Renormalization group approach to collinear parton production in nuclear matter

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INT program: Heavy Ion Physics in the EIC Era (24-2b)
Seattle, WA July 29 – August 3, 2024

Outline of the talk

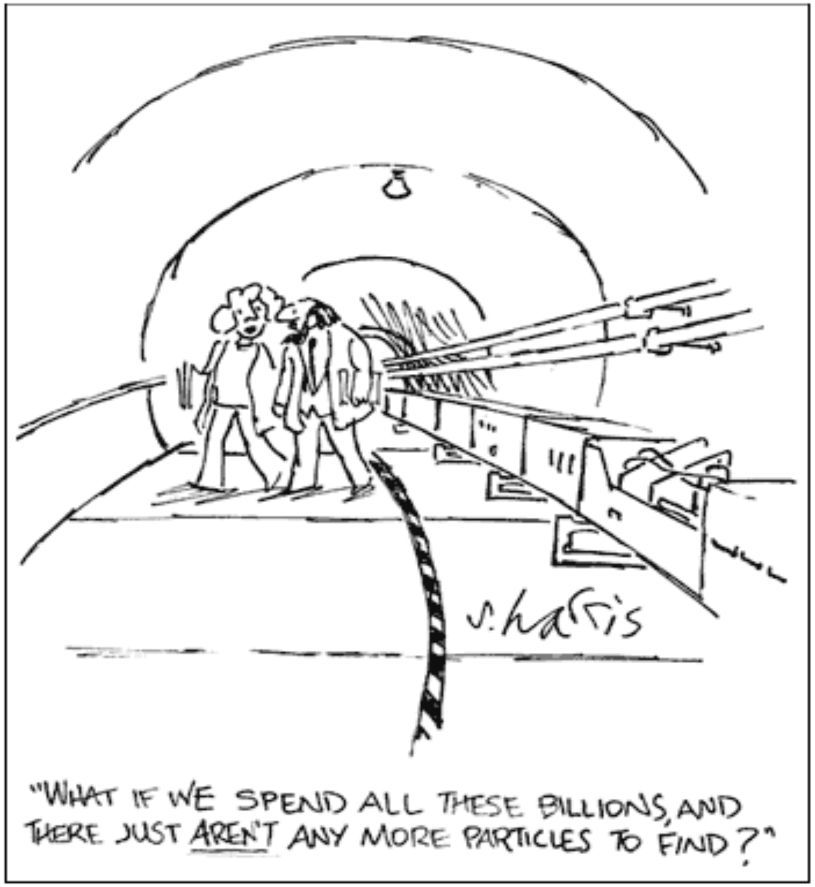
- **Introduction and motivation. Parton showers in matter and their properties**
- **In-medium DGLAP evolution. Centrality dependence of hadron production in SIDIS**
- **Renormalization group analysis of parton shower evolution in matter. Application to DIS**
- **Possible extension to heavy flavor. Conclusions**

Largely based on the following papers: 2301.11940 [hep-ph] (RG evolution); 2303.14201 [hep-ph] (mDGLAP, mSiJF centrality); 1903.06170 [hep-ph] (Full medium branching); refs therein

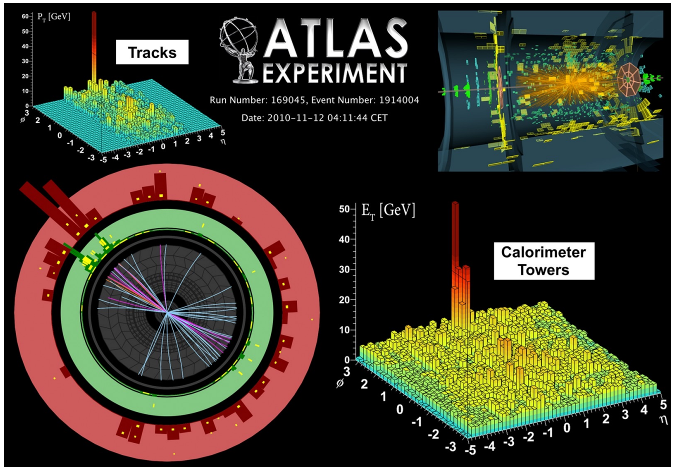
i) Thanks to the INT for hosting this program
ii) Credit for the work presented goes to my collaborators – for these works W. Ke, H. Li, Z. Liu, M. Sievert, B. Yoon (+ many others)

For hadron and jet production and modification at the EIC see the "Yellow report" *R. Abdul-Khalek et al. (2021)*

Introduction and motivation

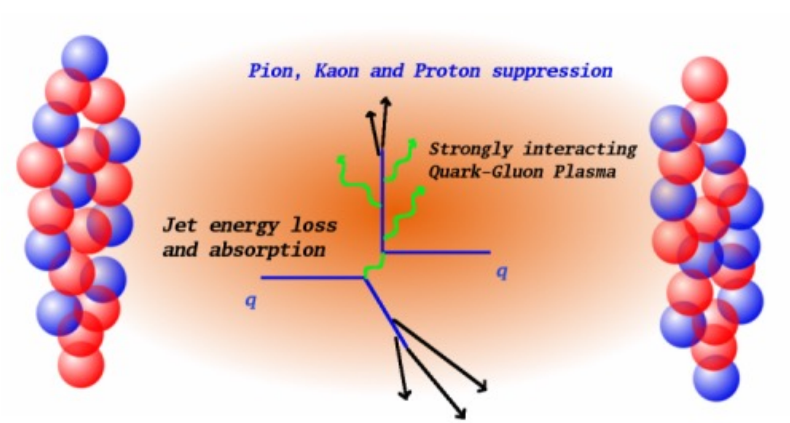


Parton showers in the vacuum and in nuclear matter



Our goal is to achieve an accurate, systematically improvable description of hadron, heavy flavor and jet production

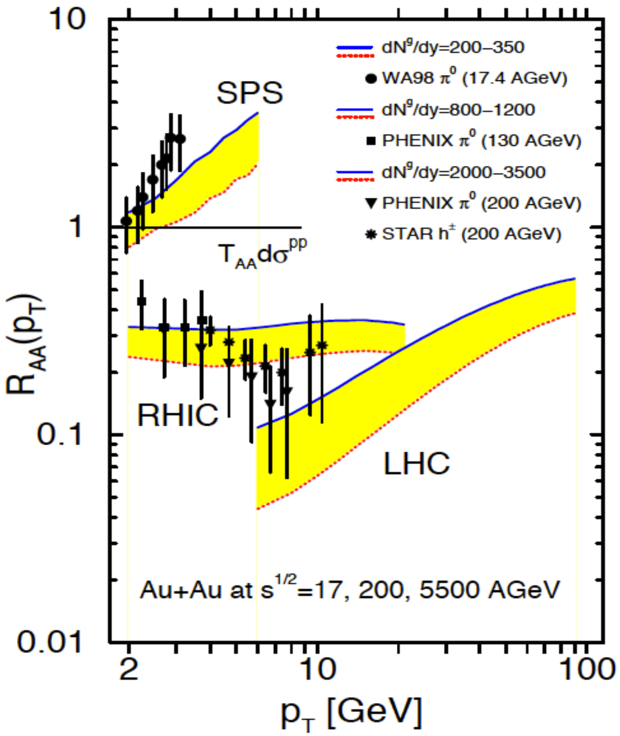
In the description of high energy processes significant effort has been devoted to logs, legs and loops



- Log - ratios of mass and energy scales, phase space, cuts. Goal is to resum
- Legs – the formation of parton shower, branchings, evolution
- Loops – virtual corrections. Goal is to include, find automated way to do some of the loops

Similar challenges exist in heavy-ion physics and similar theoretical approaches can be adapted to reactions with nuclei

Radiative energy loss processes and jet quenching



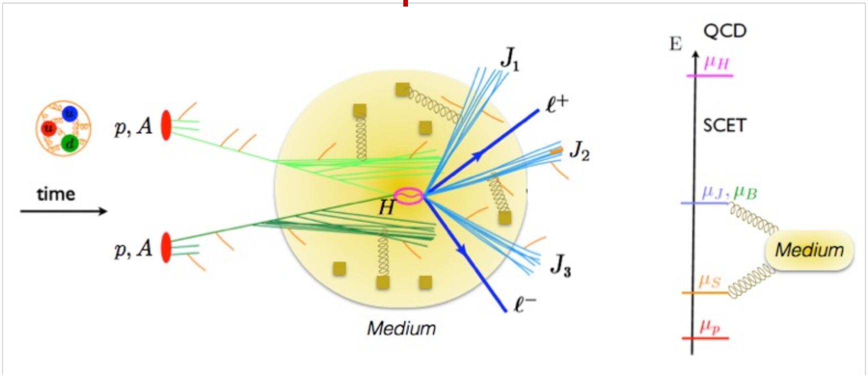
Discovery of jet quenching

RHIC (though not the first HI machine) has played a very important role in truly developing a new field – interaction of hard probes in matter. Motivated energy loss studies

M. Gyulassy et al. (1993)
B. Zakharov (1995)
R. Baier et al. (1997)
M. Gyulassy et al. (2000)
X. Guo et al. (2001)
P. Arnold et al. (2002)

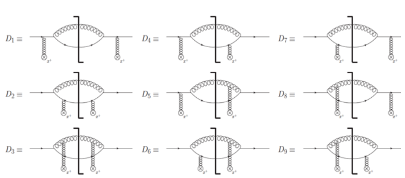
Very successful but with limitations: not systematically improvable, limited connection to established QCD techniques

QCD in the medium remains a multi-scale problem. As such, it is well suited to an EFT approach

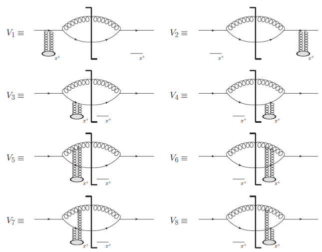


A. Idilbi et al. (2008) *G. Ovanesyan et al. (2011)*

EFTs for parton showers in matter



Single Born



Double Born

- Evaluated using EFT approaches - SCET_G , $\text{SCET}_{M,G}$
- Cross checked using light cone wavefunction approach
- Factorize from the hard part
- Gauge invariant
- Contain non-local quantum coherence effects (LPM)
- Depend on the properties of the nuclear medium

Compute analogues of the Altarelli-Parisi splitting functions
Enter higher order and resumed calculations

$$A_\perp = k_\perp,\ B_\perp = k_\perp + x q_\perp,\ C_\perp = k_\perp - (1-x) q_\perp,\ D_\perp = k_\perp - q_\perp.$$

$$\Omega_1 - \Omega_2 = \frac{B_\perp^2 + \nu^2}{p_0^+ x(1-x)},\ \Omega_1 - \Omega_3 = \frac{C_\perp^2 + \nu^2}{p_0^+ x(1-x)},\ \Omega_4 = \frac{A_\perp^2 + \nu^2}{p_0^+ x(1-x)},$$

Kinematic variables and mass dependence

$$\begin{aligned} \nu &= m & (g \to Q\bar{Q}), \\ \nu &= x\, m & (Q \to Qg), \\ \nu &= (1-x)\, m & (Q \to gQ), \end{aligned}$$

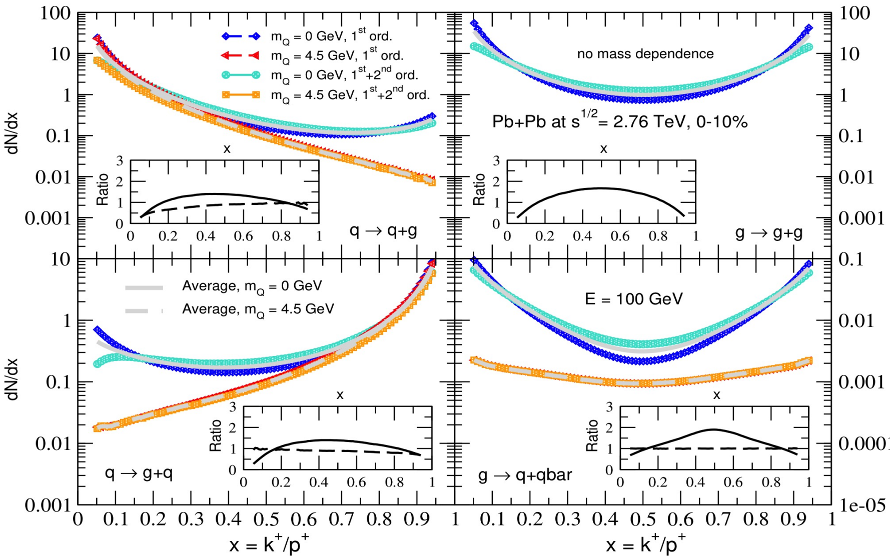
Quark to quark splitting function example

$$\left(\frac{dN^{\rm med}}{dx d^2 k_\perp}\right)_{Q\to Qg} = \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 q_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\rm med}}{d^2 q_\perp} \left\{ \left(\frac{1 + (1-x)^2}{x} \right) \left[\frac{B_\perp}{B_\perp^2 + \nu^2} \right.\right.$$
$$\times \left(\frac{B_\perp}{B_\perp^2+\nu^2} - \frac{C_\perp}{C_\perp^2+\nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{C_\perp}{C_\perp^2+\nu^2} \cdot \left(2\frac{C_\perp}{C_\perp^2+\nu^2} - \frac{A_\perp}{A_\perp^2+\nu^2} \right.$$
$$\left. - \frac{B_\perp}{B_\perp^2+\nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_3)\Delta z]) + \frac{B_\perp}{B_\perp^2+\nu^2} \cdot \frac{C_\perp}{C_\perp^2+\nu^2} (1 - \cos[(\Omega_2 - \Omega_3)\Delta z])$$
$$+ \frac{A_\perp}{A_\perp^2+\nu^2} \cdot \left(\frac{D_\perp}{D_\perp^2+\nu^2} - \frac{A_\perp}{A_\perp^2+\nu^2} \right) (1 - \cos[\Omega_4 \Delta z]) - \frac{A_\perp}{A_\perp^2+\nu^2} \cdot \frac{D_\perp}{D_\perp^2+\nu^2} (1 - \cos[\Omega_5 \Delta z])$$
$$\left. + \frac{1}{N_c^2} \frac{B_\perp}{B_\perp^2+\nu^2} \cdot \left(\frac{A_\perp}{A_\perp^2+\nu^2} - \frac{B_\perp}{B_\perp^2+\nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) \right]$$
$$\left. + x^3 m^2 \left[\frac{1}{B_\perp^2+\nu^2} \cdot \left(\frac{1}{B_\perp^2+\nu^2} - \frac{1}{C_\perp^2+\nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \ldots \right] \right\}$$

Z. Kang et al. (2016) *M. Sievert et al. (2019)*

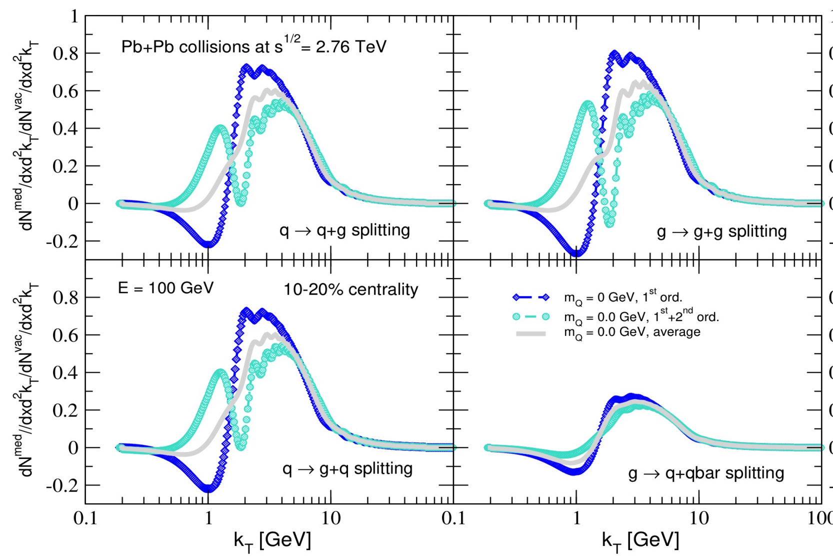
Properties of in-medium showers

Longitudinal (x) distribution



- In-medium parton showers are **softer and broader** than the ones in the vacuum
- There is even more matter-induced **soft gluon emission** enhancement

Angular (k_T) distribution – relative to vacuum



B. Yoon et al. (2019)

- **Enhancement of wide-angle radiation**, implications for reconstructed jets and jet substructure
- Limited to specific kinematic regions
- **Medium-induced scaling violations, new contributions to the jet function**

Same behavior in cold nuclear matter

Semi-Inclusive DIS

"I'm firmly convinced that behind every great man is a great computer."

In-medium evolution of fragmentation functions

Medium-induced splitting functions provide correction to vacuum showers and correspondingly modification to DGLAP evolution for FFs

$$\frac{\mathrm{d}D_q(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_q\left(\frac{z}{z'},Q\right)+P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

$$\frac{\mathrm{d}D_{\bar{q}}(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_{\bar{q}}\left(\frac{z}{z'},Q\right)+P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

$$\frac{\mathrm{d}D_g(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{g\to gg}(z',Q)D_g\left(\frac{z}{z'},Q\right) + P_{g\to q\bar{q}}(z',Q)\left(D_q\left(\frac{z}{z'},Q\right)+f_{\bar{q}}\left(\frac{z}{z'},Q\right)\right)\right\}.$$

- Fully numerical implementation, including the determination of virtual corrections
- Phenomenologically successful, e.g. verified predictions for hadron suppression in heavy ion collisions at the LHC

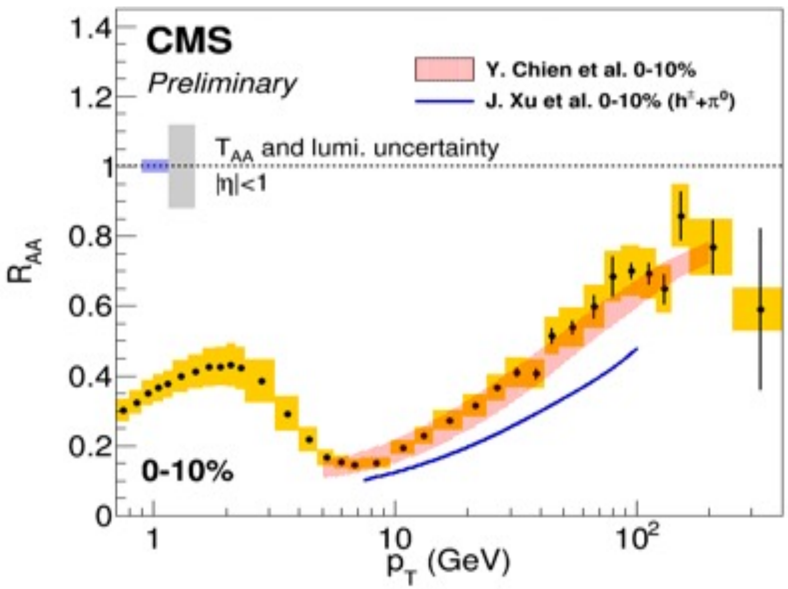
N. Chang et al. (2014)

Hybrid approach

Z. Kang et al. (2014)

Y-T. Chien et al. (2015)

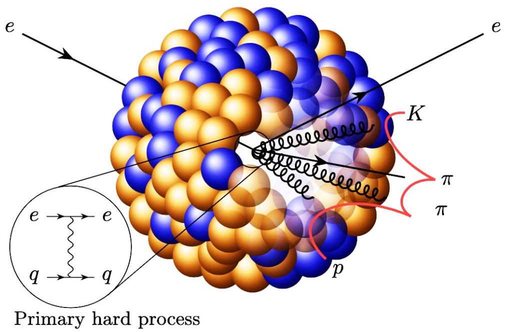
Fully medium DGLAP



Predictions vs charged hadrons at in central Pb+Pb at the LHC

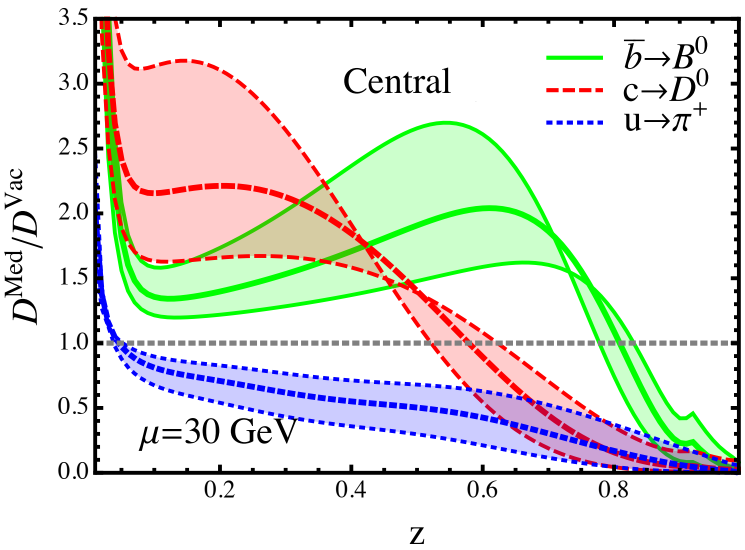
How it works in practice

Account for nuclear geometry, i.e. the production point and the path length of propagation of the hard parton in minimum bias collisions



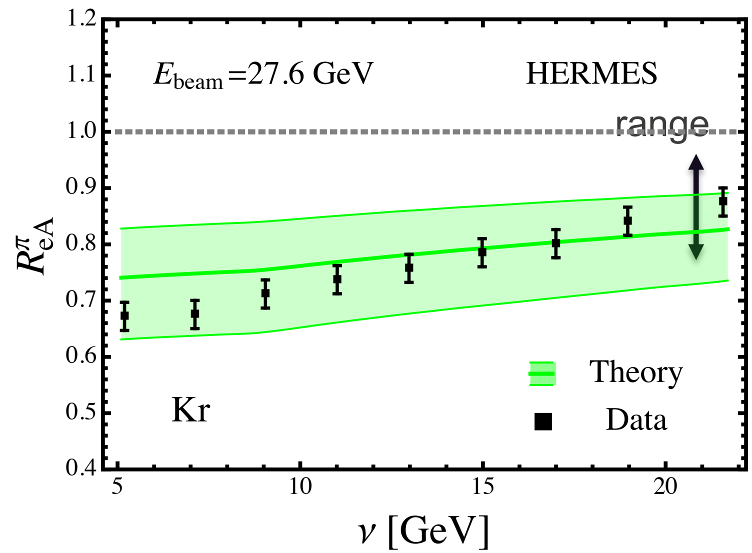
Normalization by an inclusive process is useful

$$R^{\pi}_{eA}(\nu, Q^2, z) = \frac{\left.\frac{N^{\pi}(\nu,Q^2,z)}{N^{e}(\nu,Q^2)}\right|_A}{\left.\frac{N^{\pi}(\nu,Q^2,z)}{N^{e}(\nu,Q^2)}\right|_D}$$



Full calculation in the QCD factorization formalism

$$\frac{d\sigma^{\ell N\to hX}}{dy_h d^2\mathbf{p}_{T,h}} = \frac{1}{S}\sum_{i,f}\int_0^1 \frac{dx}{x}\int_0^1 \frac{dz}{z^2} f^{i/N}(x,\mu) \times\left[\hat{\sigma}^{i\to f} + f^{\gamma/\ell}_{\rm ren}\left(\frac{-t}{s+u},\mu\right)\hat{\sigma}^{\gamma i\to f}\right] \times D^{h/f}(z,\mu)\,,$$



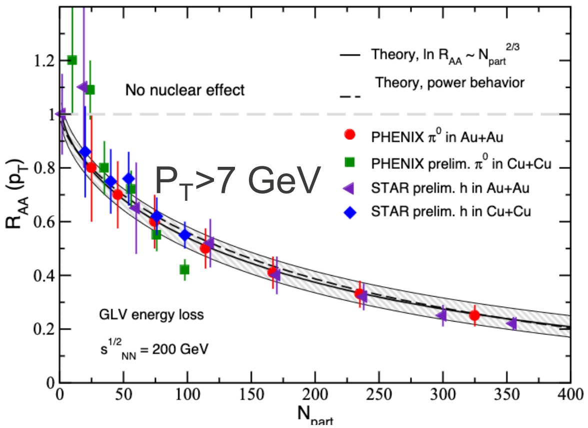
CNM transport properties for numerics – the uncertainty band can be further reduced

$$2\frac{{\mu_D}^2}{\lambda q} = 0.053\frac{GeV^2}{fm} \quad (vary \times 2,/2)$$

$$2\frac{{\mu_D}^2}{\lambda g} = 0.12\frac{GeV^2}{fm} \quad (vary \times 2,/2)$$

The concept of centrality

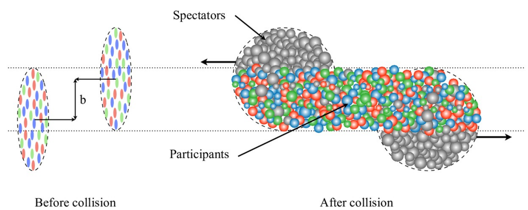
Centrality-dependent measurements are the bread and butter of heavy ion physics



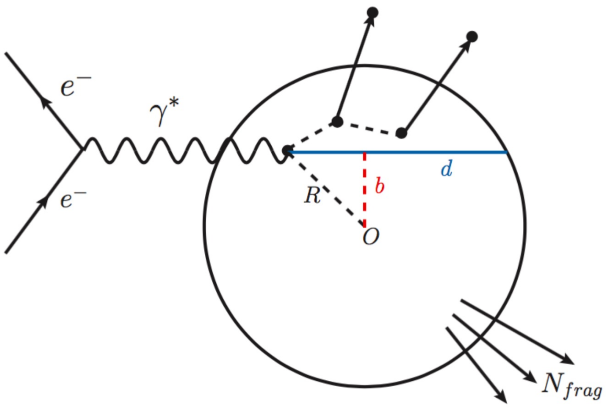
Centrality dependent hadron quenching at RHIC

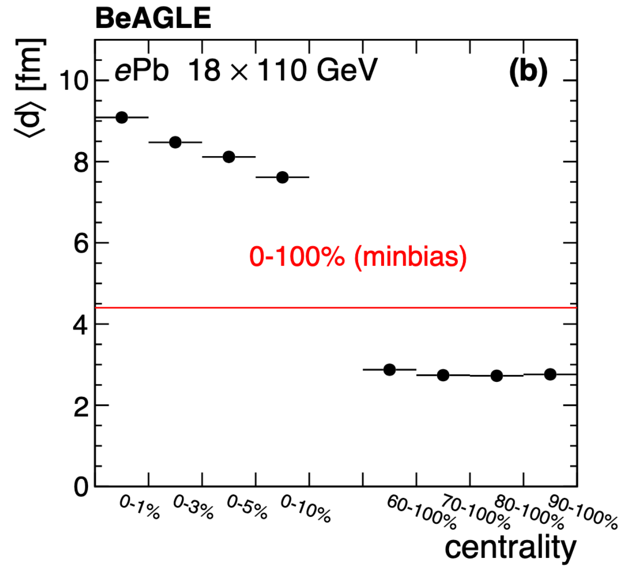
I. Vitev (2006)

A way to determine the size of the interacting system



Has never been done in DIS





Centrality dependent measurements emphasize the dynamical nature of nuclear effects

BeAGLE – centrality can be determined from the neutrons detected in the ZDC, <d>

Robust with respect to nuclear effects – shadowing, particle formation times

W. Chang et al. (2022)

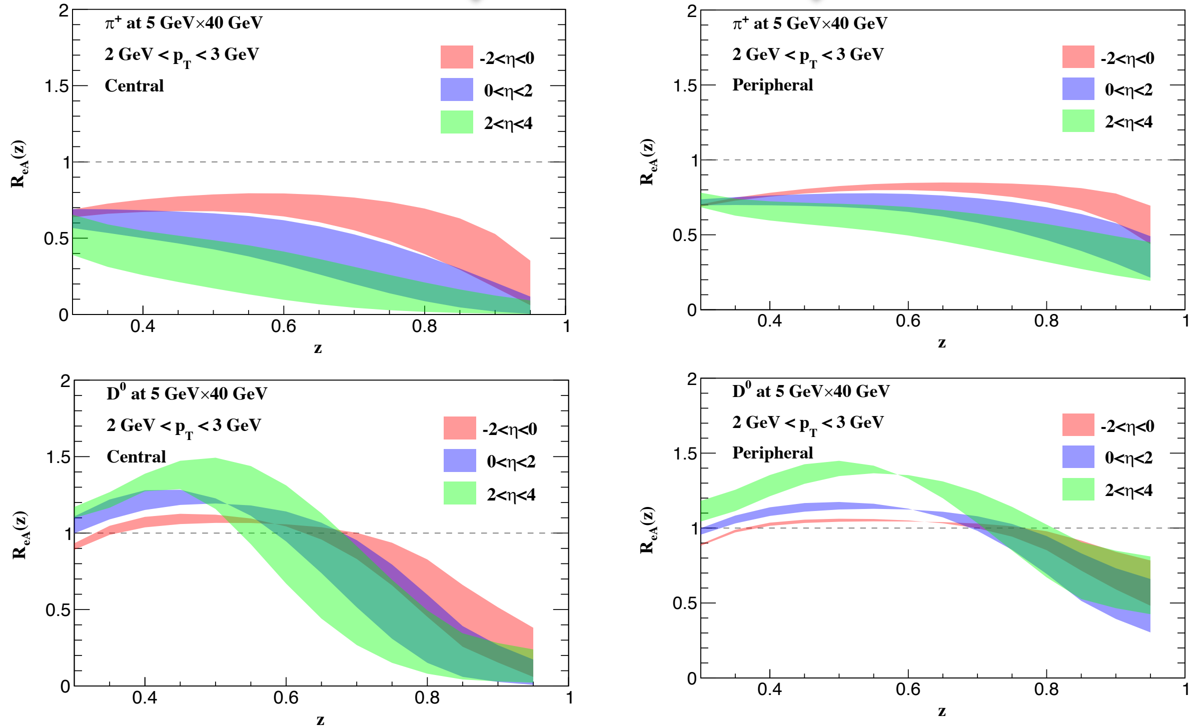
Phenomenological results – light and heavy mesons and hadronization

The observable (normalized by a large radius jet)

$$R^h_{eA}(z) = \frac{\frac{N^h(p_T,\eta,z)}{N^{\rm inc}(p_T,\eta)}\Big|_{eA}}{\frac{N^h(p_T,\eta,z)}{N^{\rm inc}(p_T,\eta)}\Big|_{ep}}$$

- Modifications to hadronization grow form backward to forward rapidity
- Transition from enhancement to suppression for heavy flavor
- Modifications to hadronization for light and heavy mesons is very different

Centrality	$0-1\%$	$0-3\ \%$	$0-10\ \%$	$60-100\ \%$	$80-100\ \%$	$90-100\ \%$	$0-100\ \%$
$\langle d\rangle[fm]$	9.09	8.48	7.61	2.88	2.71	2.71	4.40
$\langle d\rangle/\langle d\rangle_{\rm min.bias}$	2.07	1.93	1.73	0.65	0.62	0.62	1.00



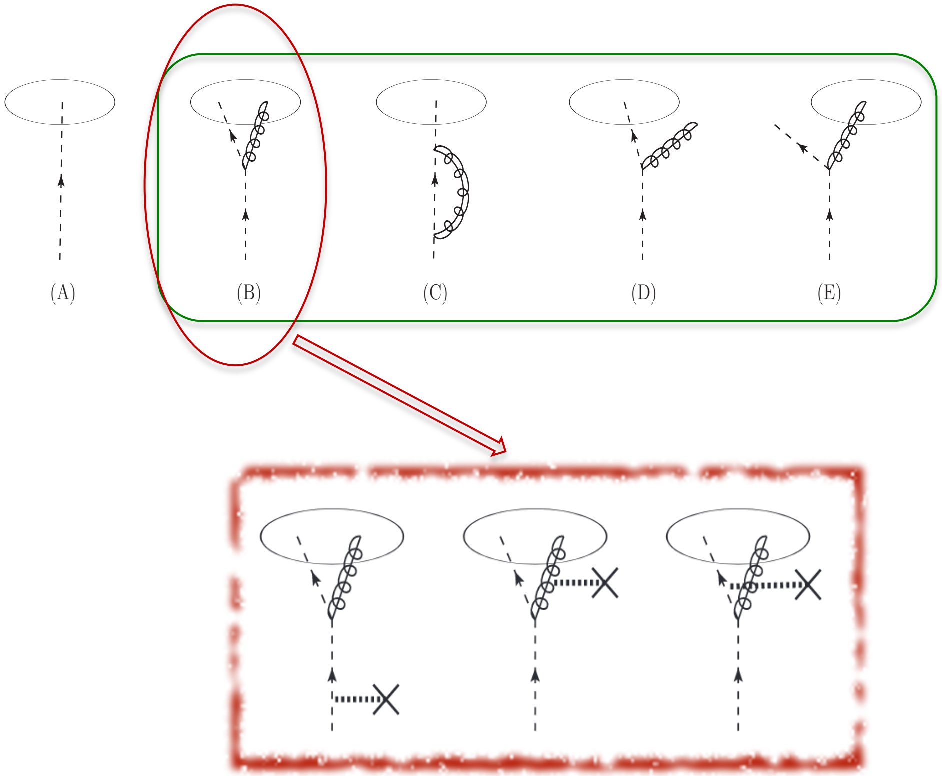
Central **Peripheral**

Analysis of light and heavy mesons and centrality will differentiate between paradigms of modifications to hadronization

A note on jet production and FO corrections

A useful modern way (though not unique) to calculate jet cross sections

SiJF up to NLO



Factorization formula

$$E_J \frac{d^3\sigma^{lN\to jX}}{d^3P_J} = \frac{1}{S}\sum_{i,f}\int_0^1 \frac{dx}{x}\int_0^1 \frac{dz}{z^2} f_{i/N}(x,\mu) \times \hat{\sigma}^{i\to f}(s,t,u,\mu) J_f(z, p_T R, \mu) \ ,$$

$$\mu_J = \omega_J \tan\frac{\mathcal{R}}{2} = (2p_T\cosh\eta)\tan\left(\frac{R}{2\cosh\eta}\right) \approx p_T R$$

In-medium jet functions

$$J_q^{\mathrm{med},(1)}(z,\omega R,\mu) = \left[\int_{z(1-z)\omega\tan(R/2)}^{\mu} dq_\perp P_{qq}(z,q_\perp)\right]_+ + \int_{z(1-z)\omega\tan(R/2)}^{\mu} dq_\perp P_{gq}(z,q_\perp)$$

- Stable in numerical implementation
- Similarly for gluon jets

Z. Kang et al. (2016)

Cross section contribution

$$d\sigma_{\mathrm{PbPb}}^{\mathrm{jet,med}} = \sum_{i=q,\bar{q},g} \sigma_i^{(0)} \otimes J_i^{\mathrm{med}}$$

Jet results and summary

- **Define nuclear modification**

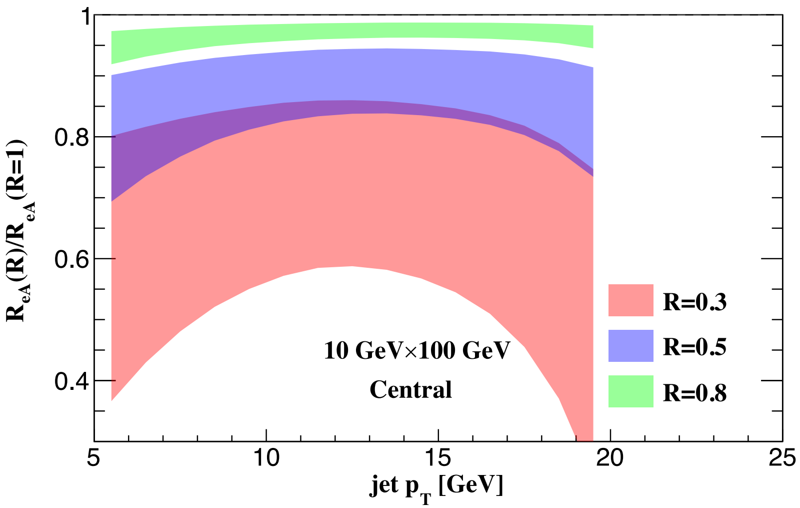
$$R_{\mathrm{eA}}(R) = \frac{1}{\Delta_b T_A(b)} \frac{\int_{\eta 1}^{\eta 2} d\sigma/d\eta dp_T|_{e+A}}{\int_{\eta 1}^{\eta 2} d\sigma/d\eta dp_T|_{e+p}}$$

- **Eliminate initial-state effects (to a few %**

$$R_R = R_{eA}(R)/\ R_{eA}(R=1)$$

H. Li et al. (2021)

Central collisions



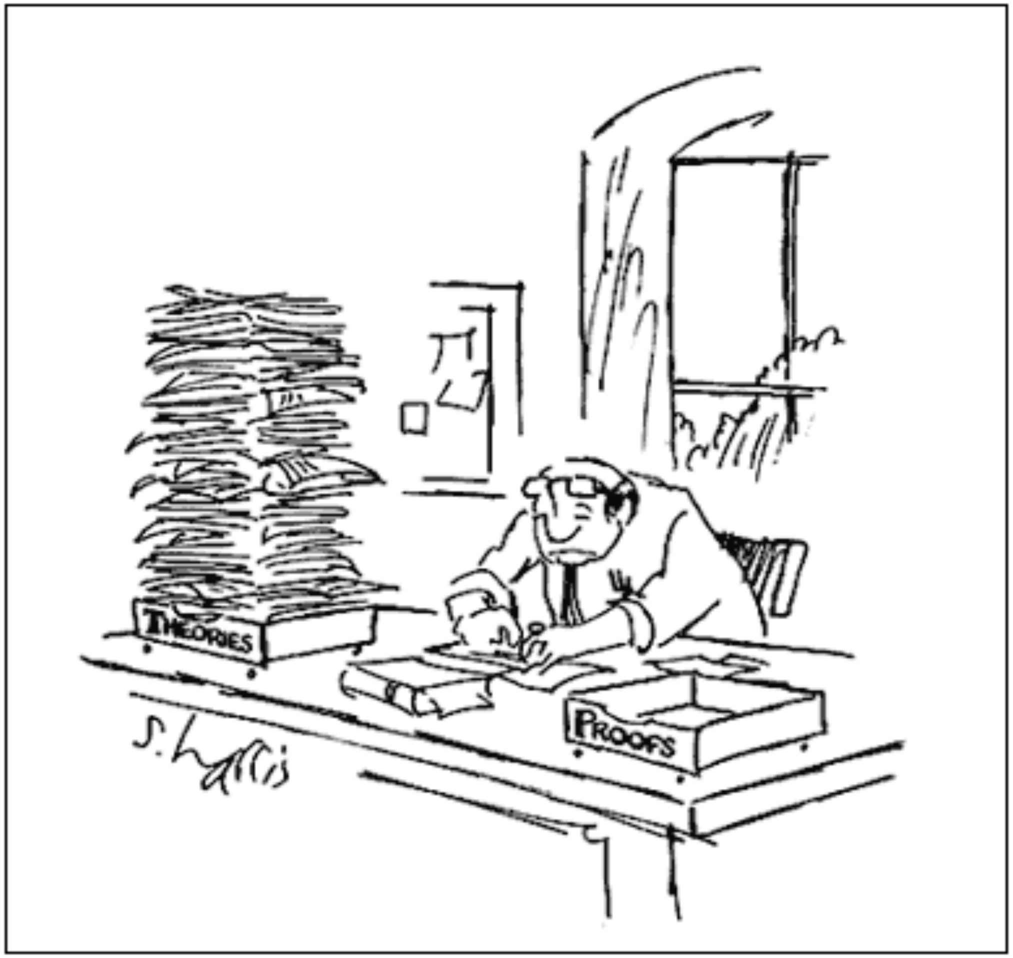
- The name of the game is to go as far forward as possible (Y) and lower CM energy, subject to being able go measure the channel. It is an optimization problem

Jets and hadrons – factors of 2-3, centrality can enhance effects, as well as differential measurements. Substructure effects generically hover around 10% even under in favorable kimematics

Summary of mDGLAP, and FO jet and jet substructure calculations for the EIC

- Light and heavy hadrons: 2007.10994
- Light/inclusive jets: 2010.05912
- Heavy flavor jets: 2108.07809
- Centrality dependence of hadron and jet modification: 2303.14201

New RG approach in evolution in matter



The renormalization group

The theory of how to connect physics at different scales. Applicable to "a number of problems in science which have, as a common characteristic, that complex microscopic behavior underlies macroscopic effects."

K. Willson (1982)
Nobel Prize speech

Origins can be traced to:

Ideas of scale transformations in QED

M. Gell-Man, I. Low (1954)

Handling infinities in field theories

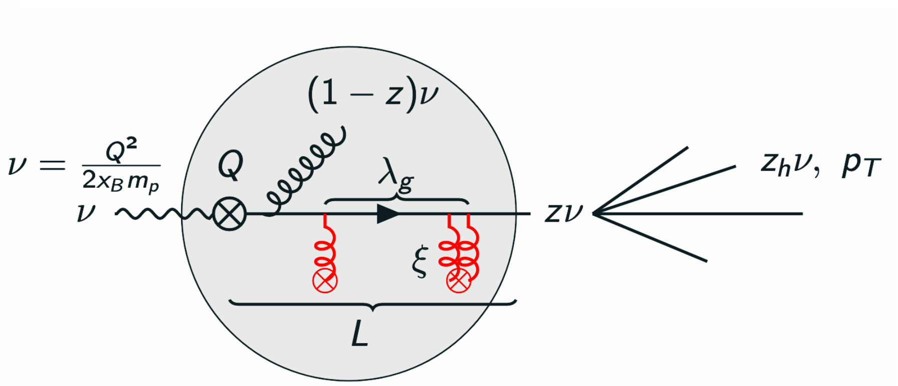
R. Feynman, J. Schwinger, S. Tomonaga, Nobel Prize (1965)

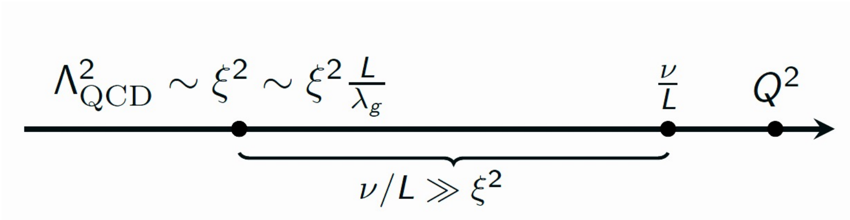
- Hydrodynamics — *V. Yakhot et al. (1986)*
- Social networks — *M. Newman et al. (1999)*
- Low energy nuclear physics — *S. Bogner et al. (2007)*
- Small-x physics — *J. Jalilian-Marian et al. (1998)*

In particle and nuclear physics – a way of making sense of inherently divergent theories

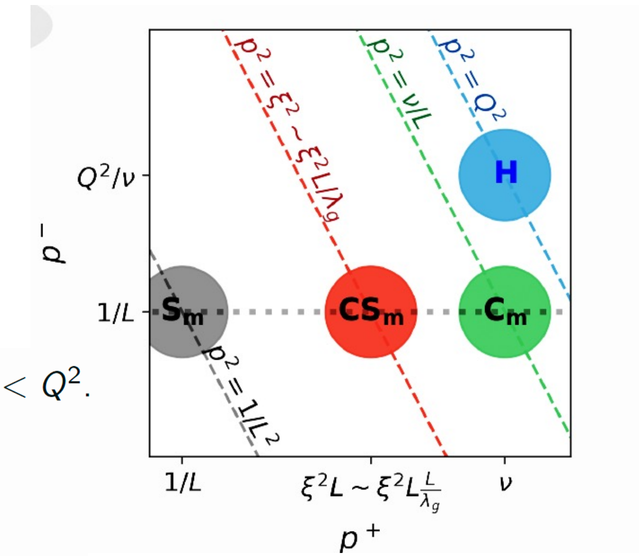
Scales in the in-medium parton shower problem

In-medium DGLAP does not tell us what kind of large logs are being resummed





- Medium-size sensitive modes have $p^- \sim \frac{1}{L} \Longrightarrow \lambda = \frac{1}{\sqrt{\nu L}}$.
  - $p_c^2 \sim q^2 \sim \nu \cdot \frac{1}{L}$ a semi-hard scale for thin medium!
  - $p_s^2 \sim 1/L^2$, non-perturbative.
- Consider eA DIS at moderately large x_B ($x_B \gtrsim 0.1$) such that $\frac{\nu}{L} \sim \frac{Q^2}{10 x_B A^{1/3}} < Q^2$.
- "The semi-hard scale $\frac{\nu}{L}$" $\gg$ "the average q_T^2 transfer $\xi^2 \frac{L}{\lambda_g}$".
- This work further assumes $L/\lambda_g = \mathcal{O}(1)$.



Modes in the virtuality plane

We encounter many ratios of scales in DIS on nuclei. Will resum large logarithms of Q/Qo and E/ξ^2L

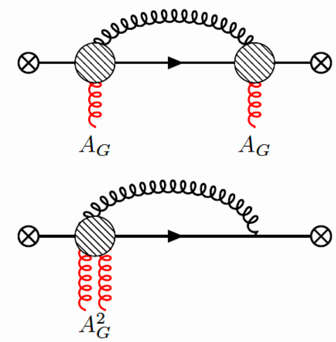
Let's revisit the calculation of semi-inclusive hadron production

- **Consider differential hadron production in ep and eA**

$$\frac{d\sigma_{ep\to h}}{dx_B\,dQ^2 dz_h} = \frac{2\pi\alpha_e^2}{Q^4}\sum_{i,j}\underbrace{e_q^2 f_{i/A}(x_B)\otimes C_{ij}^h(x,z)}_{F_{ij}(z)}\otimes d_{h/j}(z_h)$$

$$\frac{d\sigma_{eA\to h}}{dx_B\,dQ^2 dz_h} = \sum_{i,j}\frac{2\pi\alpha_e^2}{Q^4}\left[F_{ij}(z)+\Delta F_{ij}^{\rm med}(z)\right]\otimes d_{h/j}(z_h)$$

$$\Delta F_{ij}^{\rm med}(z) = F_{ik}^{(0)}\otimes P_{kj}^{\rm med(1)}$$



W. Ke et al. (2023)

Rather than evolving the fragmentation functions, we will evolve the parton shower / distribution of partons inside the shower

The invariant distribution of parton j in a shower initiated by i depends on 2 scales μ_1 and μ_2.

- Evolution μ_1 in leads to standard vacuum DGLAP
- The bare F_{ij} needs to be renormalized by a medium term that only depends on μ_2 . At one loop determined to cancel the poles in the medium bare part

$$F_{ij}(z,\mu_1^2,\mu_2^2)\to F_{ik}(y,\mu_1^2,\mu_2^2)\otimes M_{kj}\left(\frac{z}{y},\mu_2^2\right)+\mathcal{F}(z).\quad M_{kj}=M_{kj}^{(0)}+M_{kj}^{(1)}+\cdots$$

Technical aspect one: the splitting functions

In cold nuclear matter (uniform density) we can analytically integrate over the path length. We can can significantly simplify the propagator and phase structure that arises form in-medium interactions

Up to color and kinematic factors, the splitting functions have the same universal form

$i \to j$	$C_1^{ij},\ (\Delta_1^{ij})^2$	$C_2^{ij},\ (\Delta_2^{ij})^2$	$C_3^{ij},\ (\Delta_3^{ij})^2$
$q \to q$	$C_A,\ x^2$	$C_A,\ 1$	$2C_F - C_A,\ (1-x)^2$
$q \to g$	$C_A,\ 1$	$C_A,\ (1-x)^2$	$2C_F - C_A,\ x^2$
$g \to q$	$C_A,\ (1-x)^2$	$C_A,\ x^2$	$2C_F - C_A,\ 1$
$g \to g$	$C_A,\ 1$	$C_A,\ x^2$	$C_A,\ (1-x)^2$

$$P_{ij}^{(1)}(x,E,\mu_2^2) = \frac{\alpha_s^{(0)} P_{ij}(x)}{2\pi^2} L \int \frac{\mu_2^{2\epsilon} d^{2-2\epsilon}\mathbf{k}}{(2\pi)^{-2\epsilon}} \frac{\Phi\left[\frac{\mathbf{k}^2 L}{2x(1-x)E}\right]}{\mathbf{k}^2}$$
$$\times \sum_n \int \frac{\mu_2^{2\epsilon} d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{-2\epsilon}} \frac{\rho_G \alpha_s^{(0)} C_n^{ij} \Delta_n^{ij}(x)}{\pi(\mathbf{q}^2+\xi^2)^2} \frac{\mathbf{q}\cdot[\mathbf{k}+\Delta_n^{ij}(x)\mathbf{q}]}{[\mathbf{k}+\Delta_n^{ij}(x)\mathbf{q}]^2}$$

The remaining integration over the momentum exchanges with the can be performed using dim. reg. and by expanding the integrand

Final result

$$P_{ij}^{(1)}(x,E,\mu_2^2) = \frac{\alpha_s^2(\mu_2^2)\rho_G L}{8E/L} \frac{P_{ij}(x)}{[x(1-x)]^{1+2\epsilon}} \left[\frac{\mu_2^2 L}{\chi(w) E}\right]^{2\epsilon}$$
$$\times B(w) \sum_n C_n^{ij} [\Delta_n^{ij}(x)]^{2-2\epsilon} (1+\mathcal{O}(\epsilon^2))(1+\mathcal{O}(v))$$

Slowly varying functions O(one/few)

$$\int_0^w du \frac{4}{\pi} \frac{\Phi(u)}{u^{2+2\epsilon}} = B(w)[\chi(w)/2]^{-2\epsilon} + \mathcal{O}(\epsilon^2)$$

$$B(w) = \frac{4}{\pi} \int_0^w \Phi(x) \frac{dx}{x^2}, \quad \chi(w) = 2\exp\left\{\frac{1}{B(w)} \frac{4}{\pi} \int_0^w \Phi(x) \ln(x) \frac{dx}{x^2}\right\}$$

One important part here is the additional 1/x(1-x) divergence at the endpoints of the splitting function

Technical aspect one: the splitting functions

In cold nuclear matter (uniform density) we can analytically integrate over the path length. We can can significantly simplify the propagator and phase structure that arises form in-medium interactions

Up to color and kinematic factors, the splitting functions have the same universal form

$i \to j$	$C_1^{ij},\ (\Delta_1^{ij})^2$	$C_2^{ij},\ (\Delta_2^{ij})^2$	$C_3^{ij},\ (\Delta_3^{ij})^2$
$q \to q$	$C_A,\ x^2$	$C_A,\ 1$	$2C_F - C_A,\ (1-x)^2$
$q \to g$	$C_A,\ 1$	$C_A,\ (1-x)^2$	$2C_F - C_A,\ x^2$
$g \to q$	$C_A,\ (1-x)^2$	$C_A,\ x^2$	$2C_F - C_A,\ 1$
$g \to g$	$C_A,\ 1$	$C_A,\ x^2$	$C_A,\ (1-x)^2$

$$P_{ij}^{(1)}(x,E,\mu_2^2) = \frac{\alpha_s^{(0)} P_{ij}(x)}{2\pi^2} L \int \frac{\mu_2^{2\epsilon} d^{2-2\epsilon}\mathbf{k}}{(2\pi)^{-2\epsilon}} \frac{\Phi\left[\frac{\mathbf{k}^2 L}{2x(1-x)E}\right]}{\mathbf{k}^2}$$
$$\times \sum_n \int \frac{\mu_2^{2\epsilon} d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{-2\epsilon}} \frac{\rho_G \alpha_s^{(0)} C_n^{ij} \Delta_n^{ij}(x)}{\pi(\mathbf{q}^2+\xi^2)^2} \frac{\mathbf{q}\cdot[\mathbf{k}+\Delta_n^{ij}(x)\mathbf{q}]}{[\mathbf{k}+\Delta_n^{ij}(x)\mathbf{q}]^2}$$

The remaining integration over the momentum exchanges with the can be performed using dim. reg. and by expanding the integrand

Final result

$$P_{ij}^{(1)}(x,E,\mu_2^2) = \frac{\alpha_s^2(\mu_2^2)\rho_G L}{8E/L} \frac{P_{ij}(x)}{[x(1-x)]^{1+2\epsilon}} \left[\frac{\mu_2^2 L}{\chi(w) E}\right]^{2\epsilon}$$
$$\times B(w) \sum_n C_n^{ij} [\Delta_n^{ij}(x)]^{2-2\epsilon} (1+\mathcal{O}(\epsilon^2))(1+\mathcal{O}(v))$$

Slowly varying functions O(one/few)

$$\int_0^w du \frac{4}{\pi} \frac{\Phi(u)}{u^{2+2\epsilon}} = B(w)[\chi(w)/2]^{-2\epsilon} + \mathcal{O}(\epsilon^2)$$

$$B(w) = \frac{4}{\pi}\int_0^w \Phi(x)\frac{dx}{x^2}, \quad \chi(w) = 2\exp\left\{\frac{1}{B(w)}\frac{4}{\pi}\int_0^w \Phi(x)\ln(x)\frac{dx}{x^2}\right\}$$

One important part here is the additional 1/x(1-x) divergence at the endpoints of the splitting function

Technical aspect two: the subtraction of divergences

Take the flavor non-singlet distribution for simplicity

$$\Delta F_{\rm NS}^{\rm med}(z) = \int_z^1 \frac{dx}{x} F_{\rm NS}\left(\frac{z}{x}\right) P_{qq}^{\rm med(1)}(x) \quad + \quad \text{virtual term.}$$

$$P_{qq}^{\rm med(1)}(x) = A(\alpha_s, \cdots) \cdot \frac{P_{qq}^{vac(0)}(x)}{[x(1-x)]^{1+2\epsilon}} \cdot \left[\frac{\mu^2 L}{\chi z \nu}\right]^{2\epsilon} \cdot C_n \Delta_n(x)$$

Define a generalized + prescription and a subtracted function so that the integral with endpoint divergences is finite

$$\int_0^1 \frac{G(x)}{x^{1+2\epsilon}(1-x)^{2+2\epsilon}} dx = \int_0^1 \frac{\{G(x)\}_{qq}}{x(1-x)^2} dx - \frac{G(0)}{2\epsilon} + \frac{G'(1)}{2\epsilon} - G(1)\left(\frac{1}{2\epsilon}+2\right) + \mathcal{O}(\epsilon).$$

$$\{G(x)\}_{qq} = G(x) - (1-x)^2 G(0) - x(2-x)G(1) - x(x-1)G'(1).$$

The large medium induced logarithms that need to be resummed

The 1/ε divergence and $M^{(1)}$ counter term that is determined to cancel it. It arises from the soft-collinear sector $(p_{\rm cs}^2 \sim \xi^2 ... \xi^2 L/\lambda_g)$

$$\Delta F_{\rm NS} = \frac{\alpha_s^2 B(w)\rho_G L}{8\nu/L}\left(\frac{1}{2\epsilon} + \ln\frac{\mu_2^2 L}{\chi z \nu}\right) 2C_F\left(\frac{2C_A+C_F}{z} - 2C_A\frac{d}{dz}\right) F_{\rm NS}(z) + \int_0^1 \frac{dy}{y} F_{\rm NS}(y) M_{qq}^{(1)}\left(\frac{z}{y}, \mu_2, z\nu\right)$$
$$+ \frac{\alpha_s^2 B(w)\rho_G L}{8\nu/L} C_F \left[\int_0^1 \frac{\left\{\sum_n C_n^{qq}[\Delta_n^{qq}(x)]^2(1+x^2)\left[\frac{x}{z}F_{\rm NS}\left(\frac{z}{x}\right) - \frac{F_{\rm NS}(z)}{z}\right]\right\}_{qq}}{x(1-x)^2} dx + \frac{(4C_A - C_F)F_{\rm NS}(z)}{z}\right].$$

Fixed order contribution - free of divergences, no large log enhancement

Emergent analytic understanding of the in-medium shower

- **Derived a full set of RG evolution equations. The NS distribution has a very elegant traveling wave solution**

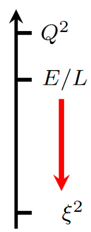
$$F_{\rm NS}(\tau, z) = \frac{F_{\rm NS}\left(0, z + 4C_F C_A \tau\right)}{(1 + 4C_F C_A \tau / z)^{1 + C_F/(2C_A)}}$$

Suitable change of variables. Also captures the density, path length and energy dependence

$$\tau(\mu^2) = \frac{\rho_G L^2}{\nu} \frac{\pi B}{2\beta_0} \left[\alpha_s(\mu^2) - \alpha_s\left(\chi \frac{z\nu}{L}\right)\right]$$

Flavor non-singlet (NS = q-qbar)

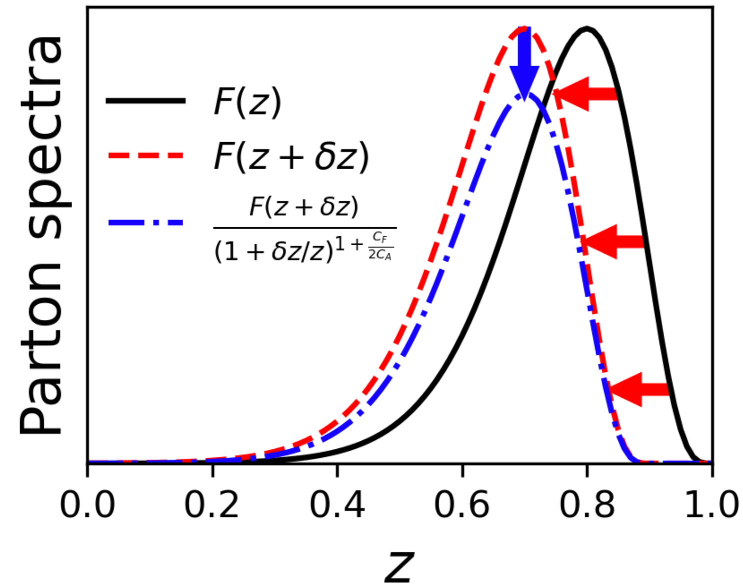
$$\frac{\partial F_{\rm NS}(\tau, z)}{\partial \tau} = \left(4C_F C_A \frac{\partial}{\partial z} - \frac{4C_F C_A + 2C_F^2}{z}\right) F_{\rm NS}$$



Flavor singlet (f = q+qbar, g)

$$\frac{\partial F_f}{\partial \tau} = \left(4C_F C_A \frac{\partial}{\partial z} - \frac{4C_F C_A + 2C_F^2}{z}\right) F_f + 2C_F T_F \frac{F_g}{z},$$

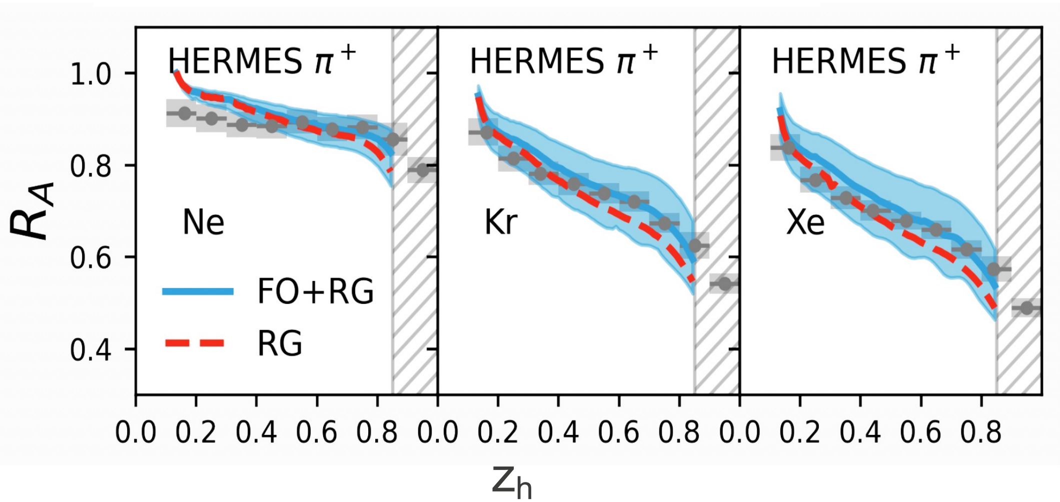
$$\frac{\partial F_g}{\partial \tau} = \left(4C_A^2 \frac{\partial}{\partial z} - \frac{2N_f C_F}{z}\right) F_g + 2C_F^2 \sum_f \frac{F_f}{z}.$$



Can directly identify parton energy loss, the nuclear size dependence of the modification, etc

Phenomenological applications of the new RG analysis to HERMES

Revisiting the HERMES data



Observable chosen to eliminate initial-state effects

$$R^{\pi}_{eA}(\nu, Q^2, z) = \frac{\left.\frac{N^{\pi}(\nu,Q^2,z)}{N^{e}(\nu,Q^2)}\right|_A}{\left.\frac{N^{\pi}(\nu,Q^2,z)}{N^{e}(\nu,Q^2)}\right|_D}$$

- RG evolution gives a good description of the data at small to intermediate z_h .
- Fixed order corrections improve the agreement at large z_h

W. Ke et al. (2023)

RG evolution advantages

- The method is systematically improvable – both higher logarithmic accuracy and fixed order terms, if higher order splitting functions are available
- Numerically, it is much faster to implement and solve in comparison to in-medium DGLAP evolution
- The proper in-medium scale separation increases predictive power
- At the level of cross sections one can identify the effects of “energy loss”

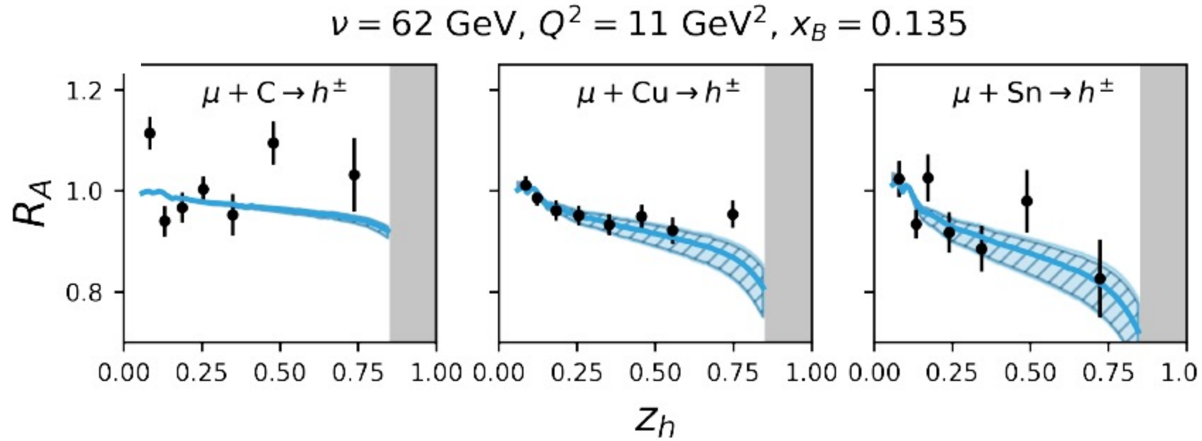
Practical concerns (not specific to the RG approach)

- The scales of the medium (lower boundary) are small and the coupling strong

Demonstration of predictive power

Addressing EMC data

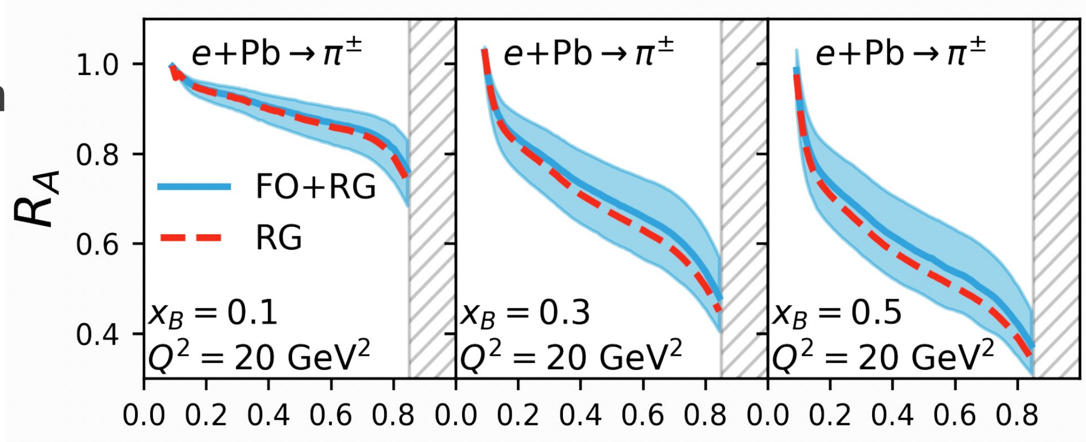
- EMC measurement for C, Cu, and Sn nuclei at similar x_B much higher $Q^2 \sim 11$ GeV^2
- Same effective Glauber gluon density used



Fixed order (FO) + RG evolution compared to EMC data

Predictions for the EIC

- The modifications to hadronization at EIC depends on kinematics x_B,Q^2
- At large x_B and (forward rapidities) the modification can be very significant



W. Ke et al. (2023)

Analytic comparison of RG and mDGLAP

We can use the insight from the RG evolution and revisit the in-medium DGLAP approach

For simplicity: fixed coupling, focus on the soft gluon emission region

- We can show that mDGLAP also resums matter-induced logarithms of the type $\ln[E/(L\mu_D^2)]$

A similar traveling wave solution

$$F^{+}_{NS}(z) = \frac{F^{-}_{NS}(z + 4C_F C_A \tau_{\rm fix})}{1 + 4C_F C_A \tau_{\rm fix}/z}$$

$$\tau_{\rm fix} = A_0 \ln \frac{2\pi E}{\mu_D^2 L}$$

Let's write mDGLAP for the flavor non-singlet distribution

$$\frac{\partial F_{\rm NS}(z)}{\partial \ln \mu^2} = \int_0^1 \mathbf{k}^2 \frac{d[P_{qq}(x,\mathbf{k}^2) + P_{qq}^{(1)}(x,\mathbf{k}^2)]}{dx d\mathbf{k}^2} \times \left[F_{\rm NS}\left(\frac{z}{x}\right) - F_{\rm NS}(z)\right] dx \,,$$

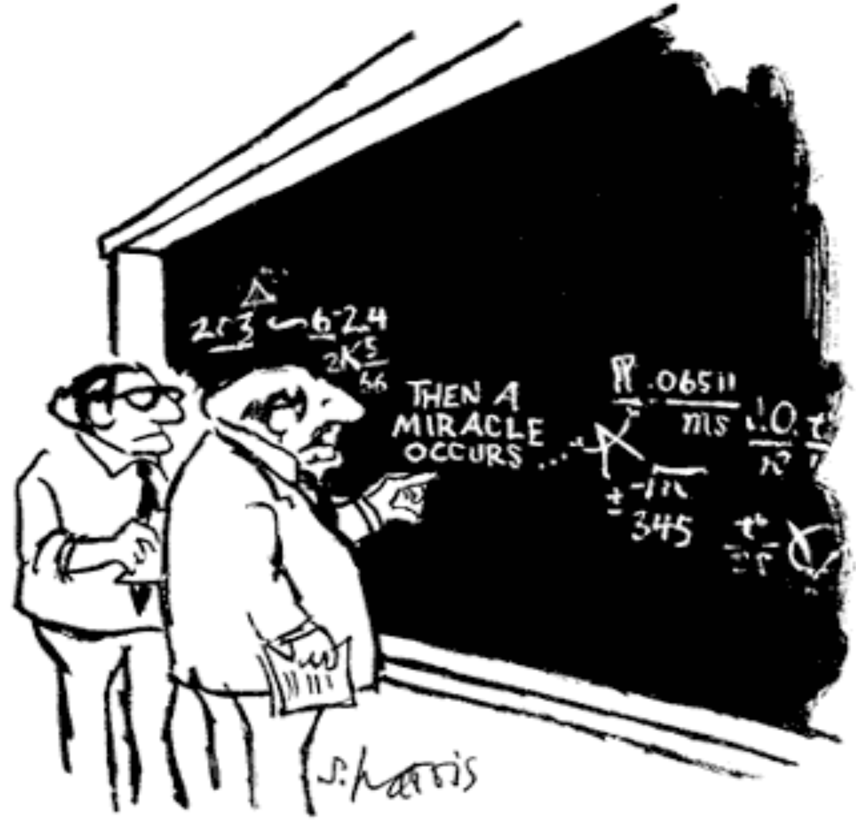
$$\frac{\partial F_{\rm NS}}{\partial \ln \mu^2} = 4C_F C_A A_0 \int_0^{1-\frac{\mu_D^2}{\mu^2}} \frac{4}{\pi}\frac{\Phi(u)}{u} \frac{\frac{x}{z}F_{\rm NS}(\frac{z}{x}) - \frac{F_{\rm NS}(z)}{z}}{(1-x)^2} dx$$
$$\approx \frac{4}{\pi}\frac{\Phi(u)}{u} 4C_F C_A A_0 \left[\frac{\partial F_{\rm NS}}{\partial z} - \frac{F_{\rm NS}}{z}\right] \ln\frac{\mu^2}{\mu_D^2}$$
$$\approx \delta\left(\mu^2 - \frac{2\pi E}{L}\right) 4C_F C_A A_0 \left[\frac{\partial F_{\rm NS}}{\partial z} - \frac{F_{\rm NS}}{z}\right] \ln\frac{\mu^2}{\mu_D^2}$$

$$A(\mu_2^2, E, w_{\rm max}) = \alpha_s^2(\mu_2^2) L^2 B(w_{\rm max}) \rho_G/(8E)$$

- "E-loss the same, dominated by soft emissions, but flavor changing processes also contribute and z-dependent

W. Ke et al., (2023)

Future directions



"I think you should be more explicit here in step two."

Other applications of the RG evolution results

- The goal is to get a simple analytic formula that shows parametrically how energy loss depends on the coupling, medium size, effective Glauber gluon density, scales

Take the soft gluon limit and also simultaneously consider light and heavy quarks

$$\frac{1}{\mathbf{k}^2 + x^2 M^2} \approx \frac{1}{\mathbf{k}^2}\frac{\theta^2}{\theta^2+\theta_D^2}, \quad \theta = \frac{\mathbf{k}}{\omega}, \quad \theta_D = \frac{M}{E}$$

- The “dead cone effect arises from the splitting-dependent mass term

$$\frac{dP_{QQ}^{\mathrm{med}(1)}}{d\omega} \otimes F_Q(E+\omega) = \int_0^\infty d\omega \frac{2C_F}{\omega} F_Q(E+\omega) \int \frac{d^{2-2\epsilon}\mathbf{k}}{(2\pi)^{-2\epsilon}} \frac{\alpha_s^{(0)}}{2\pi^2} \frac{\Phi_{\mathrm{LPM}}\left(\frac{\mathbf{k}^2+\omega^2\theta_D^2}{2\omega/L}\right)}{(\mathbf{k}^2+\omega^2\theta_D^2)}$$
$$\int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{-2\epsilon}} \frac{\alpha_s^{(0)}}{\pi} \frac{C_A \rho_G L}{(\mathbf{q}^2} \frac{2\mathbf{q}\cdot(\mathbf{k}+\mathbf{q})}{(\mathbf{k}+\mathbf{q})^2+\omega^2\theta_D^2} + \mathcal{O}\left(\frac{\xi^2}{E/L}\right)$$

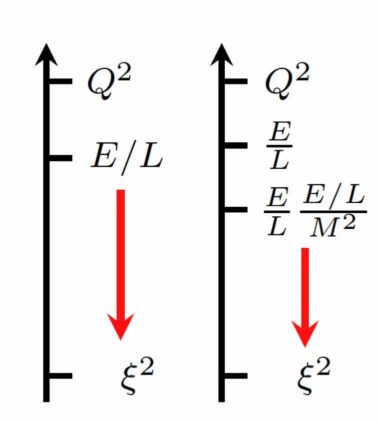
- Need to weigh by the gluon energy and integrate. To understand analytically where the heavy quark mass plays a role we look at the LPM phase

To neglect the dead cone $\quad k^2 \sim \frac{\omega}{L} \gg \omega^2\theta_D^2 \Longrightarrow \omega \ll \frac{1}{\theta_D^2 L}$

W. Ke et al. (2024 -)

Trade M in matrix element for the integration limit $\quad \Theta\left(\omega < \frac{1}{\theta_D^2 L}\right) \qquad \xi^2 L \ll \omega \ll \min\left\{\frac{1}{\theta_D^2 L}, E\right\}$

E-loss results and effects of mass



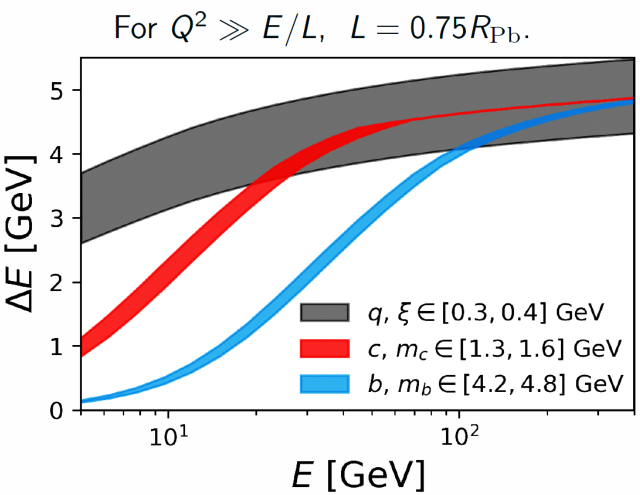
Simple formulas for energy loss and the dead cone effect

$$\Delta E_q = C_F C_A \frac{4\pi}{\beta_0}\frac{B\rho L}{2E/L}\left[\alpha_s(\xi^2) - \alpha_s\left(\frac{\chi E}{L}\right)\right]$$

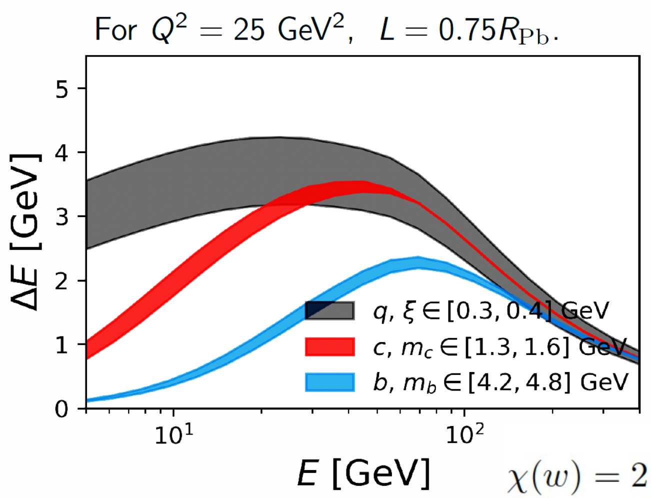
$$\Delta E_Q = C_F C_A \frac{4\pi}{\beta_0}\frac{B\rho L}{2E/L}\left[\alpha_s(\xi^2) - \alpha_s\left(\frac{\chi E}{L}\min\left\{\frac{E/L}{M^2}, 1\right\}\right)\right]$$

Numerical results

Unrestricted phase space



Restricted phase space for radiation



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$$w = Q^2 L / 2\nu$$

$$B(w) = \frac{4}{\pi}\int_0^w \Phi(x)\frac{dx}{x^2}$$

$$\chi(w) = 2\exp\left\{\frac{1}{B(w)}\frac{4}{\pi}\int_0^w \Phi(x)\ln(x)\frac{dx}{x^2}\right\}$$

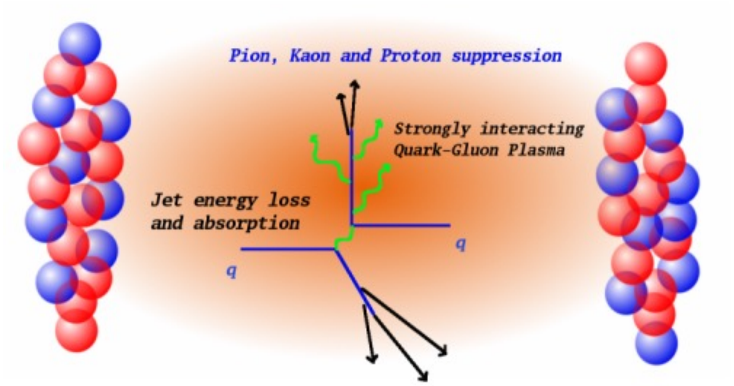
Conclusions

- Resummation of large nuclear matter induced logarithms is essential to interpret the results of reactions with nuclei
- The traditional approach to accomplish this task is through fully numerical solution to in-medium DGLAP (making use of now available full in-medium splitting function). This framework was applied to perform the first calculation of centrality-dependent hadron production in DIS with nuclei
- The intellectual commonalities between AA (HIC) and eA (DIS) made explicit through calculations of semi inclusive hadrons, jets, and jet substructure (even if only briefly mentioned here)
- Analytic insights, however, have thus far been absent. We developed an RG evolution approach that overcomes this limitation. It is fast, efficient, improvable, and represents an important rigorous development in this direction that has been absent from the literature
- Was successfully applied to phenomenology, elucidating the role of FO and resummed contributions, increasing predictive power. Also helped understand the physics contained in mDGLAP
- Multiple future applications, analytic understanding of energy loss, dead cone, differences between HIC and DIS, and application to TMD physics

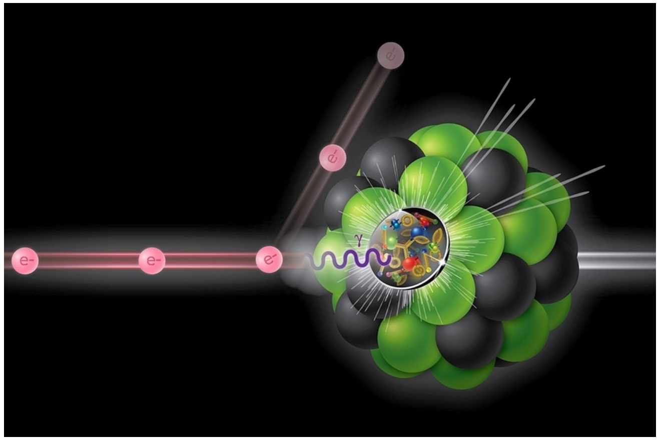
Thank you

Differences between AA and eA

- AA and eA collisions are very different. Due to the LPM effect the “energy loss” decreases rapidly. The kinematics to look for in-medium interactions / effects on hadronization very different



- Jets at any rapidity roughly in the co-moving plasma frame (Only~ transverse motion at any rapidity)
- Largest effects at midrapidity
- Higher C.M. energies correspond to larger plasma densities



- Jets are in the nuclear rest frame Longitudinal momentum matters
- Largest effects are at forward rapidities
- Smaller C.M. energies (larger only increase the rapidity gap)