# **Hunting for Humming Continuous Wave Signals —— Neutron Stars, Dark Matter, New Physics**



### Australian **National** University





Image: OzGrav | Carl Knox

Dr Ling Sun, OzGrav-ANU INT Workshop 2024, Seattle WA



# **Gravitational wave detections**





ICRR, Univ. of Tokyo/LIGO Lab/Caltech/MIT/Virgo Collaboration



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### LONG DURATION <u>. . . . . .</u>



@astronerdika

UNMODEL

 $\subset$   $\circ$ 

**NODELI** 

Image: Shanika Galaudage / LIGO-Virgo-KAGRA collaboration3





# Continuous Waves from Neutron Stars

See recent reviews: Piccinni, Galaxies 10(3) (2022) Riles, Living Reviews in Relativity 26, 3 (2023) Wette, Astroparticle Physics 153, 102880 (2023) Image: OzGrav | Carl Knox



- *• Interior structure of neutron star*
- *• Neutron star properties, e.g., mass, spin, ellipticity*
- *• Nuclear equation of state*
- *• May discover exotic states of matter*
- *• Multi-messenger studies, e.g., mass and magnetic field structure inferred from GW/EM relative phase*
- *• Testing General Relativity*
- *• … and so on*



NASA's Goddard Space Flight Center / Conceptual Image Lab





- 
- *• Signal is weak but persistent*



$$
f_{\rm GW}=2f_{\rm rotation}
$$

$$
h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} f_{\text{GW}}^2}{D} \epsilon
$$

• Non-axisymmetric deformation due to elastic stresses or magnetic field -- tiny "mountains"

$$
\left(\frac{\epsilon}{10^{-5}}\right) \left(\frac{I_{zz}}{10^{45} \text{ g cm}^2}\right) \left(\frac{f_{\text{GW}}}{100 \text{ Hz}}\right)^2 \left(\frac{1 \text{ kpc}}{D}\right)
$$

*cf. GW150914 peak h0~10-21*

 $h_0 \approx 10^{-25}$ 

$$
\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}} \leftarrow \text{ellipticity}
$$



*• Free precession around the rotation axis*



Credit: M. Kramer

 $f_{\rm GW} \sim f_{\rm rotation} + f_{\rm precession}$  $f_{\rm GW} \sim 2 f_{\rm rotation} + 2 f_{\rm precession}$ 

### **Continuous waves from neutron stars**



*• r-modes —— long-lasting oscillations in the fluid that makes up most of the star —— a fluid wave travelling around the star and driven by the Coriolis force due to rotation (see Rossby waves)*

Credit: C. Hanna and B. Owen



$$
f_{\text{GW}} \sim 4 f_{\text{rotation}} / 3
$$
  
\n  
\n
$$
h_0 \simeq 3.6 \times 10^{-26} \left(\frac{\alpha}{10^{-3}}\right) \left(\frac{f_{\text{GW}}}{100 \text{ Hz}}\right)^3 \left(\frac{1 \text{ kpc}}{D}\right)
$$



### *• Deformation due to matter accretion in the binary system*



- *• Accretion is a natural method of powering GW emission*
- *• Torque-balance theory —— accretion spins the star up; GW emission slows it down*
- *• Signal frequency might be wandering slightly due to accretion*



Credit: Mark Myers, OzGrav-Swinburne

### **Continuous waves from neutron stars**

 $f_{\rm GW} \sim 2 f_{\rm rotation}$ 

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# **Continuous waves signal models**

<sup>10</sup> Wette, Astroparticle Physics 153, 102880 (2023)



- *• Intrinsic parameters, e.g., spin frequency, time derivatives of the frequency*
- *• Extrinsic parameters, e.g., sky location, orientation, orbital motion (if applicable), etc.*



*• Secular spin down of the neutron star* 

 $f_{\rm gw} \propto f_{\rm gw}^n$ 



- *• EM (n~3)*
- *• GW (n=5)*
- *• r-mode (n=7)*
- *• Signal frequency is modulated by the motion of the detector*
- *• Signal amplitude is modulated by the antenna pattern*

### **Modulation of continuous wave signals**

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*(There is extra modulation if the source is in a binary orbit)*

### **Continuous waves search types**



Sieniawska & Bejger, Universe 2019, 5(11), 217 and the set of the se

### **Continuous waves search methods**



Credit: K. Wette, G2401246

*• Fully coherent searches provide* 

*the best sensitivity but allow for* 

*least flexibility in signal models;* 

- Strain
- *• Semi-coherent methods sacrifice some sensitivity but are more computationally efficient and can allow for more flexible signal models*

*also most expensive*



### **Continuous waves search methods**

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See Riles, Living Reviews in Relativity 26, 3 (2023) for a comprehensive review of search methods

- *• Can increase the coherent time as follow-ups in hierarchical searches to balance sensitivity vs computing cost*
- *• But there are difficulties with different signal model assumptions, e.g., spinwandering effect*



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- *• Trade-off between breadth and depth*
- 



Wette, Astroparticle Physics 153, 102880 (2023)

![](_page_15_Picture_12.jpeg)

# **TARGETED: 236 known pulsars**

Also see O3 narrowband search: O3 HLV, Abbott+ (LVK) ApJ 932, 133 (2022)

![](_page_15_Figure_7.jpeg)

- *• For 23 pulsars, resulting upper limits have surpassed EM measured spin-down limits.*
- *• For 9 pulsars, their spin-down limits have been surpassed for the first time.*
- 

*• For Crab & Vela, our limits are factors of ~100 and ~20 more constraining than the spin-down limits, respectively.* 

![](_page_15_Picture_11.jpeg)

![](_page_15_Picture_4.jpeg)

![](_page_15_Figure_5.jpeg)

![](_page_16_Picture_12.jpeg)

- *• Searched at once and twice the spin frequency 62 Hz*
- *• First time reach below GW spin-down limit for this star by more than a factor of 2 and limit GWs (l=m=2 mode) to account for <14% of the spin-down energy budget.*

![](_page_16_Figure_4.jpeg)

# **TARGETED: Energetic young pulsar PSR J0537-6910**

### **X-ray pulsar, largest spin-down luminosity, frequent and strong glitches; Use a NICER timing ephemeris**

![](_page_16_Figure_6.jpeg)

**[NICER — Neutron star Interior Composition Explorer]**

- *• Inter-glitch braking index suggests that r-mode oscillations may be important to GW emission.*
- *• Search in a narrow band 86—97 Hz*
- *• Allow for the r-mode driven spin-down scenario for a lowermass neutron star with soft EoSs*

O3 HL, Abbott+ (LVK), ApJ 922, 71 (2021)

![](_page_16_Picture_11.jpeg)

### **DIRECTED: Young supernova remnants & Milky Way center**

![](_page_17_Figure_3.jpeg)

[Also see open data searches, e.g., Ming et al., 2024]

*• Young supernova remnants may have larger ellipticity and* 

![](_page_17_Figure_1.jpeg)

![](_page_18_Picture_12.jpeg)

# **DIRECTED: Scorpius X-1**

O3 HL, Abbott+ (LVK), PRD 106, 062002 (2022) O3 HL, Abbott+ (LVK), ApJL 941, L30 (2022)

Re-analyses with corrected orbital ephemeris: Whelan+ ApJ 949 117 (2023) Vargas+Melatos arXiv:2310.19183 (2023)

 $2\times10^{-26}$ 

![](_page_18_Figure_11.jpeg)

- *• Scorpius X-1 is the most X-ray-luminous low-mass X-ray binary*
- *• Several methods have been used, reaching physically interesting regimes*
- *• Better understanding of the spin-wandering effect can facilitate more sensitive searches*

![](_page_18_Picture_4.jpeg)

 $10^{-24}$ 

 $5\times10^{-25}$ 

 $h_0^{\text{eff}}$  upper limit  $2\times10^{-25}$  $10^{-25}$ 

![](_page_19_Picture_5.jpeg)

# **BLIND ALL-SKY: Isolated neutron stars**

![](_page_19_Figure_2.jpeg)

- Need to account for the modulation due to the binary orbit
- *• Binary orbital parameters: orbital period [3, 45] days and projected semimajor axis [2, 40] light-sec*

![](_page_20_Picture_9.jpeg)

### **BLIND ALL-SKY: Neutron stars in binary systems**

![](_page_20_Figure_3.jpeg)

O3a HL, Abbott+ (LVK), PRD 103, 064017 (2021) [Also see open data analyses, e.g., Covas et al., (2022), Covas et al., (2024)]

![](_page_20_Figure_7.jpeg)

# **Challenges in searches and interpretations**

### *• Large parameter space*

- ‣ *The full possible frequency range, and/or sky positions*
- ‣ *Large range of frequency time derivatives*
- ‣ *Breadth vs depth trade-off*

![](_page_21_Figure_7.jpeg)

![](_page_21_Picture_19.jpeg)

### *• Long-duration integration*

‣ *Computationally challenging*

### *• Uncertainty in signal models*

- ‣ *Lower limit of neutron star ellipticity*
- ‣ *Improved theoretical modelling of r-modes*
- **Credit: ESA Credit: ESA Credit: ESA Credit: ESA**
- ‣ *Spin-wandering has been seen in EM observations fluctuation of magnetospheric or superfluid torques; fluctuation of accretion torque in binaries*
- ‣ *Glitches*
- ‣ *Other imperfectness in signal models*
- ‣ *Unknown effects*

### *• Uncertainty in interpretations*

‣ *Uncertainties associated with the source properties, e.g., distance, orientation, age, and signal models*

### *• Need better theoretical understanding, e.g.*

# Probe the Dark Sector with GW Detectors

![](_page_22_Picture_1.jpeg)

![](_page_22_Picture_2.jpeg)

### **Probes of dark matter with GW detectors**

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![](_page_23_Figure_1.jpeg)

### **Astrophysical probes via GW observations**

![](_page_24_Picture_16.jpeg)

![](_page_24_Picture_17.jpeg)

![](_page_24_Figure_8.jpeg)

Goddard Space Flight Center/NASA

![](_page_24_Picture_10.jpeg)

- *• If neutron stars were to contain dark matter, there would be imprints in the star's tidal deformability, which may be accessible to GW observation.*
- *• Compact objects can form with an astrophysical or dark matter origin. Dissipative dark matter can allow new formation channels for compact objects.*
- *• Primordial black holes are also dark matter candidates. Sub*solar-mass black hole inspirals are continuous GW sources.
- *• Astrophysical probes of ultralight boson condensates around black holes become possible via GW observation, by only assuming a coupling through gravity.*

![](_page_24_Picture_14.jpeg)

Superradiance: Brito, Cardoso, Pani

# **Searches and constraints on primordial black holes**

![](_page_25_Picture_5.jpeg)

- 
- Quasi-continuous-wave search methods are being developed and improved to carry out more *sensitive searches to primordial black hole inspirals.* e.g., [Horowitz+ 2020, Miller+ 2021, Alestas+ 2024, Andres-Carcasona+ 2024, Velcani 2024]

![](_page_25_Figure_1.jpeg)

# **Ultralight bosons**

![](_page_26_Picture_9.jpeg)

![](_page_26_Picture_12.jpeg)

- *• Alternative beyond Standard Model theoretical frameworks*  predict the existence of new ultralight boson particles, including *scalar (spin 0), vector (spin 1), and tensor (spin 2) fields.*
- *• QCD axion (well motivated to solve the strong CP problem), string axion, dark photon, etc. —— They are also dark matter candidates.*
- *• Their model-dependent weak couplings to the Standard Model (if at all) and the vanishingly small mass make them extremely difficult to detect by conventional lab experiments.*
- Now we can appeal to the new experimental field and use GW detectors to search *for them, taking advantage of their universal character of gravitational couplings.*

Artist's impression of an axion. Image: Science Photo Library / Andrade, Ramon / 3dciencia

# **Superradiant instability**

![](_page_27_Figure_1.jpeg)

![](_page_27_Picture_7.jpeg)

$$
\equiv 2\pi \lambda_{\mu} \equiv h/(m_b c) \qquad r_g \equiv GM/c^2
$$
  
 Boson mass

![](_page_27_Picture_6.jpeg)

[Arvanitaki+ (2010), Arvanitaki & Dubovsky (2011)]

### **Continuous waves from boson clouds**

![](_page_28_Figure_1.jpeg)

![](_page_28_Picture_14.jpeg)

—— Probe a parameter space that is inaccessible by conventional *lab particle physics experiments; may detect new particles* 

*• A massive vector boson cloud oscillates around a black hole and produces gravitational waves* 

Credit: W. East (<https://www2.perimeterinstitute.ca/personal/weast/research.html>)

![](_page_28_Picture_9.jpeg)

![](_page_28_Picture_10.jpeg)

### *• What can we learn?*

![](_page_28_Picture_4.jpeg)

See e.g., [Arvanitaki+ (2010), Yoshino & Kodama (2014), Arvanitaki+ (2015), Arvanitaki+ (2017), Brito+ (2017), Baryakhtar+ (2017), East (2018)]

![](_page_28_Picture_12.jpeg)

![](_page_28_Picture_13.jpeg)

# **Searches for individual galactic sources (examples)**

![](_page_29_Picture_17.jpeg)

### *• Searches for boson clouds around unknown BHs*

- ‣ *An all-sky search tailored for scalars in O3 data* [Abbott+ PRD 105, 102001 (2022)]  *semiquantitative constraints on the possible presence of emitting boson clouds in our Galaxy, e.g., systems*  ${younger}$  *than*  $\sim 10^3$  ${yrs}$  *are disfavored in the whole Galaxy for*  ${boson}$  ${\rm masses}\sim [2.5,\,10]\times 10^{-13}\, {\rm eV}$  for a maximum BH mass of  $50M_\odot$  and  $\sim [1.2,\,10]\times 10^{-13}\, {\rm eV}$ for a maximum BH mass of  $100M_\odot$  (using Kroupa mass distribution with PDF  $\propto m^{-2.3}$ )
- ‣ *Constraints derived from all-sky/Milky Way center continuous wave searches, e.g.,*  [Dergachev+ (2019), Palomba+ (2019), Abbott+ PRD 106, 042003 (2022)]

**Unknown black holes or black holes with unknown history are not ideal in order to obtain robust constraints Remnant black holes in GW merger events are better targets!** [Arvanitaki+ (2017)]

![](_page_29_Picture_13.jpeg)

 $\triangleright$  **E.g., boson masses** [6.4, 8.0] × 10<sup>-13</sup> eV are disfavored assuming a BH age *of 5* ×  $10^6$  yrs and [6.3, 13.2] ×  $10^{-13}\,\mathrm{eV}$  assuming a BH age of  $10^5$  yrs

![](_page_29_Picture_16.jpeg)

### *• A dedicated search for scalars targeting Cygnus X-1 in O2*

[Sun+ PRD 101, 063020 (2020), PRD 102, 089902 (2020), Collaviti+ (2024)]

![](_page_29_Picture_4.jpeg)

![](_page_30_Picture_4.jpeg)

# **CBC remnant black holes — Horizon distance (scalar)**

![](_page_30_Figure_1.jpeg)

### 1*.*0  $\cdot\,10^4$ 27 °10 *<sup>f</sup> <sup>&</sup>gt;* <sup>10</sup>°<sup>8</sup> Hz/s 0*.*8  $\cdot$   $10^2$  $\begin{picture}(180,170)(-170,-170) \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}} \put(100,170){\line(1,0){150}}$ 13 11 0*.*6 °° $-10^{0}$ 10 10  $\begin{array}{c} \text{10--2} \\ \text{10--2} \\ \text{11--2} \\ \end{array}$ .<br>.  $\bm{\times}$ 0*.*4 10°<sup>2</sup>  $\overline{\mathbb{X}}$ °**10** 0*.*2  $-10^{-4}$  $J \cdot \angle$ **Promising with future detectors**  $\frac{1}{10^{-6}}$  $0.0^{+}$  $10^0$   $10^1$   $10^2$   $10^3$   $10^4$  $M_i$  $M_i = 60 M_{\odot}$   $\chi_i = 0.70$ **[GW150914] can reach ~160 Mpc**

![](_page_30_Figure_3.jpeg)

![](_page_31_Picture_12.jpeg)

Jones+ PRD 108, 064001 (2023) Sensitivity estimates based on latest numerical relativity studies [2 aLIGO, 1 CE, 1ET, observation duration depends on emission timescale  $\tau_{\rm GW}$ ]

![](_page_31_Figure_10.jpeg)

Horizon Distance  $[obj]$ 

# **CBC remnant black holes — Horizon distance (vector)**

- *• Vector signals have much stronger radiation power but last much shorter (hours—months)*
- *• Vector clouds around CBC remnants can potentially be reached by current-generation detectors*
- *• Plots are showing an optimally matching scenario —— max GW strain when the cloud is saturated*
- *• Can probe a small range of boson masses for each given BH target; sensitive to non-optimally matching cases —— signals are slightly weaker but last longer*

![](_page_31_Figure_5.jpeg)

### **Searches via Direct Interactions**

![](_page_32_Picture_9.jpeg)

![](_page_32_Picture_10.jpeg)

*• GW detectors are extremely sensitive to displacements — can be used as direct dark matter detectors through the field's weak coupling to normal matter.*

![](_page_32_Figure_2.jpeg)

![](_page_32_Picture_4.jpeg)

Caltech/MIT/LIGO Lab

O3LHV, Abbott+ (LVK), PRD 105, 063030 (2022) PRD 109, 089902 (2024)

- *• Dark photons may directly couple to the baryons in the test masses and cause an oscillatory force on the detector*
- *• Constraints on the coupling strength of dark photons to baryons in the mirrors using two methods*

# **Searches via Direct Interactions**

![](_page_33_Figure_1.jpeg)

![](_page_33_Picture_11.jpeg)

*• KAGRA mirrors are made from different materials (sapphire test masses and fused silica auxiliary mirrors), enhancing a potential vector DM signal*

*• Projected sensitivity for 1-year observation —— KAGRA is more sensitive than LIGO/Virgo in low mass range (< 10Hz) by using auxiliary length channels for the B-L coupling* 

![](_page_33_Picture_10.jpeg)

Michimura+ PRD 102, 102001 (2020)

*• Constraints are less stringent noise level and measurement time) —— demonstrating the* 

*than those derived from previous experiments (limited by current applicability of the method to the lower-mass vector DM search*

```
m_A[eV/c^2]
```
![](_page_33_Figure_7.jpeg)

![](_page_33_Figure_4.jpeg)

# **Challenges in searches and interpretations**

![](_page_34_Picture_20.jpeg)

### *• Uncertainty in signal models*

- ‣ *Signal morphology highly depends on theories*
- ‣ *Minor features not taken into account in simplified signal models*
- ‣ *Uncertainties and approximations in theoretical and numerical studies*
- ‣ *Not well-understood effects*

### *• Uncertainty in interpretations*

- ‣ *Uncertainties associated with the source properties, e.g., black hole age and spin, PBH systems*
- ‣ *Uncertainties of the property distribution in the population (more relevant for blind search and stochastic background studies)*

![](_page_34_Picture_7.jpeg)

### *• Large parameter space*

- ‣ *The full possible frequency range, and/or sky positions*
- ‣ *Large range of frequency time derivatives*
- ‣ *Breadth vs depth trade-off*

### *• Long-duration integration*

‣ *Computationally challenging*

### *• Need better theoretical understanding, e.g.*

- ‣ *Coupling and self-interaction strength*
- ‣ *More accurate waveforms*
- ‣ *Theoretically preferred parameter space*
- *• Multiple types of searches for multiple types of continuous-wave signals*
- *• Opened up new avenues for studying dark matter and new physics*
- *• No detections yet but probing physically interesting regions*
- *• Continuous-wave signal processing techniques continue to develop and improve*
- *• Inputs from astronomers and theorists are essential to improve future studies*
- *• With further improved detectors in the near future, new discovery is at the horizon!*

![](_page_35_Picture_7.jpeg)

![](_page_35_Picture_8.jpeg)

![](_page_35_Picture_9.jpeg)

![](_page_35_Picture_10.jpeg)

# Thanks! Questions?

![](_page_36_Picture_2.jpeg)

### **GW signal timescales (scalar)**

• Fastest growing scalar level  $(s = n = 0, j = l = m = 1)$ 

![](_page_37_Figure_2.jpeg)

$$
\alpha \equiv \frac{r_g}{\lambda_\mu} = \frac{GM}{c} \frac{m_b}{\hbar}
$$

$$
\tau_{\text{inst}}^{(\text{s})} \approx 27 \text{days} \left( \frac{M}{10 \ M_{\odot}} \right) \left( \frac{0.1}{\alpha} \right)^9 \frac{1}{\chi_i}
$$

$$
\tau_{GW}^{(s)} \approx 6.5 \times 10^4 \text{ yr} \left(\frac{M}{10 M_{\odot}}\right) \left(\frac{0.1}{\alpha}\right)^{15} \frac{1}{\chi}
$$

![](_page_37_Picture_7.jpeg)

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![](_page_38_Picture_13.jpeg)

![](_page_38_Figure_1.jpeg)

# **GW signal timescales (vector)**

# **Population and stochastic background studies**

![](_page_39_Picture_6.jpeg)

### *• Constraints obtained from black hole spin measurements*

*e.g. [Arvanitaki et al. 2017, Brito et al. 2017, Baryakhtar et al. 2017, Cardoso et al. 2018, Ng et al., PRD 2021, Ng et al. PRL 2021]*

- *• Exclusion regions in the BH mass-spin plane for a massive scalar field (l=m = 1,2,3) for a time scale of 50 Myr*
- *• There are systematics and uncertainties associated with spin measurements*

![](_page_39_Figure_3.jpeg)

![](_page_40_Picture_6.jpeg)

### *• Constraints from searches for stochastic GW background*

*e.g. [Tsukada et al. 2019, Tsukada et al. 2021, Yuan et al. 2022]*

![](_page_40_Figure_3.jpeg)

- *• Constraints on vectors (O1+O2 aLIGO data) —— excluding vector boson mass ~10-13 eV*
- *• Assumptions are made for BH population and spin distribution*

# **Population and stochastic background studies**

- *• Unknown age and history*
- *• Systematics affecting the spin measurements*
- *• Not well understood impact from the active environment*
- *• Relatively low BH mass*
- *• Search challenges due to the binary motion*

![](_page_41_Figure_1.jpeg)

*• Clean environment, no impact from binary motion • Unknown location (need an all-sky blind search) • Contingent on BH populations • Unknown age, spin, etc.*

![](_page_41_Picture_10.jpeg)

![](_page_42_Picture_4.jpeg)

### *• There may be plenty of detectable sources in our galaxy (if the bosons exist)*

*[Zhu et al., PRD 102, 063020 (2020)]*

![](_page_42_Figure_3.jpeg)

 $10^{-16}$ .

 $10^{-17}$  :

 $10^{-18}$  =

 $10^{-19}$ .

 $GeV$ 

# **Relax the assumption — consider self interactions (scalar)**

- *• Self-interaction leads to smaller clouds and faster growth of the next level*
- *• Constraints from Cygnus X-1 O2 search and future prospects considering boson*  $\widetilde{f}$ *self-interaction (assuming a BH age of 105 year)*
- *• There are potential observational prospects of level transition signals*
- *• Further numerical studies in high boson mass regime that capture full dynamics of the cloud will facilitate future searches*

![](_page_43_Figure_7.jpeg)

*• Test weak coupling of kinetically mixed dark photon —— With a non-vanishing kinetic mixing, the superradiant cloud also dissipates energy through electromagnetic radiation.*

# **Relax the assumption — consider weak couplings (vector)**

![](_page_44_Picture_7.jpeg)

![](_page_44_Figure_1.jpeg)

*• Test the dark Higgs-Abelian sector —— Depending on the relevant coupling strengths, the presence of the Higgs boson may lead to additional frequency evolution/dark radiation or an explosive bosenova forming strings* 

![](_page_44_Picture_6.jpeg)

Jones+ in prep (2024)

![](_page_44_Figure_4.jpeg)

# **Imprints on hierarchical mergers**

- *• Hierarchical mergers in dense stellar clusters can create intermediate/massive BHs*
- *• Recoil velocity can kick the remnant BH out of the dense stellar environment*
- *• Low-spin progenitor BHs lead to lower recoil velocity of the remnant*
- *• Superradiance can rapidly spin down BHs and may keep more BHs in the dense stellar environment*

![](_page_45_Figure_6.jpeg)

*• Simulate merging BH populations by evolving a model cluster population (such that it visually matches the observed nuclear star cluster population after evolution)*