NOVEL ENERGY LOSS MECHANISM IN THE CHIRAL MEDIA

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Heavy Ion Physics in the EIC Era @ INT

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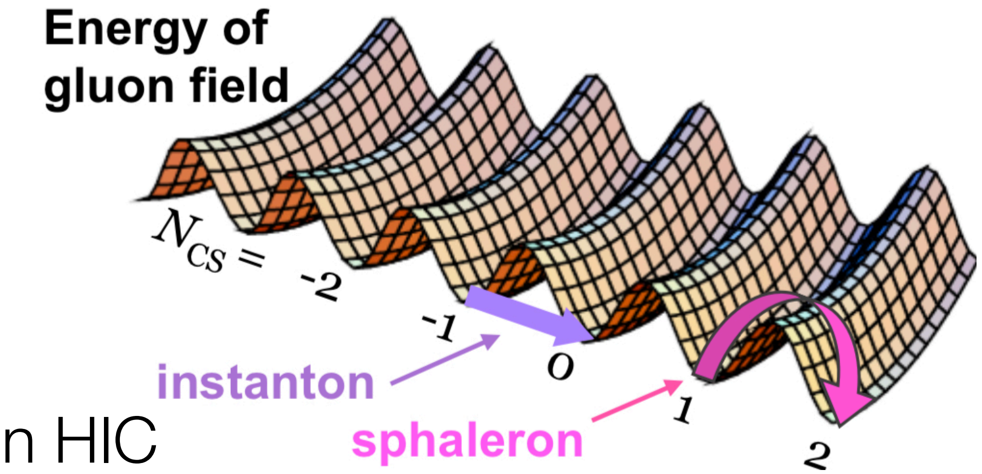
NUCLEAR MATTER IS CHIRAL

Hot nuclear matter/quark-gluon plasma:



Sphaleron transitions generate domains with chiral imbalance ⇒ Chiral magnetic effect

If the domain size is a few fm ⇒ charge separation in HIC

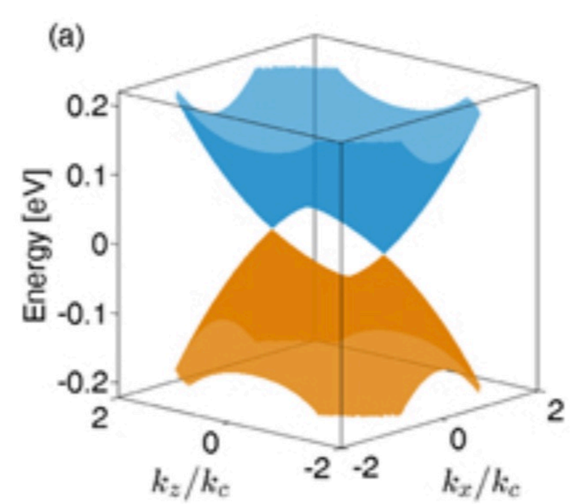
Cold nuclear matter:

Long-range topological order from the lattice calculations.

Low-dimensional long-range topological charge structure in the QCD vacuum

I. Horváth,[1] S. J. Dong,[1] T. Draper,[1] F. X. Lee,[2,3] K. F. Liu,[1] N. Mathur,[1] H. B. Thacker,[4] and J. B. Zhang[5]

Emergent chiral domains in Weyl semimetals:



FRAMEWORK: AXION ELECTRODYNAMICS

$$\mathcal{L}_{\mathrm{MCS}} = \mathcal{L}_{\mathrm{QED}} + c_A \theta(x) \vec{E} \cdot \vec{B}$$

Sikivie (84), Wilczek (87), Carroll et al (90)

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0\,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - c\,\boldsymbol{\nabla}\theta \cdot \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + c(\partial_t \theta\, \boldsymbol{B} + \boldsymbol{\nabla}\theta \times \boldsymbol{E})\,,$$

Anomalous Hall Effect (not in this talk)

Chiral magnetic effect: $\boldsymbol{j} = \sigma_\chi \boldsymbol{B}$

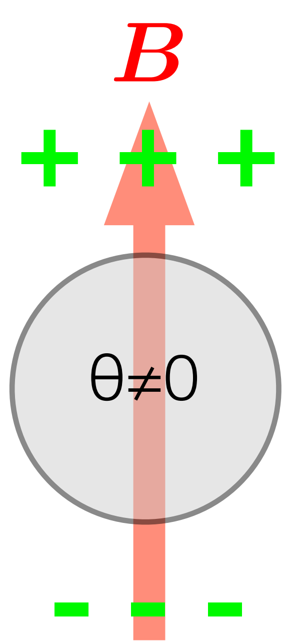
P-odd, T-odd

P-even, T-odd

⇒Breaks Parity!

Kharzeev, Zhitnitsky (2007),
Kharzeev, McLerran, Warringa (2008)

Charge separation:



Critical assumption:
existence of chiral domains.

This talk is about how the new current affects photon and gluon radiation.

1. Classical Fermi Model
2. QED
3. QCD

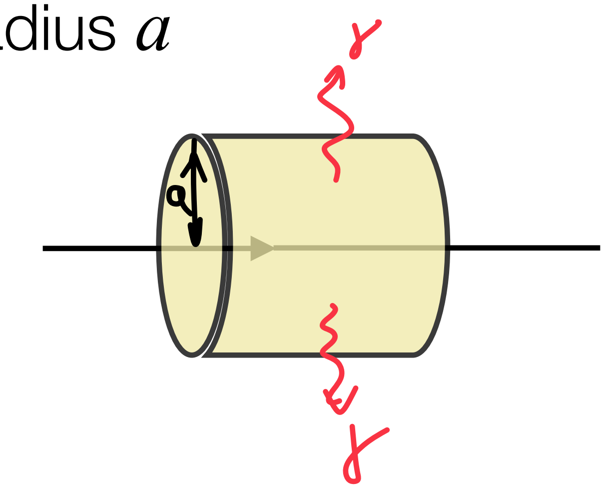
FERMI'S MODEL OF ENERGY LOSS

MARCH 15, 1940 PHYSICAL REVIEW VOLUME 57

The Ionization Loss of Energy in Gases and in Condensed Materials*

ENRICO FERMI
Pupin Physics Laboratories, Columbia University, New York, New York
(Received January 22, 1940)

The energy loss rate = flux of the Poynting vector out of cylinder of radius a coaxial with the particle path:



$$-\frac{d\varepsilon}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a \,\mathrm{Re} \int_0^{\infty} (E_{\phi\omega} B^*_{z\omega} - E_{z\omega} B^*_{\phi\omega}) d\omega$$

Maxwell equations $\boldsymbol{\nabla} \times \boldsymbol{B}_\omega = -i\omega \boldsymbol{D}_\omega + j_\omega$ etc.

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

Energy loss: $a \to 0$ UR limit: $-\frac{d\varepsilon}{dz} = \frac{q^2}{4\pi v^2}\omega_p^2 \ln \frac{v}{a\omega_p}$

(small) Cherenkov radiation contribution emerges at $a \to \infty$ if $v > 1/\sqrt{\epsilon(0)}$

EM FIELDS OF POINT CHARGE IN CHIRAL MEDIUM 1

Hansen, KT, 2012.06089

EM field of a point charge with large enough constant velocity v

$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{D} + \sigma_\chi \boldsymbol{B} + qv\hat{\boldsymbol{z}}\delta(z - vt)\delta(\boldsymbol{b})\,,$$

impact parameter

$$\nabla \cdot \boldsymbol{D} = q\delta(z - vt)\delta(\boldsymbol{b})\,,$$

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}\,,$$

$$\nabla \cdot \boldsymbol{B} = 0\,,$$

$$\boldsymbol{B}(\boldsymbol{r}, t) = \int \frac{d^2k_\perp d\omega}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t} \sum_\lambda \boldsymbol{\epsilon}_{\lambda\boldsymbol{k}} \frac{q\hat{\boldsymbol{z}} \cdot \boldsymbol{\epsilon}^*_{\lambda\boldsymbol{k}} \lambda k}{k_\perp^2 + \omega^2(1/v^2 - \epsilon) - \lambda\sigma_\chi k}\,,$$

$$\boldsymbol{E}(\boldsymbol{r}, t) = \int \frac{d^2k_\perp d\omega}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t} \left(\sum_\lambda \boldsymbol{\epsilon}_{\lambda\boldsymbol{k}} \frac{iq\omega\hat{\boldsymbol{z}} \cdot \boldsymbol{\epsilon}^*_{\lambda\boldsymbol{k}}}{k_\perp^2 + \omega^2(1/v^2 - \epsilon) - \lambda\sigma_\chi k} + \hat{\boldsymbol{k}} \frac{q}{ivk\varepsilon} \right)\,,$$

EM FIELDS OF POINT CHARGE IN CHIRAL MEDIUM 2

ω-Fourier components can be computed analytically e.g.:

$$B_{\phi\omega}(\boldsymbol{r}) = \frac{q}{2\pi}\frac{e^{i\omega z/v}}{k_1^2 - k_2^2}\sum_{\nu=1}^{2}(-1)^{\nu+1}k_\nu(k_\nu^2 - s^2)K_1(bk_\nu)$$

with $k_\nu^2 = s^2 - \frac{\sigma_\chi^2}{2} + (-1)^\nu \sigma_\chi \sqrt{\omega^2\epsilon + \frac{\sigma_\chi^2}{4}}$ and $s^2 = \omega^2\left(\frac{1}{v^2} - \epsilon(\omega)\right)$

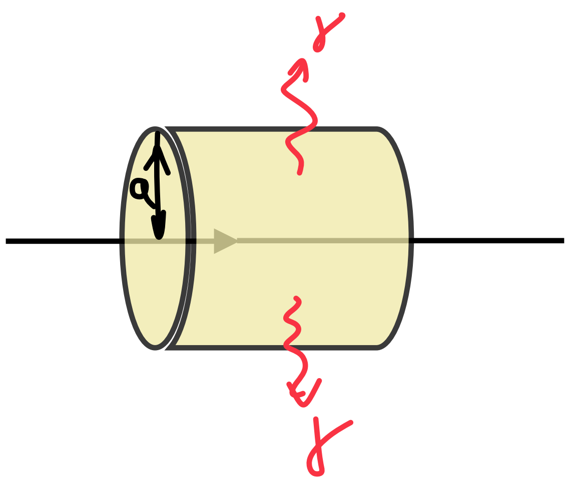
Radiation if $k_{1,2}^2 < 0$

- Cherenkov radiation: $\sigma_\chi=0$ and $k^2=s^2<0$ — small contribution to the total energy loss
- **Chiral Cherenkov radiation**: $\sigma_\chi\neq 0$ and $k^2<0$, (even when $s^2>0$)

FERMI'S MODEL WITH CHIRAL MAGNETIC CURRENT

Hansen, KT, 2012.06089

$$-\frac{d\varepsilon}{dz} = 2\pi a \int_{-\infty}^{\infty} (E_\phi B_z - E_z B_\phi) dt = 2a \,\mathrm{Re} \int_0^{\infty} (E_{\phi\omega} B^*_{z\omega} - E_{z\omega} B^*_{\phi\omega}) d\omega$$



UR limit $\gamma \gg 1$ at $a \to 0$ gives energy loss

$$-\frac{d\varepsilon}{dz} = \frac{q^2}{4\pi v^2}\left(\omega_p^2 \ln \frac{v}{a\omega_p} + \frac{1}{4}\gamma^2\sigma_\chi^2\right)$$

increases as E^2

Chiral Cherenkov radiation emerges at $a \to \infty$ even if $\epsilon = 1$

$$\frac{dW}{d\omega} = -\frac{d\varepsilon}{dz\omega d\omega}\bigg|_{a\to\infty} = \frac{q^2}{4\pi}\left\{\frac{1}{2}\left(1 - \frac{1}{v^2}\right) + \frac{\sigma_\chi}{2\omega} + \frac{(1+v^2)\sigma_\chi^2}{8v^2\omega^2} + \dots\right\}, \quad \omega < \sigma_\chi \gamma^2$$

Power of chiral Cherenkov radiation $P = \frac{q^2}{4\pi}\frac{\sigma_\chi^2\gamma^2}{4}$

*In the UR limit, energy loss is due to the **chiral** Cherenkov radiation.*

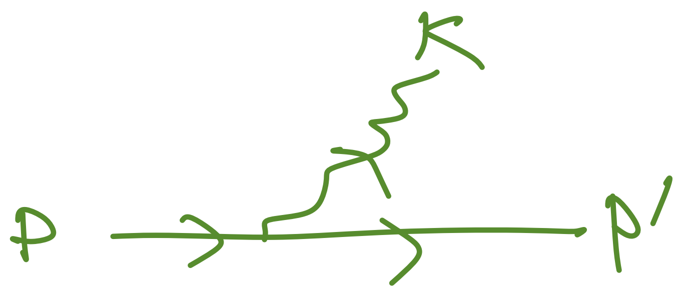
CHIRAL CHERENKOV IN QFT

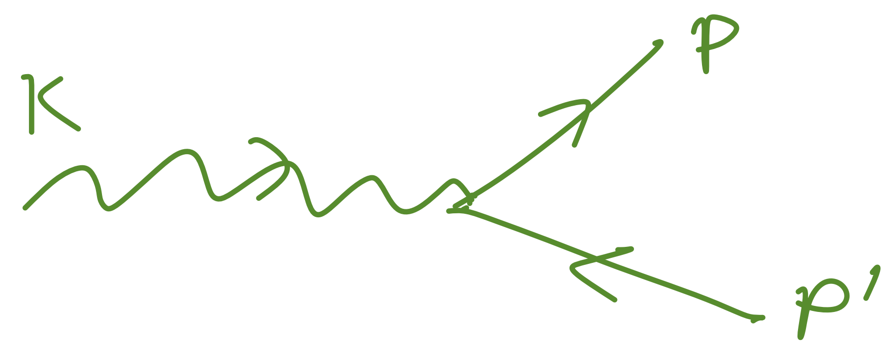
KT, 1702.07329

In radiation gauge: $\nabla^2 \boldsymbol{A} = \partial_t^2 \boldsymbol{A} - \sigma_\chi \boldsymbol{\nabla} \times \boldsymbol{A}$

The dispersion relation $k^2 = -\lambda \sigma_\chi |\boldsymbol{k}|$ $\rightarrow$ photon becomes space- or timelike

λ =helicity





$k^2 = (p \pm p')^2 = 2m(m \pm \varepsilon)$ forbidden in vacuum, but allowed in chiral medium

Pair production: $k^2 > 0 \Rightarrow \lambda \sigma_\chi < 0$

Photon radiation: $k^2 < 0 \Rightarrow \lambda \sigma_\chi > 0$

CHIRAL CHERENKOV RADIATION IN QED

KT, 1702.07329

$$\mathcal{M} = -eQ\bar{u}(p')\gamma^\mu u(p)\epsilon^*_\mu \times 4\pi\varepsilon x(1-x)\delta(q_\perp^2 + \kappa_\lambda)$$

$x = \dfrac{\omega}{\varepsilon}$ $\qquad\kappa_\lambda(z) = x^2m^2 - (1-x)x\lambda\sigma_\chi\varepsilon$ can become negative!

Chiral Cherenkov effect: photon radiation at $\vartheta \sim \sqrt{|\sigma_\chi|/\omega}$

Kappa is negative if $\lambda\sigma_\chi > 0$ and $x < x_0 = \dfrac{1}{1+m^2/(\lambda\sigma_\chi\varepsilon)}$ $\Rightarrow$ $\omega < \omega^* = \dfrac{\lambda b_0\varepsilon^2}{\lambda b_0\varepsilon + m^2}$

- Photon radiation rate:

$$\frac{dW_+}{dx} = \frac{\alpha Q^2}{2\varepsilon x}\left\{\sigma_\chi\varepsilon\left(\frac{x^2}{2} - x + 1\right) - m^2 x\right\}\theta(x_0 - x)$$

Vanishes as $\hbar\to 0$

Quantum anomaly!

$$\frac{dW_-}{dx} = 0\,.$$

Classical limit: x→0 (no recoil)

- Total rate of energy loss:

$$-\frac{d\varepsilon}{dz} = \int_0^1 \frac{dW_+}{dx}x\varepsilon dx = \frac{1}{3}\alpha Q^2\sigma_\chi\varepsilon$$

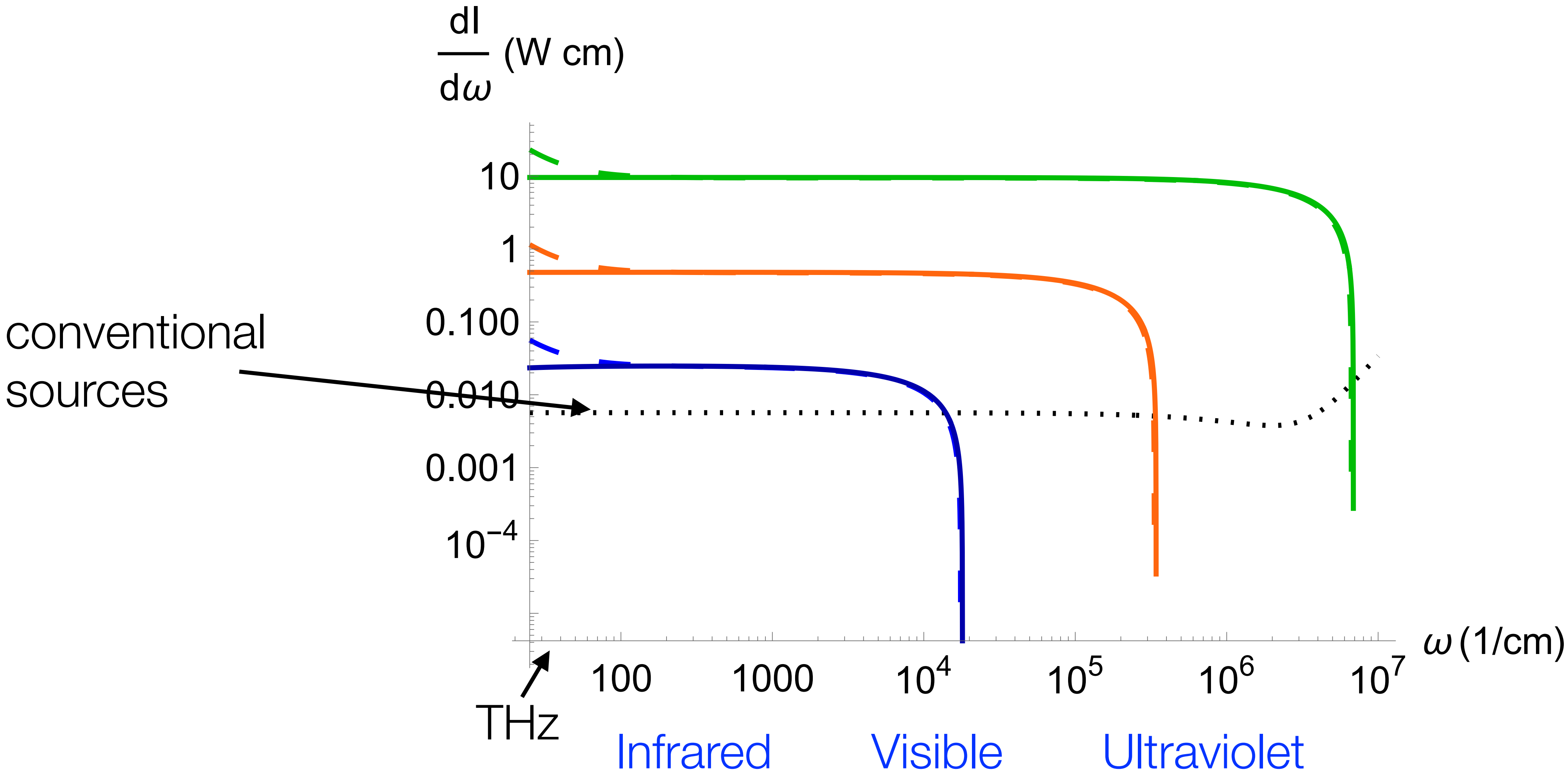
Thus the recoil reduces the energy loss $\gamma^2 \to \gamma$

CHIRAL CHERENKOV RADIATION IN A WEYL SEMIMETAL

Magnetic Weyl semimetals as a source of circularly polarized THz radiation

Jeremy Hansen, Kazuki Ikeda, Dmitri E. Kharzeev, Qiang Li, Kirill Tuchin (May 17, 2024)

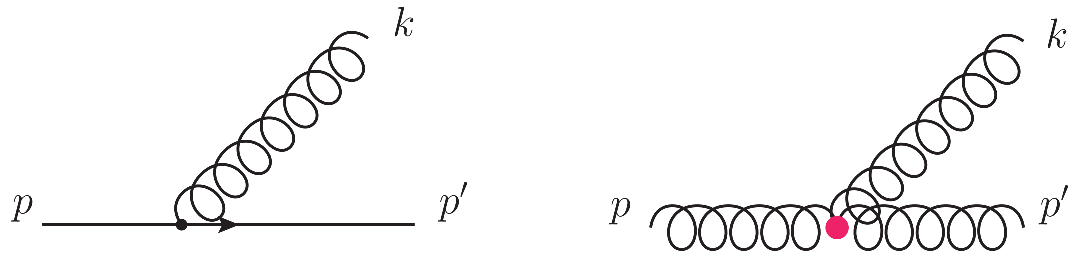
e-Print: 2405.11076 [cond-mat.mtrl-sci]



CHIRAL CHERENKOV IN QCD

Hansen KT, 2405.0897

Non-Abelian θ-term: $S_\theta = c_A \int \partial_\mu \theta \epsilon^{\mu\nu\rho\sigma} \left(\frac{1}{2} A^a_\nu \partial_\rho A^a_\sigma - g \frac{2i}{3} \frac{1}{4} i f^{abc} A^a_\nu A^b_\rho A^c_\sigma \right) d^4x$



$$\sum_{ss'} |\mathcal{M}_{q\to qg_R}|^2 = 4\left[EE' - m^2 - \frac{(\boldsymbol{k}\cdot\boldsymbol{p})(\boldsymbol{k}\cdot\boldsymbol{p}')}{\boldsymbol{k}^2}\right],$$

$$\sum_{ss'} |\mathcal{M}_{q\to qg_L}|^2 = 0\,.$$

$$i\mathcal{M}^A_{g_R\to g_R g_R} = \frac{k_\perp}{x(1-x)}\,,$$

$$iM^A_{g_L\to g_R g_L} = \frac{(1-x)k_\perp}{x}\,,$$

$$iM^A_{g_L\to g_L g_R} = \frac{x k_\perp}{(1-x)}\,,$$

$$iM^A_{g_L\to g_R g_R} = \mathcal{O}(k_\perp^2/E, b_0/E)\,,$$

$b_0 \equiv \sigma_\chi$

$$iM^B_{g\to gg} = \frac{-b_0 k_\perp(\lambda(1-x) + \lambda' x + \lambda_0)}{x(1-x)E}$$

ENERGY LOSS DUE TO CHIRAL CHERENKOV

Hansen KT, 2405.0897

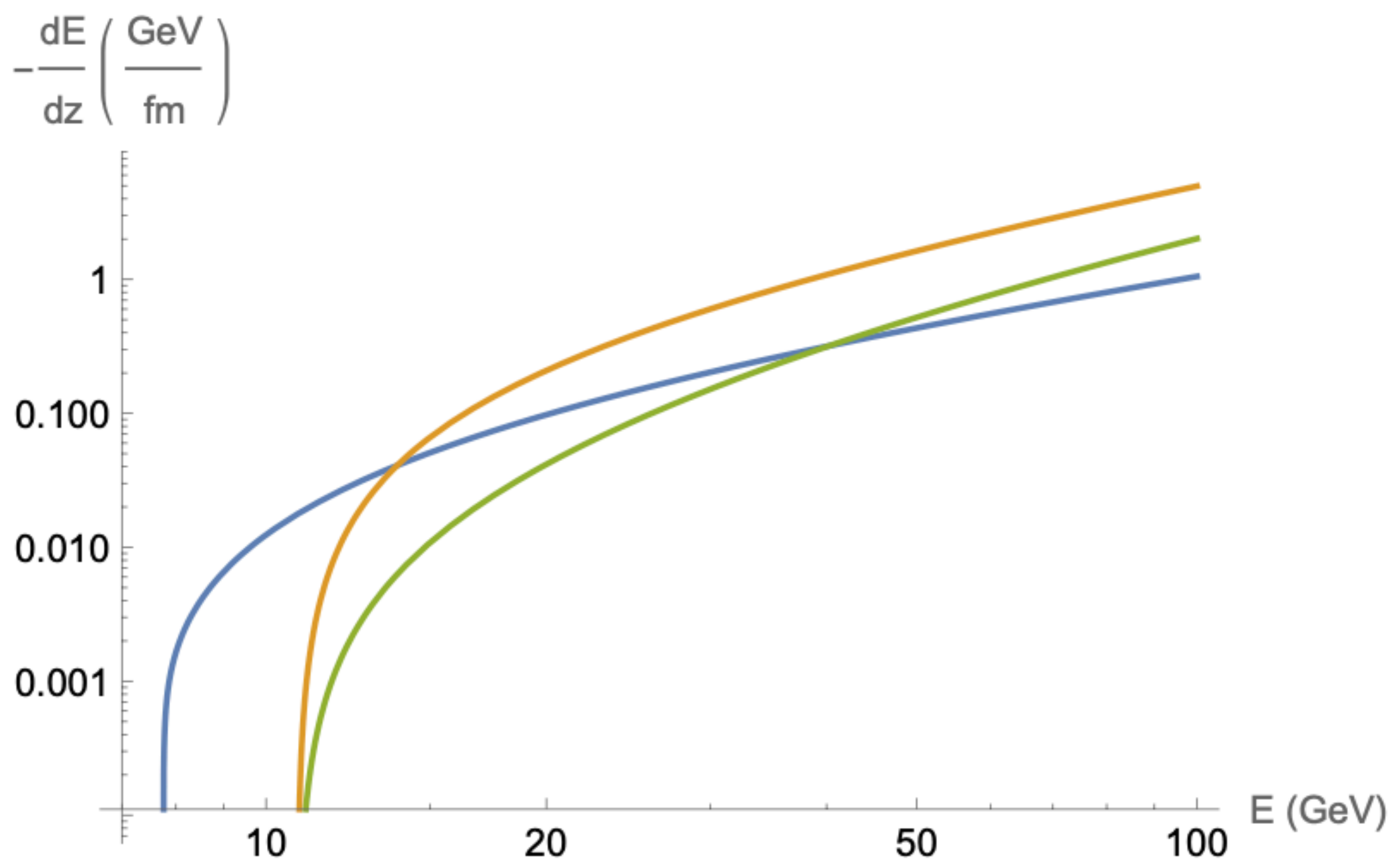


Fig. 3. The rate of energy loss due to the Color Chiral Cherenkov radiation for $q \to qg$ (blue), $g_R \to gg$ (orange), and $g_L \to gg$ (green). Parameters: $g = 2$, $T = 300$ MeV, $b_0 = 50$ MeV.

$$-\frac{dE_{q\to qg}}{dz} = \frac{4\alpha_s g^2 b_0 E}{9},$$

$$-\frac{dE_{g_R\to gg}}{dz} = \frac{3\alpha_s g^2 b_0 E}{4}\left(\ln\frac{b_0 E}{\omega_p^2} - 1\right),$$

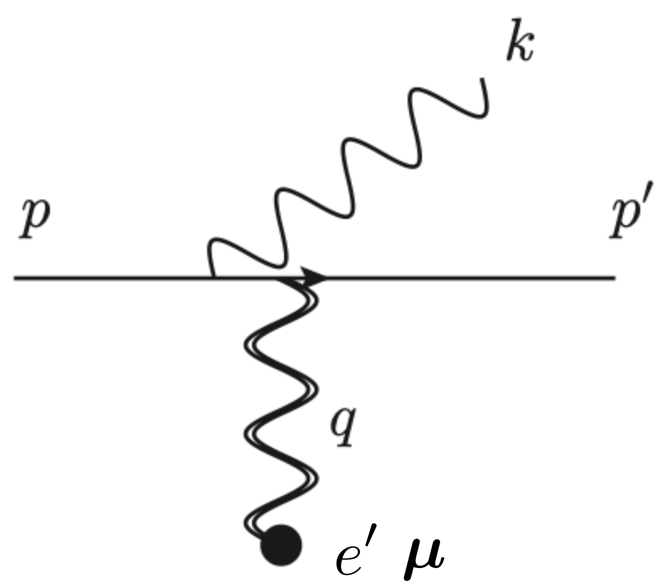
$$-\frac{dE_{g_L\to gg}}{dz} = \frac{3\alpha_s g^2 b_0 E}{4}\left(\ln\frac{b_0 E}{\omega_p^2} - \frac{17}{6}\right).$$

$b_0 \equiv \sigma_\chi$

What about bremsstrahlung?

RADIATIVE ENERGY LOSS

Hansen, KT, 2203.13134



Photon propagator (static limit):

$$D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2}\,,$$

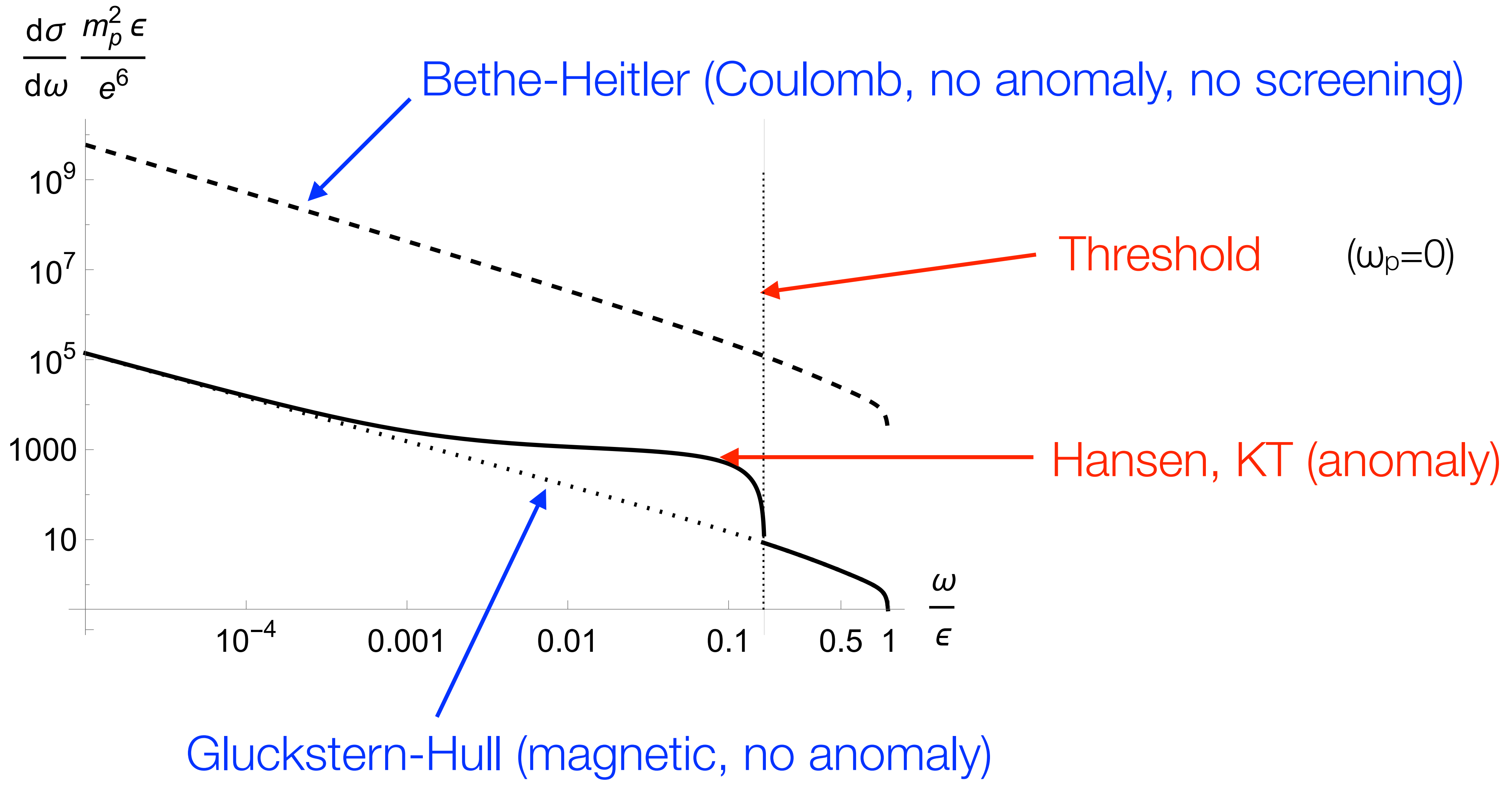
$$D_{0i}(\boldsymbol{q}) = D_{0i}(\boldsymbol{q}) = 0\,,$$

$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$

resonance!

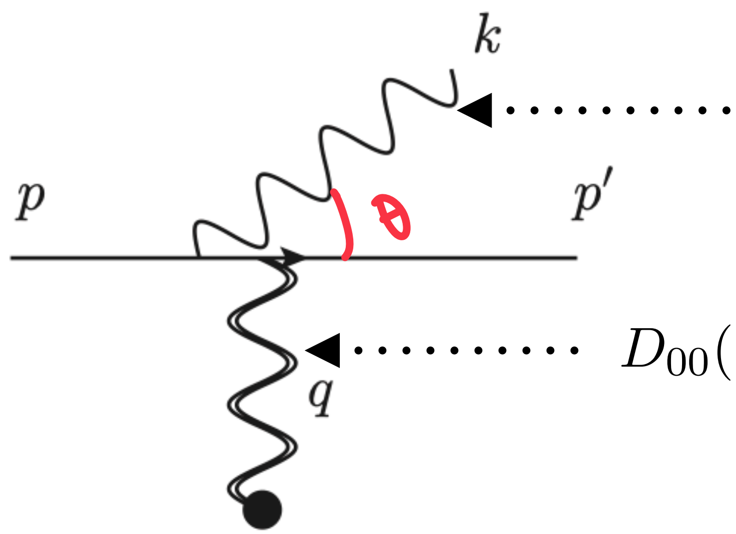
- D_{ij} couples only to the magnetic moment of the target $\boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{\nabla} \times (\boldsymbol{\mu}\delta(\boldsymbol{x}))$

MAGNETIC MOMENT CONTRIBUTION TO PHOTON BREMSSTRAHLUNG



ELECTRIC MONOPOLE CONTRIBUTION TO PHOTON BREMSSTRAHLUNG

Hansen, KT, 2307.05761



$$\omega^2 = \boldsymbol{k}^2 + k^2 = \boldsymbol{k}^2 - \lambda b_0 |\boldsymbol{k}|$$

$$D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2}$$

- Fermion propagator: $\frac{1}{2p \cdot k - k^2 + iE/\tau} = \frac{1}{\omega E \left(\frac{m^2}{E\omega} \frac{\omega - \omega^*}{E - \omega^*} + \theta^2 + \frac{i}{\omega \tau} \right)}$ with $\omega^* = \frac{\lambda b_0 E^2}{\lambda b_0 E + m^2}$

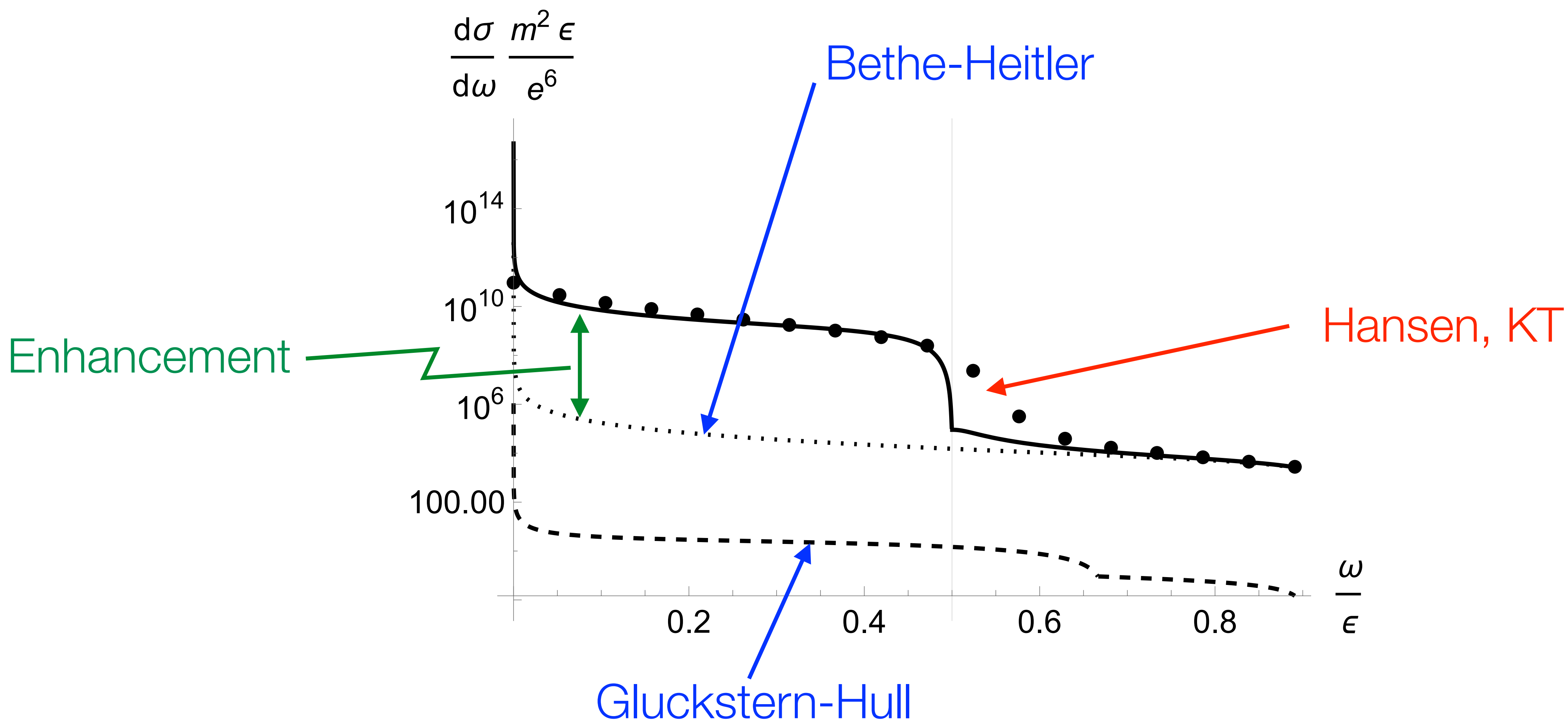
  $b_0 \lambda > 0$

- The resonance emerges when $\omega < \omega^*$ due to the anomaly in the photon dispersion relation.

- The photon propagator has similar behavior: $\boldsymbol{q}^2_{\rm min} = \frac{1}{4} \frac{\omega^2 E^2}{E'^2} \left[\frac{m^2(\omega - \omega^*)}{\omega E(E - \omega^*)} + \theta^2 \right]^2$

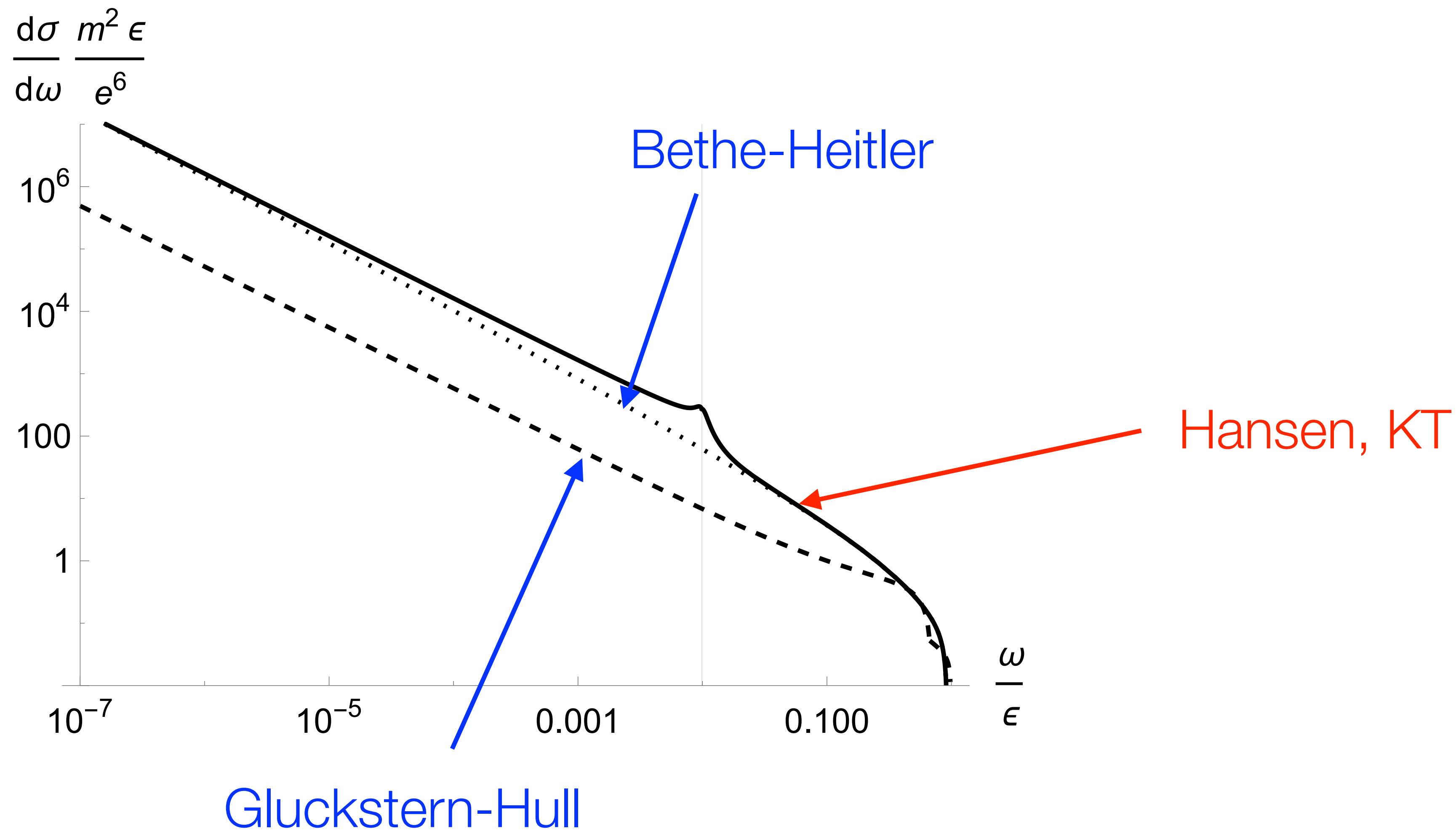
WEAK SCREENING $\mu \ll m$

μ-Debye mass



- Only one photon polarization ($b_0\lambda>0$) is enhanced!
- At higher energy - must take LPM into account.

STRONG SCREENING $\mu \gg m$



- The effect of anomaly is reduced by screening.

SUMMARY

- Chiral anomaly induces Chiral Cherenkov radiation which enhances energy loss
- Chiral Cherenkov is local effect (unlike chiral separation)
- It certainly affects jets in QGP, and maybe at EIC.
- Interdisciplinary applications: Weyl semimetals, dark matter (axions).