Challenges of measuring the Neutron-Star equation of state using gravitational wave observations

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INT: EOS Measurements with Next-Gerneration GW Detectors
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NP3M

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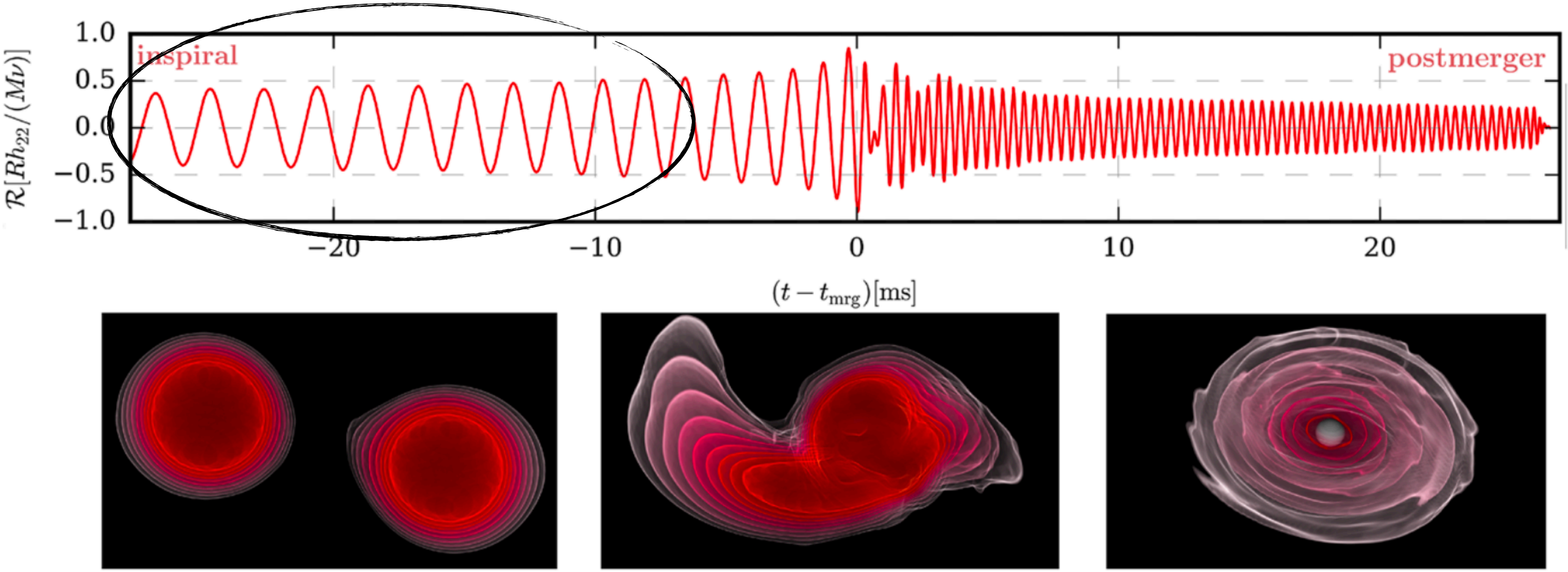
Cold Dense Matter in a Neutron Star Core

How GW signals reveal properties of cold dense matter?

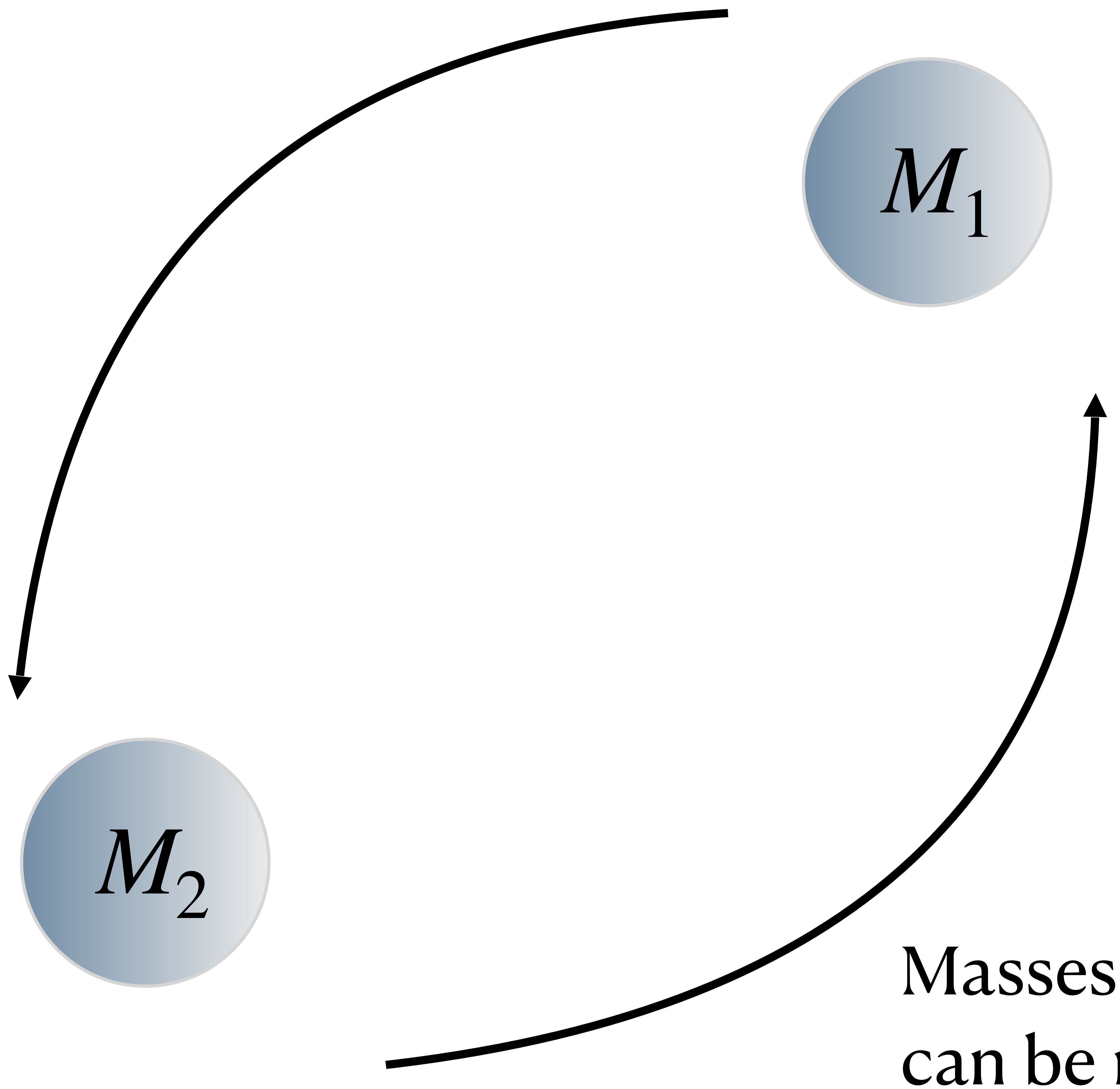
Detection GW170817



Dietrich et al. 2021

Cold Dense Matter in a Neutron Star Core

How inspiral-phase signals reveal properties of cold dense matter?



Masses (M_1 and M_2)
can be measured at early inspiral phase

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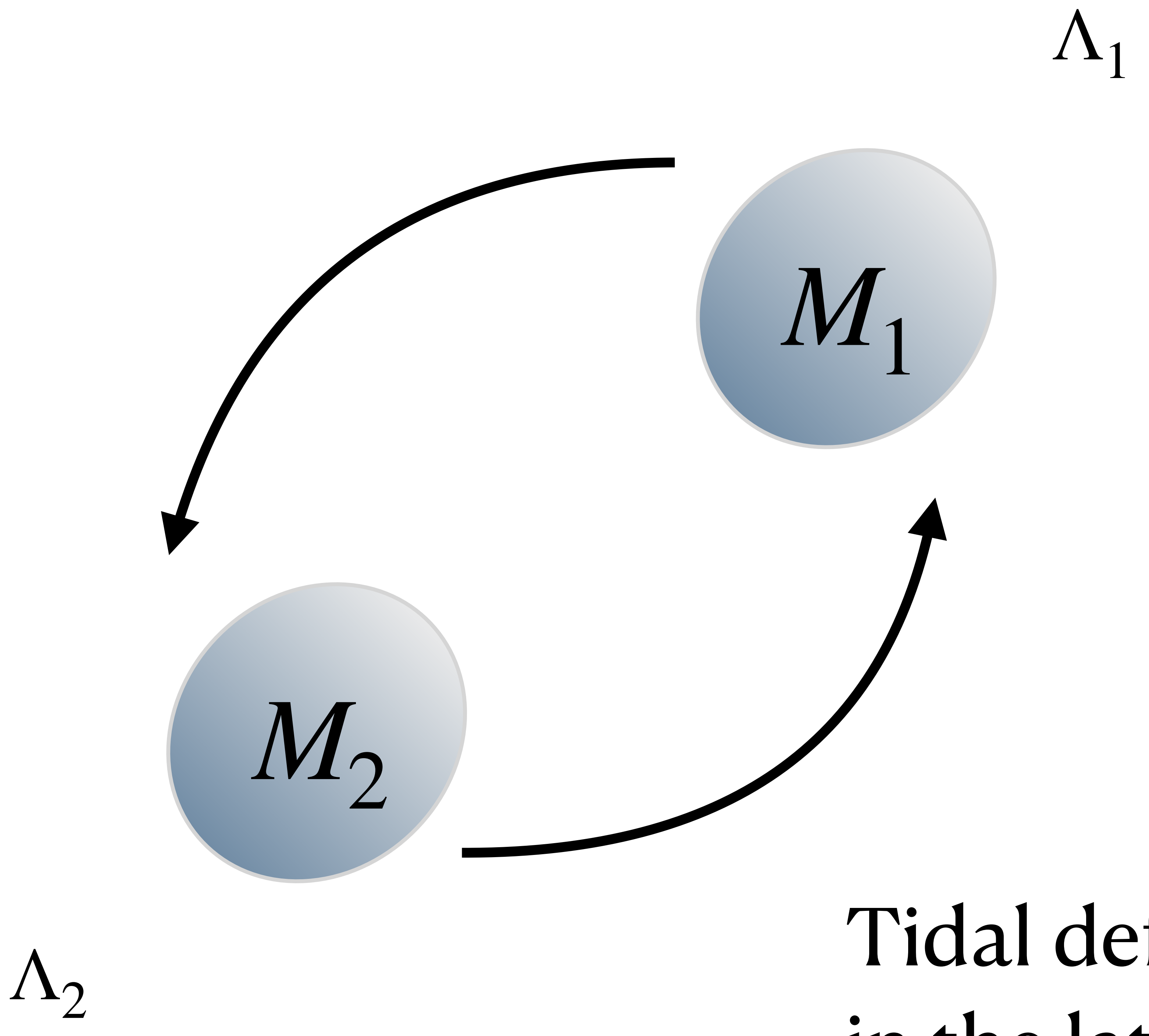
Cold Dense Matter in a Neutron Star Core

How inspiral-phase signals reveal properties of cold dense matter?

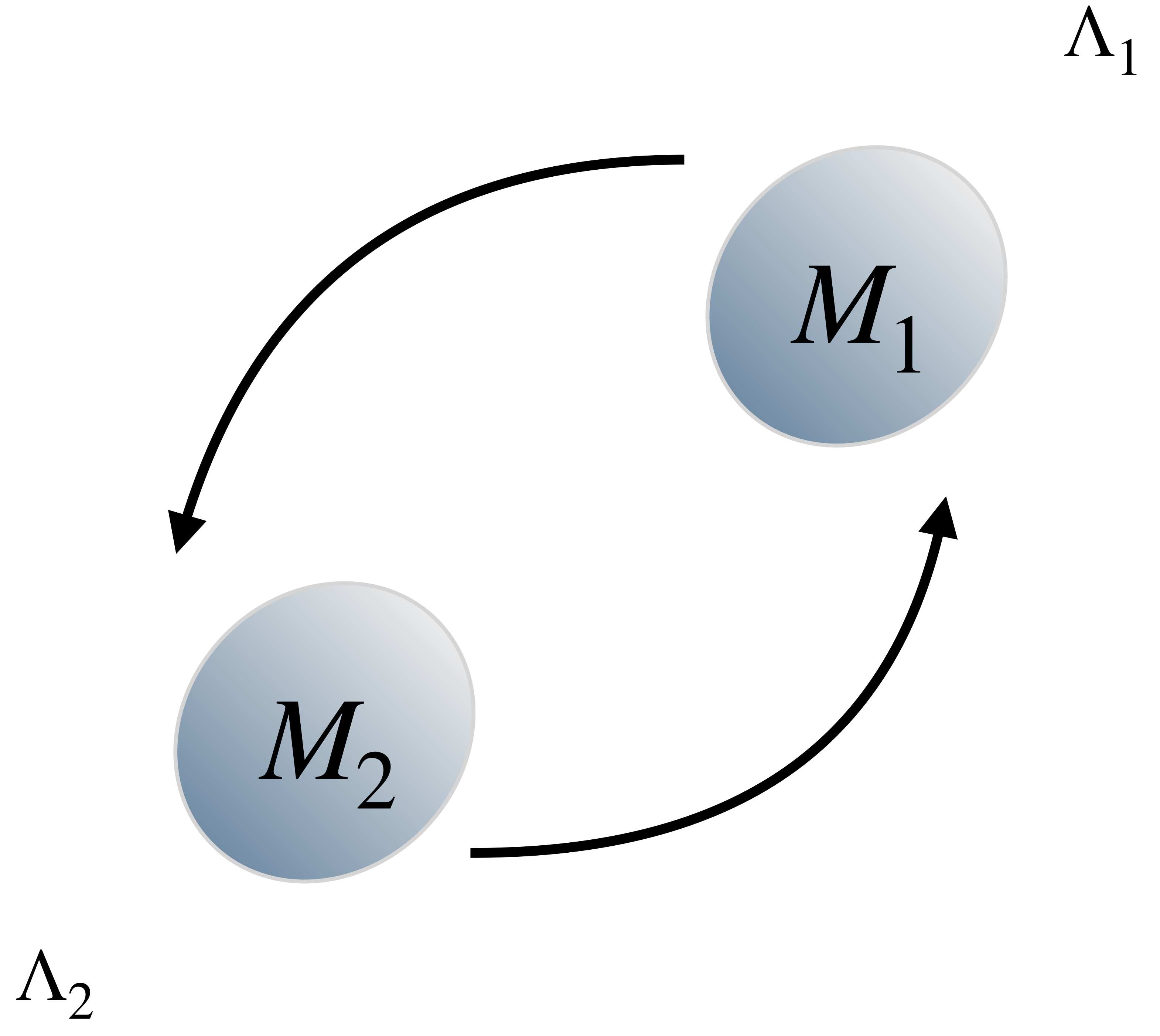
Tidal deformability!



Tidal deformability (Λ_1 and Λ_2) in the late inspiral phase

Problems

Degeneracy in GW inference



- Early inspiral phase: M_1, M_2
  (Assuming no spin)

- Late inspiral phase:

$$\tilde{\Lambda} = \frac{16}{13}\eta_1^4(12 - 11\eta_1)\ \Lambda_1 + \frac{16}{13}\eta_2^4(12 - 11\eta_2)\ \Lambda_2$$

$$\eta_i = M_i/(M_1 + M_2)$$

Outline

- How gravitational wave (GW) observations constrain equation of state (EoS)?
- Spectral Parametrazation
  - What's the challenge?
- EoS insensitive relations
  - What's the challenge?
- Future Work

Spectral Parametrization

Parametrizing EoSs

$$\Gamma \equiv \frac{\epsilon + p}{p}\frac{dp}{d\epsilon}$$

↓ Adiabatic index

$$\ln \Gamma = \sum_k \gamma_k \, x^k$$

Spectral Parametrization

Parametrizing adiabatic index

$$\Gamma \equiv \frac{\epsilon + p}{p}\frac{dp}{d\epsilon} = \exp\left(\sum_k \gamma_k\, x^k\right),$$

Adiabatic index

Free parameters:
Spectral coefficient

$$x = \ln \frac{p}{p_0}$$

Pressure at saturation density (n_{sat}) , fixed

$$\ln \Gamma = \sum_k \gamma_k\, x^k$$

A spectral EoS is represented by $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \ldots)$ → 0

Lindblom 2010

Spectral Parametrization

An Example

$$\Gamma \equiv \boxed{\frac{\epsilon + p}{p}\frac{dp}{d\epsilon} = \exp\left(\sum_k \gamma_k\, x^k\right)}$$

Example:

$(\gamma_0, \gamma_1, \gamma_2, \gamma_3) = (0.982, 0.128, -0.039, 0.003)$

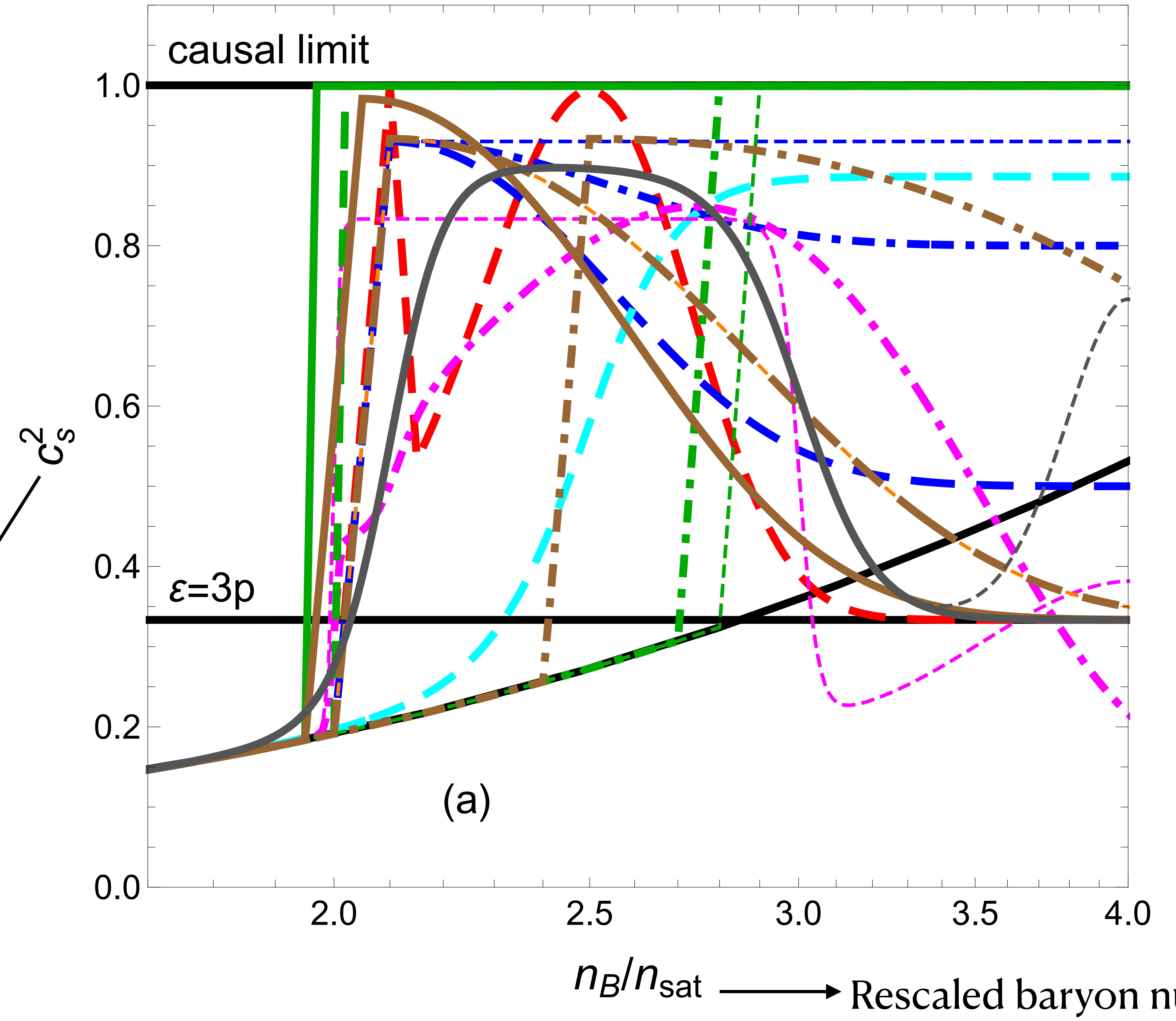
$$\frac{\epsilon + p}{p}\frac{dp}{d\epsilon}$$
$$= \exp\left(0.982 + 0.128x - 0.039x^2 + 0.003x^3\right)$$

- Gravitational waveform template $h(t;\ M_i\,, \Lambda_i)$ can be calculated for a given spectral EoS
- If waveform template $\nsim$ observed GW signal, try another set of γ
- After finding the best fit of γ, we find the best fit EoS
- M, R, Λ can be calculated using that EoS, which avoids the degeneracy problem and constrains the radius

Spectral Parametrization

Challenges: what if the true EoS cannot be well approximated?

"Toy" EoSs



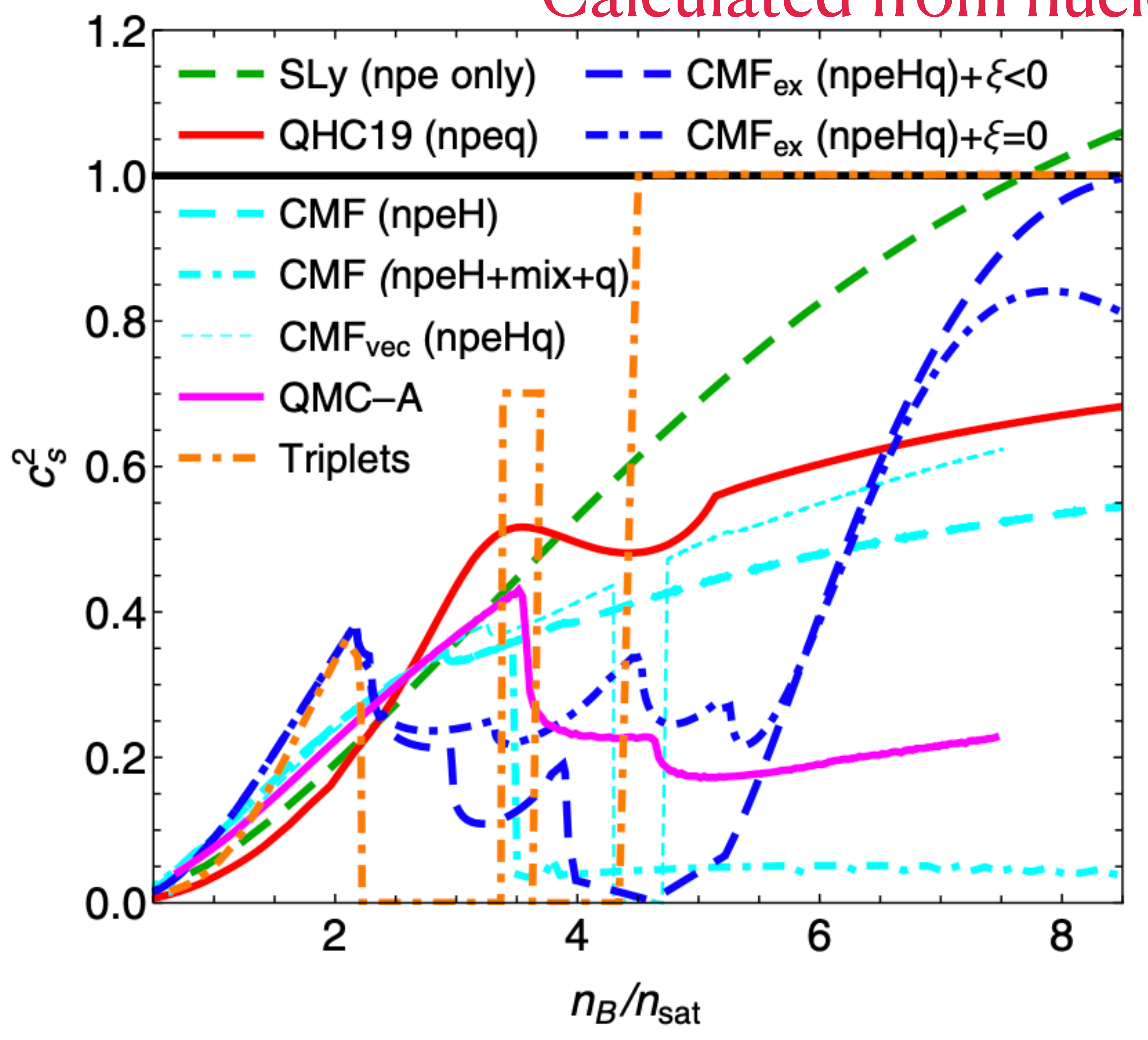
$dp/d\epsilon$

Rescaled baryon number density

Tan et al. PRL (2020)

Why?

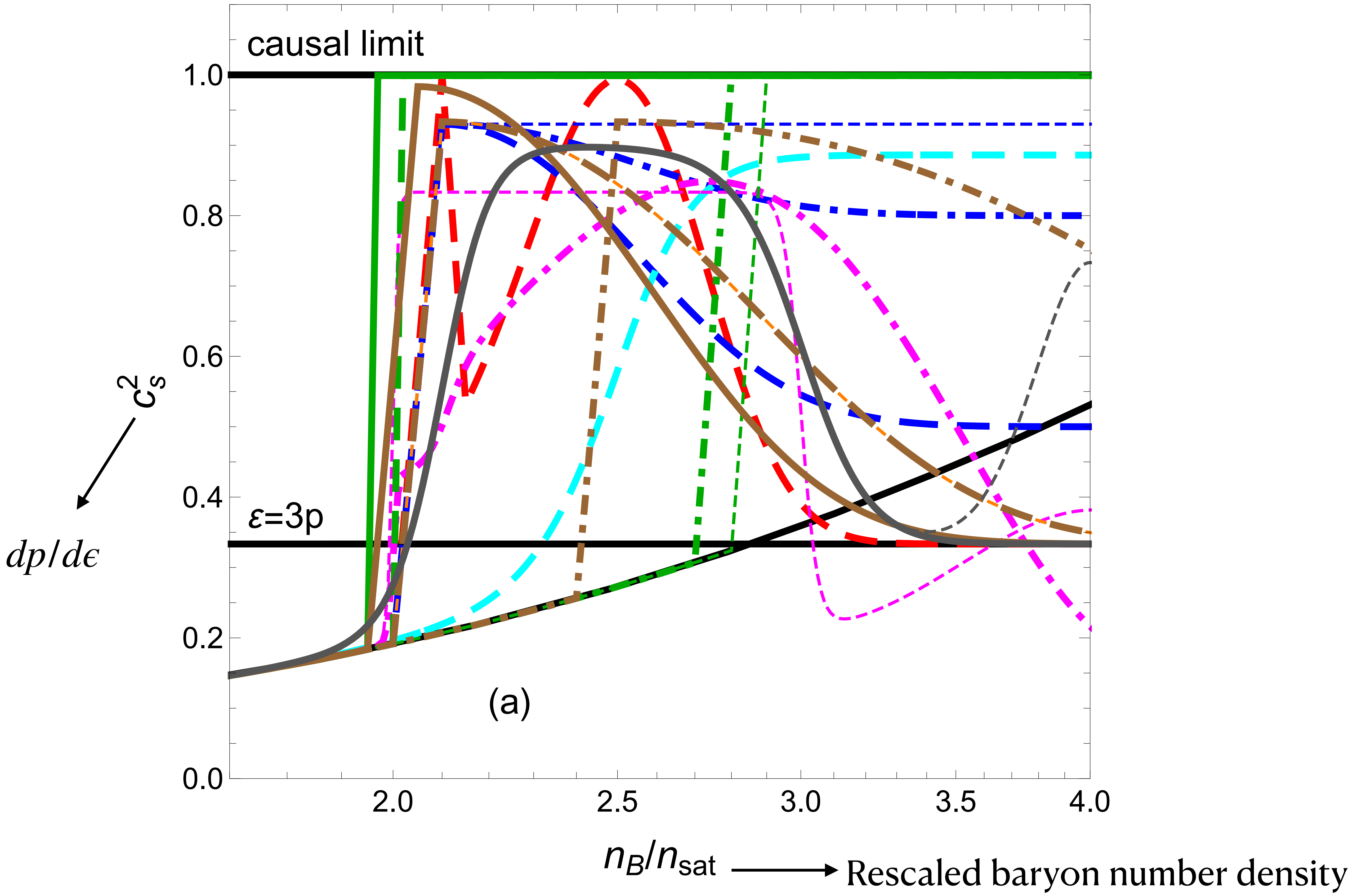
Calculated from nuclear model



Tan et al. PRD (2022)

Spectral Parametrization

Challenges: what if the EoS is not that smooth

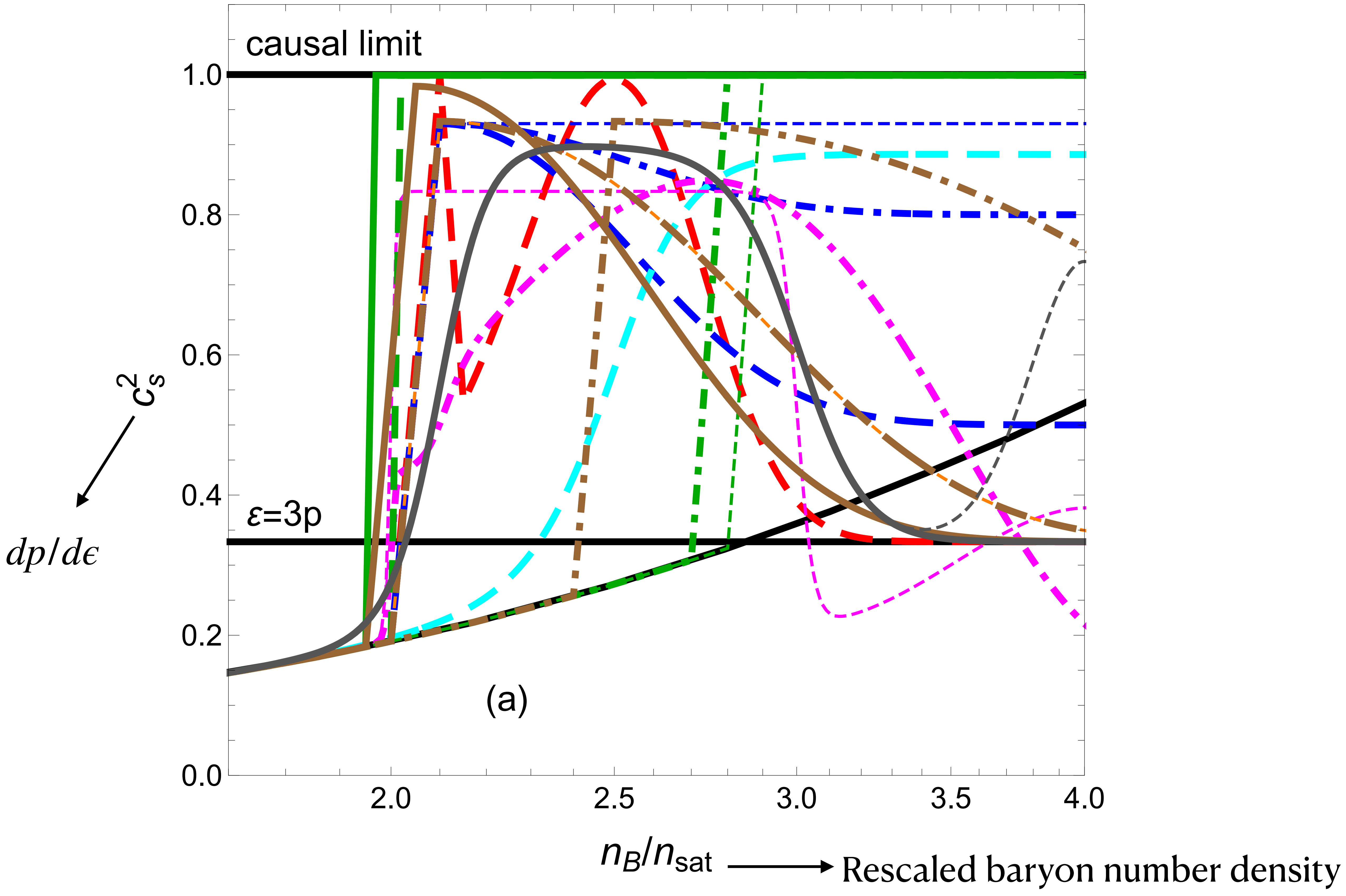


Tan et al. PRL (2020)

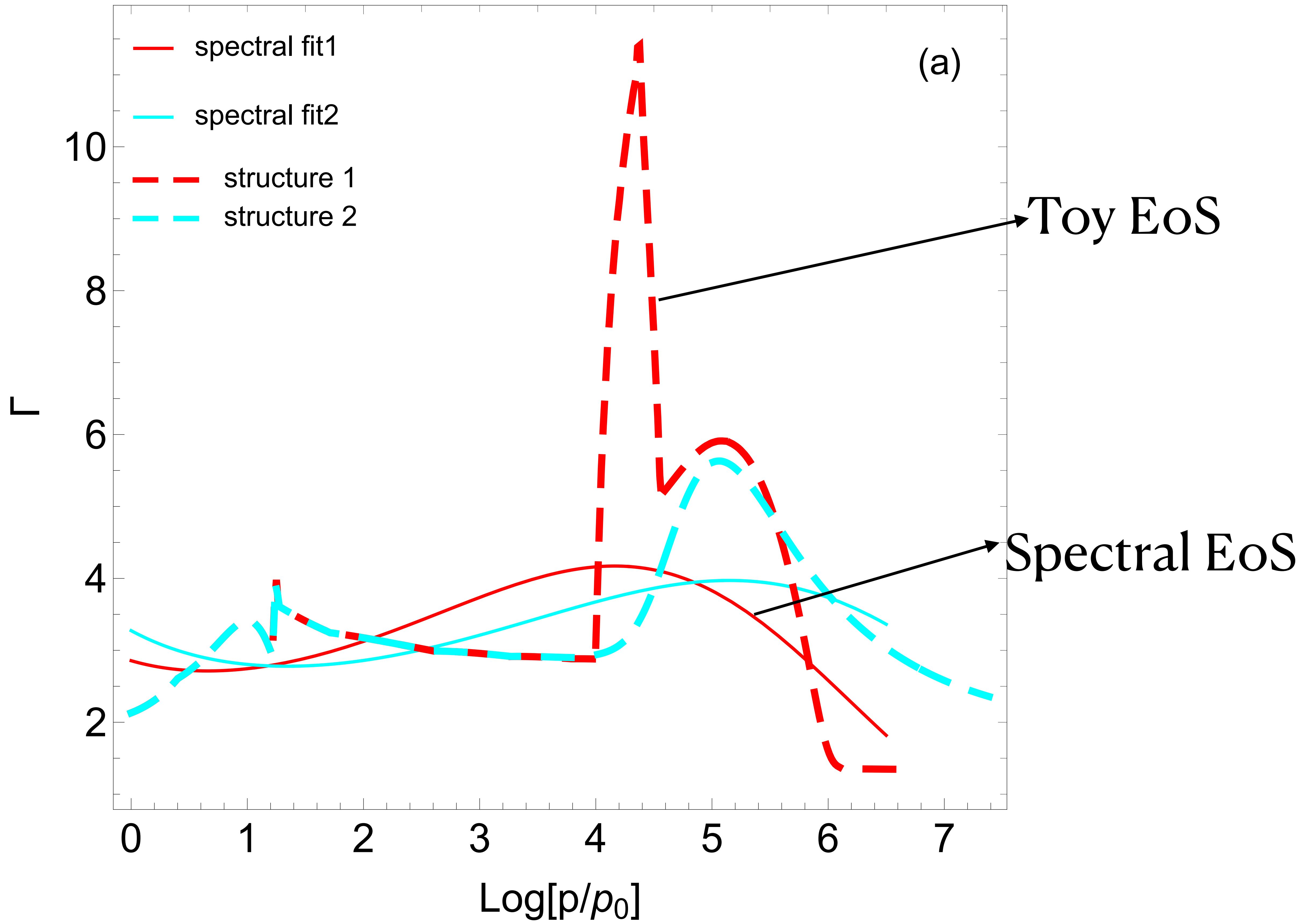
- Phase transition (PT)
- $$\Gamma \equiv \frac{\epsilon + p}{p}\frac{dp}{d\epsilon} = \exp\left(\sum_k \gamma_k x^k\right)$$

Spectral Parametrization

Challenges: what if the EoS is not smooth



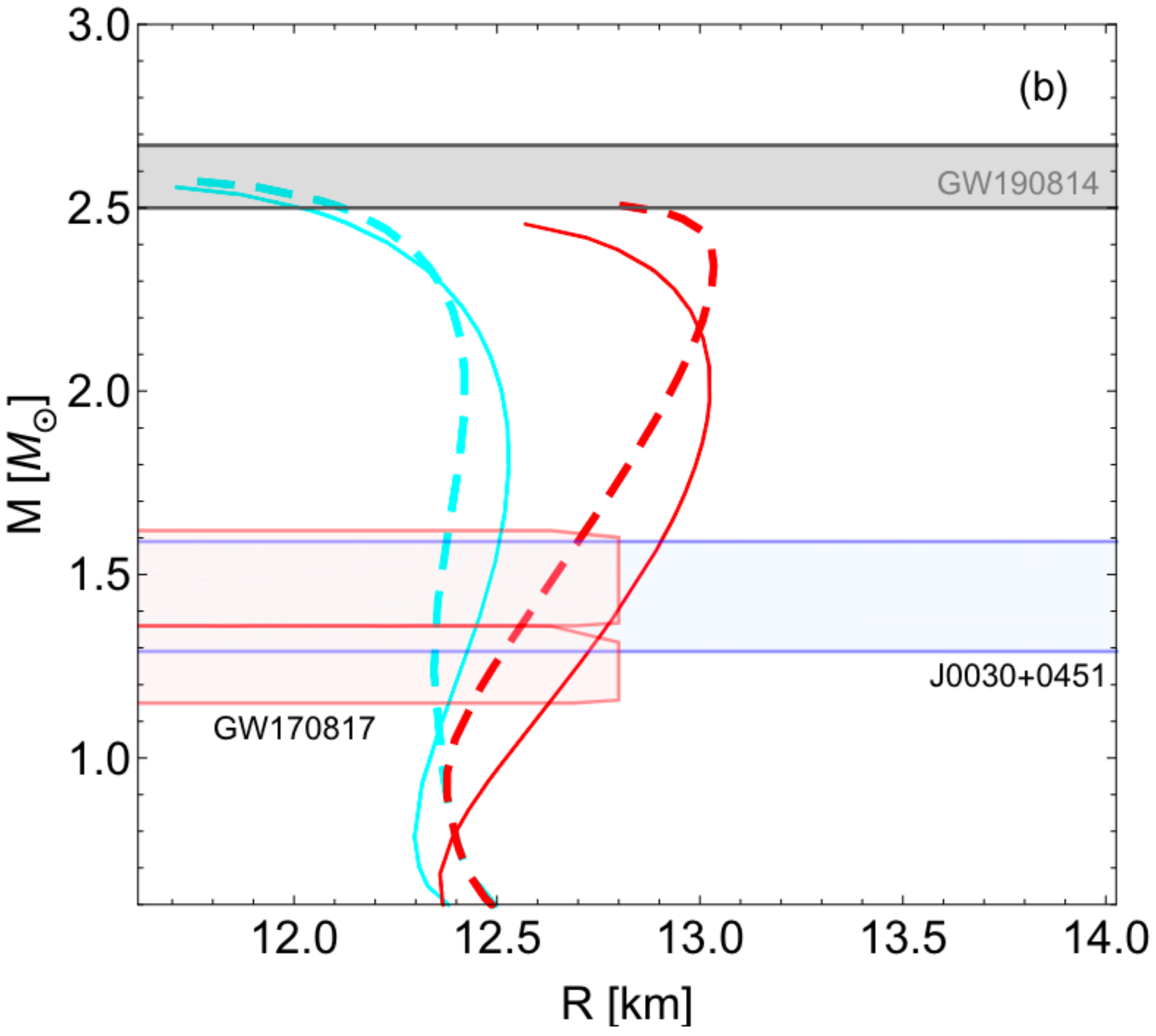
Tan et al. PRL (2020)



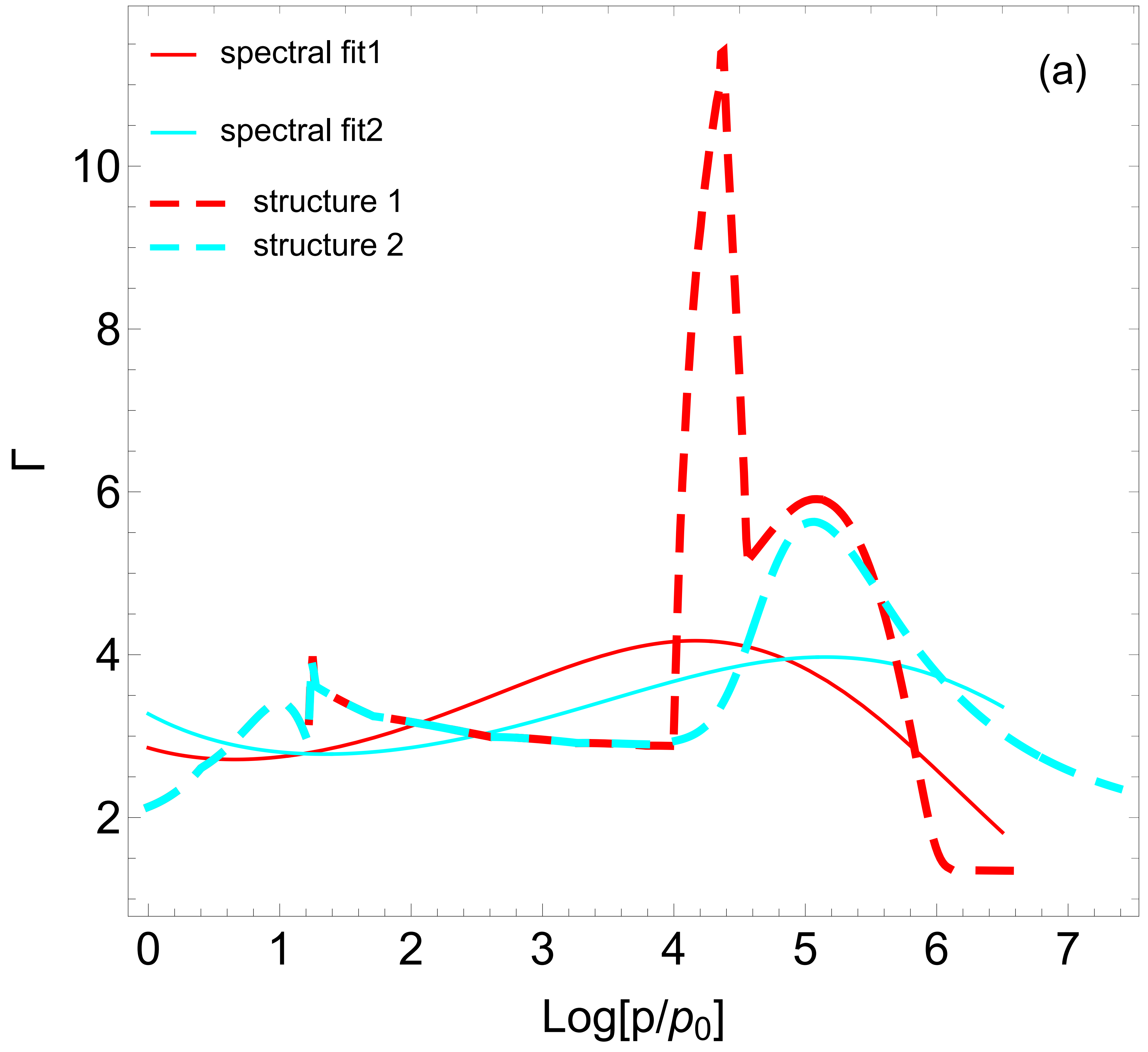
Tan et al. PRL (2020)

Spectral Parametrization

Challenges: what if the EoS is not smooth



Tan et al. PRL (2020)



Tan et al. PRL (2020)

Spectral Parametrization

GW190814: A NS or a BH

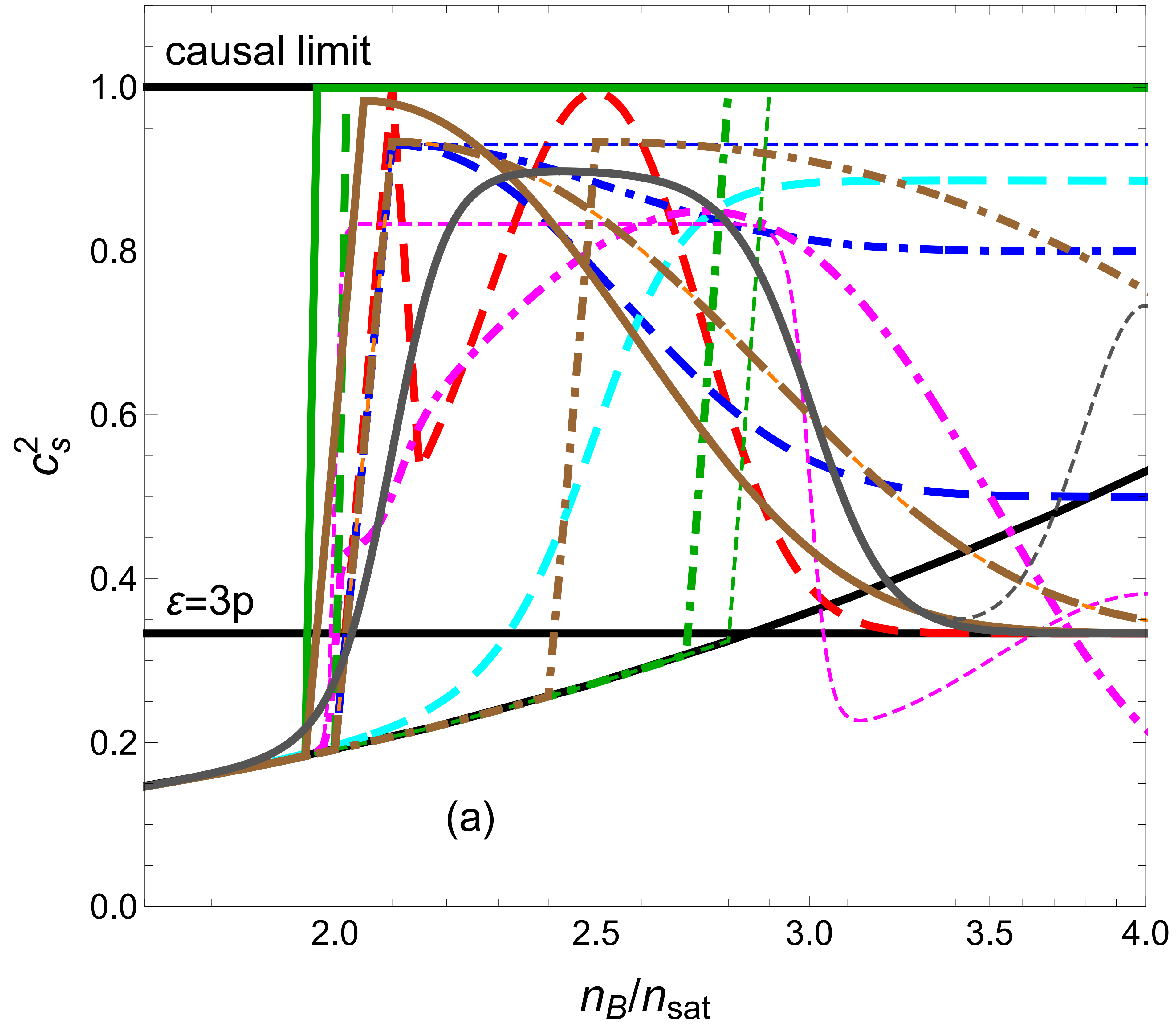
Spectral EoS constraint

$M \approx 2.6 M_\odot$ (GW190814) $\longrightarrow\!\!\!\times\!\!\!\longleftarrow$ $M_{max} \lesssim 2.43 M_\odot$ (90%)

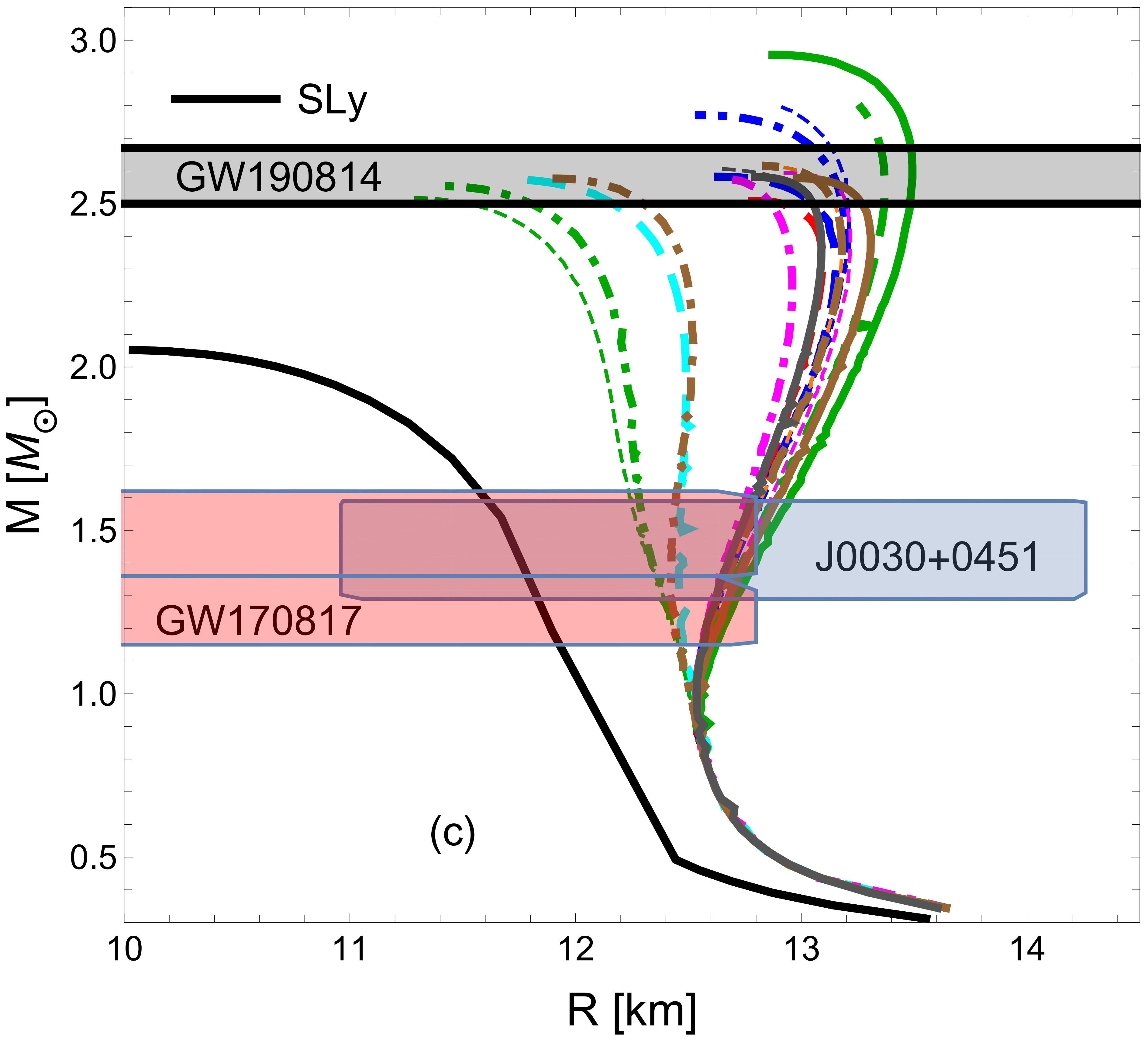
GW170817, LVC 2018

Spectral Parametrization

GW190814: A NS or a BH



Tan et al. PRL (2020)



Tan et al. PRL (2020)

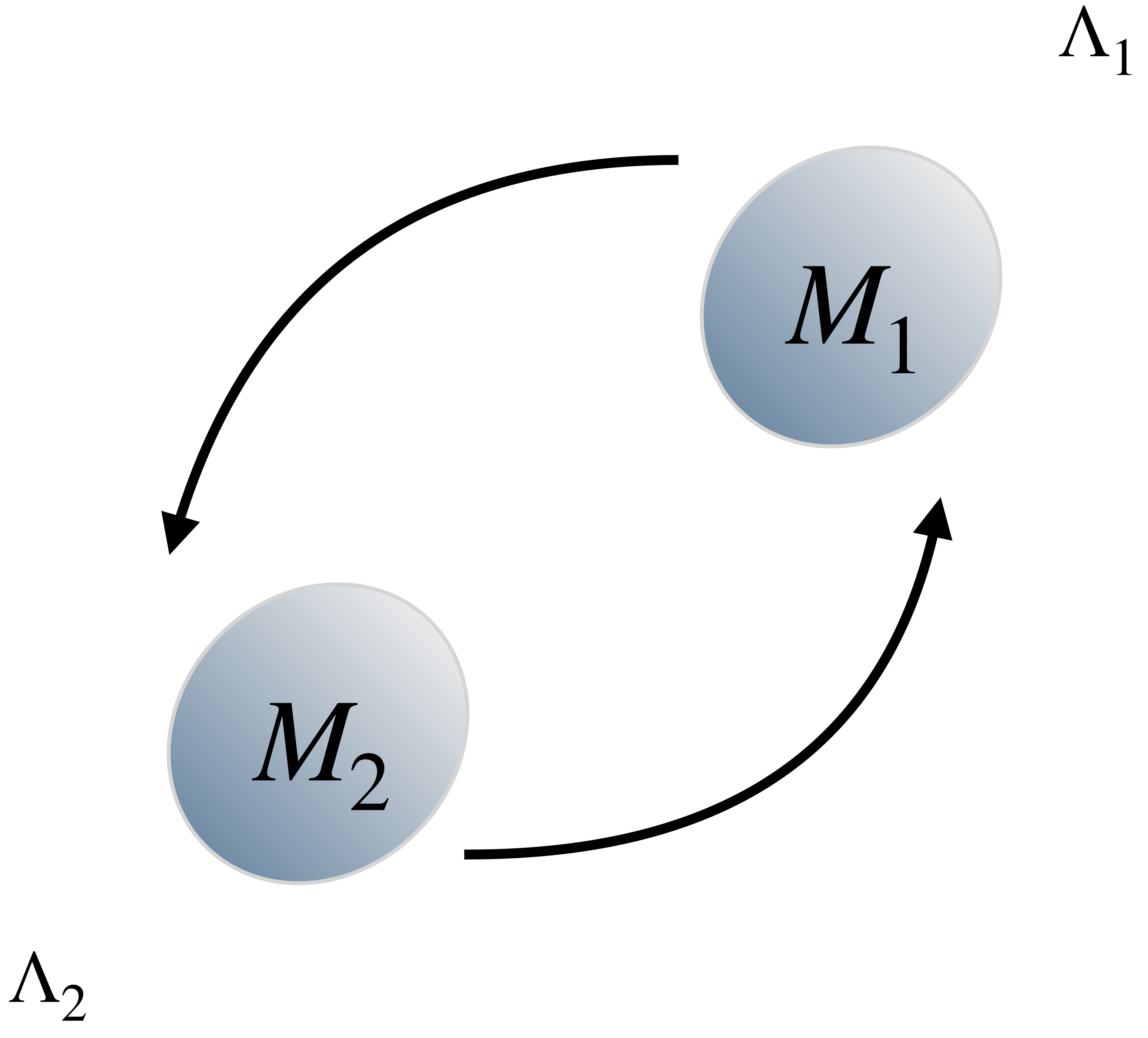
Spectral Parametrization

Summary

- EoSs with a phase transition are not well approximated by spectral EoS
- We can miss the opportunity to detect a phase transition

EoS-insensitive relations

Introducing chirp deformability



- Early phase of the inspiral: M_1, M_2
  (Assuming no spin)

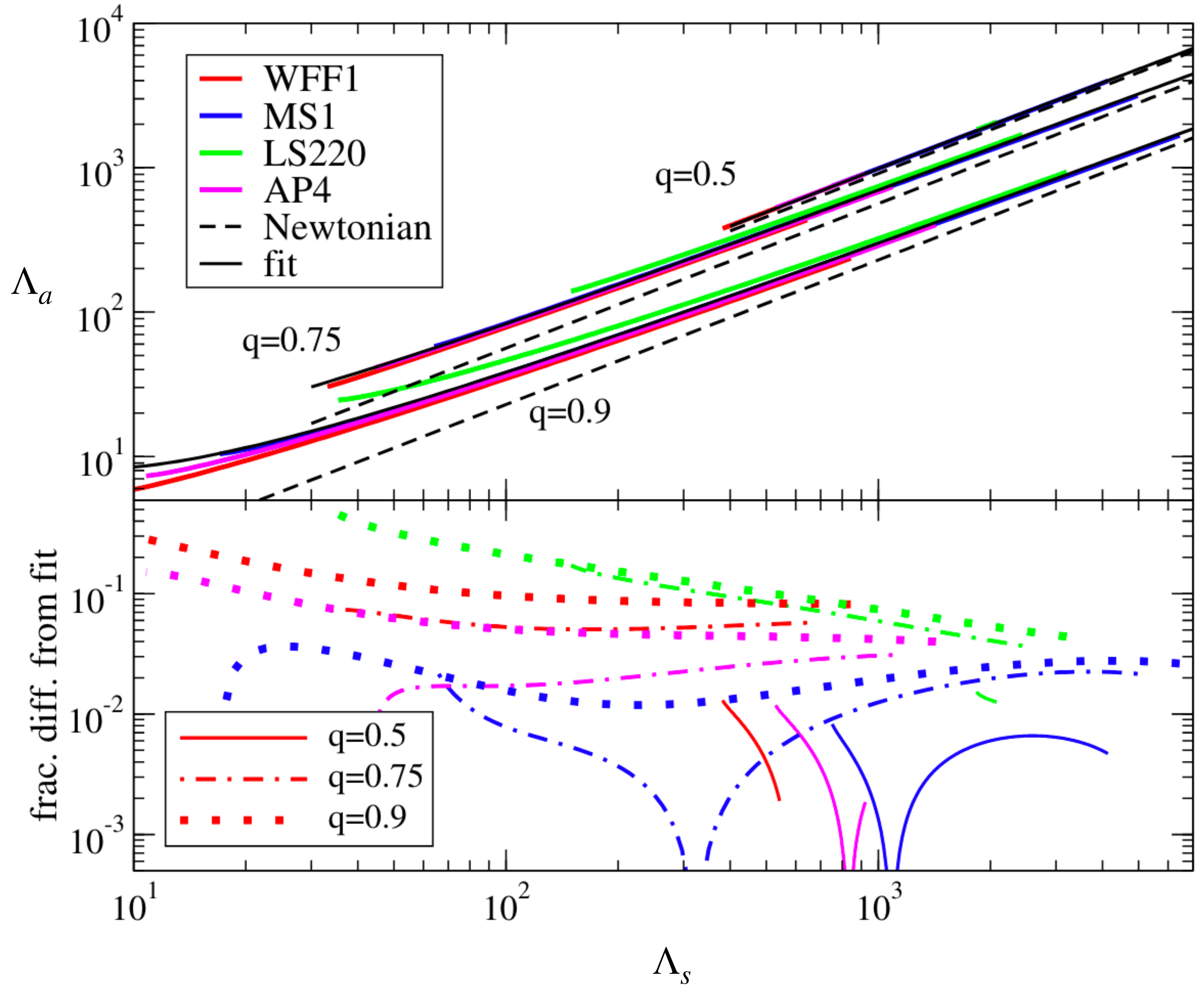
- Late inspiral phase:

$$\tilde{\Lambda} = \frac{16}{13}\left(\eta_1^4(12 - 11\eta_1)\Lambda_1 + \eta_2^4(12 - 11\eta_2)\Lambda_2\right)$$

$$\eta_i = M_i/(M_1 + M_2)$$

EoS-insensitive relations

Binary Love relation: $\Lambda_a(\Lambda_s)$



Yagi and Yunes 2016

- Symbols
  - $\Lambda_a = (\Lambda_1 - \Lambda_2)/2$
  - $\Lambda_s = (\Lambda_1 + \Lambda_2)/2$
  - $q = M_1/M_2$
- $\mathcal{O}(10\%)$ error to the fit
- Break degeneracy with the fitted $\Lambda_a(\Lambda_s)$ and $\tilde{\Lambda}(\Lambda_1, \Lambda_2)$

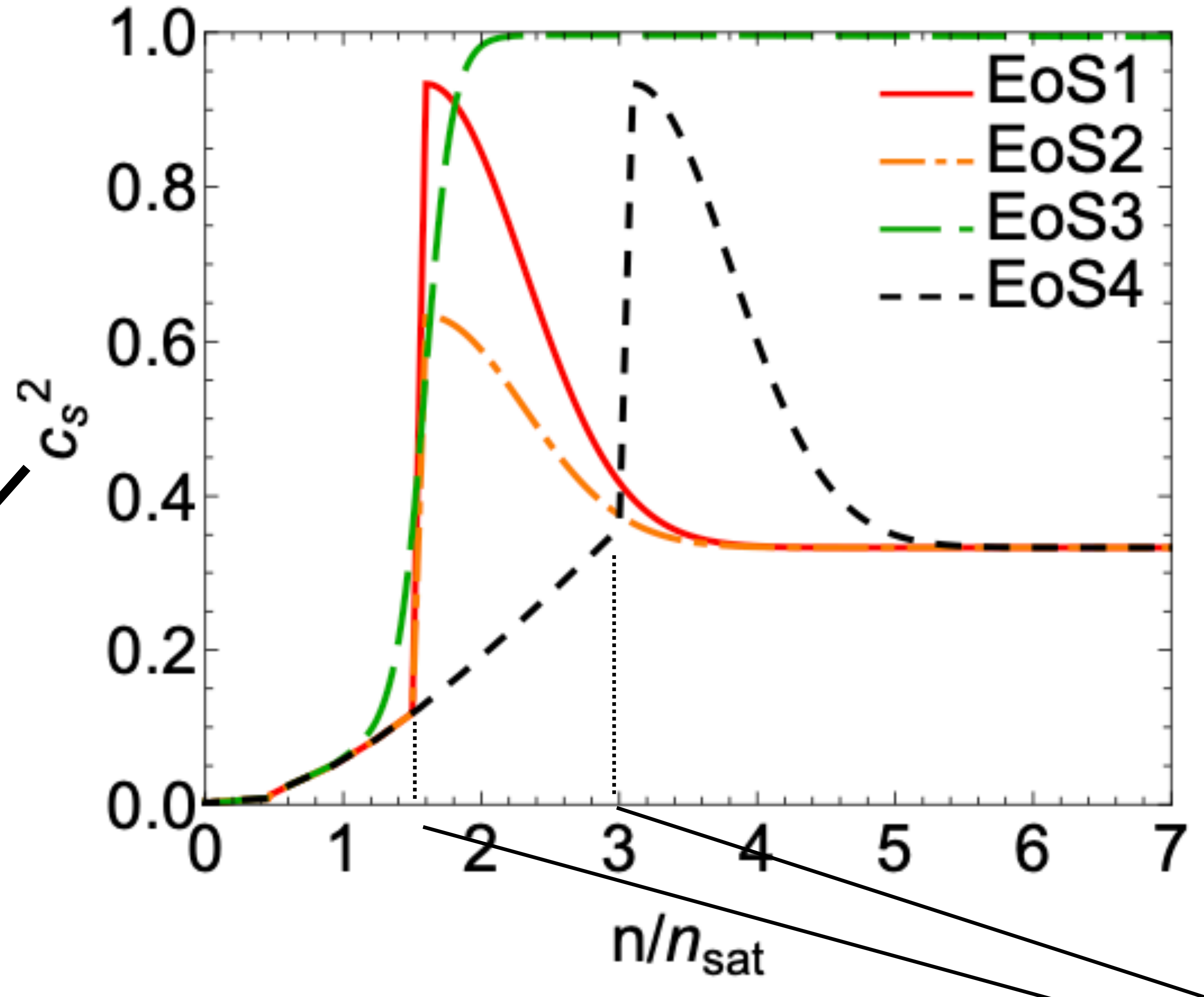
EoS-insensitive relations

What will happen if we consider phase transitions?

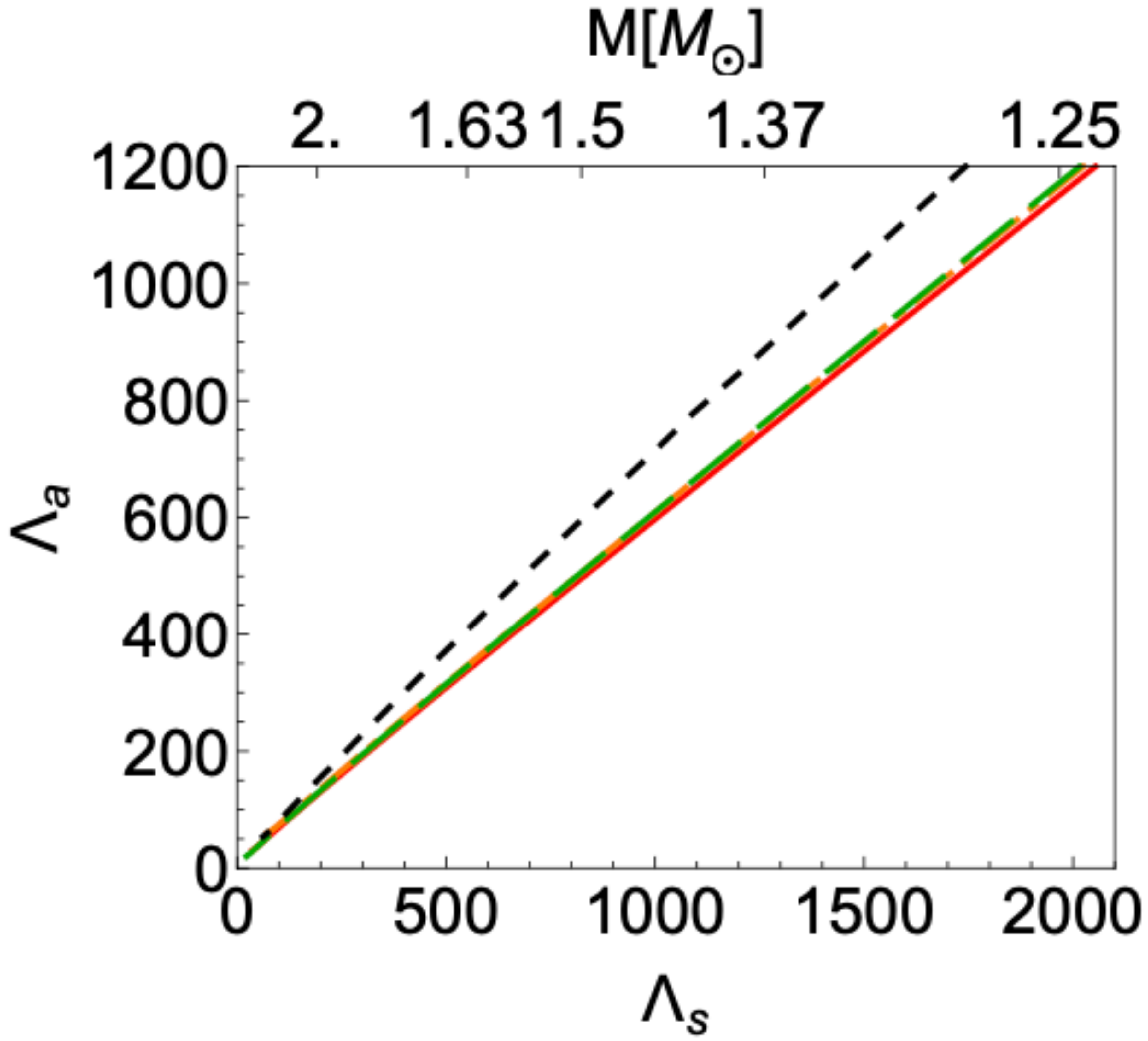
EoS-insensitive relations

Binary Love relation: $\Lambda_a(\Lambda_s)$

$\frac{dp}{d\epsilon}$



Tan et al. (2021)



n_{PT} → Seems we can tune the slope with n_{PT}

Tan et al. (2021)

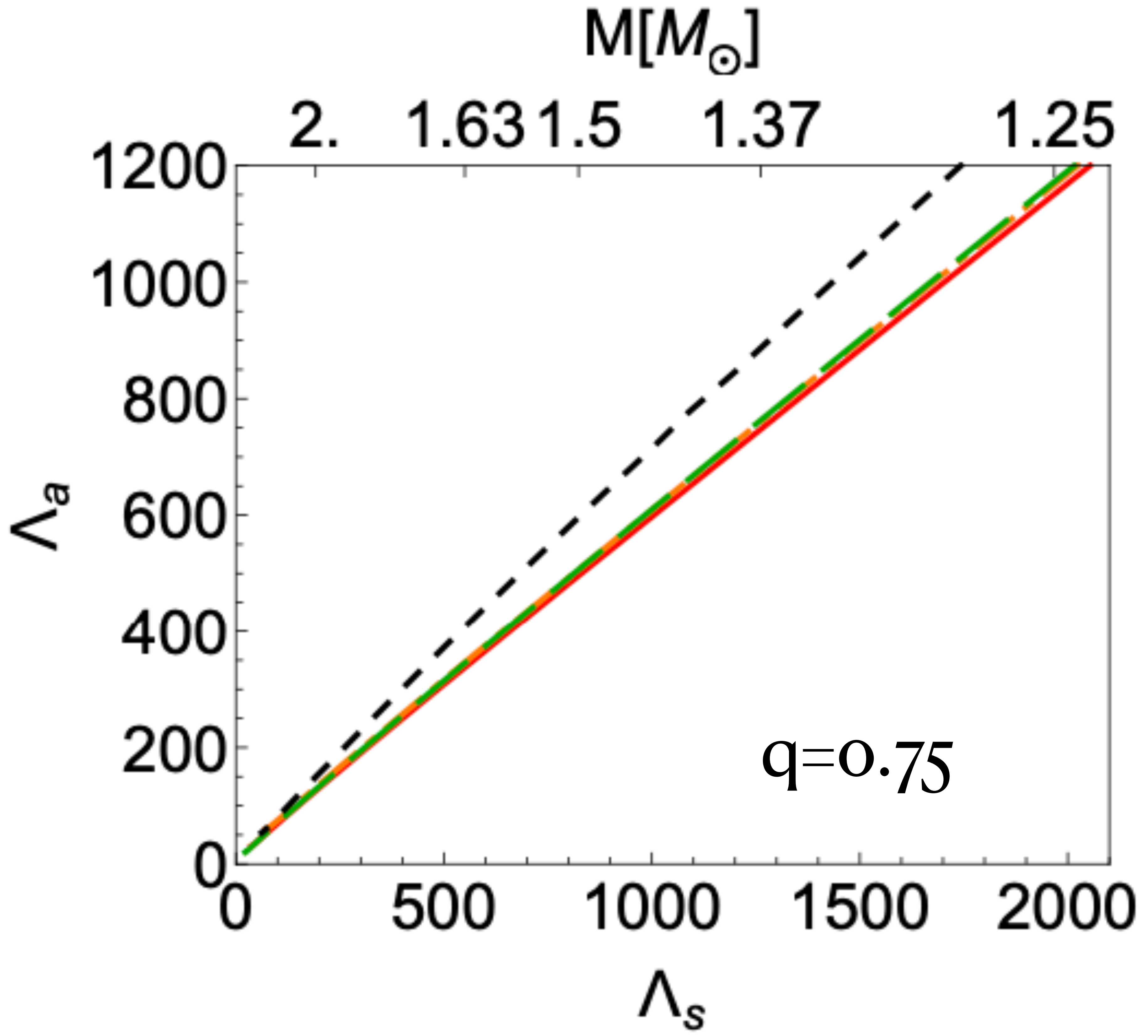
EoS-insensitive relations

Slope of binary Love relation

M_1/M_2 $\quad$ M_1/R_1

$$\frac{\Lambda_a}{\Lambda_s} = \frac{1-q^5}{1+q^5} - 10C_1\left(\frac{1}{dM_1/dR_1}\right)\frac{q^4(q-1)}{(q^5+1)^5} + \mathcal{O}(C_1^2)$$

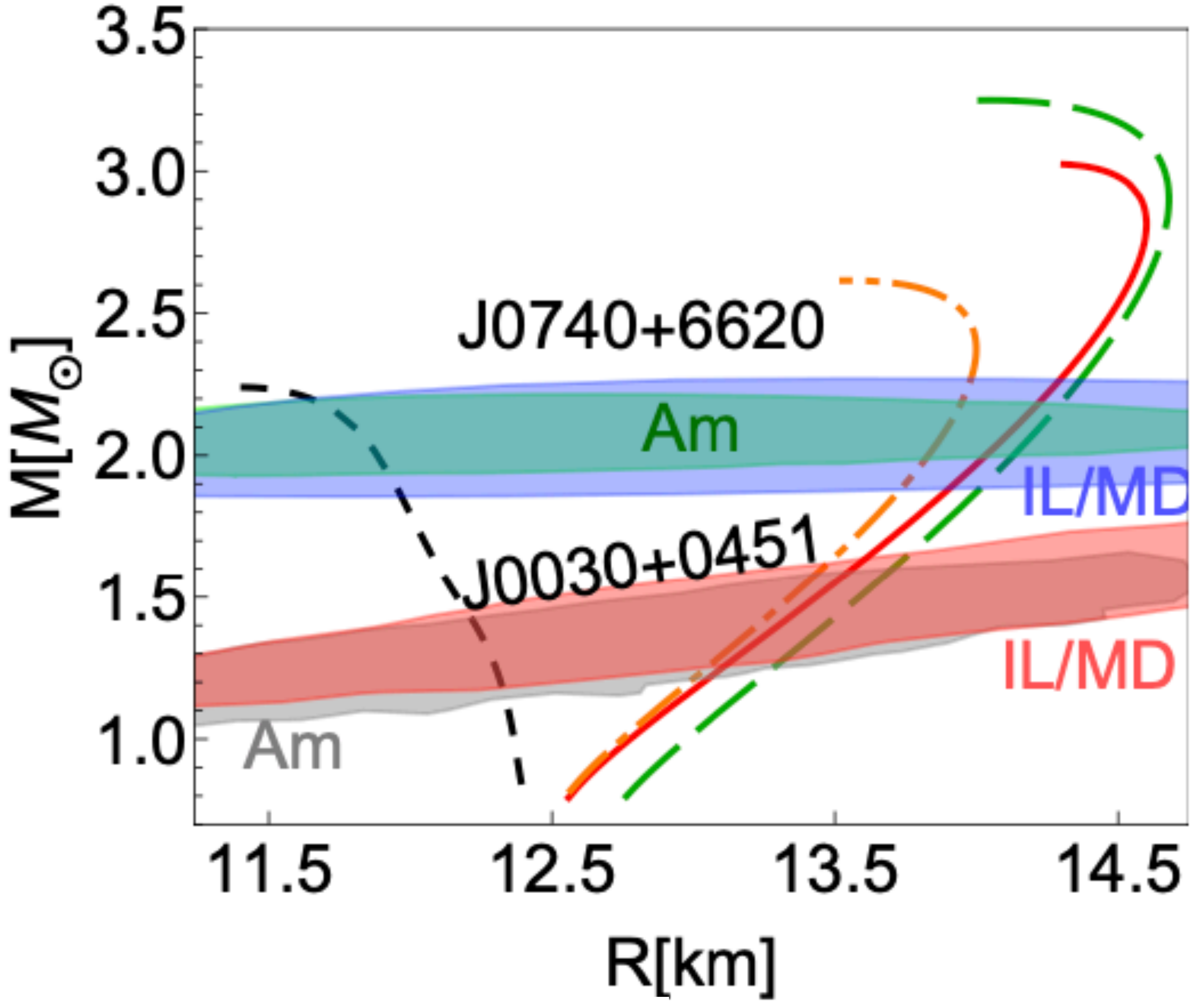
EoS independent $\quad$ EoS dependent



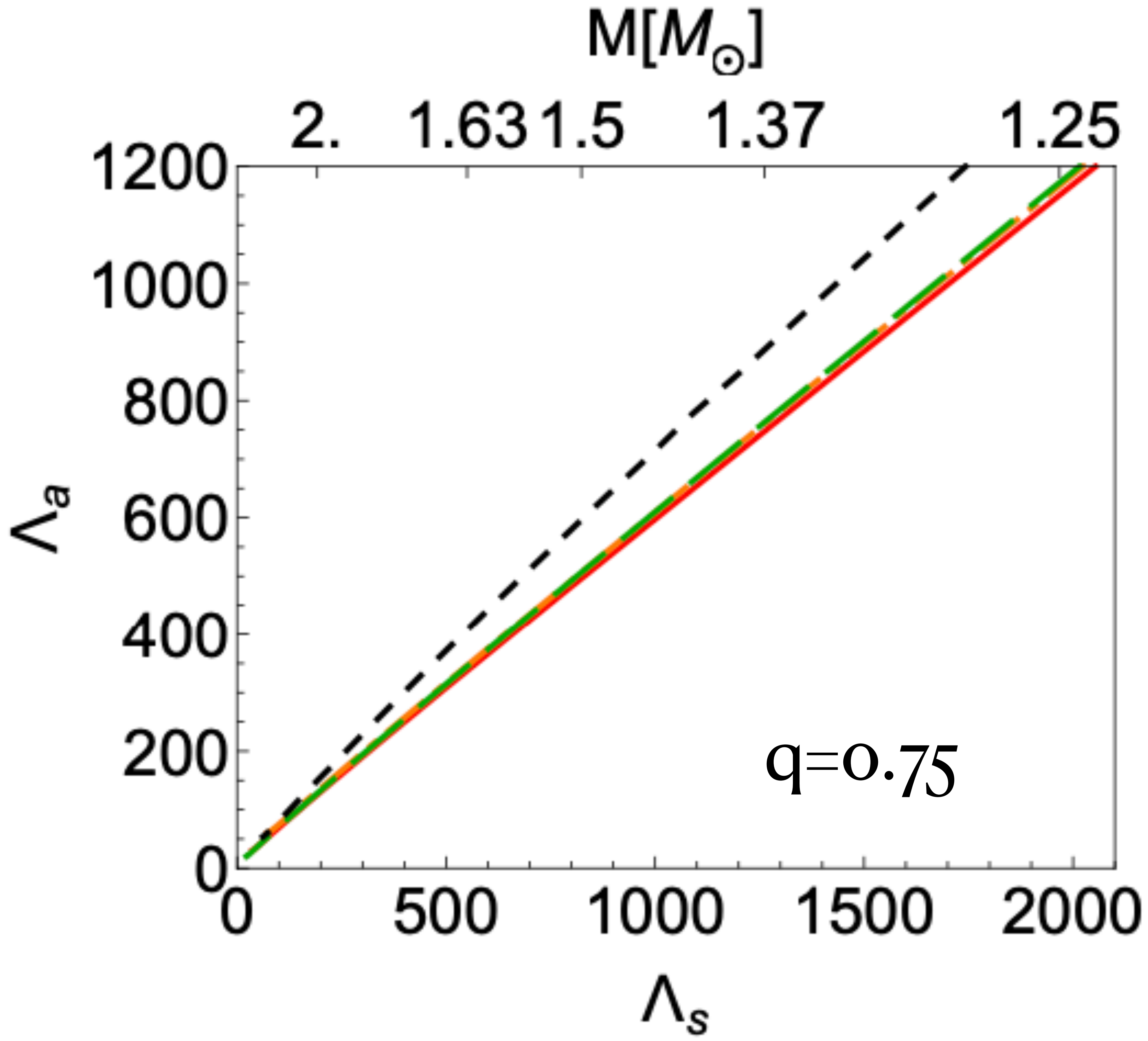
Tan et al. (2021)

EoS-insensitive relations

Slope of binary Love relation



Tan et al. (2021)



Tan et al. (2021)

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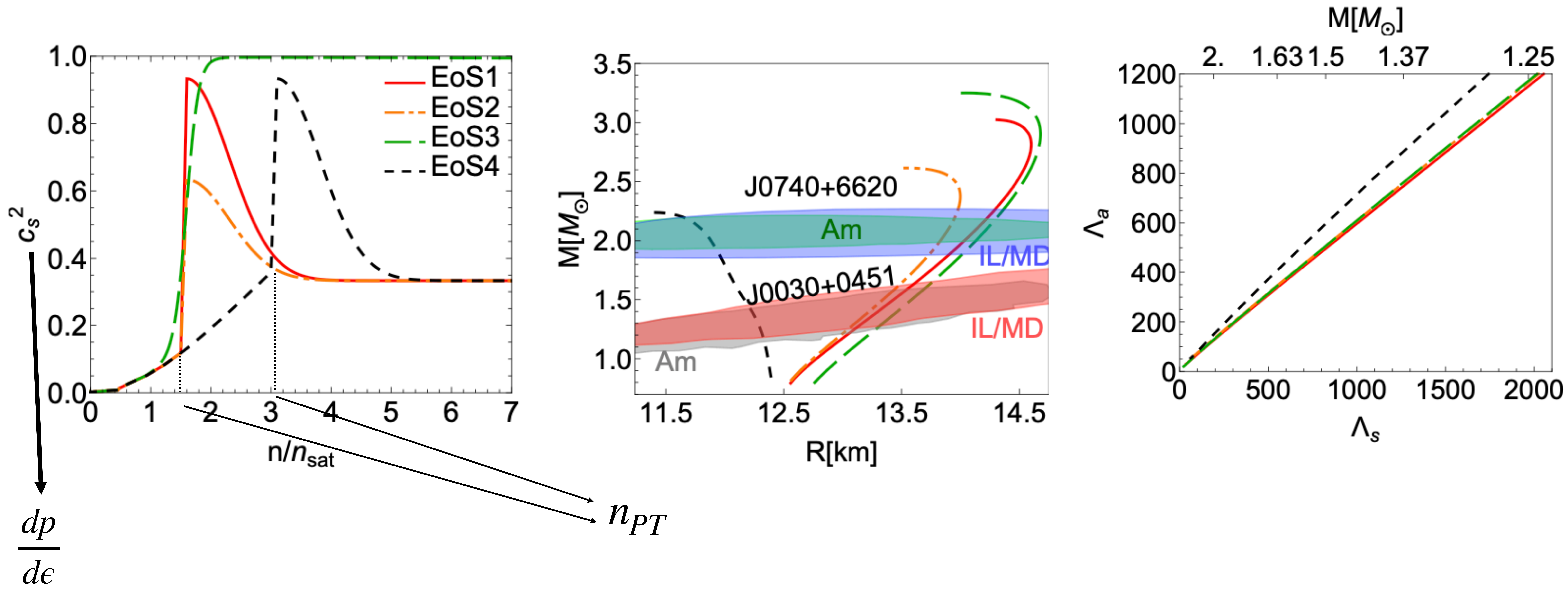
EoS-insensitive relations

Why does n_{rise} change the slope of M-R curve?



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EoS-insensitive relations

Summary

- Binary Love relations contain information of phase transitions, do not assume the relation is universal and use it blindly.

Future Work

- Parametrizing EoS: What if the true EoS cannot be well represented?
- EoS insensitive relation: more problems

Future Work

Next generation detector

$$5\text{PN: } \tilde{\Lambda} = \frac{16}{13}\left(\eta_1^4(12 - 11\eta_1)\Lambda_1 + \eta_2^4(12 - 11\eta_2)\Lambda_2\right)$$

$$6\text{PN: } \delta\Lambda = \eta_1^4(-\frac{15895}{28} + \frac{4595}{28}\eta_1 + \frac{5715}{14}\eta_1^2 - \frac{325}{7}\eta_1^3)\Lambda_1 + (1 \leftrightarrow 2)$$