A new class of three-nucleon forces In Chiral EFT

Based on: arXiv:2411.00097

with

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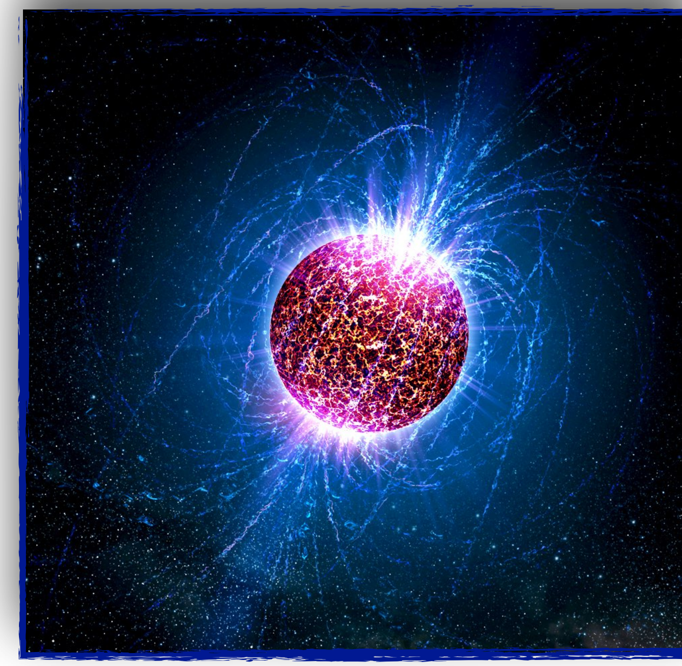
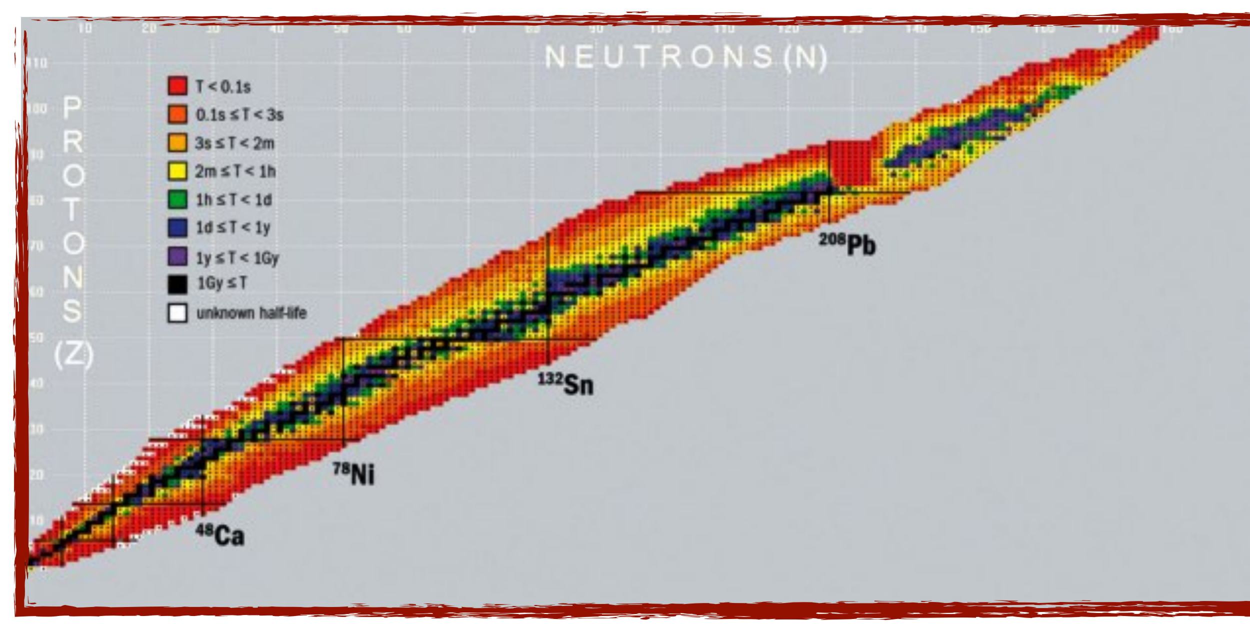
INSTITUTE for NUCLEAR THEORY

Neutron Stars



Nuclei



Chiral 3 nucleon forces

- Play an important role in nuclei & dense matter
  - Contribute to binding energies
  - Needed to describe saturation

From quarks to nucleons

Modern view of nucleon forces

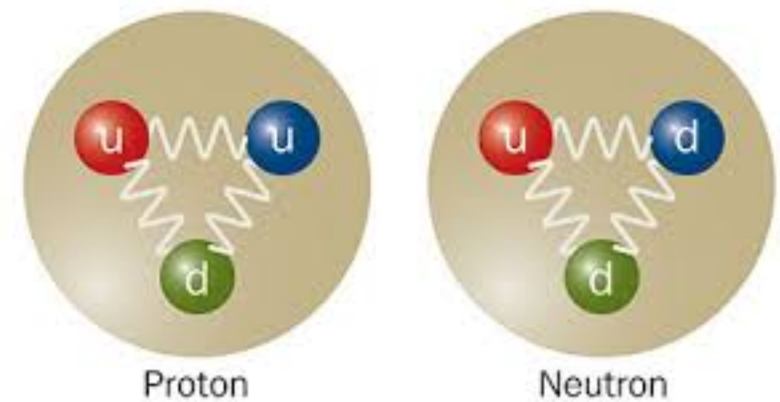
M_{QCD}
1 GeV

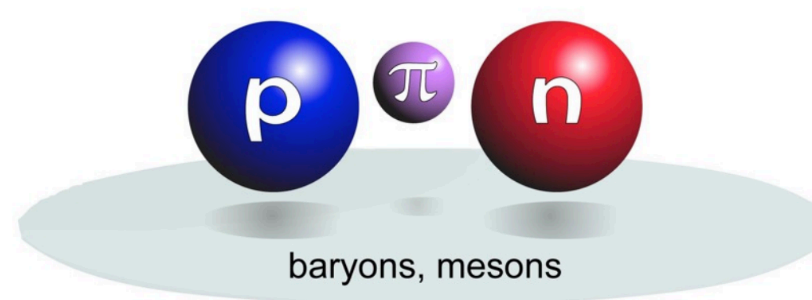
SM (QCD)

In terms of quarks, gluons, leptons

Chiral Effective Theory

In terms of nucleons, pions, leptons

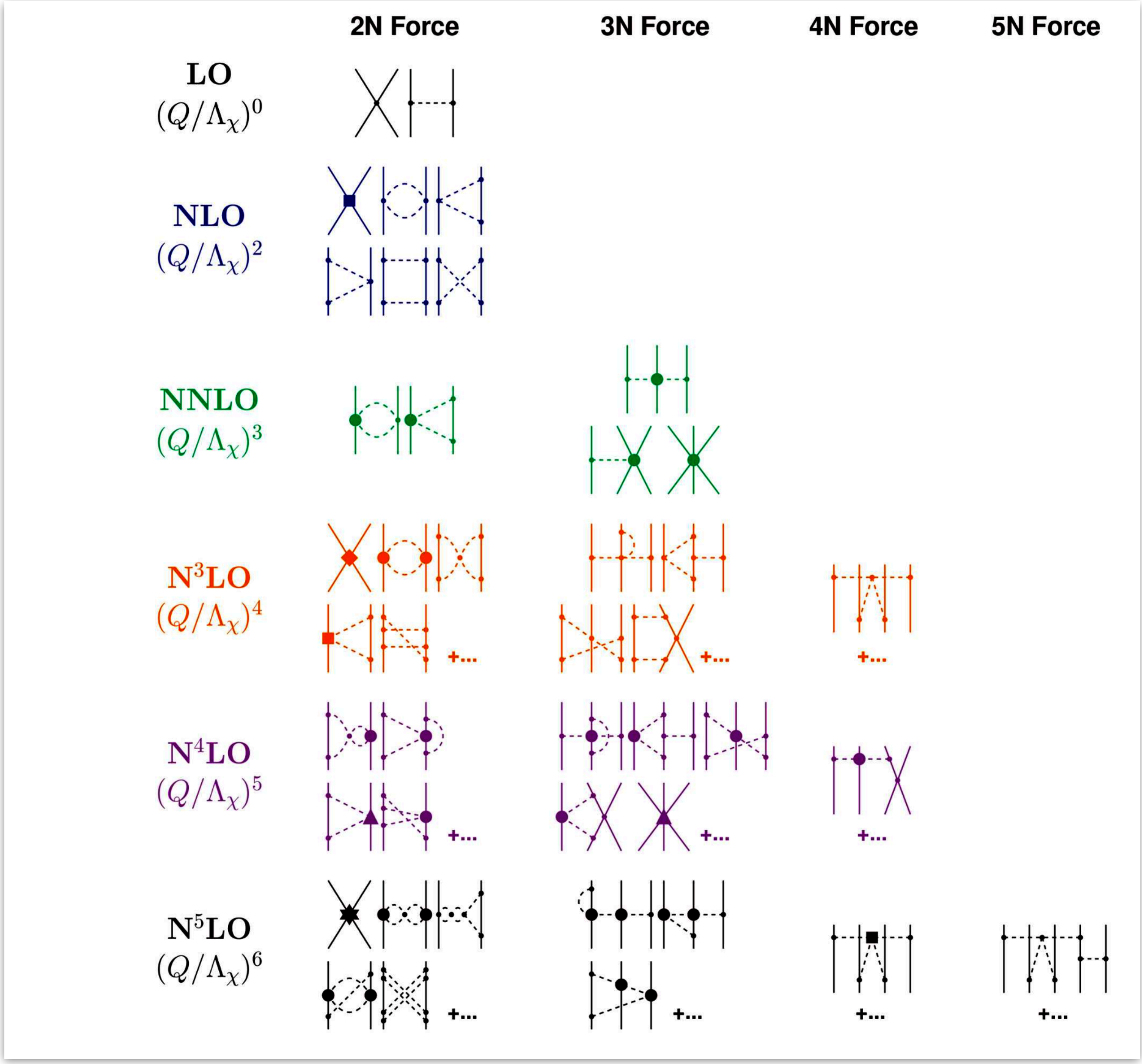




- Possible hadronic interactions determined by (chiral) symmetries
- Expansion in $Q/\Lambda_\chi \sim m_\pi/\Lambda_\chi$, where $\Lambda_\chi \sim 1$ GeV
- Interactions come with (unknown) coupling constants
  - *Power counting needed to determine their importance*

Chiral Effective Theory

Nucleon forces in Weinberg counting



Hierarchy of nuclear forces up to N5 LO in ChiPT. Solid lines represent nucleons and dashed lines pions. Entem, Machleidt, Y. Nosyk, (arXiv:1703.05454)

Chiral Effective Theory

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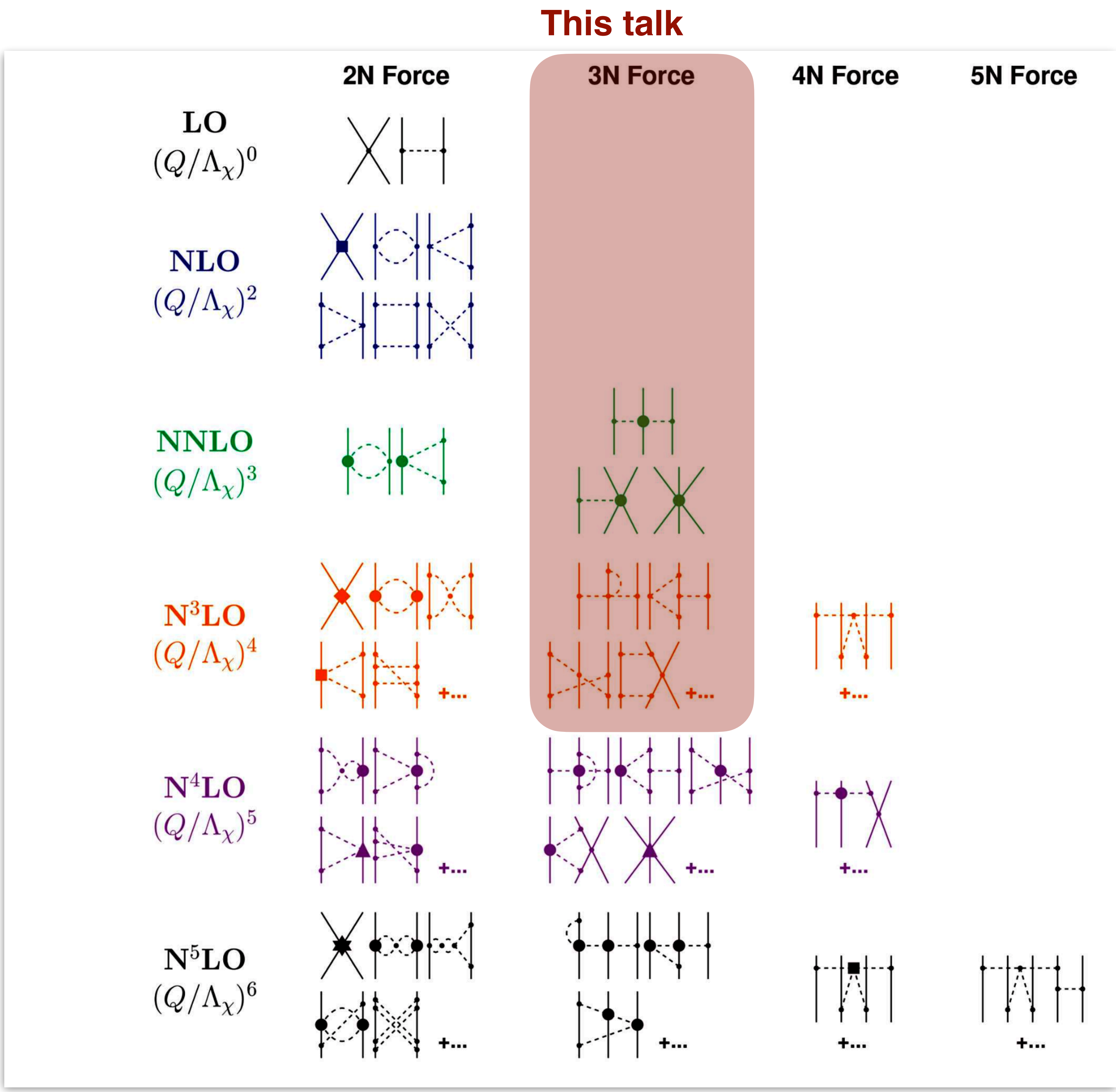

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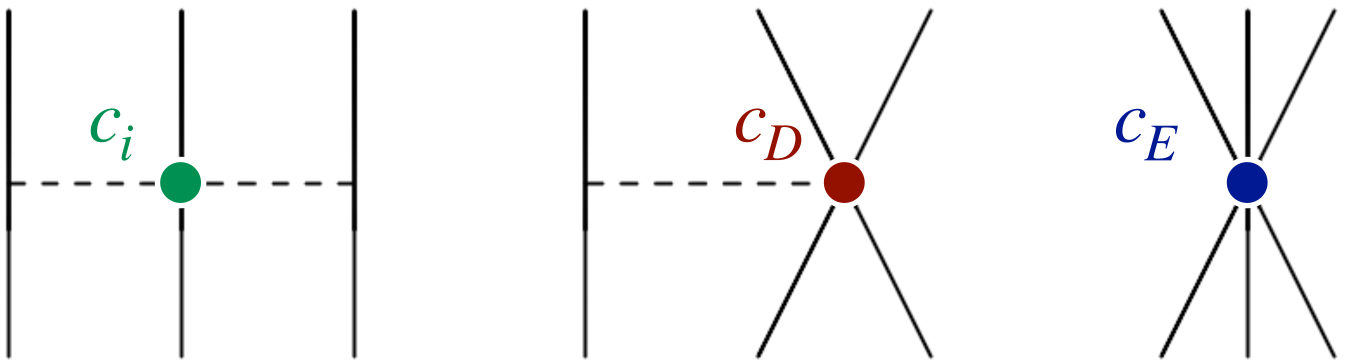
Conventional three-body force

N2LO contributions

- Consists of
  - One- and two-pion exchange
  - Short-distance contributions



Van Kolck '94; Epelbaum Nogga, Glöckle Hamada, Meissner, '02

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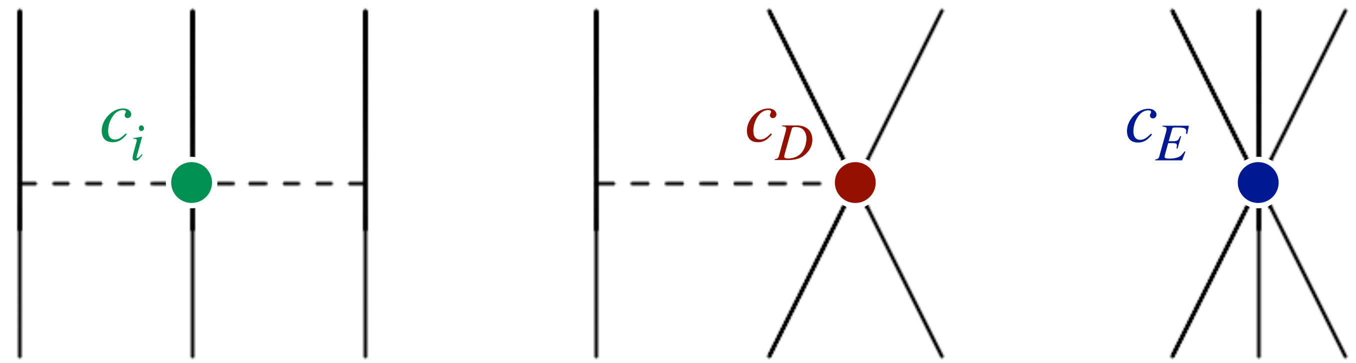
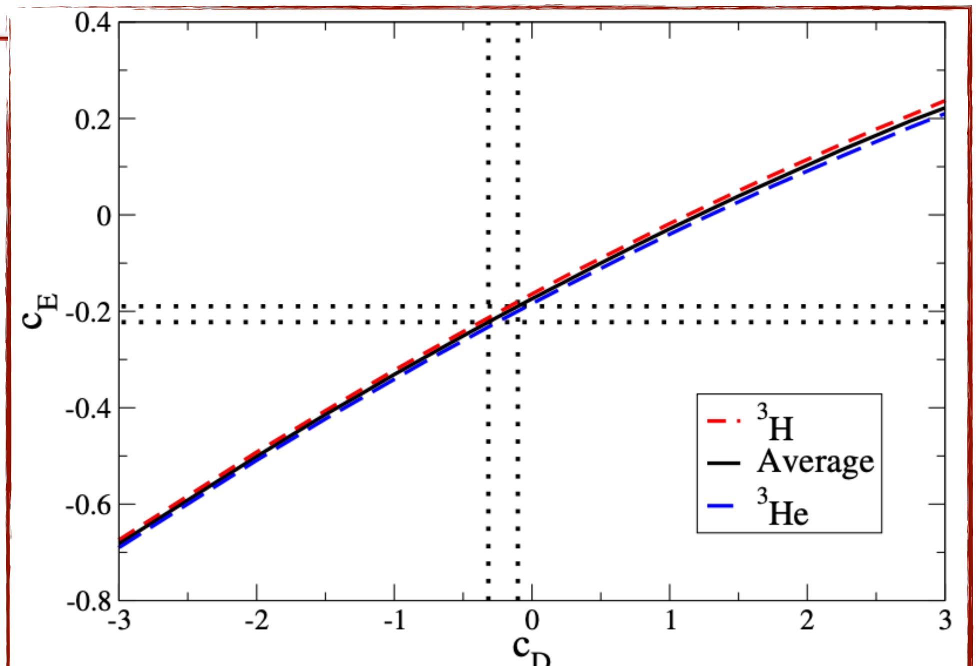


- LECs determined by
  - c_i: πN scattering
    Hoferichter et al Phys. Rept. '15; PRL '15; Phys. Lett. B '16
  - $c_{D,E}$:
    - Nd scattering
    - light nuclei
    - tritium β decay

See Martin's talk



Gazit et al '09

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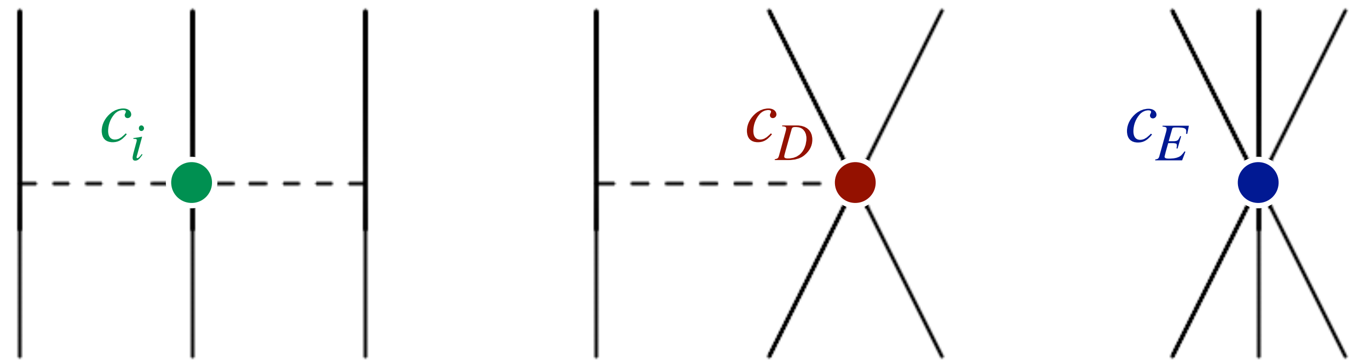

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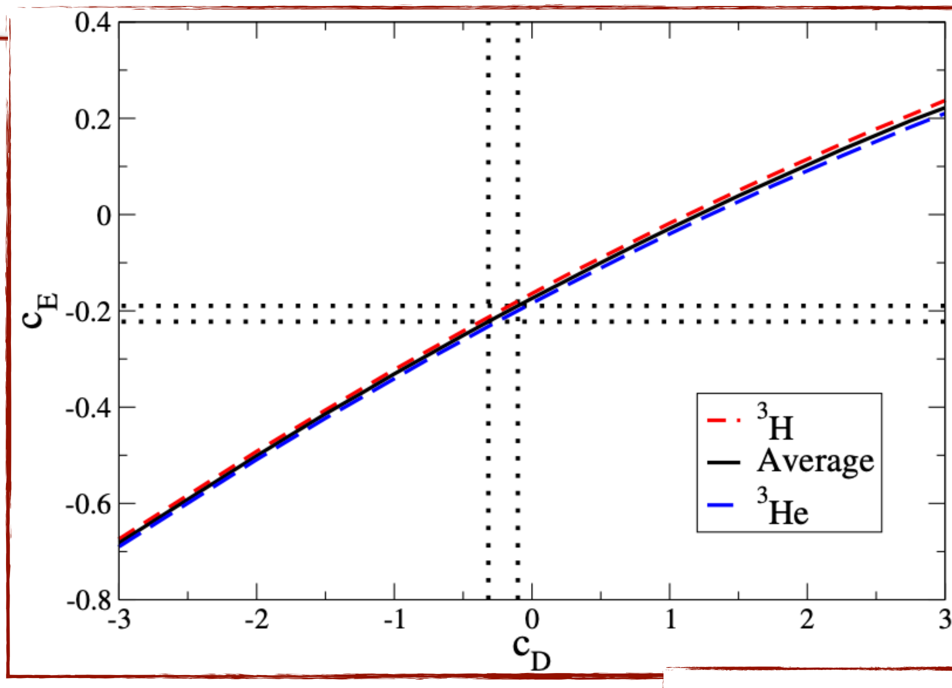


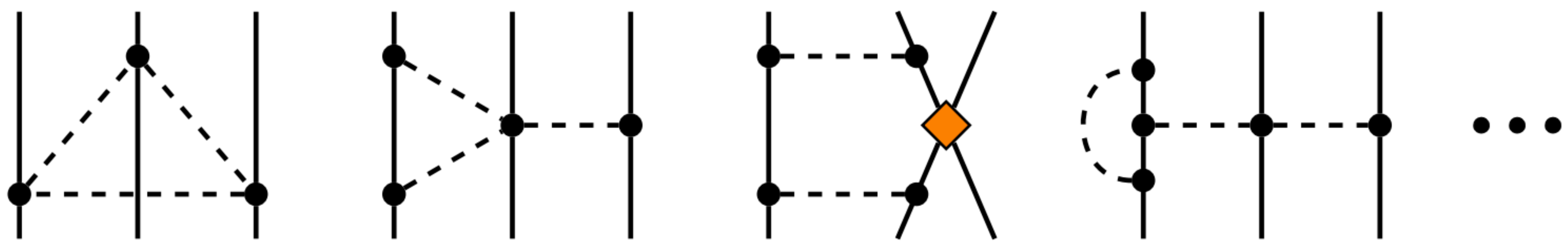
Gazit et al '09

- ***Only c_1 and c_3 contribute in neutron matter***

Van Kolck '94; Epelbaum Nogga, Glöckle Hamada, Meissner, '02

Conventional three-body force

N3LO contributions

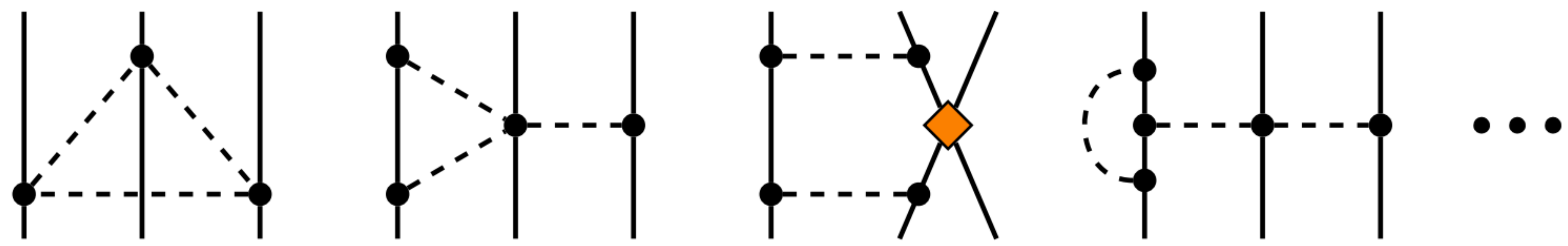


- Consists of
    - Loop diagrams with LO vertices
    - Tree graphs involving relativistic corrections

Bernard, Epelbaum, Krebs, Meissner ’08,’11; Ishikawa, Robilotta ’07,

Conventional three-body force

N3LO contributions



- Consists of
  - Loop diagrams with LO vertices
  - Tree graphs involving relativistic corrections

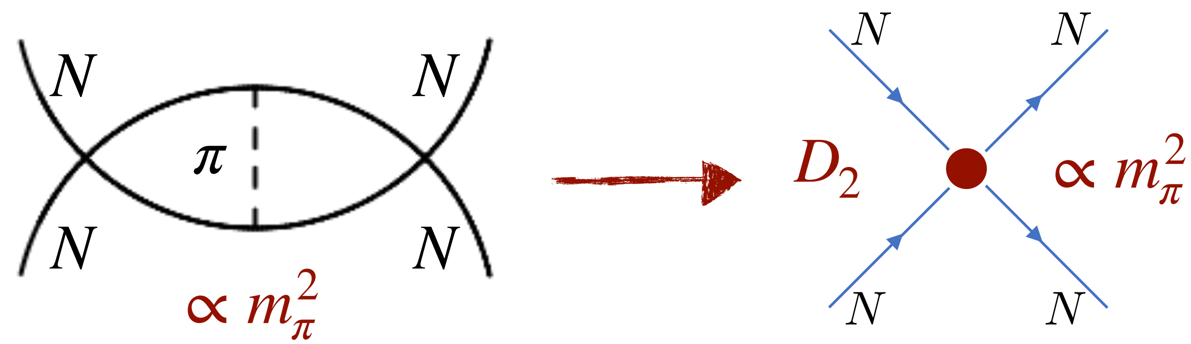
- ***No new LECs***

Bernard, Epelbaum, Krebs, Meissner ’08,’11; Ishikawa, Robilotta ’07,

Beyond Weinberg counting

Beyond Weinberg counting

- Bubble diagrams in ${}^1S_0$ lead to divergences
- Requires short-distance interactions
- Can be absorbed in $C_0' = C_0 + m_\pi^2 D_2$

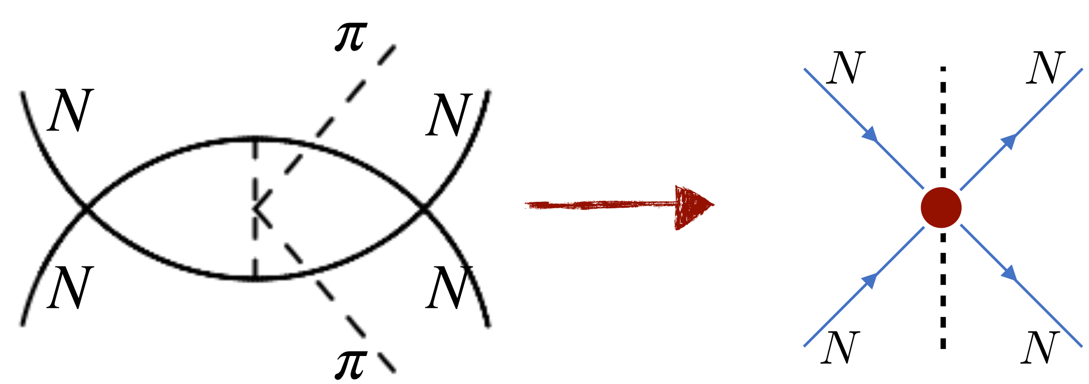


Kaplan, Savage, Wise, ’96; Beane, Bedaque, Savage, van Kolck, ’03, Nogga, Timmermans, van Kolck, ’05, Long, Yang, ’12;

Beyond Weinberg counting

Borasoy, Griesshammer, '01,'03

- Similar diagrams induce $\pi^2 NN$ couplings
- ***Cannot*** be absorbed in C_0
- Depend on m_π or momenta:



$m_\pi^2, \quad E_\pi E'_\pi, \quad \vec{q}_\pi \cdot \vec{q}'_\pi \qquad D_2, \quad E_2, \quad F_2$

$$\mathscr{L} = -\frac{1}{4} D_2 \, (N^T P_i N)^\dagger \, (N^T P_i N) \, \langle \chi_+ \rangle \; + \; \frac{1}{4} \left[E_2 \langle (v \cdot u)^2 \rangle + F_2 \langle u \cdot u - (v \cdot u)^2 \rangle \right] \; (N^T P_i N)^\dagger \, (N^T P_i N)$$

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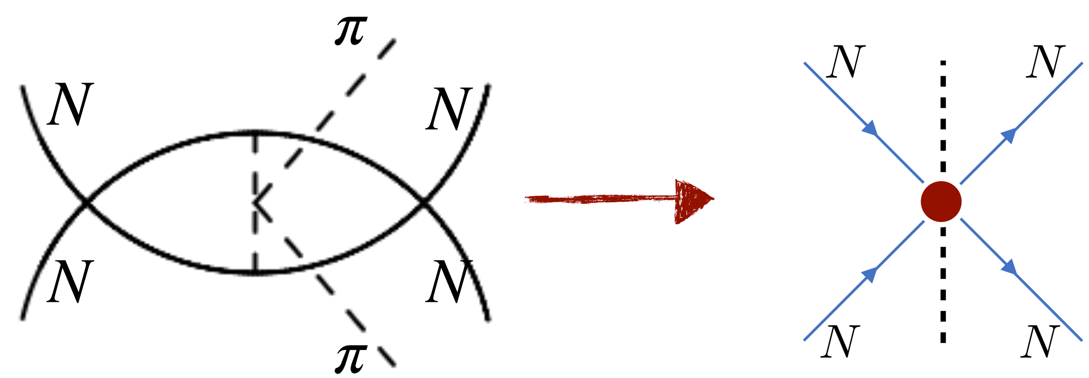

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D_2, E_2, F_2 currently poorly known

- Estimate from renormalization group:

$$\frac{d}{d\ln\mu}\left[\frac{X}{\tilde{C}_0^2}\right] = \gamma_X \left(\frac{m_N}{4\pi f_\pi}\right)^2, \qquad X \in \{D_2, E_2, F_2\}$$

$$\gamma_{D_2} = g_A^2/4$$
$$\gamma_{E_2} = -(1+g_A^2)/3,$$
$$\gamma_{F_2} = -g_A^2/3$$

- Suggests LO size $X \simeq C_0^2/5 \sim 1/(5\,F_\pi^4)$
  - Compared to N2LO Weinberg $X \sim 1/(F_\pi^2 \Lambda_\chi^2)$

Kaplan, Savage, Wise, ’96; Beane, Bedaque, Savage, van Kolck, ’03, Nogga, Timmermans, van Kolck, ’05, Long, Yang, ’12;

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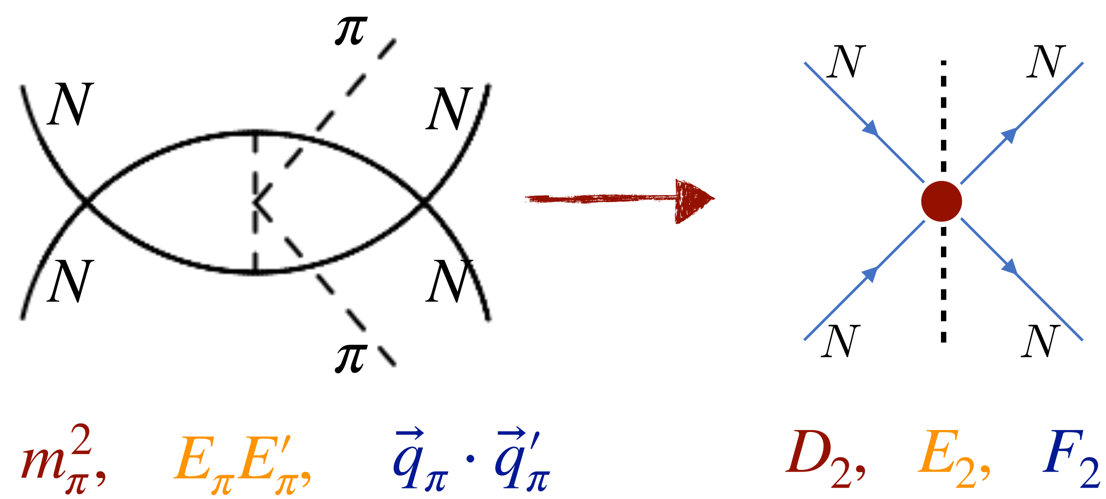

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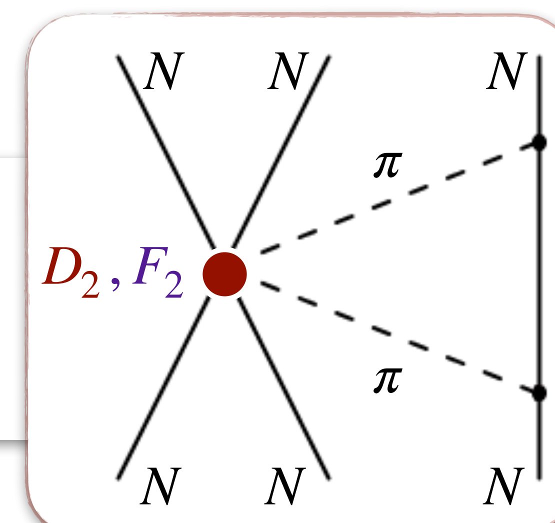
- $\pi^2 NN$ interactions do not affect NN
- Induce 3-nucleon forces through loops



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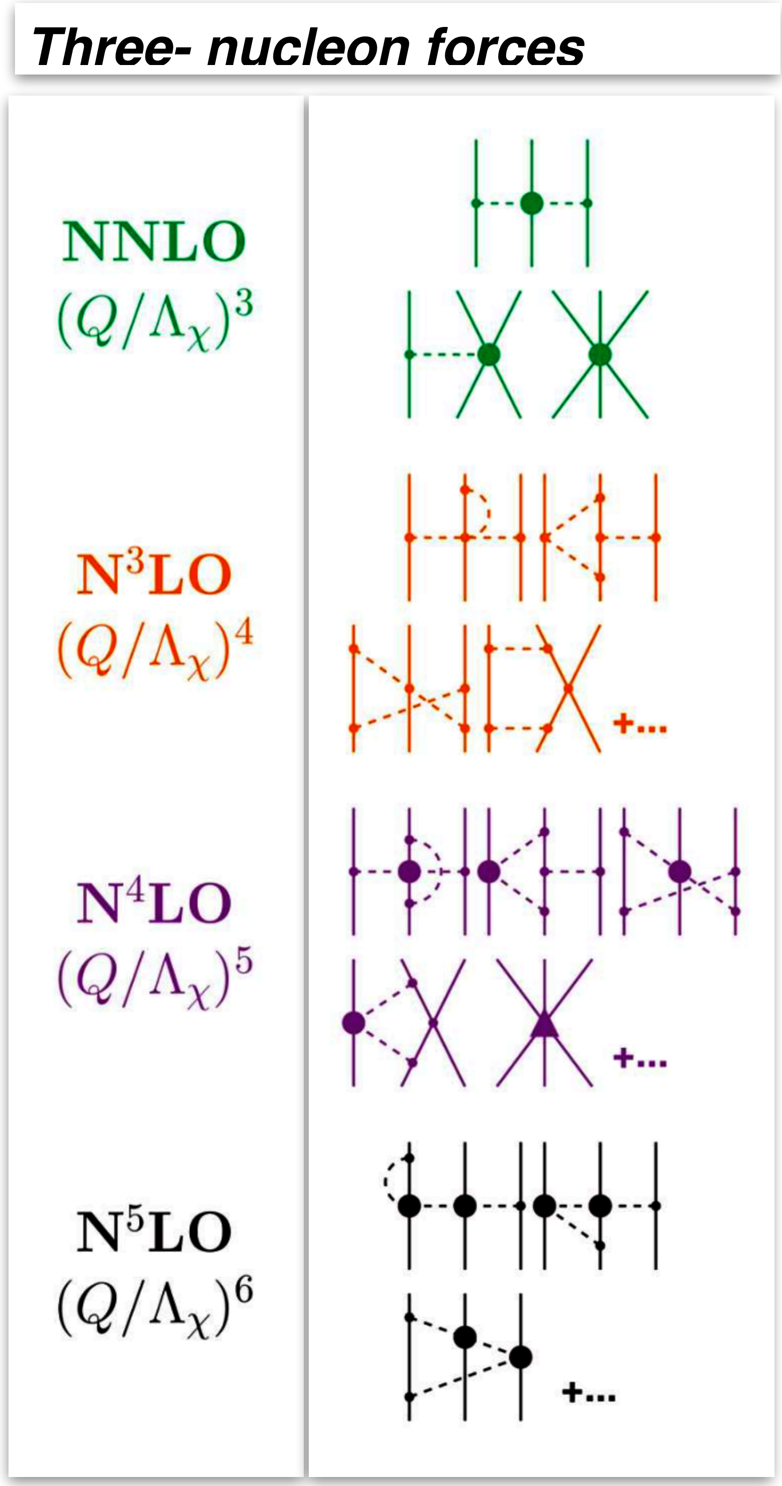

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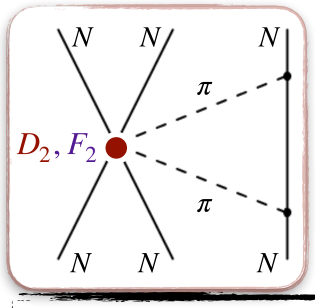
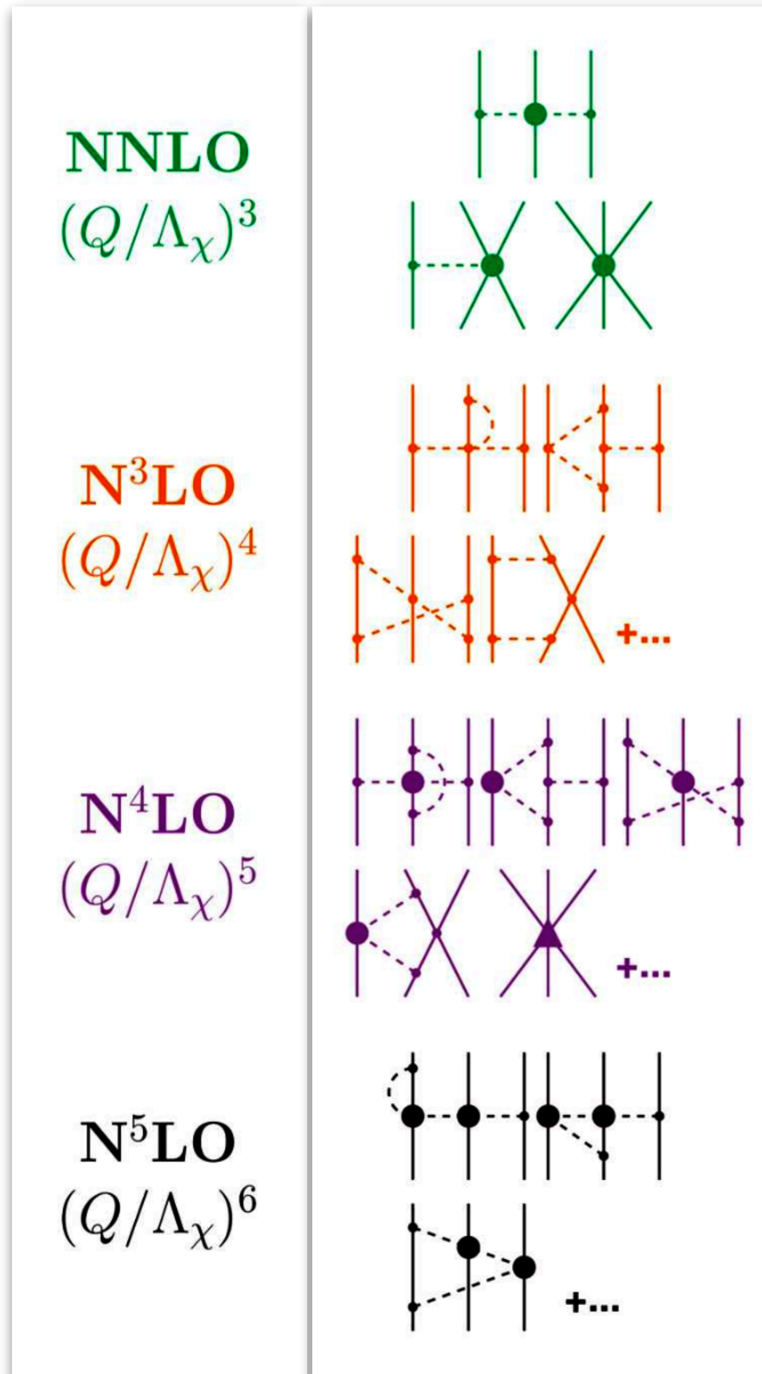


Weinberg's counting

Beyond Weinberg counting

Three- nucleon forces



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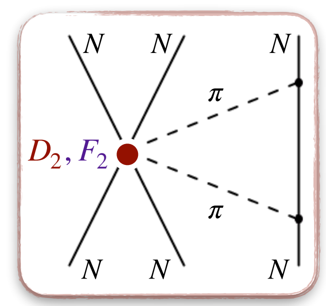
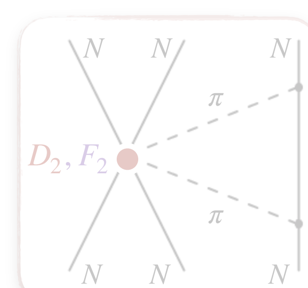


Consistency requirement



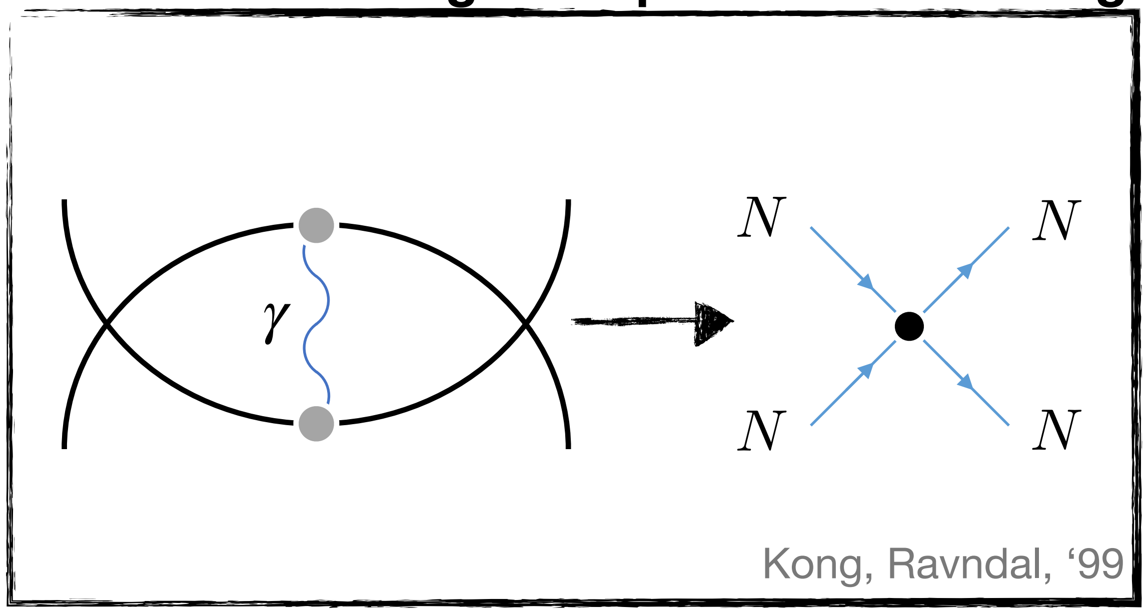
Weinberg's counting

Beyond Weinberg counting

Electroweak/BSM interactions

Electroweak and BSM interactions affected by the same RG arguments:

Charge-independence Breaking



Beyond Weinberg counting

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See Emanuele/Jordy's Talks on Thursday

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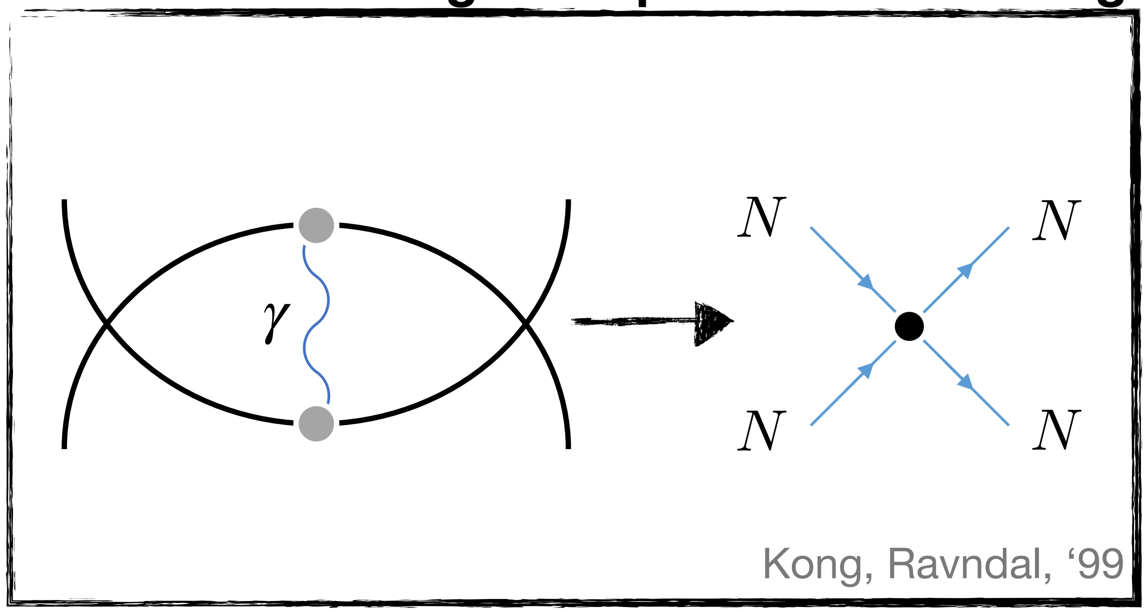
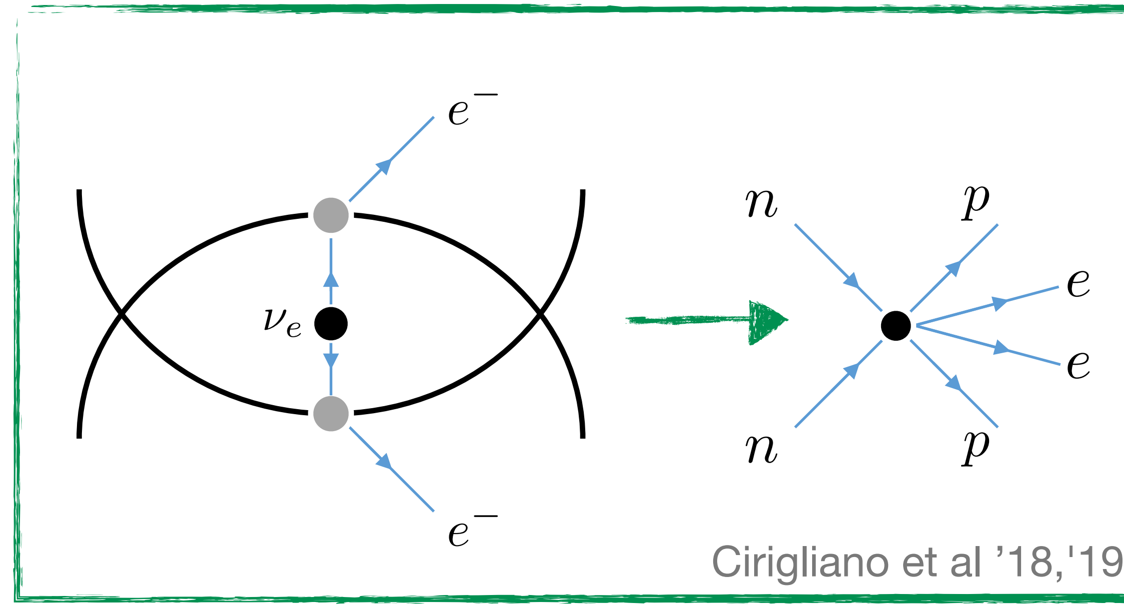
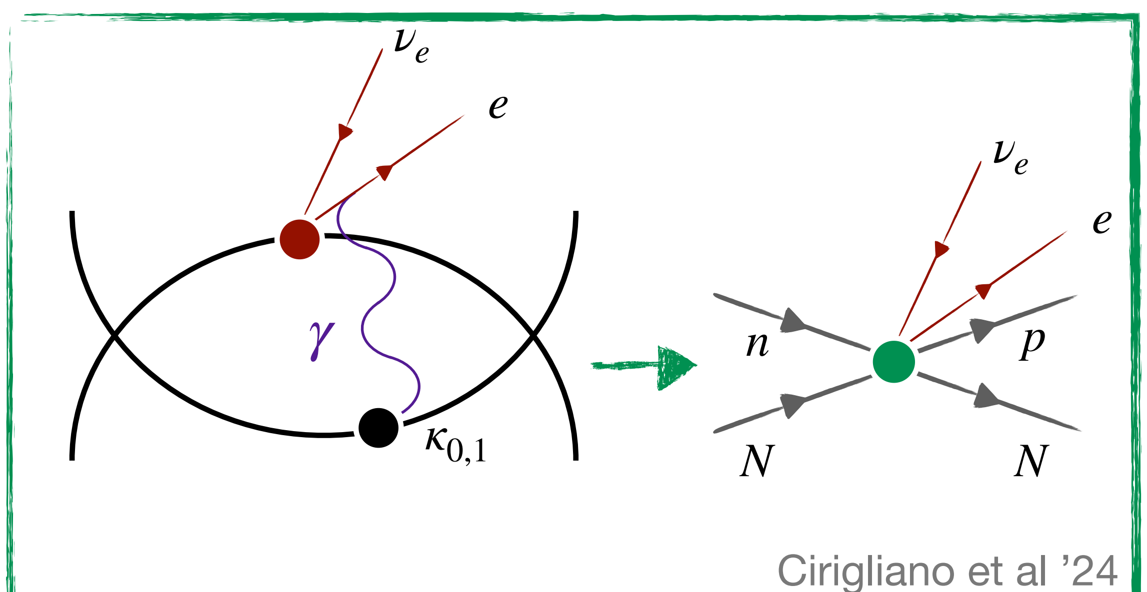
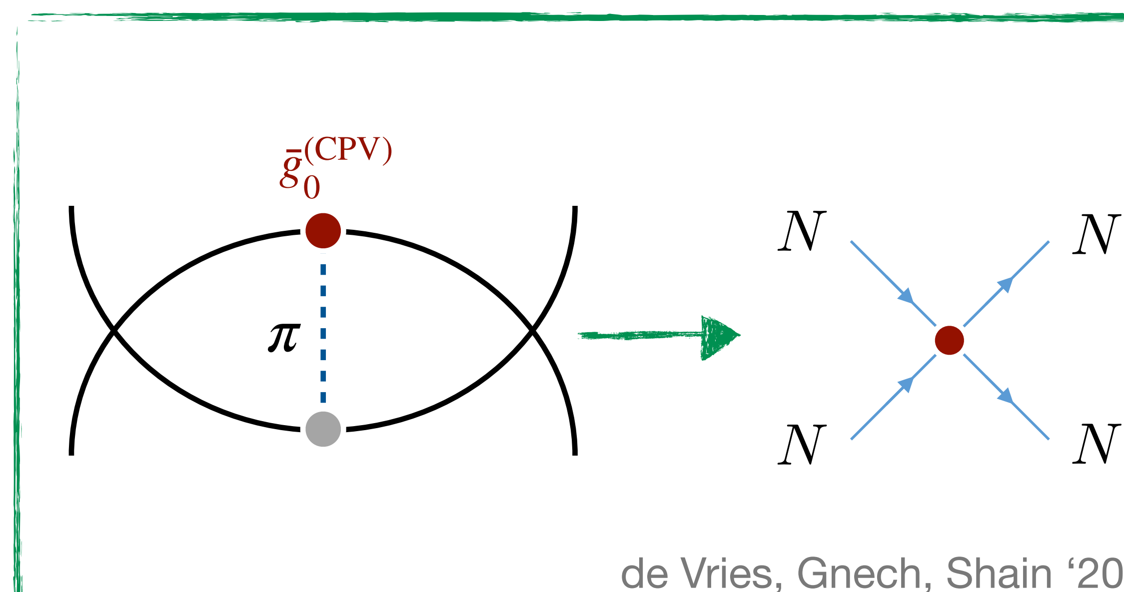


$0\nu\beta\beta$





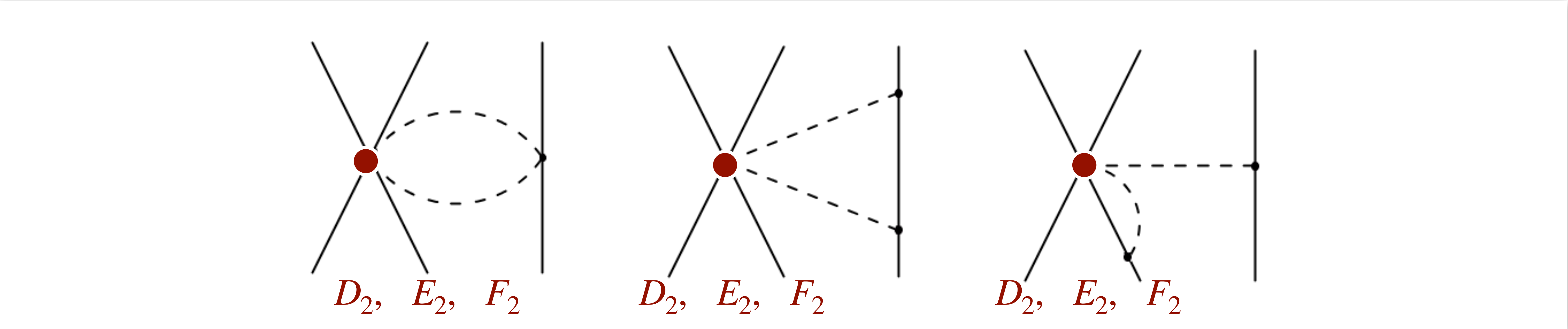
Radiative corrections in β decay



Electric Dipole Moments

Beyond Weinberg counting

Induced potential



Beyond Weinberg counting

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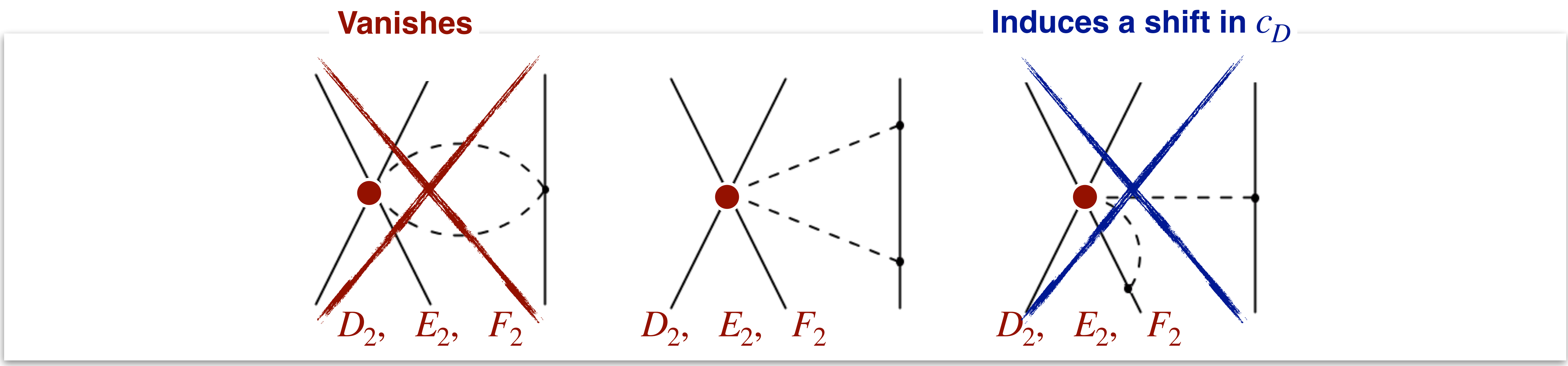

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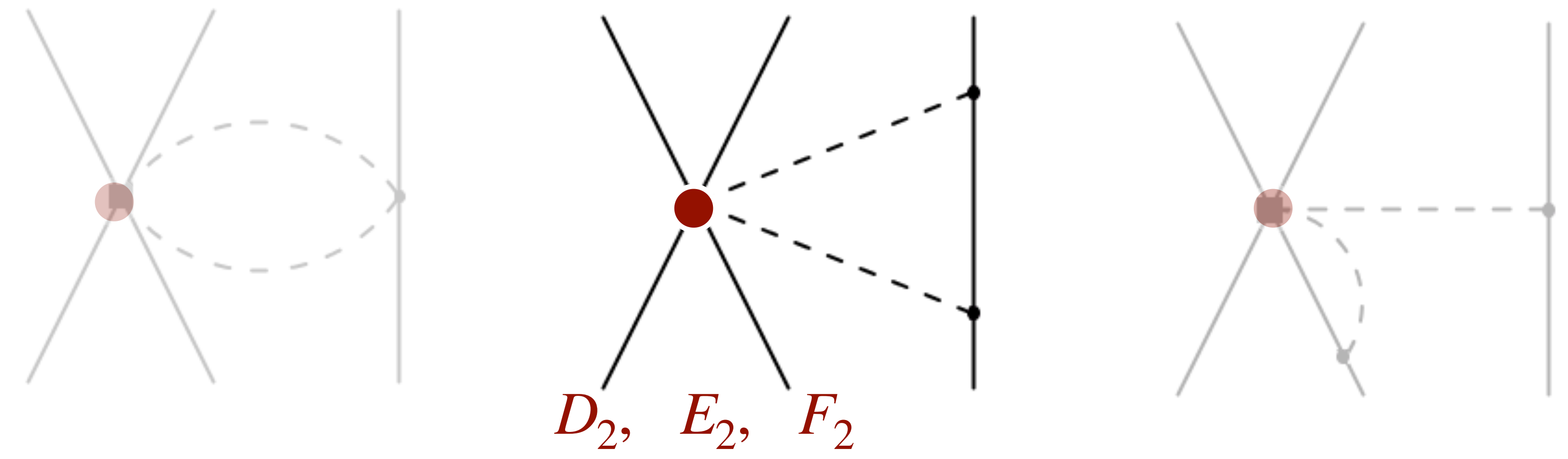

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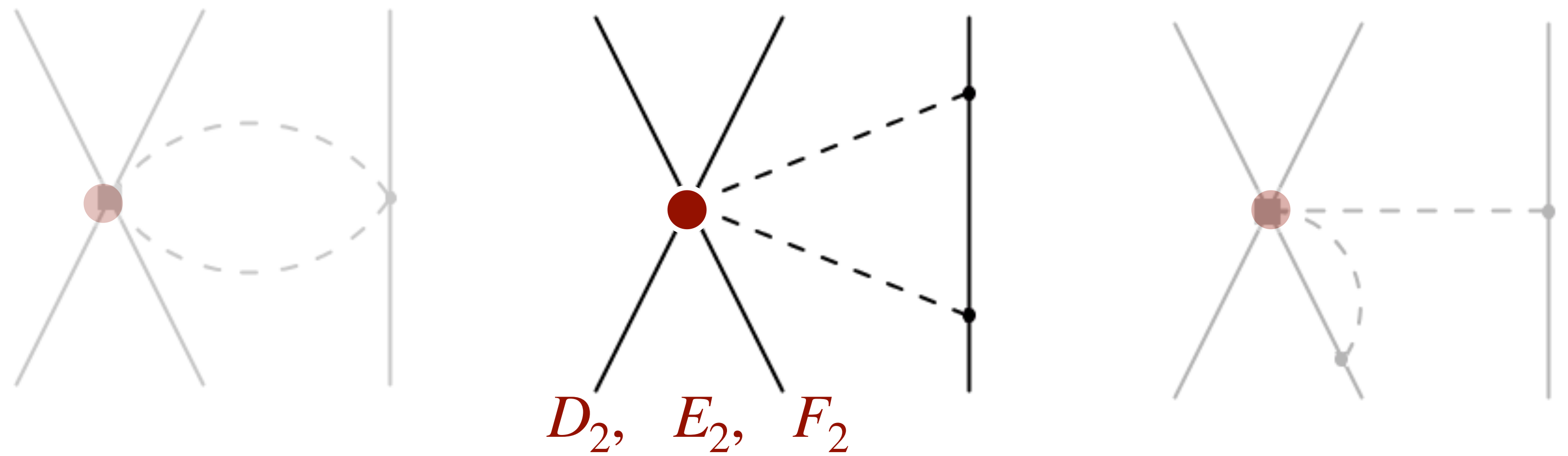

Three-nucleon potential from $m_\pi^2 D_2$

$$V(\vec{q}_1, \vec{q}_2, \vec{q}_3) = \frac{9 g_A^2 D_2 m_\pi^3}{512 \pi f_\pi^4} F\left(\frac{\vec{q}_3^{\,2}}{4 m_\pi^2}\right) \left(1^{(i)} 1^{(j)} - \vec{\sigma}^{(i)} \cdot \vec{\sigma}^{(j)}\right) \quad , \qquad F(b) = \frac{2}{3}\left(1 + \left(\frac{1}{2\sqrt{b}} + \sqrt{b}\right) \tan^{-1}(\sqrt{b})\right) .$$

Beyond Weinberg counting

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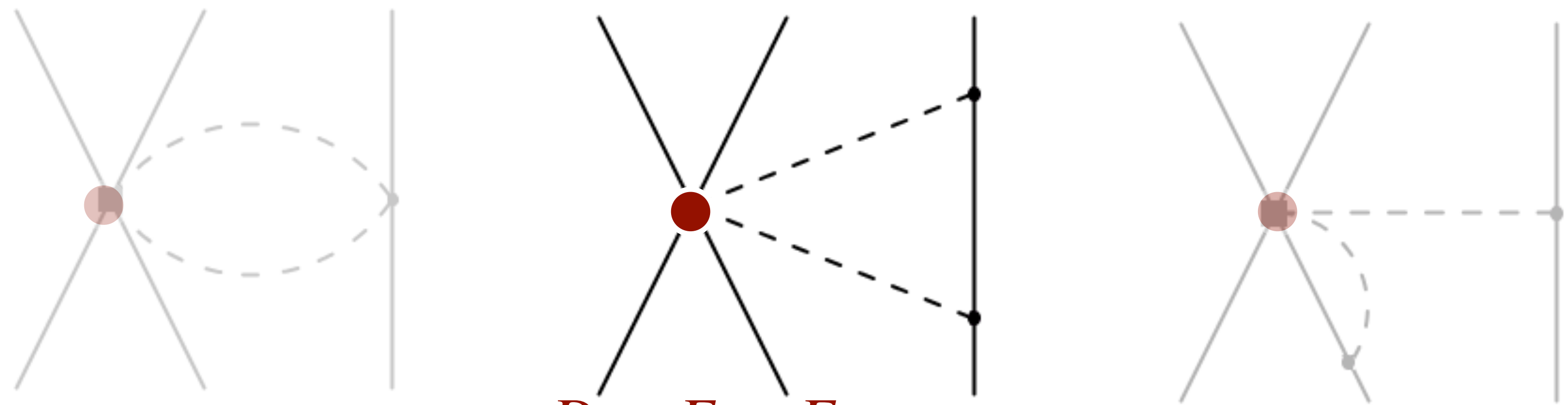

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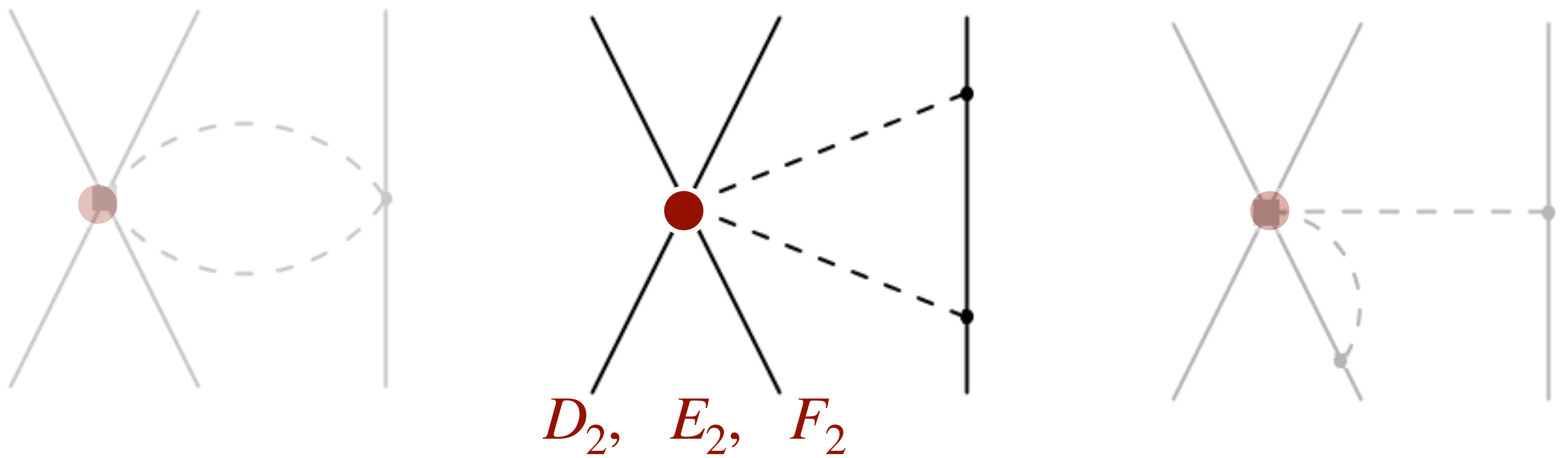
Contributions from $\sim E_\pi^2 \, E_2$

- E_2 induces same structure
  - With $m_\pi^2 \to \left(\vec{q}^{\,2} / m_N\right)^2$

Beyond Weinberg counting

Induced potential



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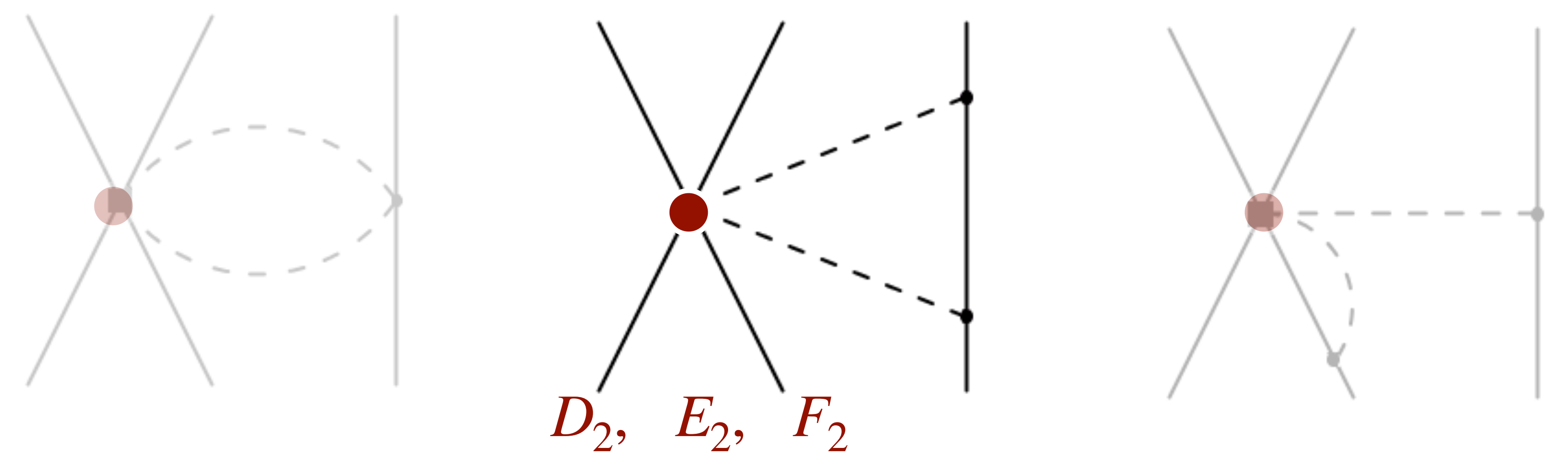
Contributions from $\sim \vec{q}^{\,2} F_2$

- F_2 induces same structure as D_2
  - With additional factors of $\vec{q}^{\,2}$

Beyond Weinberg counting

Induced potential



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Contributions from $\sim E_\pi^2\, E_2$

- E_2
  - With $m_\pi^2 \to \left(\vec{q}^{\,2}/m_N\right)^2$

Negligible for $n \lesssim 2 n_{\rm sat}$

Contributions from $\sim \vec{q}^{\,2} F_2$

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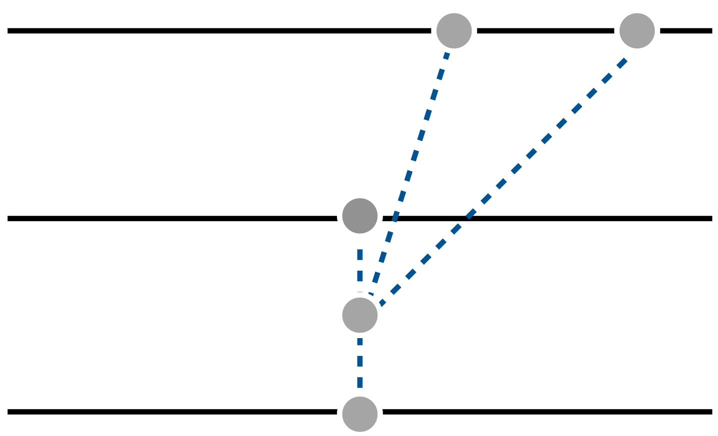
  - With additional factors of $\vec{q}^{\,2}$

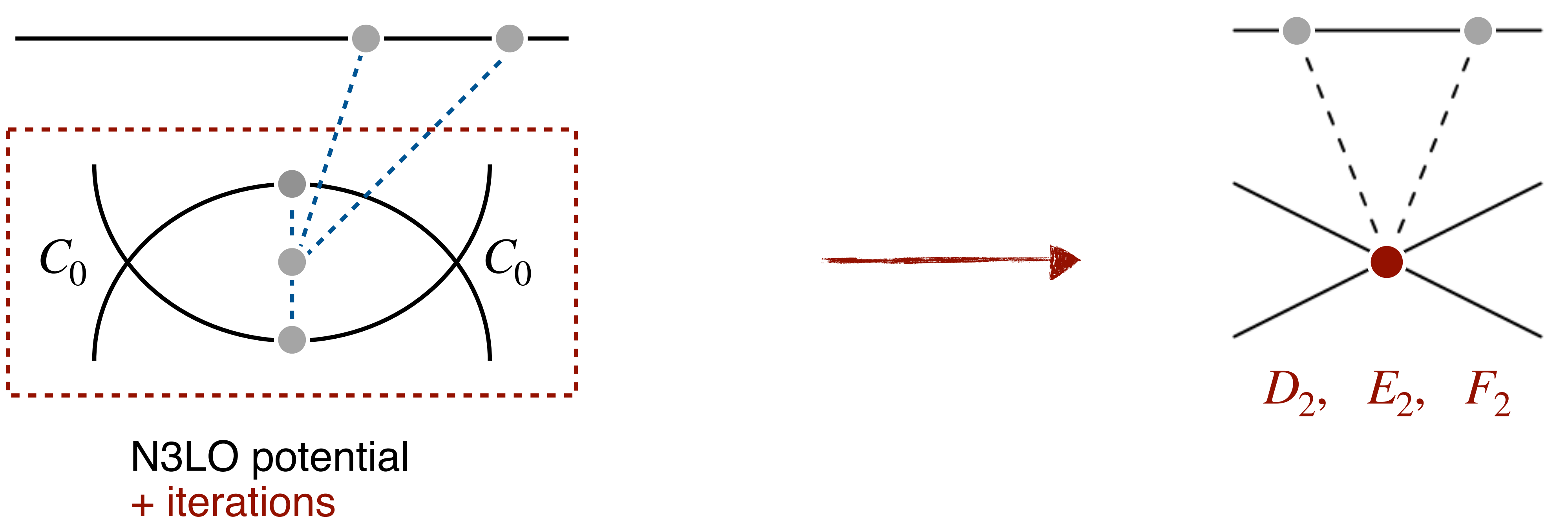
Connection to usual N3LO graphs



N3LO potential

Bernard, Epelbaum, Krebs, Meissner ’08,’11; Ishikawa, Robilotta ’07,

Connection to usual N3LO graphs



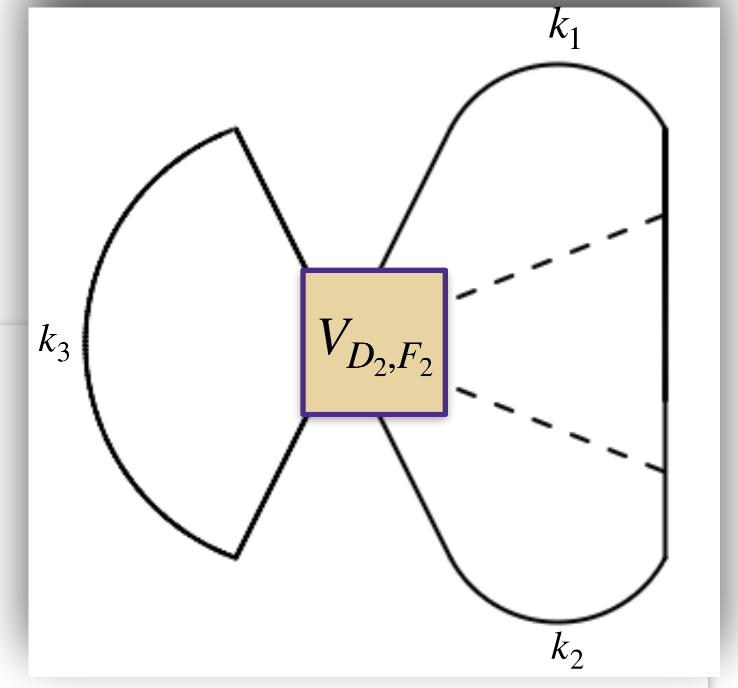
- Part of the ‘conventional’ N3LO potential is connected to D_2, E_2, F_2
  - Generates the divergent diagrams that induce D_2, E_2, F_2
  - Need to be considered simultaneously for a consistent calculation

Bernard, Epelbaum, Krebs, Meissner ’08,’11; Ishikawa, Robilotta ’07,

Impact in dense matter

Effect in dense matter

Hartree-Fock estimate

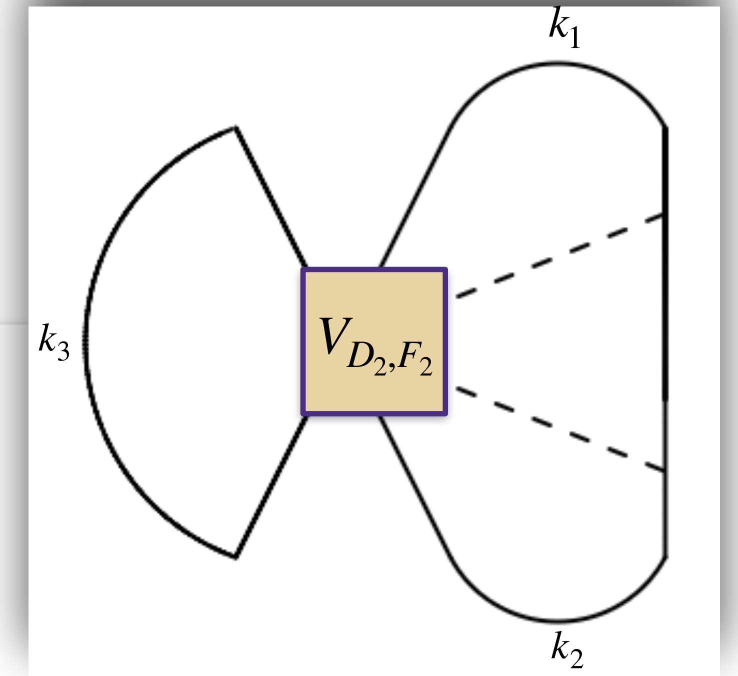


- Expectation value of V_{D_2,F_2} in Fermi gas state
  - Diagrammatically; contract nucleons in all possible ways
  - Finite, no need for additional regulators

$$\langle \mathscr{H}(0) \rangle = \int_{\vec{p}_1,\vec{p}_2,\vec{p}_3 \leq k_f} \left[V^{ijk}_{ijk}(0,0,0) - V^{ikj}_{ijk}(0,\vec{p}_{32},\vec{p}_{23}) + V^{jki}_{ijk}(\vec{p}_{21},\vec{p}_{32},\vec{p}_{13}) + V^{kij}_{ijk}(\vec{p}_{31},\vec{p}_{12},\vec{p}_{21}) - V^{kji}_{ijk}(\vec{p}_{31},0,\vec{p}_{13}) - V^{jik}_{ijk}(\vec{p}_{21},\vec{p}_{12},0) \right]$$

Effect in dense matter

Hartree-Fock estimate



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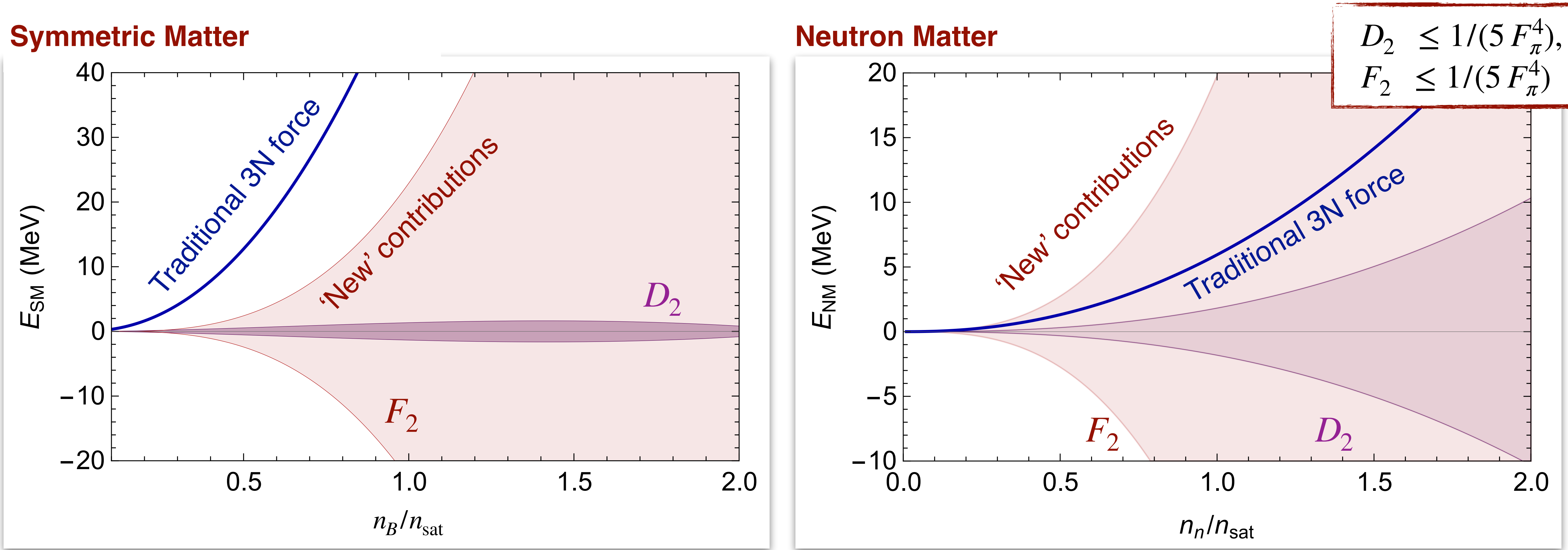
$$\langle \mathscr{H}(0) \rangle = \int_{\vec{p}_1,\vec{p}_2,\vec{p}_3 \leq k_f} \left[V_{ijk}^{ijk}(0,0,0) - V_{ijk}^{ikj}(0,\vec{p}_{32},\vec{p}_{23}) + V_{ijk}^{jki}(\vec{p}_{21},\vec{p}_{32},\vec{p}_{13}) + V_{ijk}^{kij}(\vec{p}_{31},\vec{p}_{12},\vec{p}_{21}) - V_{ijk}^{kji}(\vec{p}_{31},0,\vec{p}_{13}) - V_{ijk}^{jik}(\vec{p}_{21},\vec{p}_{12},0) \right]$$

- Need the size of the LECs
  - Use expectation from RGE for a first estimate:

$$D_2 \leq 1/(5\,F_\pi^4), \qquad F_2 \leq 1/(5\,F_\pi^4)$$

New class of three-nucleon forces

Effects in dense matter

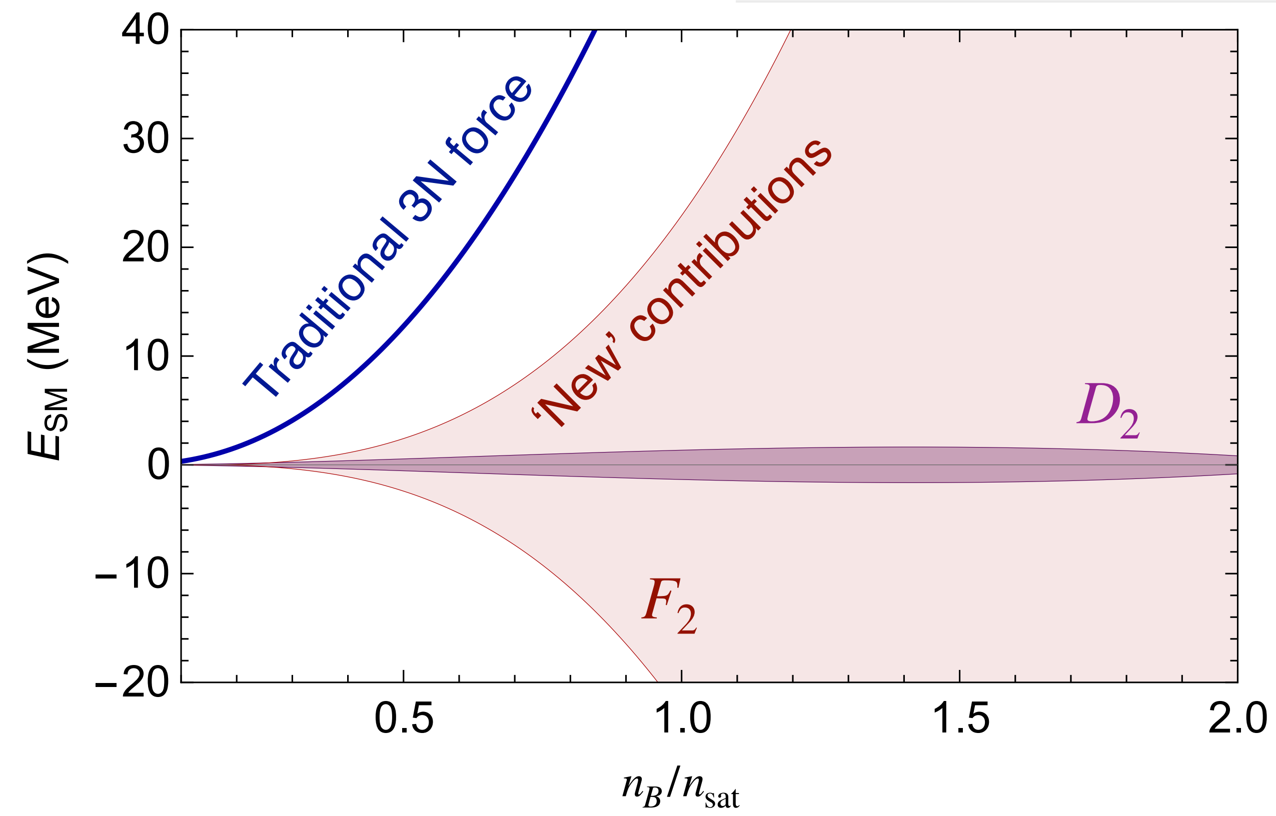


- Sizable contributions
- D_2, F_2 induce a stronger density dependence

New class of three-nucleon forces

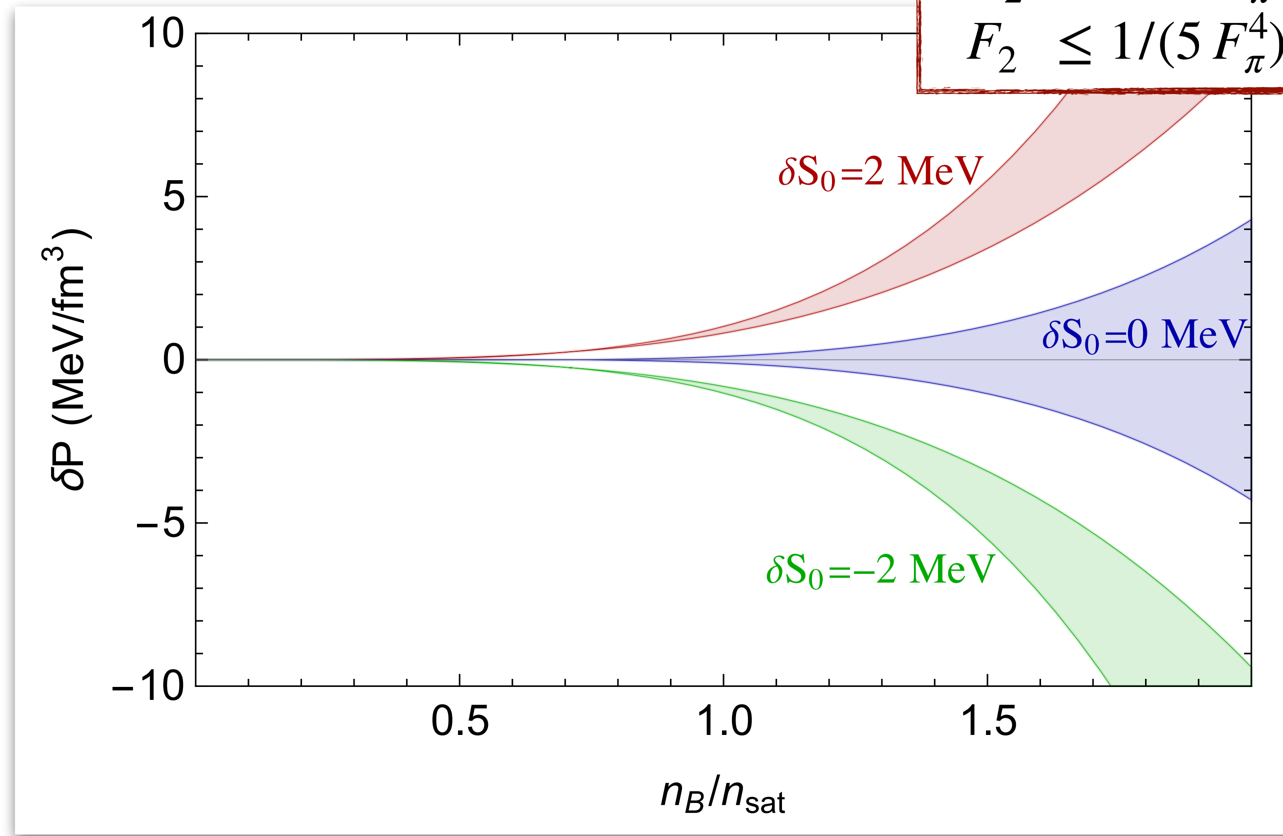
Effects in dense matter

Symmetric Matter



Pressure in Symmetric Matter

$D_2 \leq 1/(5\,F_\pi^4)$,
$F_2 \leq 1/(5\,F_\pi^4)$



- Sizable contributions
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New class of three-nucleon forces

Effects in dense matter

Symmetric Matter **Pressure in Symmetric Matter**

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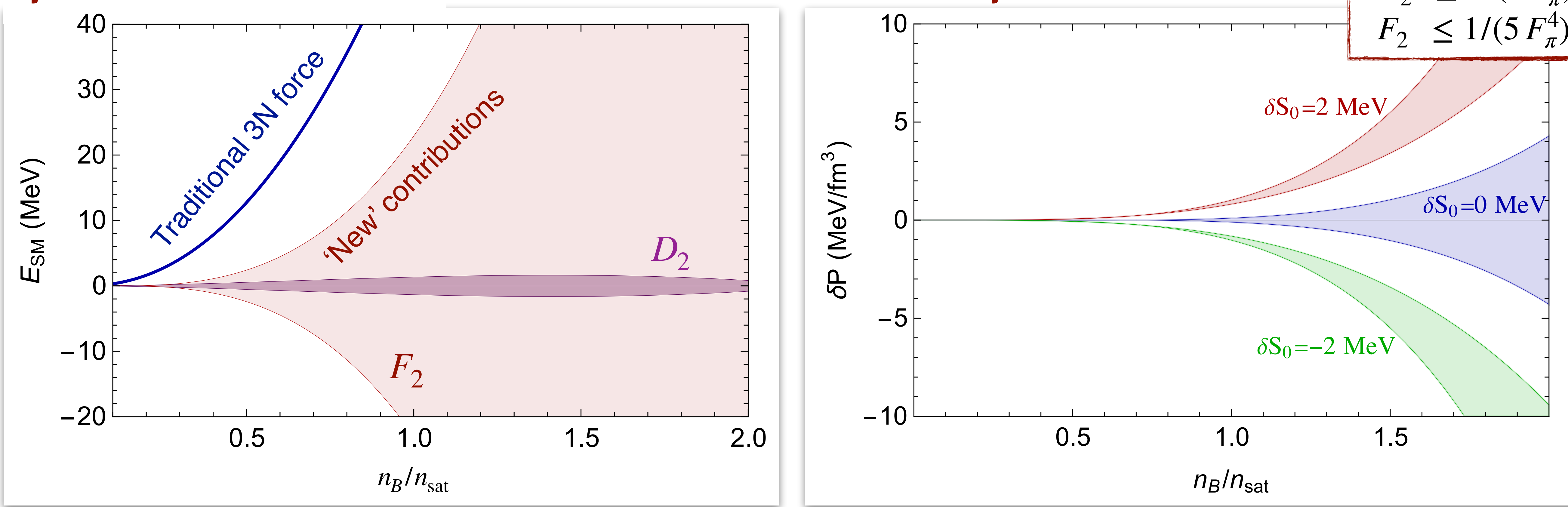


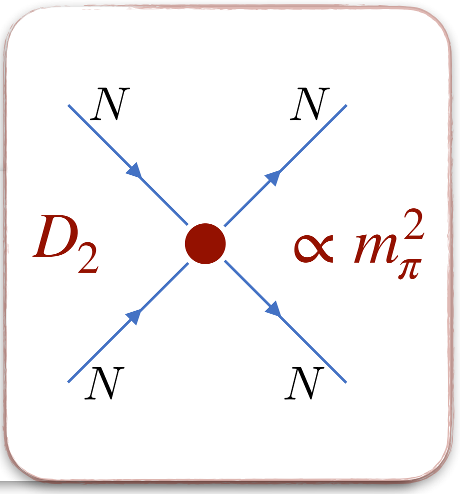
- Sizable contributions
- D_2, F_2 induce a stronger density dependence

- Scheme and regulator dependent
  - Roughly $\sim 1/3$ smaller using dispersive regulator scheme [for $\Lambda = 500$ MeV at n_{sat}]
  - Requires consistent combination with `usual' N3LO three-nucleon force
- Crucially depend on the size of D_2, F_2

Impact Beyond the Standard Model

Effects on BSM scenarios

- D_2 induces m_π dependence of NN interactions



Effects on BSM scenarios

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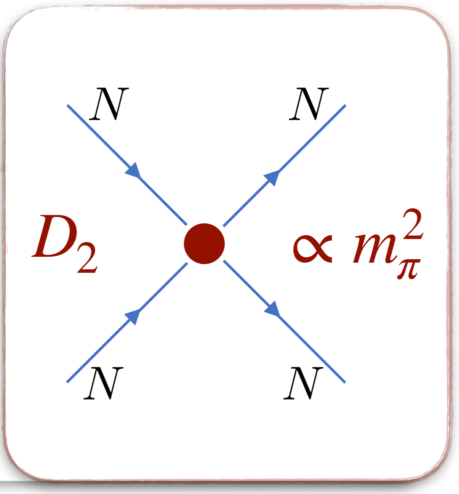
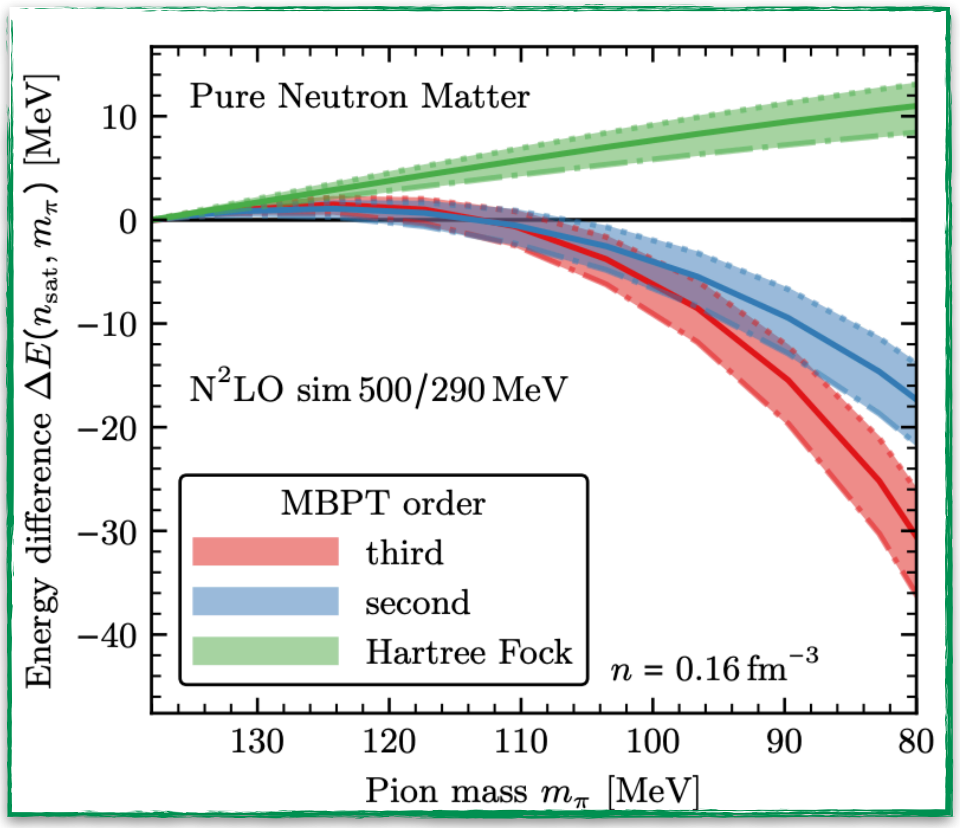


BSM scenarios can affect the quark masses

- Variations of fundamental constants
  - Lead to time dependent $m_q(t)$
- **Axion scenarios**
  - Axion could condense in dense matter like neutron stars
  - Would change $m_\pi(\theta = 0) \rightarrow m_\pi(\theta = \pi) \simeq 80\,\mathrm{MeV}$

See Sanjay's talk

- Can be probed through their effect on the nuclear force
  - Requires m_π dependence of the nuclear force and D_2



Kumamoto, Huang, Drischler, Baryakhtar, Reddy, '24

Determining the LECs

How to determine D_2, F_2

From theory:

- First principles determination using Lattice QCD
  - Currently only calculations at unphysical m_π

e.g. Beane, Bedaque, Orginos, Savage, ’06; Beane et al ’15;

How to determine D_2, F_2

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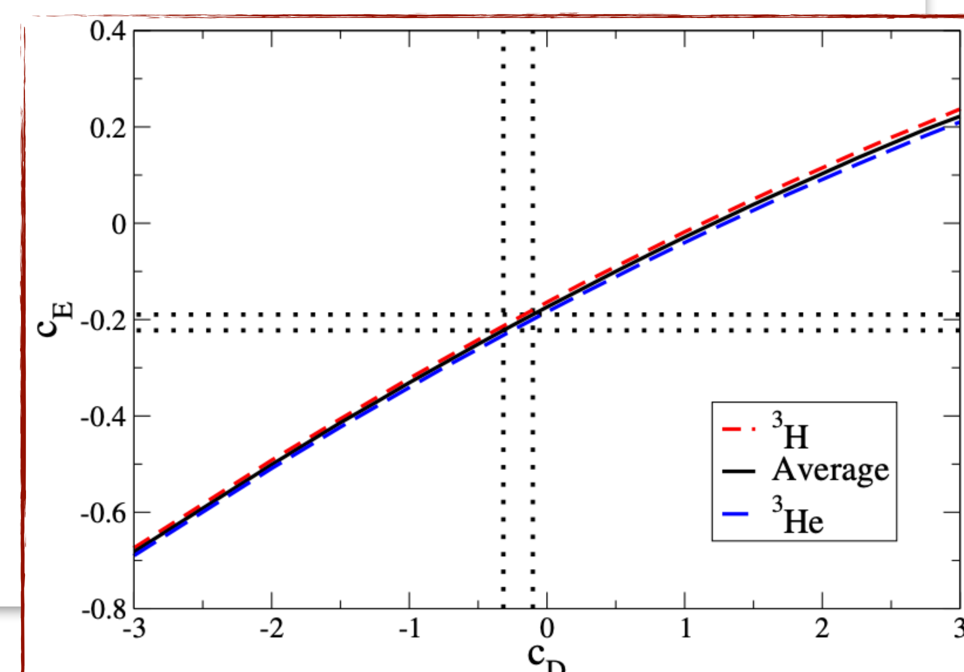
e.g. Beane, Bedaque, Orginos, Savage, ’06; Beane et al ’15;

From experiment:

- Determine D_2, F_2 together with $c_{D,E}$ from
    - Light systems:
        - Nd scattering
        - Binding energies
        - tritium β decay
    - Properties of dense matter
    - Properties of neutron stars
    - π-nucleus scattering

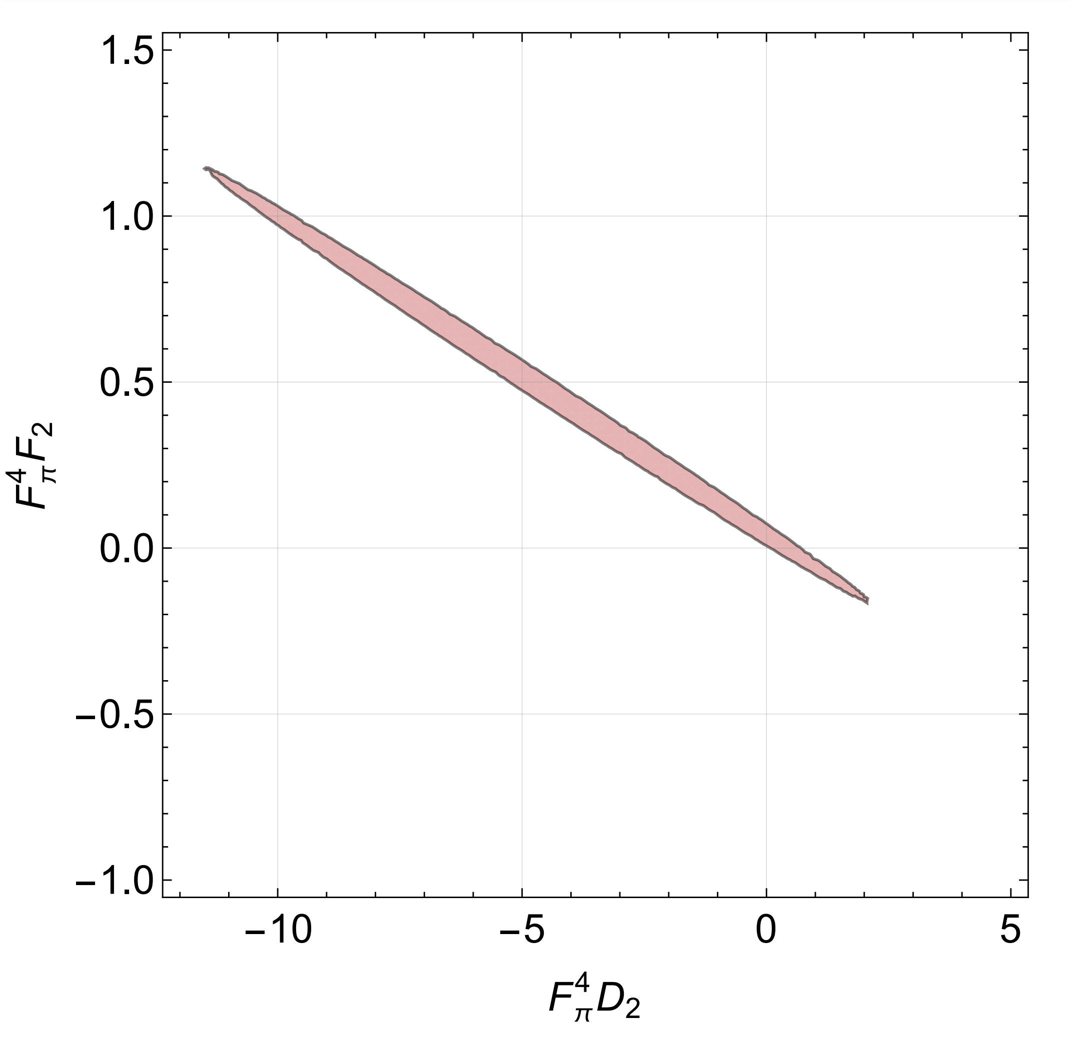


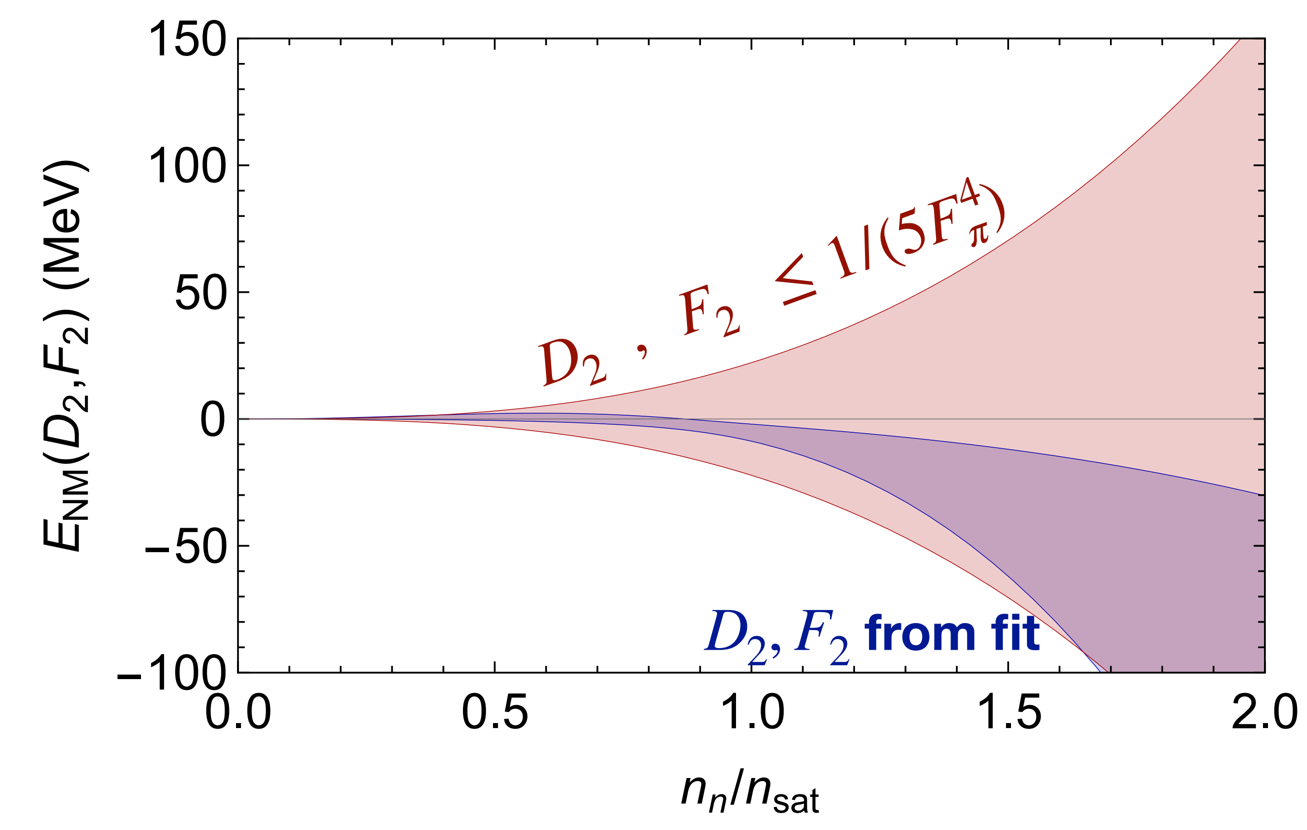
Gazit et al ‘09

How to determine D_2, F_2

Properties of dense matter

- ***Naive*** implementation
  - Combines HF estimates of 3-nucleon force with 2nucleon contributions
  - Fits to properties of dense matter near saturation

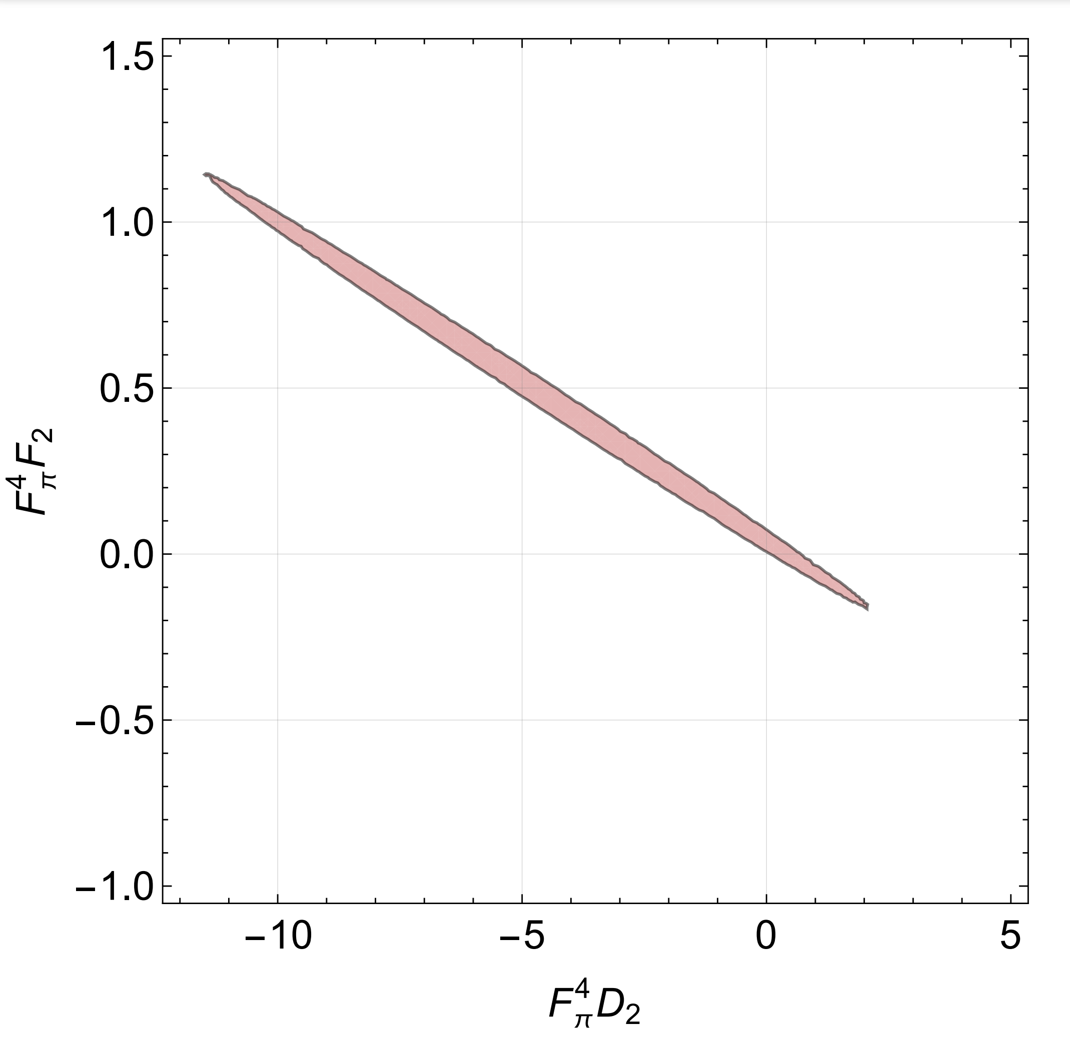


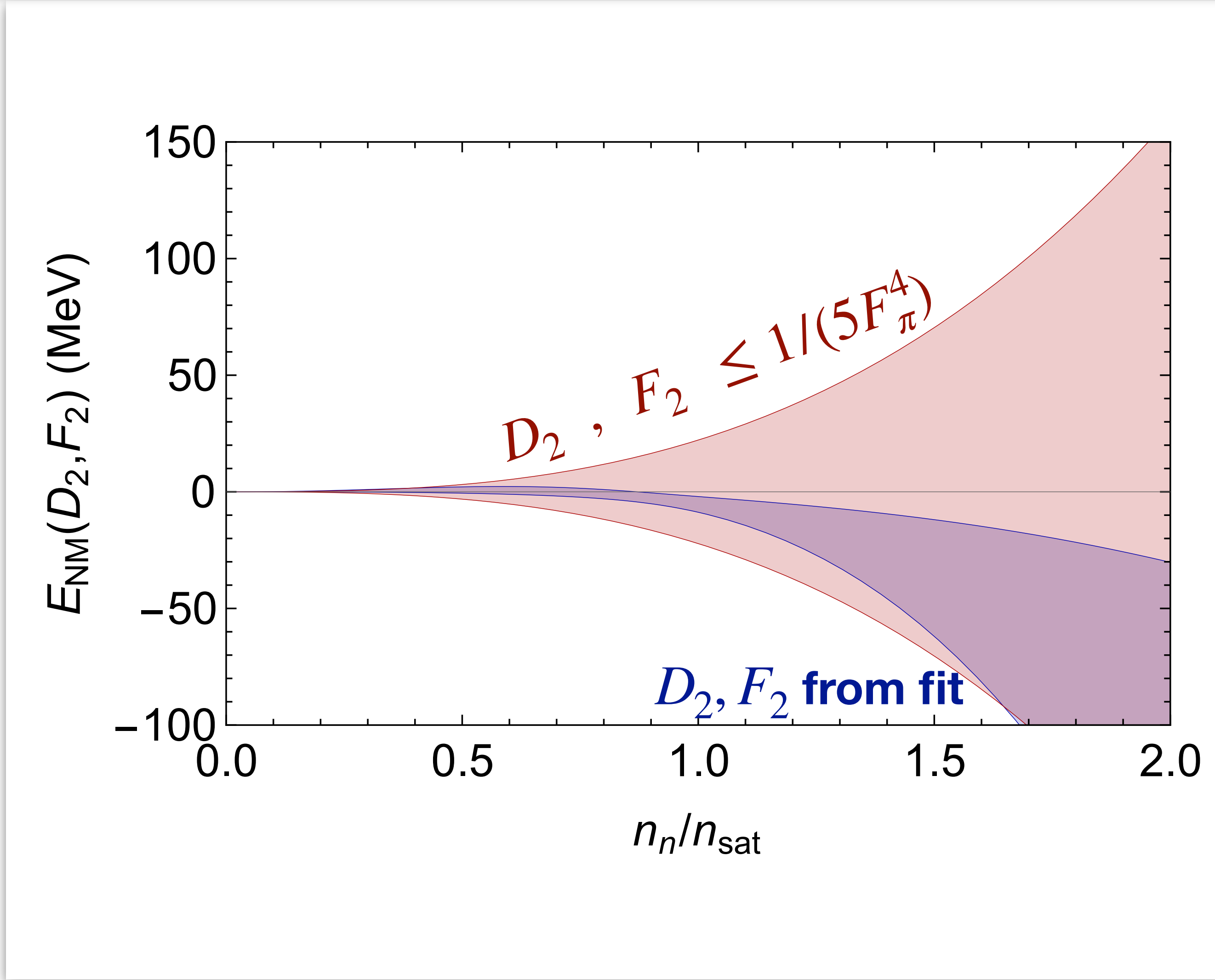


How to determine D_2, F_2

Properties of dense matter

Naive (inconsistent) implementation



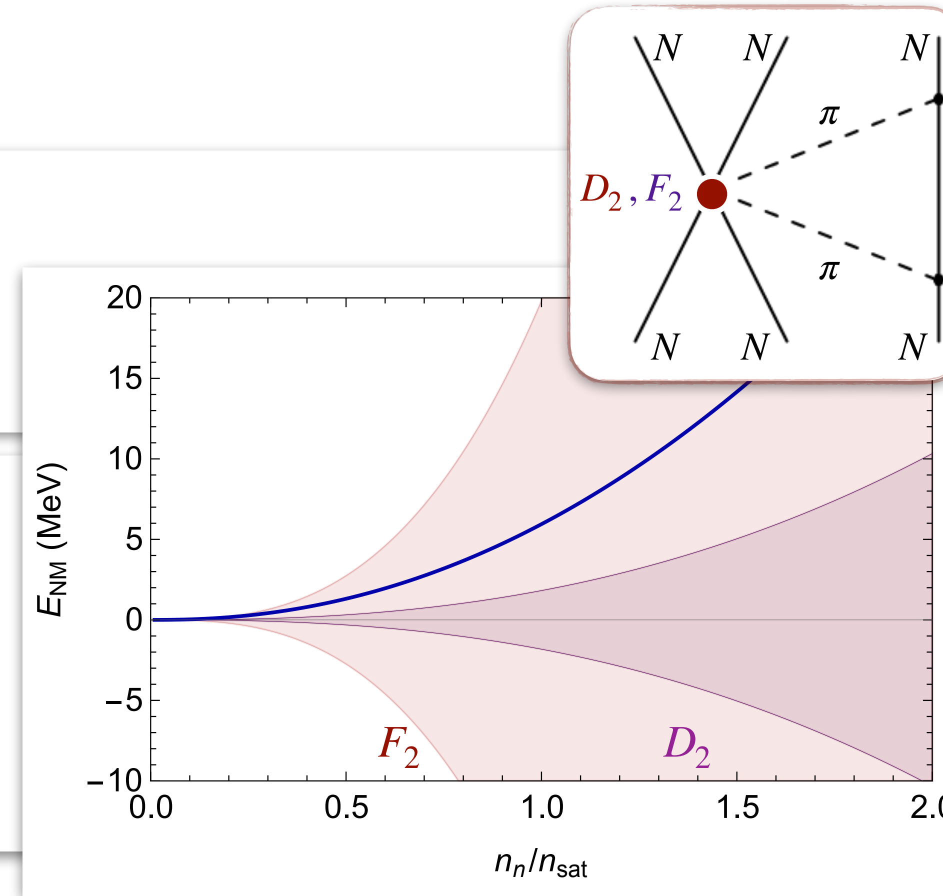


- ***Inconsistent:*** 2- and 3-nucleon forces using different regulators, Hartree-Fock estimates…
- Does suggest dense matter can help pin down D_2, F_2

- A ***consistent*** determination is work in progress (w/ C. Drischler, M. Kumamoto, M. Dawid, S. Reddy)

Summary

- Renormalization requires $\pi^2 NN$ interactions at LO
- Induce 3-nucleon forces through loops



- Significant contributions in dense matter
  - Important for neutron stars (equation of state)
  - Nuclei
- Crucially depends on the value of the LECs
  - Need to be fit to data

Questions/outlook

- Are these the only `new' terms?
  - Additional contributions other channels?

  de Vries, Gnech, Shain, '20

- Determining the LECs
  - From light nuclei or properties of dense matter
- Needed to asses the impact on
  - Dense systems
  - BSM scenarios & m_π dependence on nuclear force

Energy/particle at $m_\pi \neq m_\pi^{(\mathrm{phys})}$