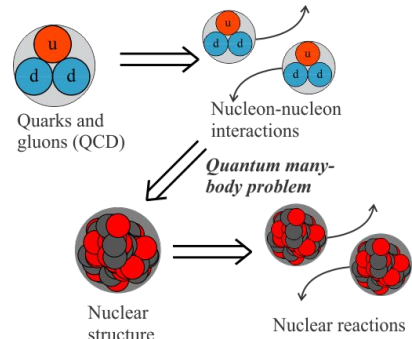
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In this talk, focus on this

In principle, how would we do this?

Model for interaction between particles



Leading order (LO)

NLO

N³LO

$$V_{NN} = c_1(\Lambda) \text{---} + c_2(\Lambda) \text{---} + c_3(\Lambda) \text{---} + \dots$$

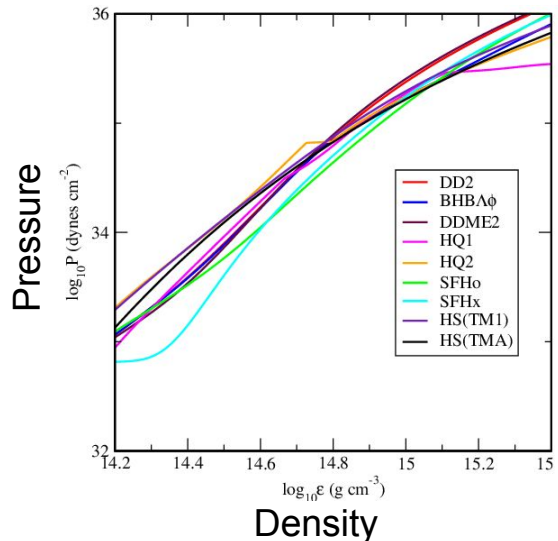
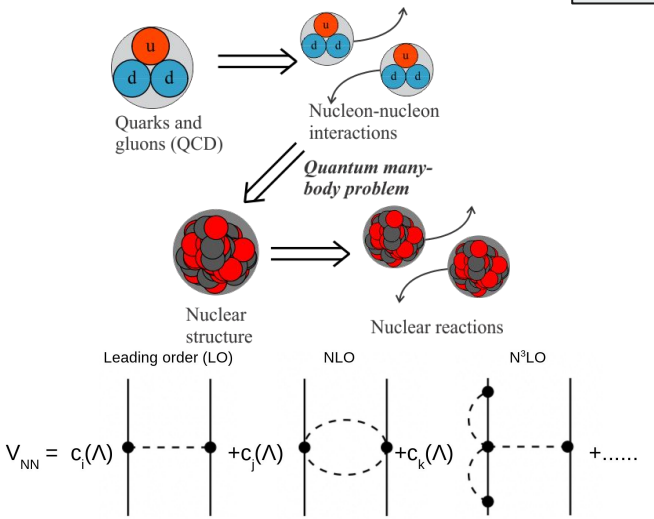
The diagram shows the expansion of the nucleon-nucleon potential V_{NN} in terms of orders of the chiral expansion. It consists of four terms separated by plus signs, each with a coefficient $c_i(\Lambda)$. The first term is a single horizontal dashed line between two vertical lines representing nucleons. The second term is a horizontal dashed line with a loop (two vertical lines connected by a dashed line) between two vertical lines. The third term is a horizontal dashed line with a triangle loop (two vertical lines connected by two dashed lines) between two vertical lines. The fourth term is a horizontal dashed line with a more complex loop structure between two vertical lines. The expansion ends with '+.....'.

In principle, how would we do this?

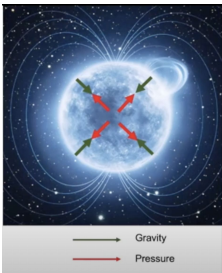
Model for interaction between particles

$$H|\psi\rangle = E|\psi\rangle$$

The Equation of State



In principle, how would we do this?



Tolman–Oppenheimer–Volkoff (TOV) equation

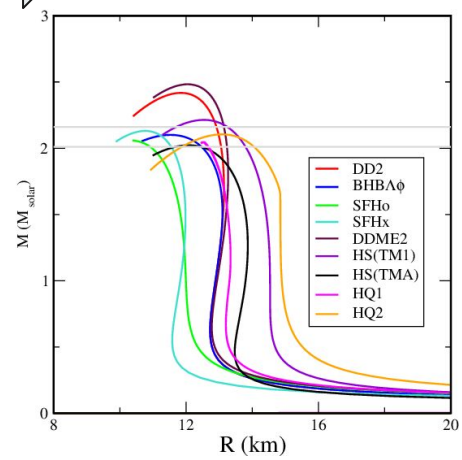
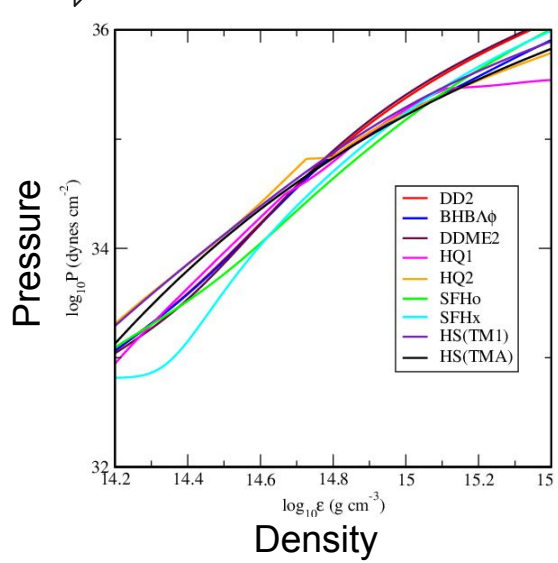
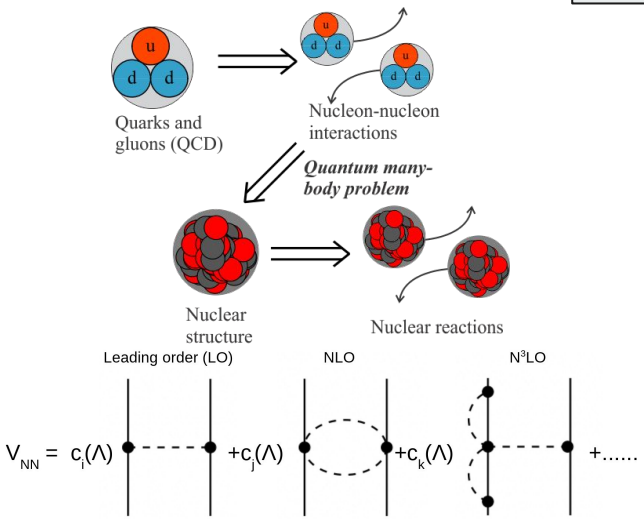
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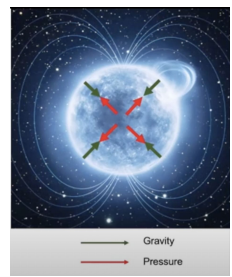
$$G^{\mu\nu} = \kappa T^{\mu\nu}$$

Neutron star observables



In principle, how would we do this?

Bayesian inference requires $\sim 10^7$ model evaluations



Tolman–Oppenheimer–Volkoff (TOV) equation

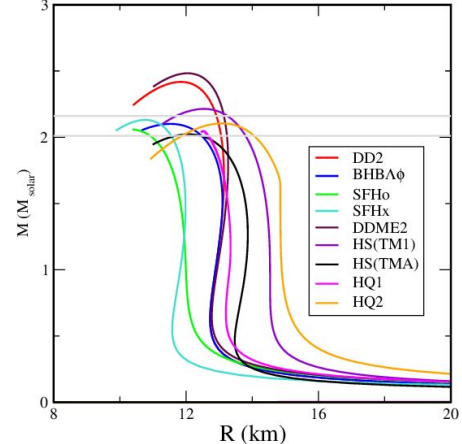
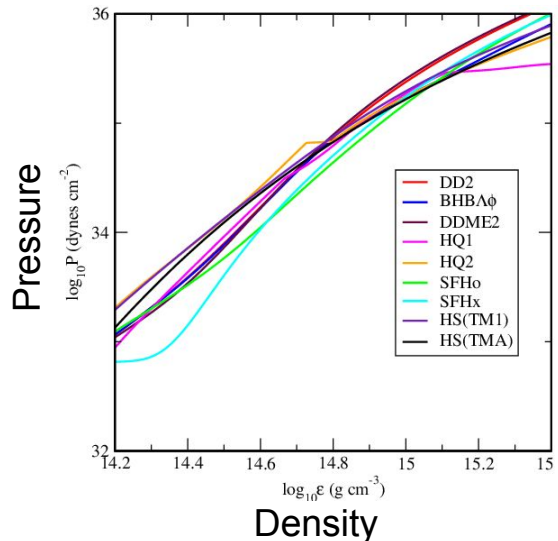
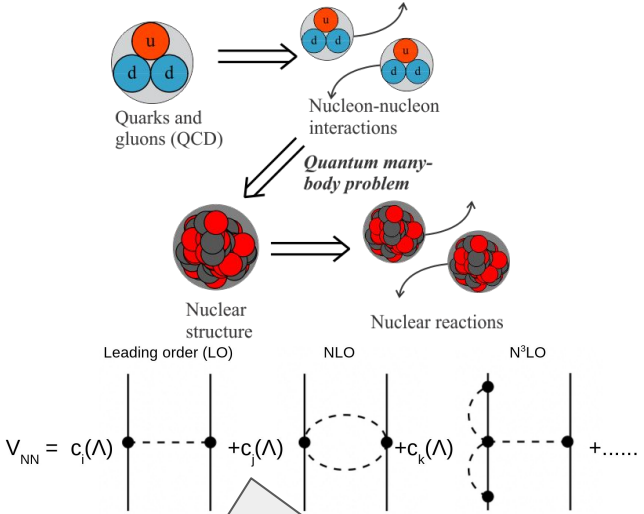
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Neutron star observables

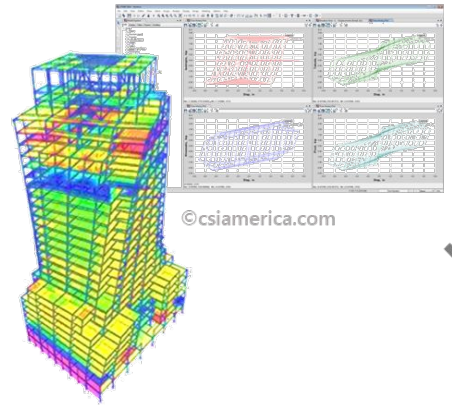


Bayesian Parameter Estimation

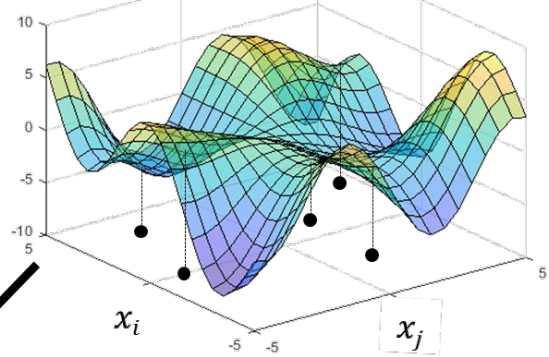
The solution: Use emulators to accelerate calculations

Emulators mimic the behaviour of the full-scale model at a small fraction of its computational cost

Simulation model

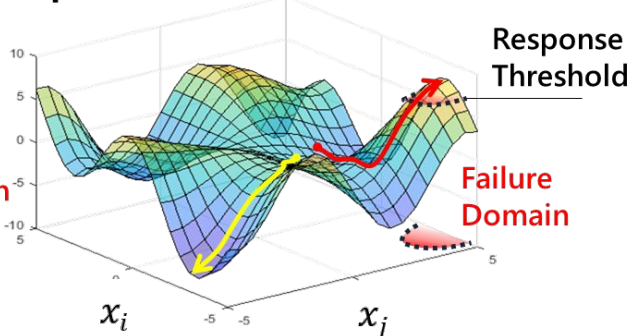
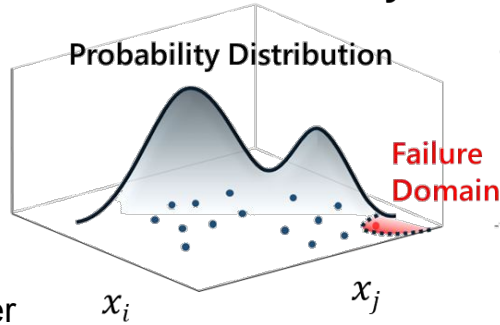


Surrogate model

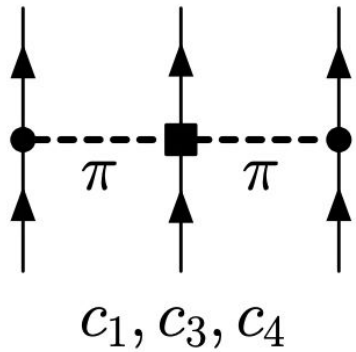


Training

UQ Analysis / Optimization

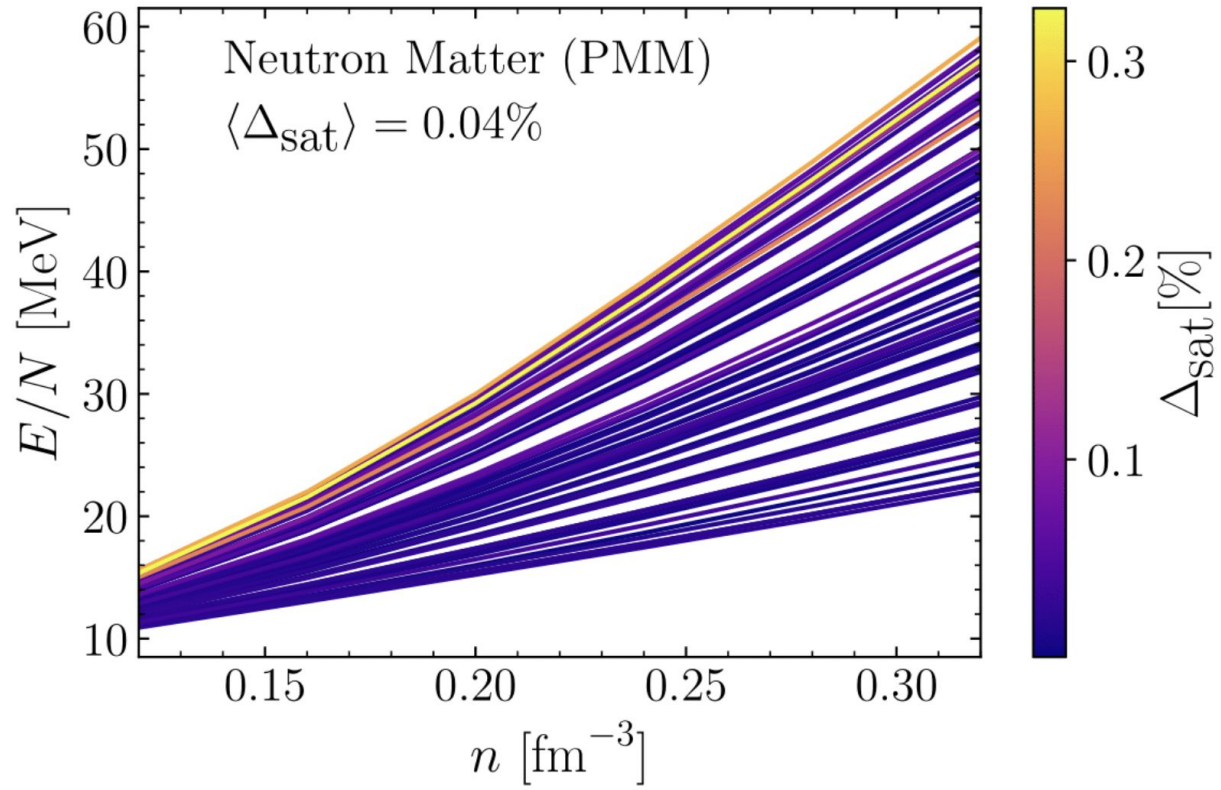


Emulators for neutron matter calculations



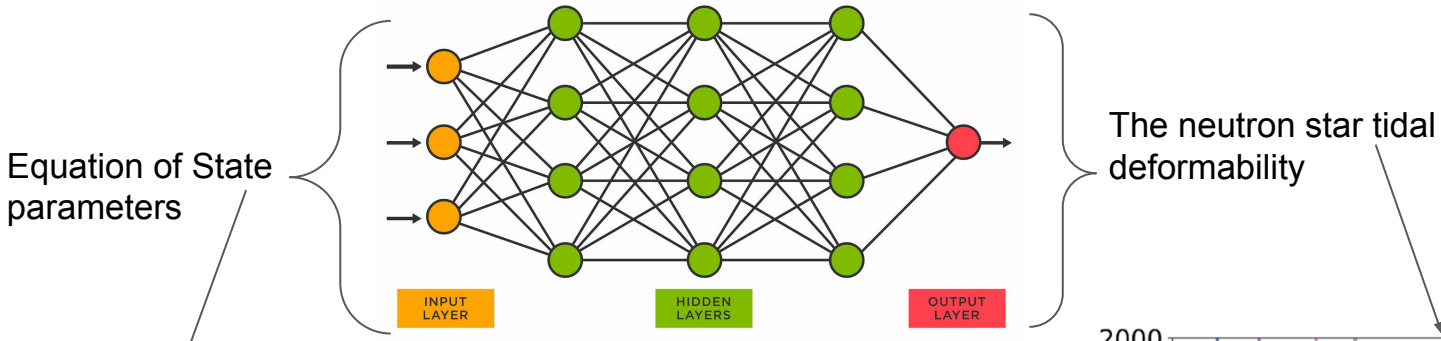
- Use Many-Body Perturbation Theory (MBPT) as 'High Fidelity Model'
- Approximate MBPT predictions with lowest eigenvalue of a 2x2 matrix

$$M = M_0 + c_1 M_1 + c_3 M_3$$



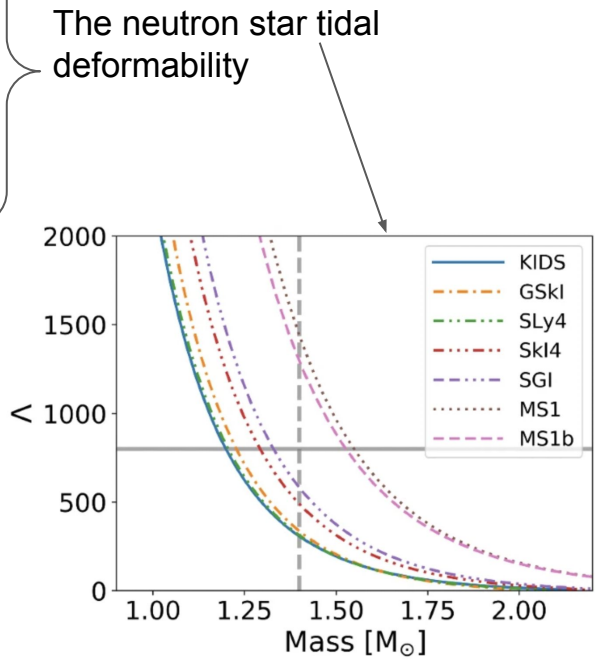
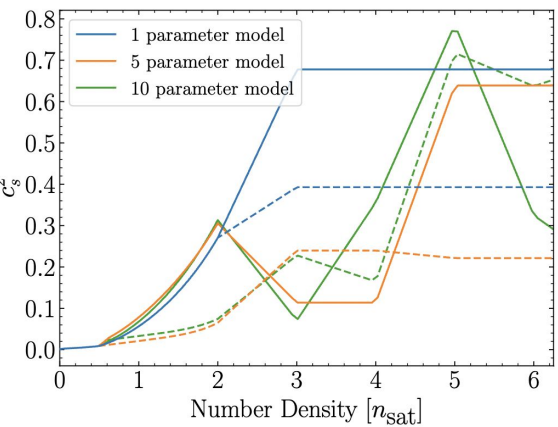
Emulators for the TOV equations

We use an ensemble of feedforward neural networks to emulate solutions to the TOV equations



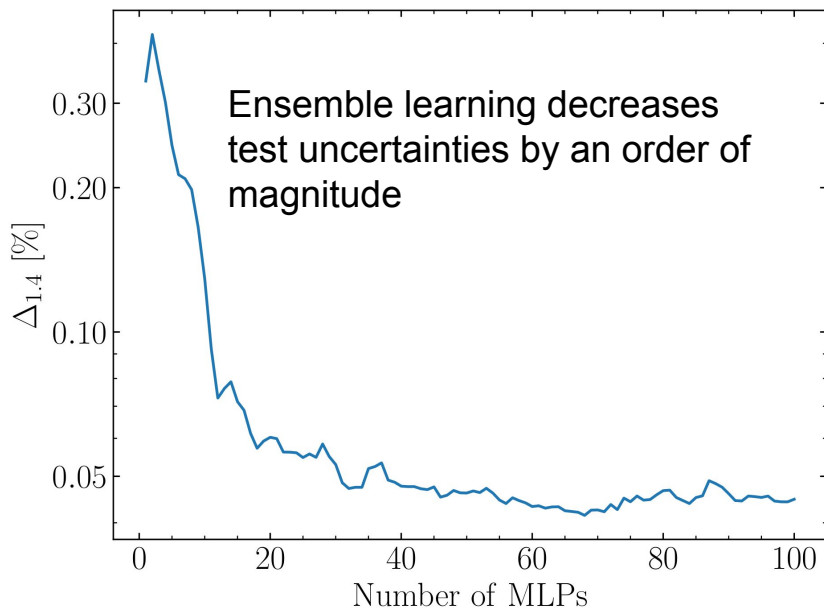
(x100)

We generated 200K samples, randomly split into training and test data



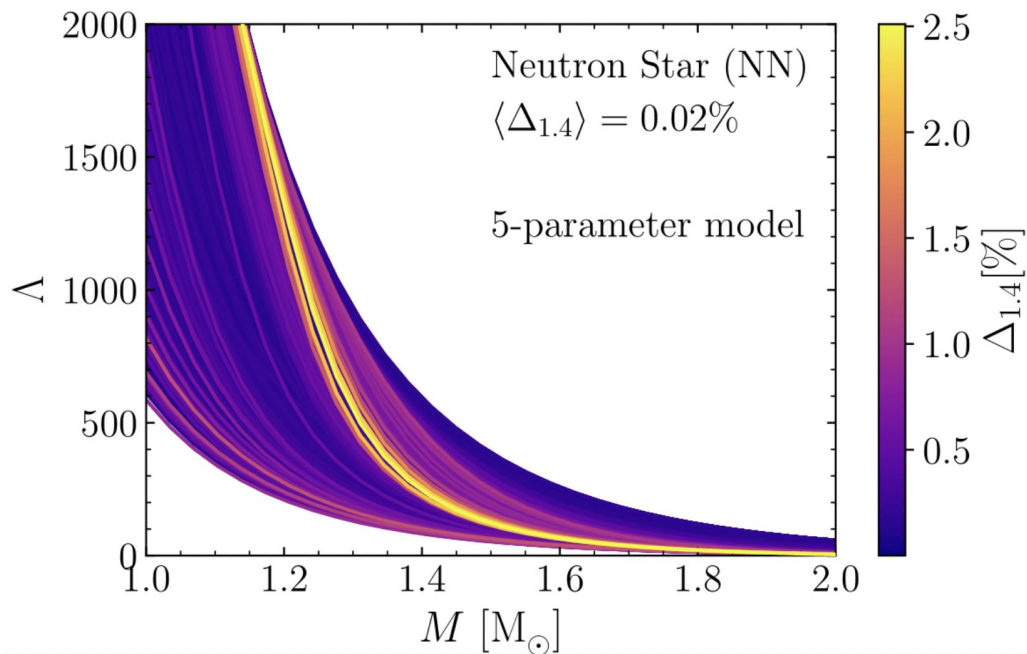
Reed, **RS**, et al., arXiv:2405.20558

Emulators for the TOV equations



- Average uncertainty on test samples is 0.02 %
- 0.01% of test samples are outliers (uncertainty > 2%)

$$\Delta[\%] = \left| \frac{\Lambda_{\text{TOV}} - \Lambda_{\text{NN}}}{\Lambda_{\text{TOV}}} \right| \times 100$$



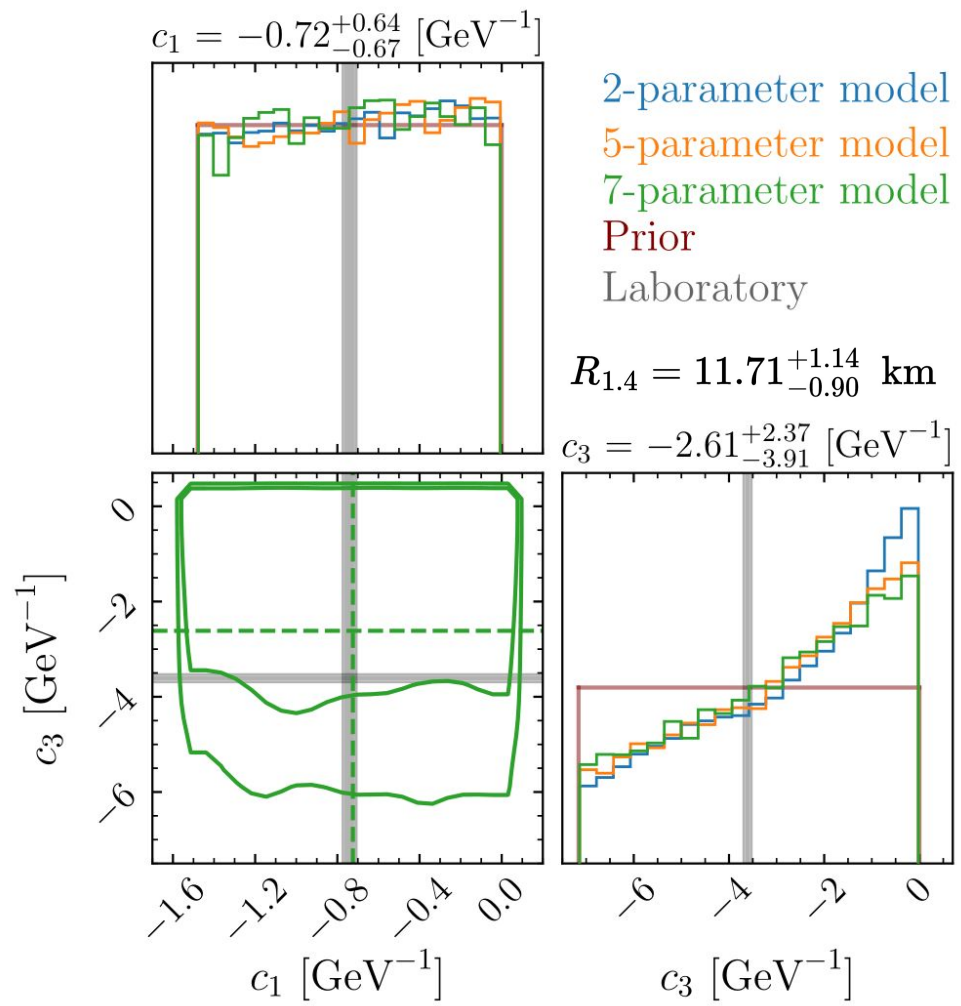
Putting everything together

$$p(\theta|d, H) = \frac{p(d, H|\theta)p(\theta|H)}{p(d|H)}$$

Estimate the posterior using MCMC; Use emulators to evaluate Likelihood

For the data, we use:

- GW170817 + $2 M_{\text{solar}}$ constraint



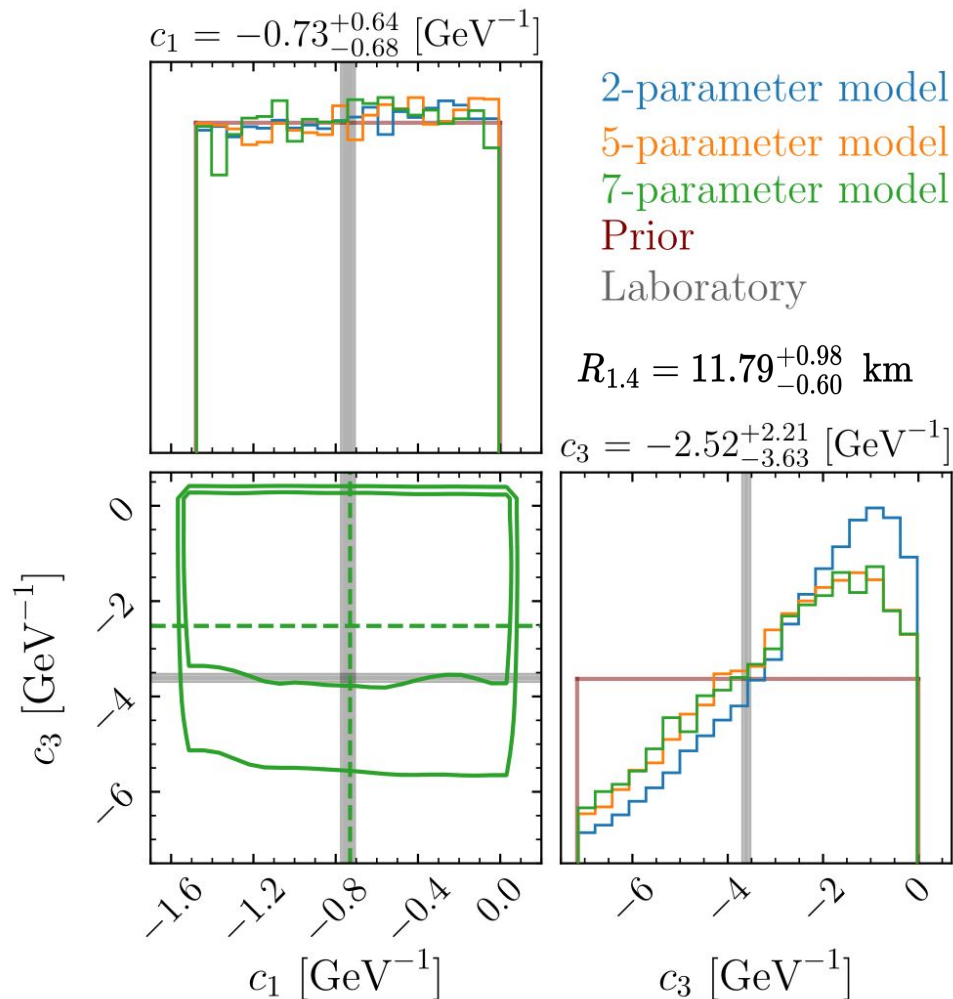
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For the data, we use:

- GW170817 + $2 M_{\text{solar}}$ constraint
- + NICER's J0030 observation
- + NICER's J0740 observation
- + NICER's J0437 observation

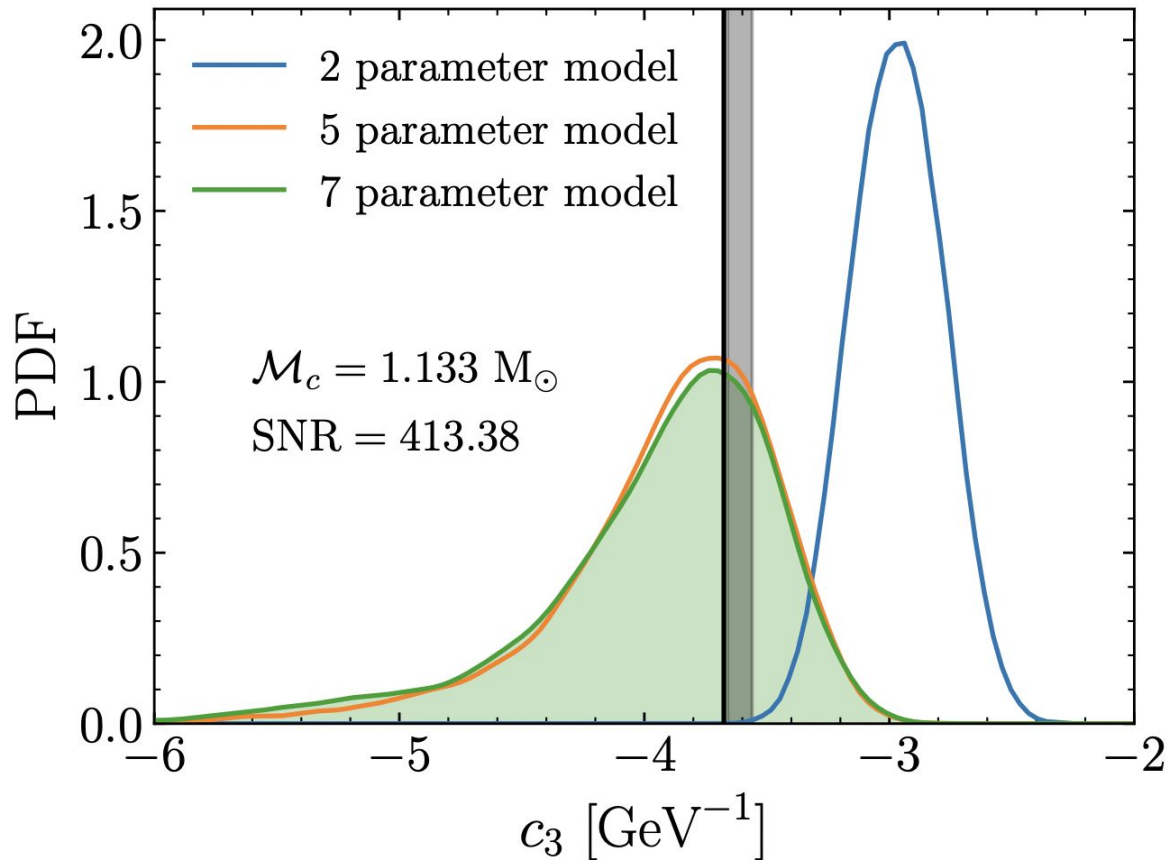


We can do much better with next-generation detectors!

Simulation with CE-40, CE-20, and ET for 1 year's worth of observation

For source population, assume uniform distribution on component masses with random pairing into binaries

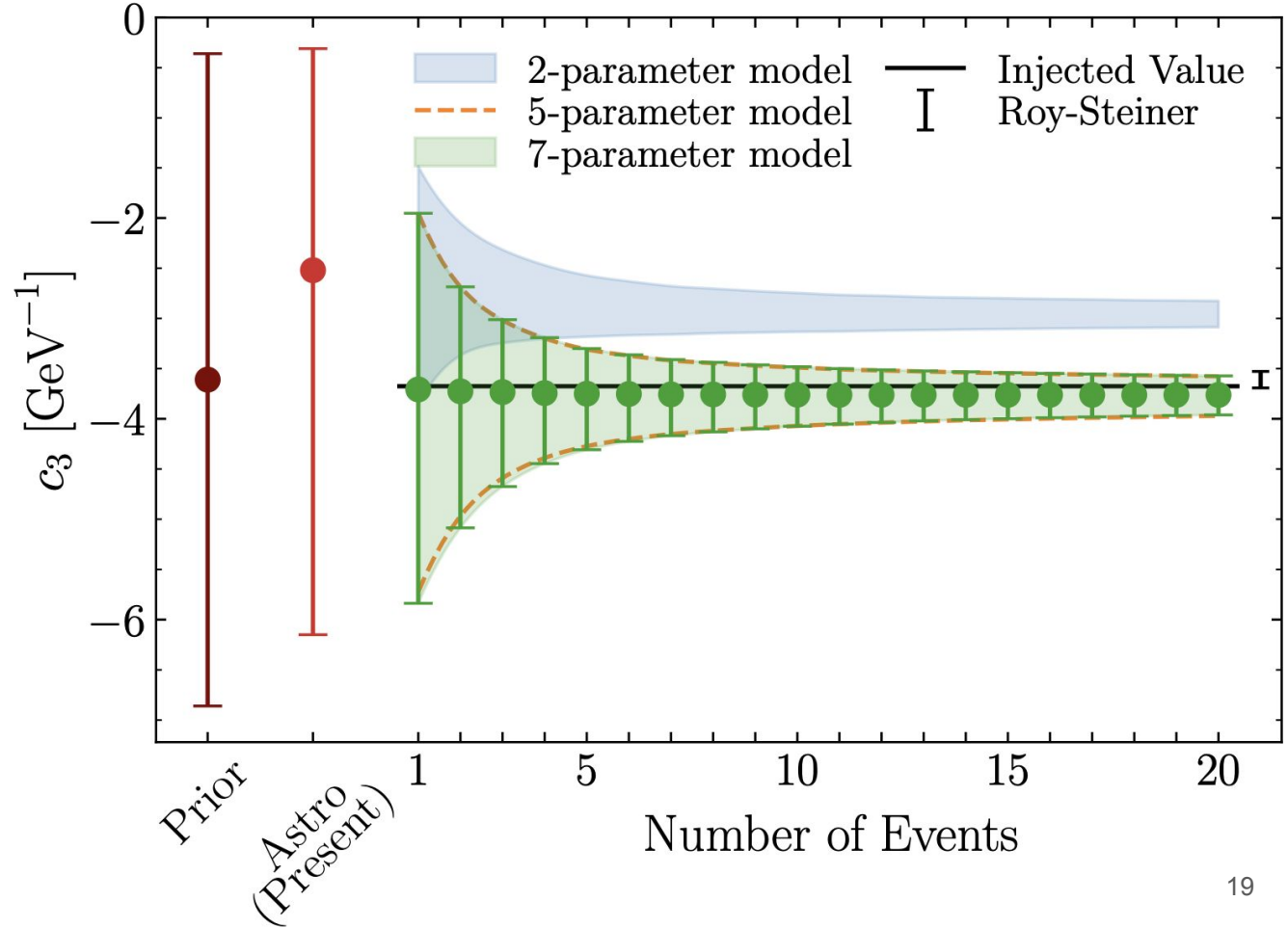
From the generated 427 events, select the 20 loudest events; $\text{SNR} > 225$. Analyse each event within the Fisher Matrix approximation



We can do much better with next-generation detectors!

Within 1 year, next-generation observatories can measure c_3 at a level comparable with laboratory data.

Marginalization over high-density EOS necessary to avoid systematic uncertainties



Conclusion and Outlook

We have demonstrated how to use astrophysical NS observations, especially next-generation observations, to constrain microphysical parameters governing the nuclear interaction

Future work:

- Improved nuclear interactions at higher orders in the EFT expansion
- Explore EFTs with heavier degrees of freedom, such as the Delta resonance
- Going beyond the Fisher Matrix approximation: Implement emulators in GW analysis pipelines such as PyCBC
- Incorporate systematics carefully: Waveform modeling, High density EOS extrapolations, emulator uncertainties, etc.



Thank You!