Universal properties of weakly bound two-neutron halo nuclei

Dam Thanh Son (University of Chicago)
Quantum Few- and Many-Body Systems in Universal Regimes
Institute for Nuclear Theory, October 24, 2024

References

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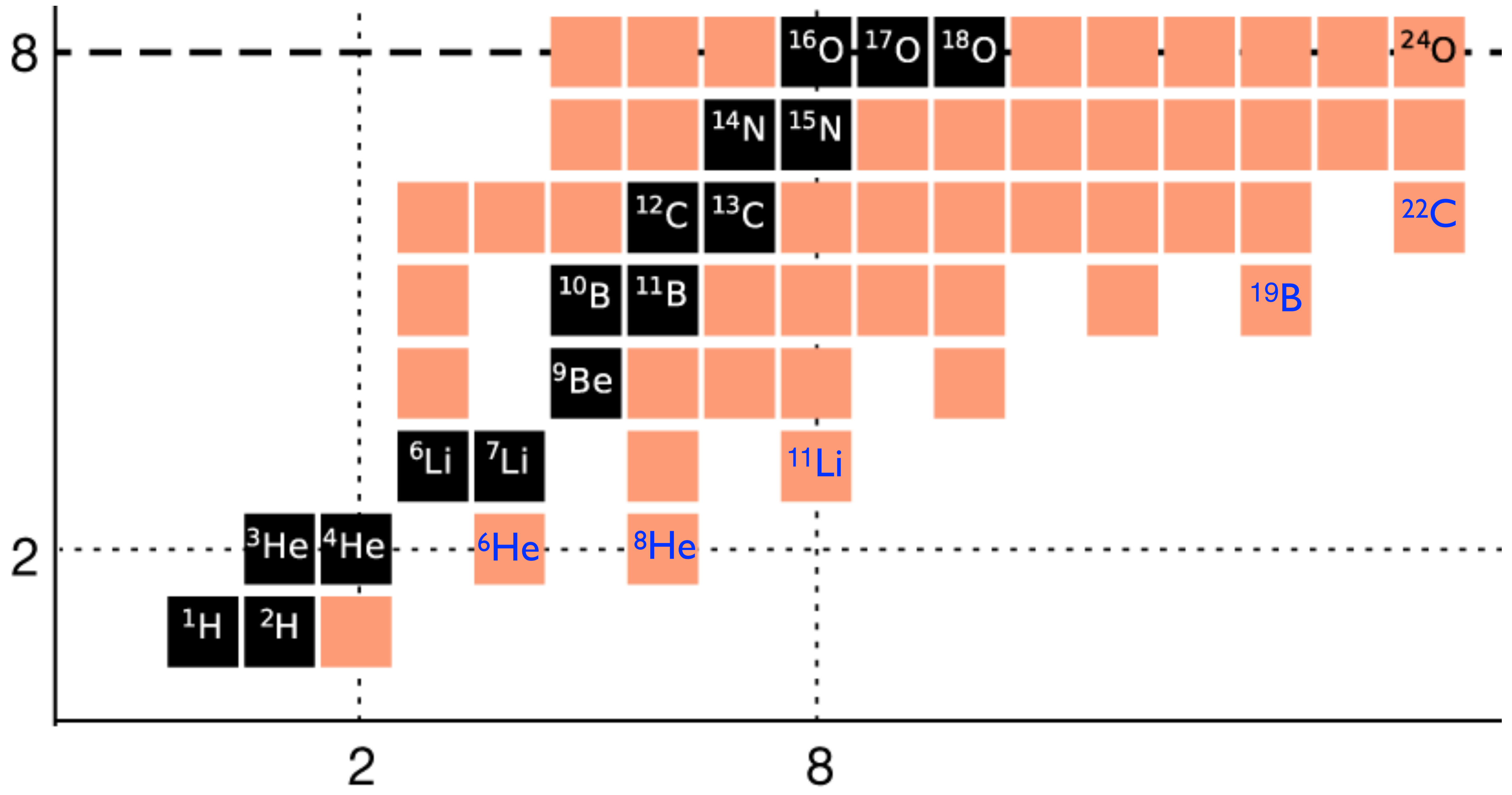
Davi Costa, Masaru Hongo, DTS, to appear



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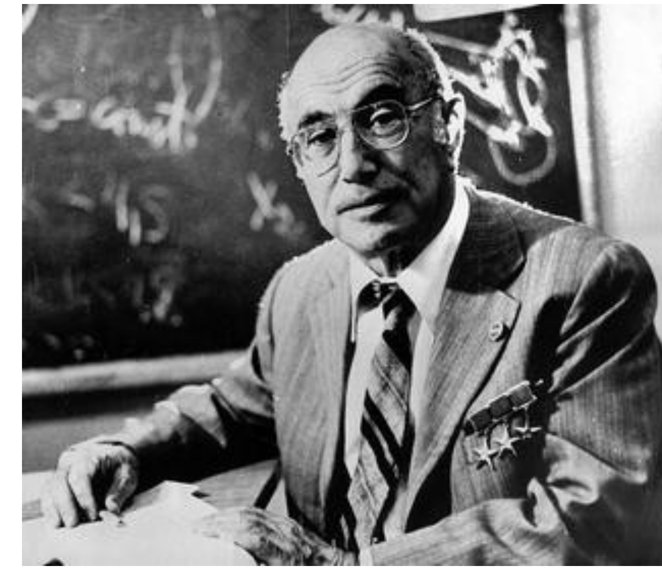

SOVIET PHYSICS JETP VOLUME 11, NUMBER 4 OCTOBER, 1960

THE EXISTENCE OF NEW ISOTOPES OF LIGHT NUCLEI AND THE EQUATION OF STATE OF NEUTRONS

Ya. B. ZEL'DOVICH

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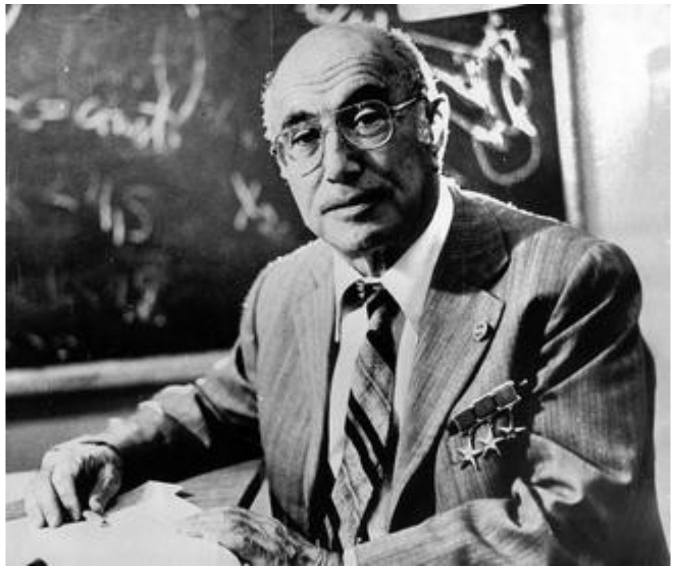

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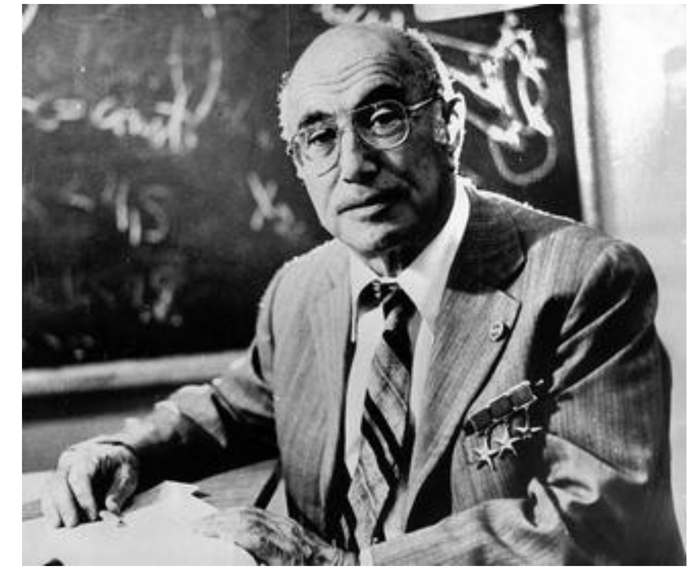

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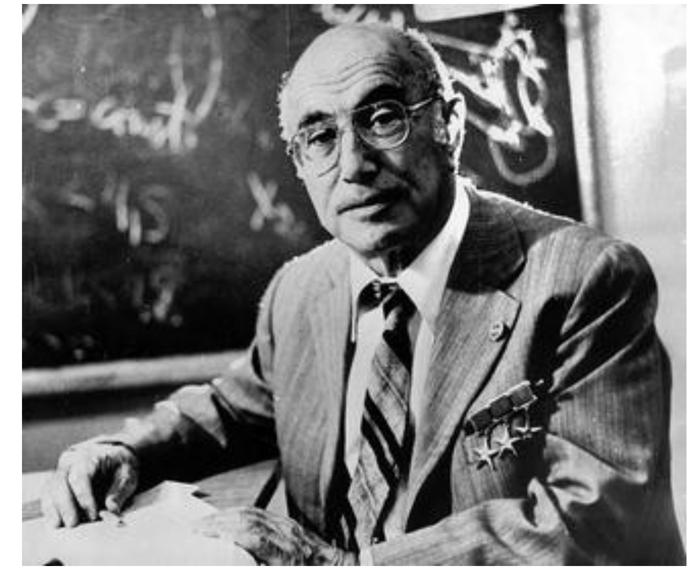

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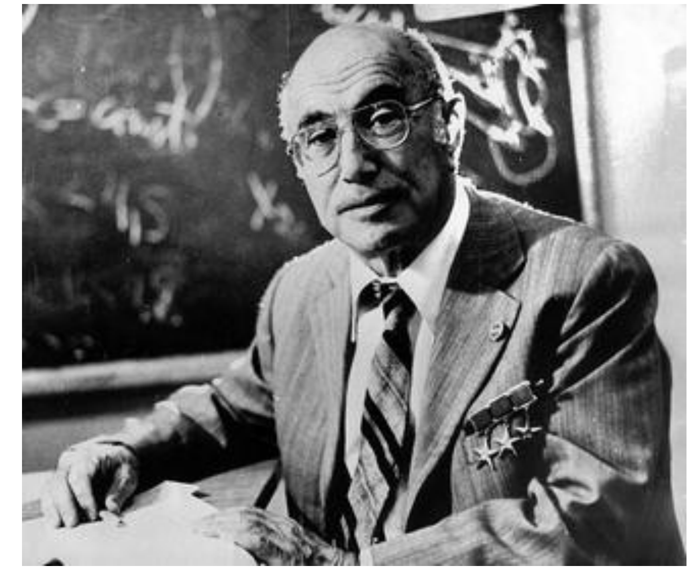

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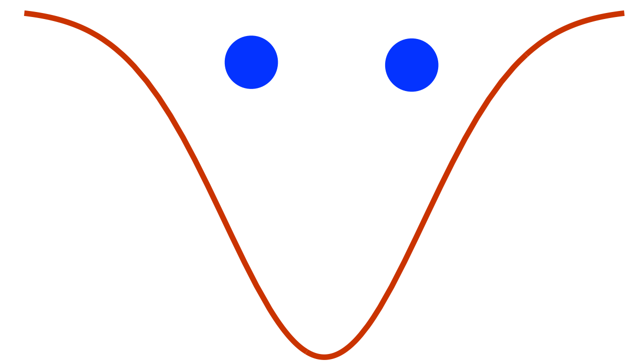
- One attractive model of two-neutron Borromean nuclei: Efimov effect
- large neutron-neutron, core-neutron scattering lengths, modeled by zero-range interaction
- Is Efimov effect necessary?

Core-neutron s-wave resonance needed?



- Two particles with zero-range resonant interaction, in Gaussian potential of an infinitely massive core:

$$H = -\frac{1}{2}(\nabla_1^2 + \nabla_2^2) - V_0(e^{-r_1^2/2} + e^{-r_2^2/2}) - c_0\delta(\vec{r}_1 - \vec{r}_2)$$

- core-neutron scattering length diverges when $V_0 = V_0^{\mathrm{cn}} = 0.671$
- At which V_0^{3body} 3-body bound state first appears? How close this value is to V_0^{cn}?

Variational calculation

- $\psi(\vec{r}_1, \vec{r}_2) = \dfrac{e^{-\alpha(r_1^2+r_2^2)}}{|\vec{r}_1 - \vec{r}_2|}$

  satisfies Bethe-Peierls boundary condition

- variational bound $V_0^{\rm 3body} \le 0.417 < \dfrac{2}{3} V_0^{\rm cn}$

- better variational ansatz: $V_0^{\rm 3body} \le 0.3285 < \dfrac{1}{2} V_0^{\rm cn}$

- 3-body bound state appears long before 2-body one

Two regimes

- When the core-neutron scattering length is large: Efimov effect
- But 3-body bound state can exist without the Efimov effect

strength of core-n attraction

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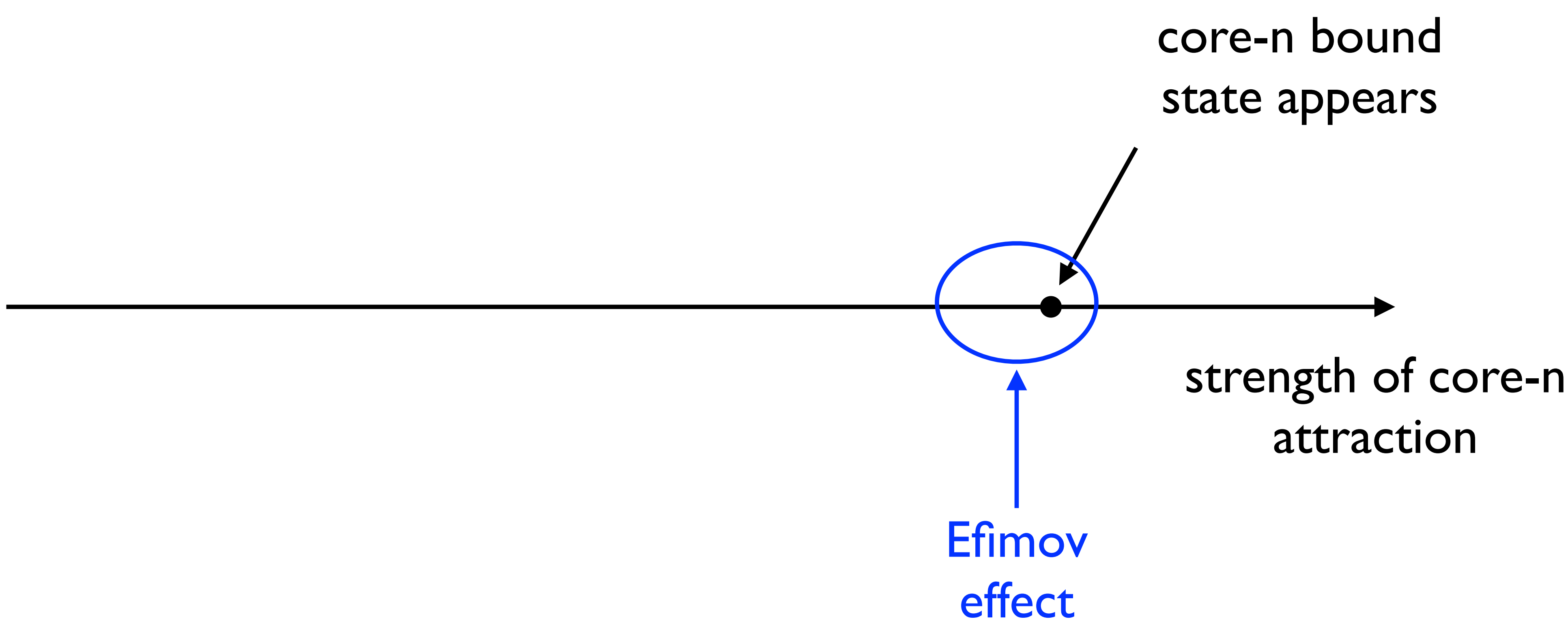

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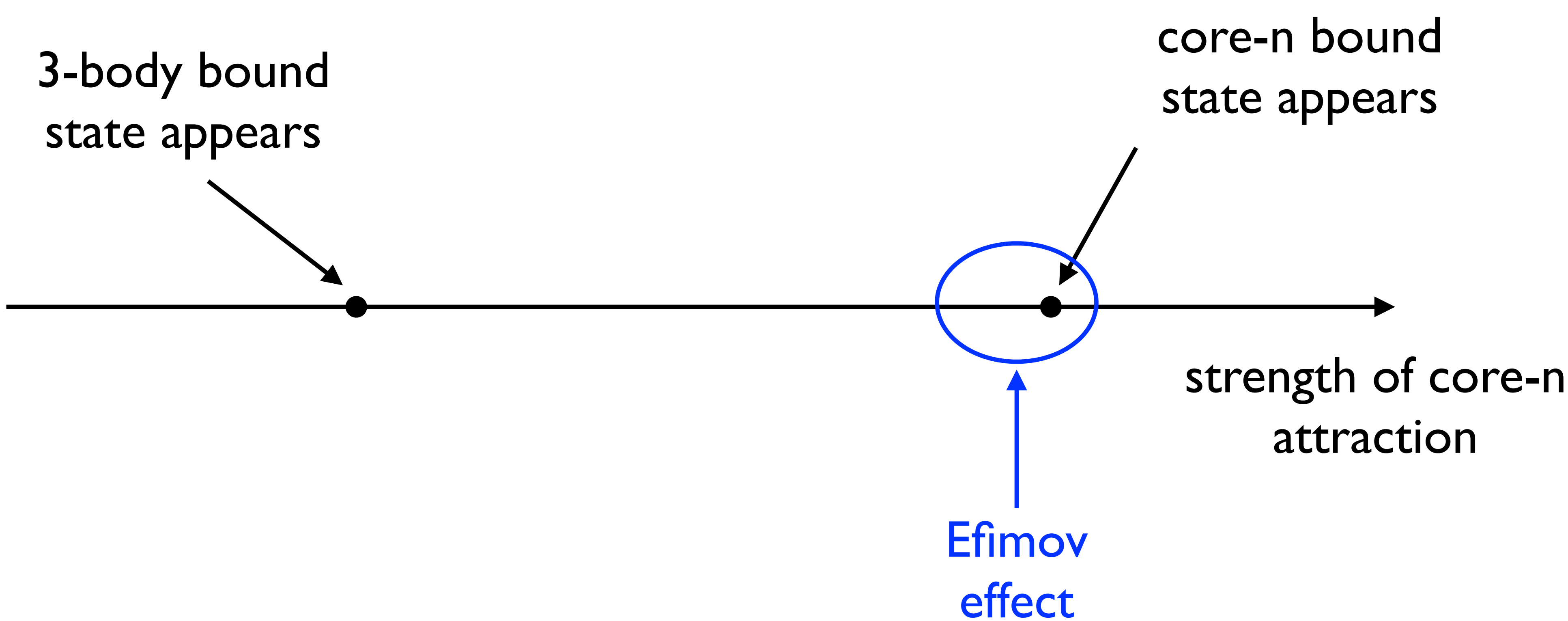

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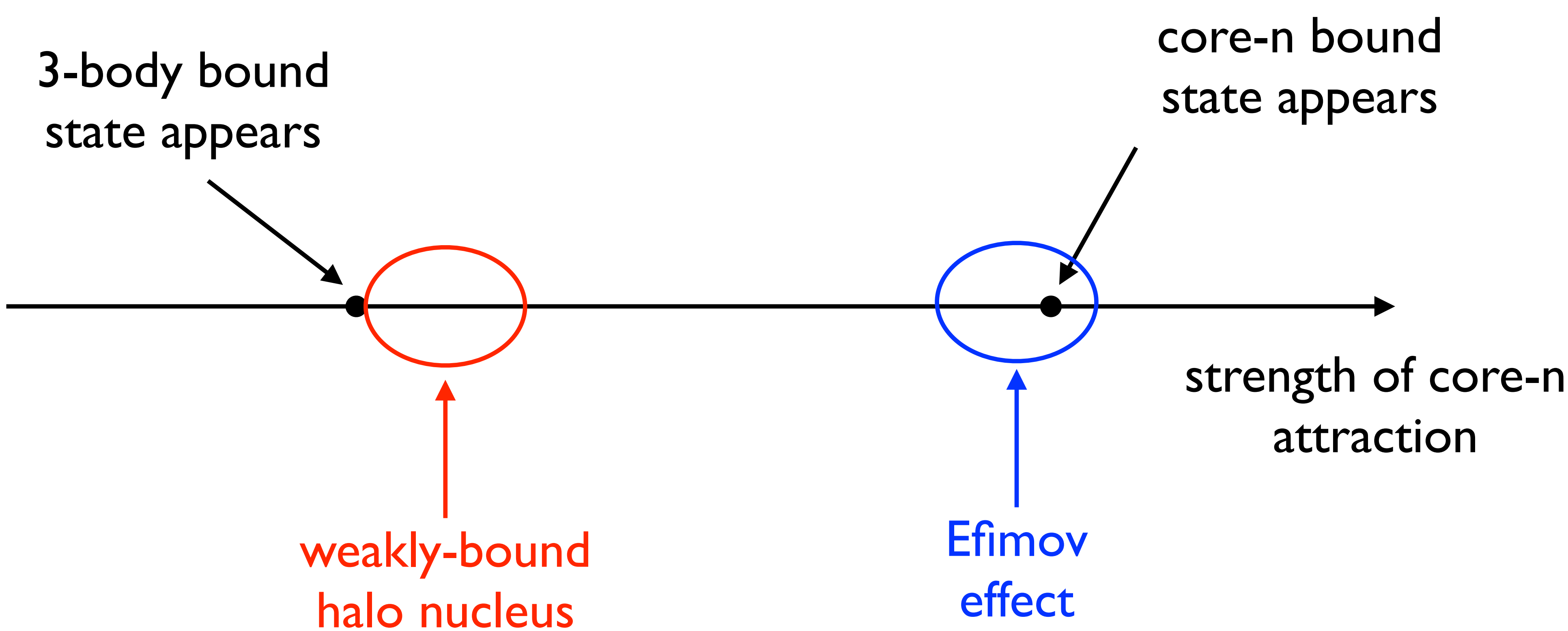

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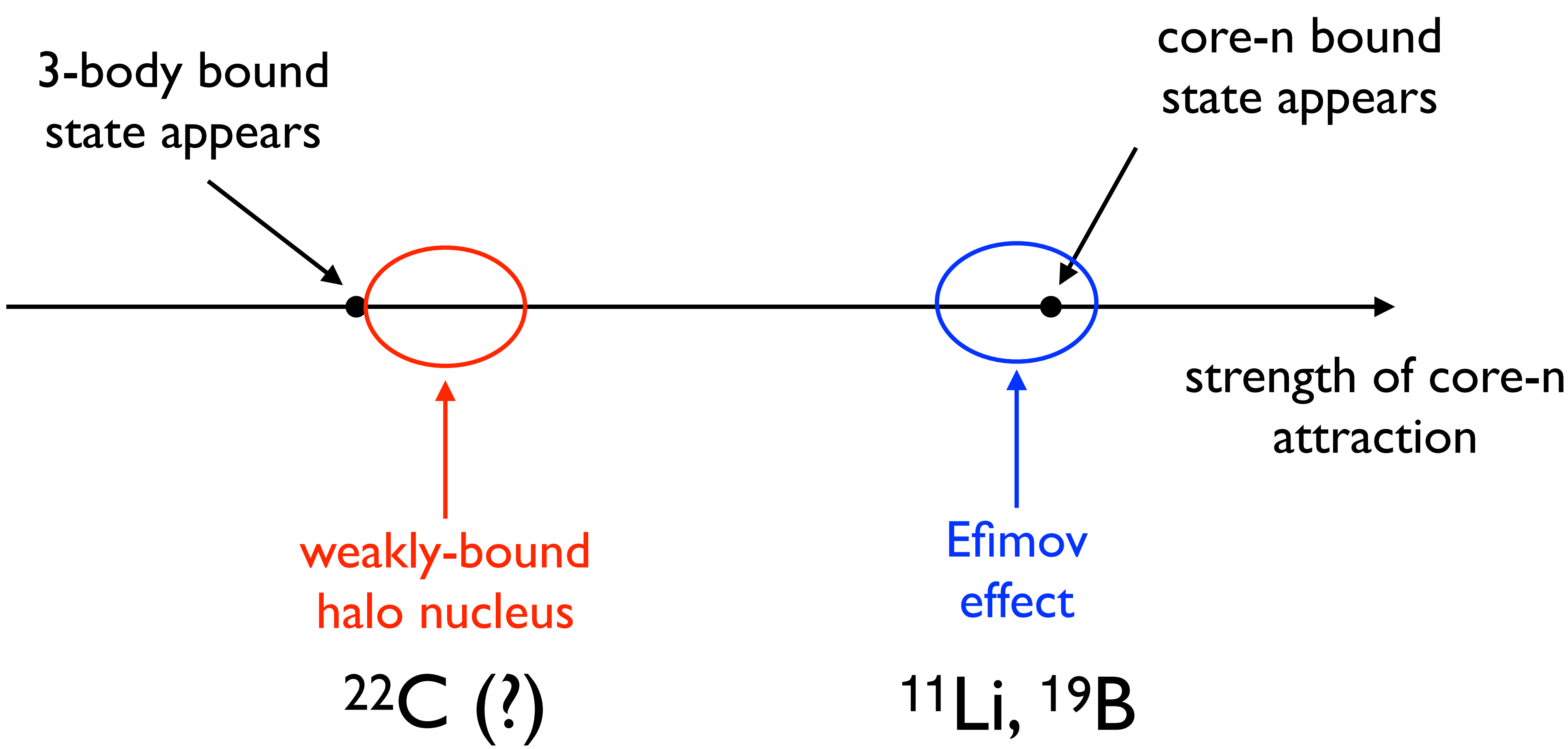


Examples

- ^{22}C has large matter radius Togano et al 2016 → small binding energy
- $|a(n^{20}C)| < 2.8$ fm Mosby et al 2013
- Hypertriton Λpn: total binding energy 2.35 MeV, $a_{pn} \approx 5.4$ fm
- but most estimate for the Λn scattering length is < 3 fm, and typically $|a| < r_{\text{eff}}$

Two fine tunings

- Weakly bound 2-neutron halos with two small energy scales:
- neutron-neutron virtual energy

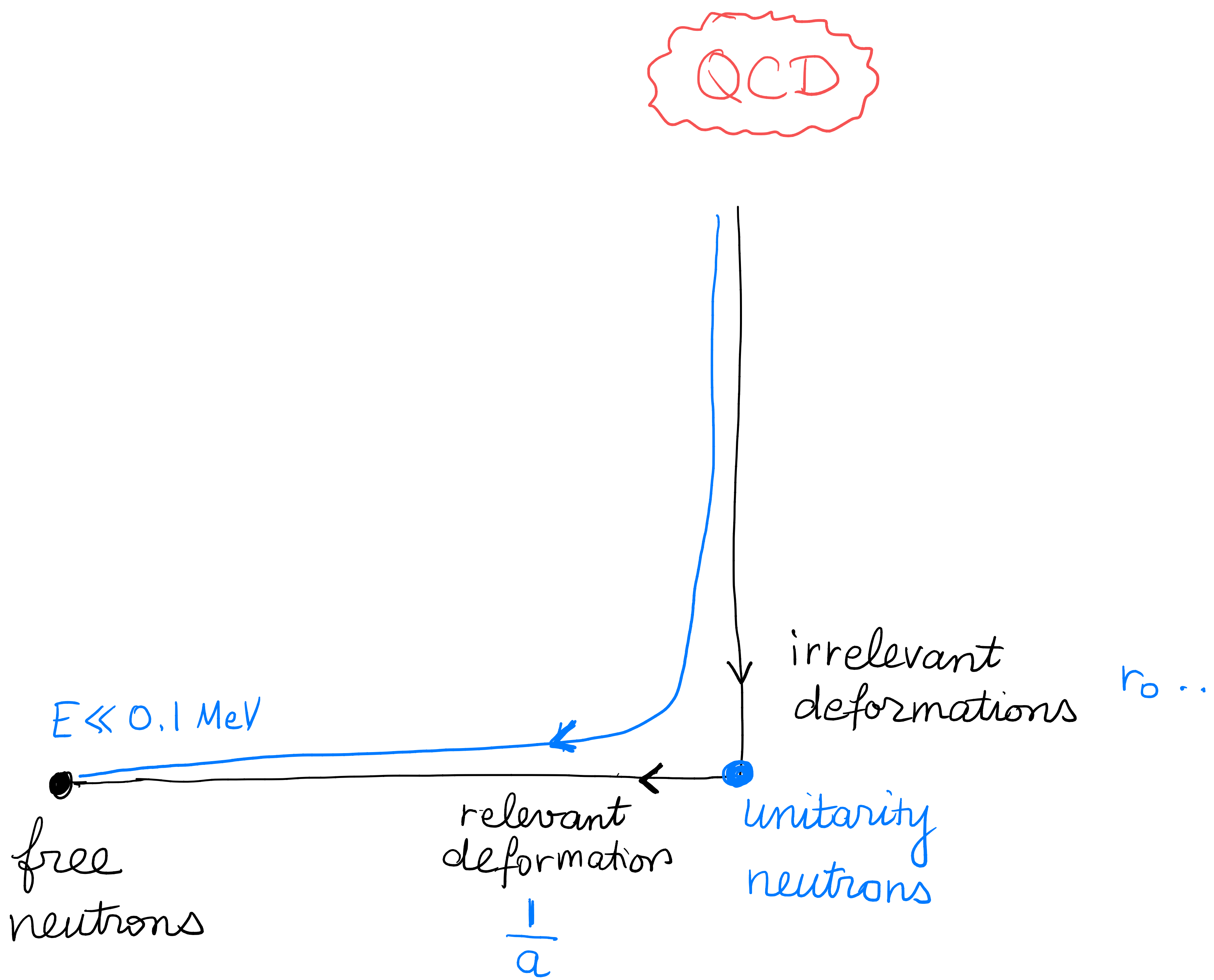
$$a \approx -19 \text{ fm} \qquad \epsilon_n = \frac{\hbar^2}{m_n a^2} \approx 0.12 \text{ MeV}$$

- 2-neutron separation energy

$$B(^{22}\text{C}) \sim 0.1 \text{ MeV}$$

- Appropriate approach: effective field theory (if no other small energy scale)

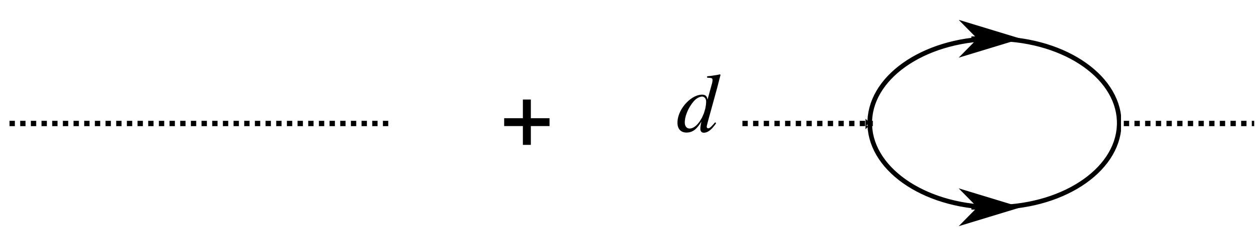
Neutrons and fixed points



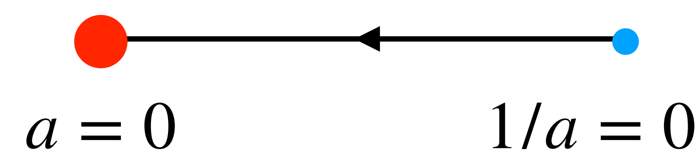
Neutrons sector

- $\mathscr{L}_{\text{neutron}} = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - c_0\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$
- Introducing auxiliary field d ("dimer")
- $\mathscr{L}_{\text{neutron}} = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger d - d^\dagger\psi_\downarrow\psi_\uparrow + \frac{d^\dagger d}{c_0}$
- Fine-tuning c_0

$$G_d(\omega, \mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega - \frac{1}{a}}}$$



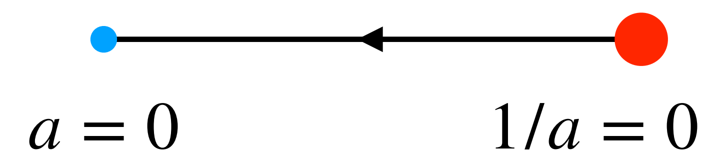
Free fixed point



- $$S = \int dt\, d\mathbf{x}\, \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2} \right) \psi$$
- Nonrelativistic power counting $[x] = -1, \;\; [t] = -2$
- $[\psi] = \dfrac{3}{2}, \;\; [\psi_\uparrow \psi_\downarrow] = 3$
- $[\psi_\downarrow^\dagger \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow] = 6$

$a\, \psi_\downarrow^\dagger \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow$ is an irrelevant deformation:
fixed point is stable

Unitary fixed point



- When $a = \infty$ $\quad [\psi] = \frac{3}{2}$ but

$$\langle d(t,\vec{x})d^\dagger(0,\vec{0})\rangle \sim \frac{1}{t^2}\exp\left(\frac{ix^2}{4t}\right) \qquad [d] = 2$$

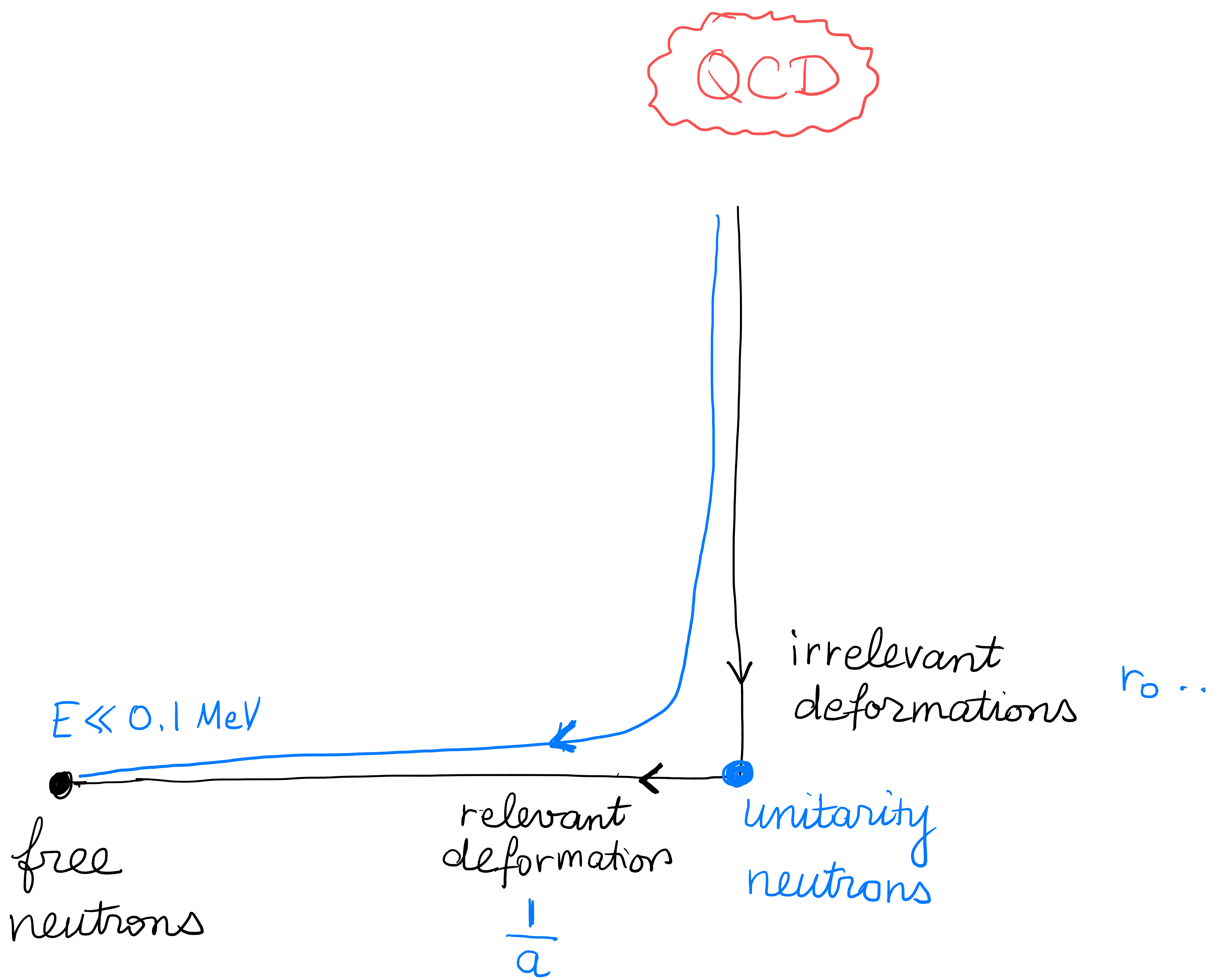
Operator product expansion:

$$\psi_\uparrow(\vec{x})\psi_\downarrow(\vec{0}) = \frac{d(\vec{0})}{|\vec{x}|} + \cdots$$

- $[d^\dagger d] = 4$ so $\frac{1}{a}d^\dagger d$ is a relevant deformation

Fixed point is unstable

Neutrons and fixed points



EFT for weakly-bound halo nuclei: degrees of freedom

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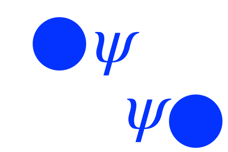

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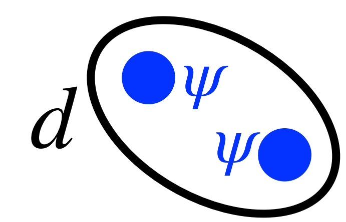

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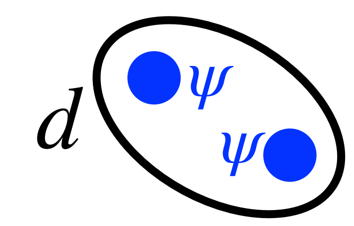

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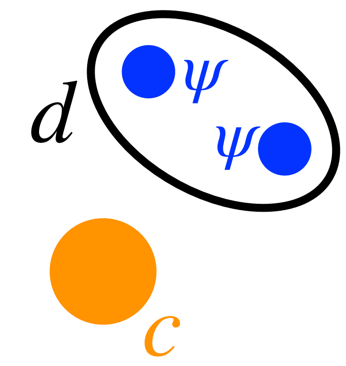

EFT for weakly-bound halo nuclei: degrees of freedom

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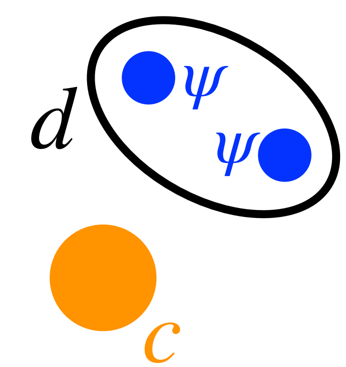
  - halo nucleus h *as independent field* (small binding energy B)



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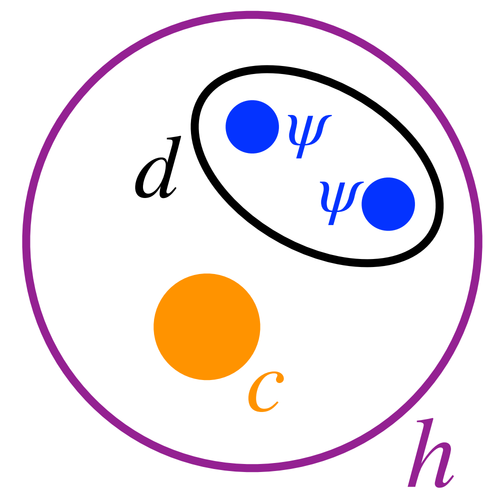

EFT for weakly-bound halo nuclei: degrees of freedom

- neutron ψ, forming dimer d
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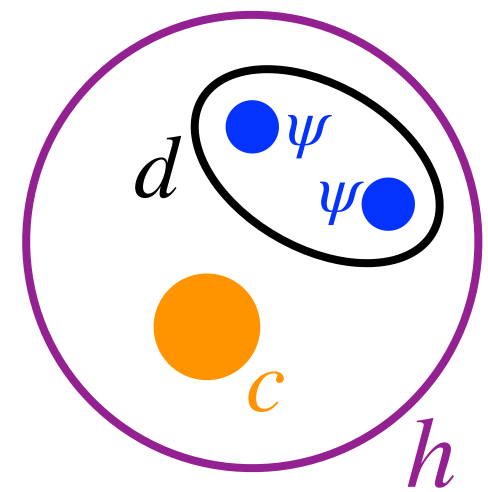
- Interaction: $h^\dagger dc + d^\dagger c^\dagger h$



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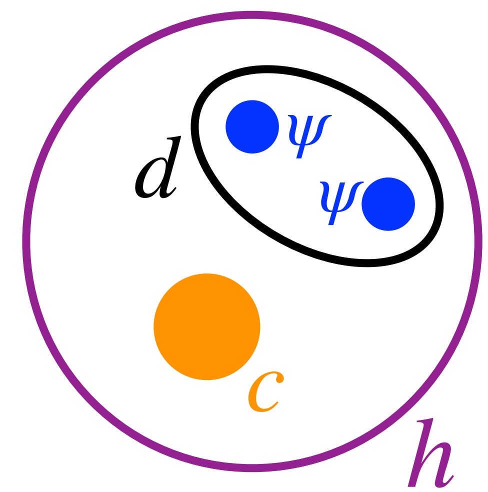
  - dimension: $\frac{3}{2}+\frac{3}{2}+2=5$: marginal

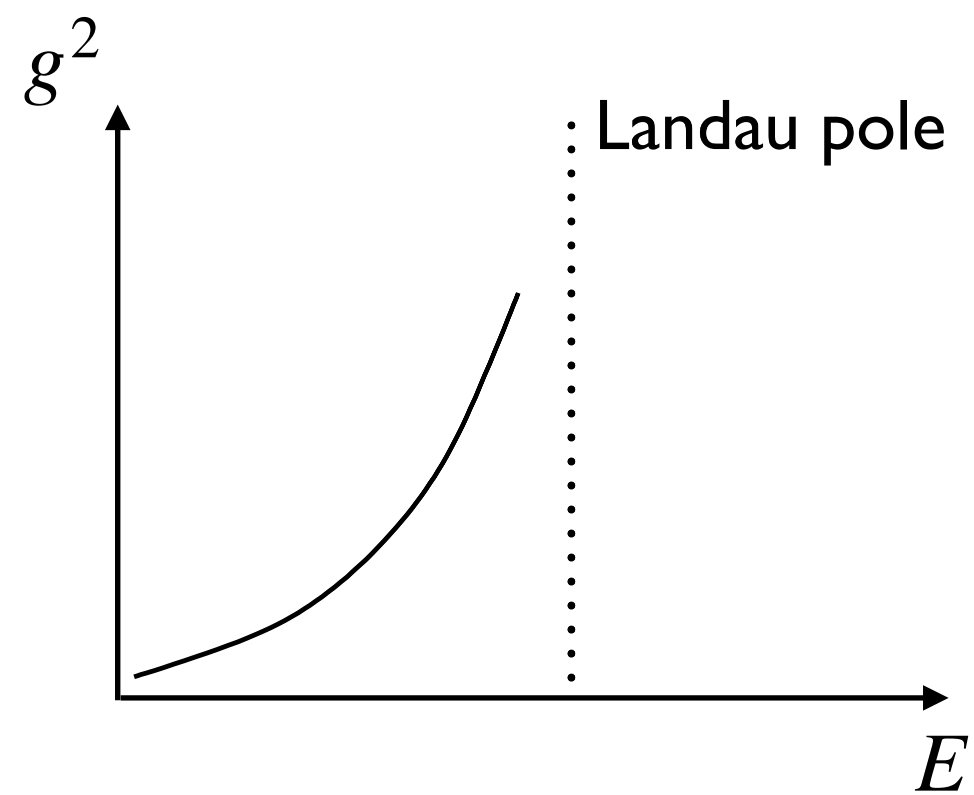


Effective Lagrangian

$$\mathscr{L} = h^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_h} + B\right)h + c^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_\phi}\right)c + g(h^\dagger c d + c^\dagger d^\dagger h)$$

$$+\mathscr{L}_{\text{neutron}}$$

g runs logarithmically



Universality?

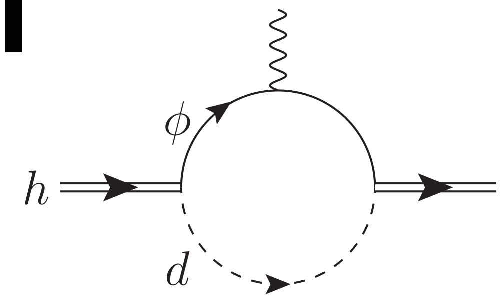
- Is the 3-body system universal?

  Can physical quantities be written as

$$O = B^{\Delta_O} F\left(\frac{B}{\epsilon_n}\right)$$

- Answer: almost, up to the logarithmically running coupling

Charge and matter radii



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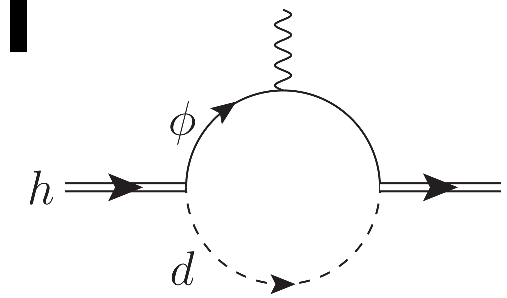


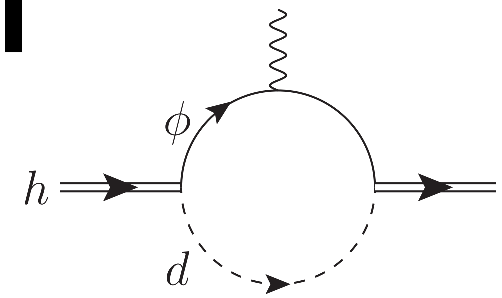
- Charge radius $\langle r_c^2 \rangle = \frac{4}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{g^2}{B} f_c(\beta),$

$$\beta = \sqrt{\frac{\epsilon_n}{B}}$$

$$A = A_{\text{core}}$$

$$f_c(\beta) = \frac{1}{1-\beta^2} - \frac{\beta \arccos \beta}{(1-\beta^2)^{3/2}}$$

Charge and matter radii



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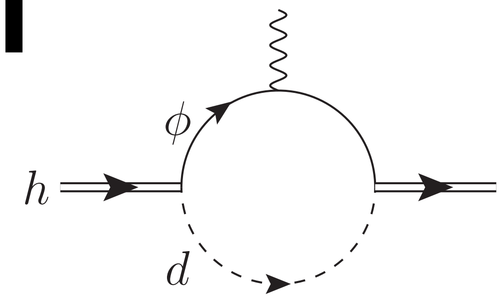
$$f_n(\beta) = \frac{1}{\beta^3} \left[\pi - 2\beta + (\beta^2 - 2) \frac{\arccos \beta}{\sqrt{1-\beta^2}} \right]$$

Charge and matter radii



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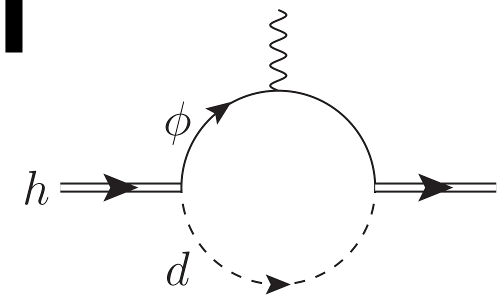
- Each contains g^2, but universal ratio

$$\frac{\langle r_m^2 \rangle}{\langle r_c^2 \rangle} = \frac{A}{2} \left[1 + \frac{f_n(\beta)}{f_c(\beta)} \right] = \begin{cases} \frac{2}{3} A & B \gg \epsilon_n \\ A & B \ll \epsilon_n \end{cases}$$

Charge and matter radii



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Hongo, DTS 2201.09912
Naidon 2302.08716

E1 dipole strength function

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- $$\frac{dB(E1)}{d\omega}(\omega) \sim \sum_n |\langle n|(\mathbf{r}_c - \mathbf{R}_{cm})|0\rangle|^2 \delta(E_n - \omega)$$

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- $\frac{dB(E1)}{d\omega}(\omega) \sim \sum_n |\langle n|(\mathbf{r}_c - \mathbf{R}_{cm})|0\rangle|^2 \delta(E_n - \omega)$
- can be mapped to current-current correlation

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega}(\omega) \sim \frac{1}{\omega^2} \mathrm{Im} \langle JJ \rangle(\omega)$$

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- Feynman diagram ~ inelastic scatterings

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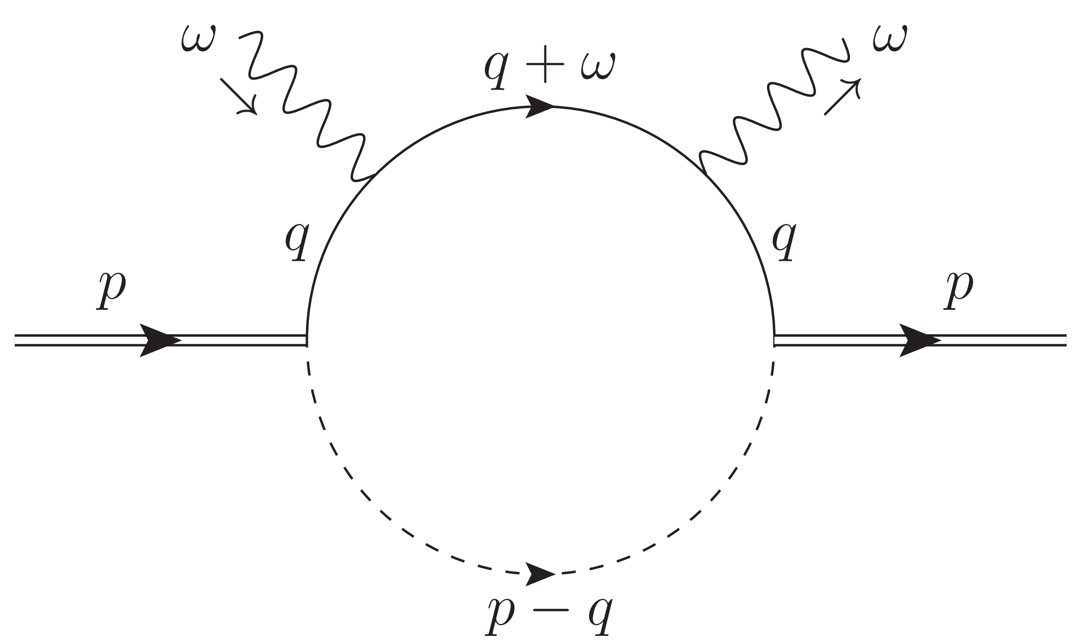
- can be mapped to current-current correlation

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega}(\omega) \sim \frac{1}{\omega^2} \mathrm{Im}\,\langle JJ\rangle(\omega)$$

- Feynman diagram ~ inelastic scatterings



Dipole strength in the unitarity limit

- When neutrons are in the unitarity limit $a = \infty$

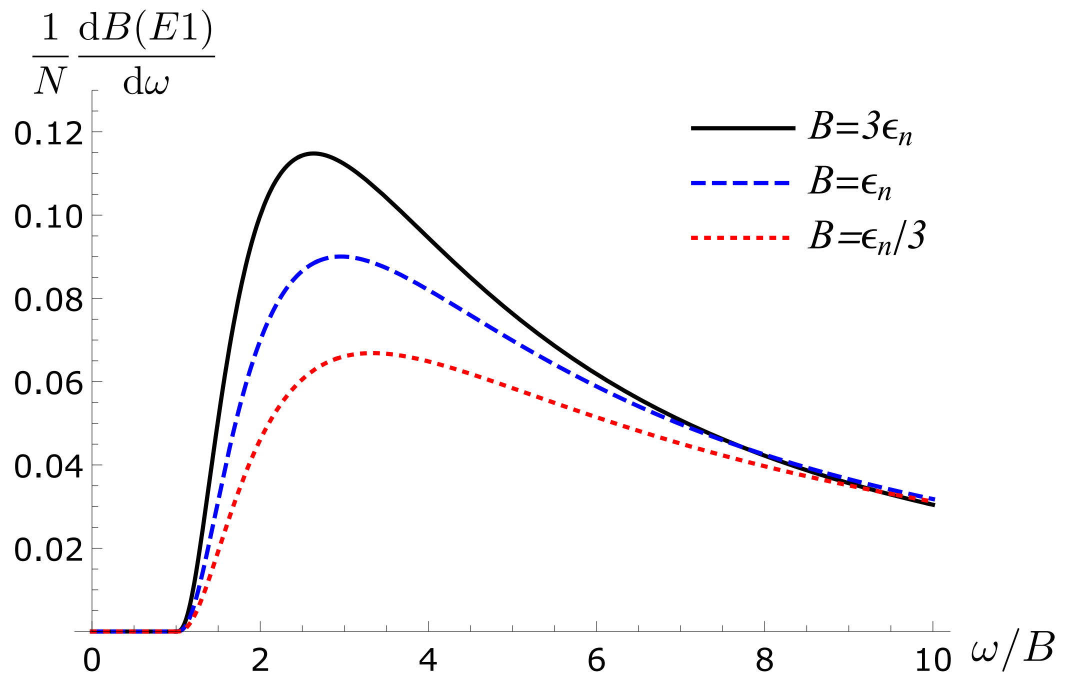
$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} \sim g^2 \frac{(\omega - B)^2}{\omega^4}$$

Result for dipole strength

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} \sim g^2 \frac{(\omega - B)^2}{\omega^4} f_{E1}\left(\sqrt{\frac{\epsilon_{nn}}{\omega - B}}\right)$$

$$f_{E1}(x) = 1 - \frac{8}{3}x(1+x^2)^{3/2} + 4x^2\left(1 + \frac{2}{3}x^2\right)$$

M.Hongo, DTS 2201.09912



consistency check: sum rules

$$\int_0^\infty \mathrm{d}\omega \, \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \langle r_c^2 \rangle,$$

$$\int_0^\infty \mathrm{d}\omega \, \omega \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{3}{A(A+2)},$$

Corrections to EFT

- Corrections to EFT: irrelevant terms EFT
  - Effective range in *n-n* scattering:

    $$r_0 d^\dagger \left(\mathrm{i}\partial_t - \frac{1}{4}\nabla^2\right) d$$ dimension 6

    D.Costa, M.Hongo, DTS to appear
  - *s*-wave core-neutron scattering

    $a_{cn} c^\dagger \psi^\dagger \psi c$ also dimension 6
- Corrections in r_0 and a_{cn} should be computed perturbatively

Core-n resonance?

- A p-wave core-neutron resonance can be treated perturbatively:

  introduce resonance as a free field $\vec{\chi}$

  $$[\vec{\chi}\,\phi\,\overrightarrow{\nabla}\psi] = \frac{11}{2} > 5$$

- He-6 can be treated within EFT?

Conclusion

- Weakly bound two-neutron halo nuclei can be described by an EFT. Renormalizable with 1 log-running coupling
- Ratios of radii and shape of E1 dipole function are universal, analytically computable
- Corrections: perturbative in *nn* effective range, core-neutron scattering length
- Applications beside ^{22}C? ^{6}He? Cold atom realization?
- Unstable systems