Analysis of nucleon excited states from lattice QCD with the Bayesian Reconstruction

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(χQCD collaboration)

Inverse Problems and UQ in Nuclear Physics
INT Workshop 24-88W
July 10th, 2024

Outline

- Introduction
  - Finite-volume spectrum from lattice QCD
- How to extract finite-volume spectrum from lattice QCD
  - Multi-exponential fit
  - Variational methods (GEVP)
  - An example: Roper state from lattice QCD
- Application of the Bayesian Reconstruction
- Conclusion and outlook

The path integral in the Euclidean space - lattice QCD

- Observables from the path integral (Minkowski):

Perturbative expansions

$$\langle O \rangle = \frac{\int \mathscr{D}\phi O e^{iS_{\rm M}(\phi)}}{\int \mathscr{D}\phi e^{iS_{\rm M}(\phi)}} \longleftarrow Z_M(J) = \int \mathscr{D}\phi e^{i\int d^4x \mathscr{L}_M}$$

Wick rotation $\Updownarrow$ $t \to -i\tau$, $k^0 \to ik_0^E$

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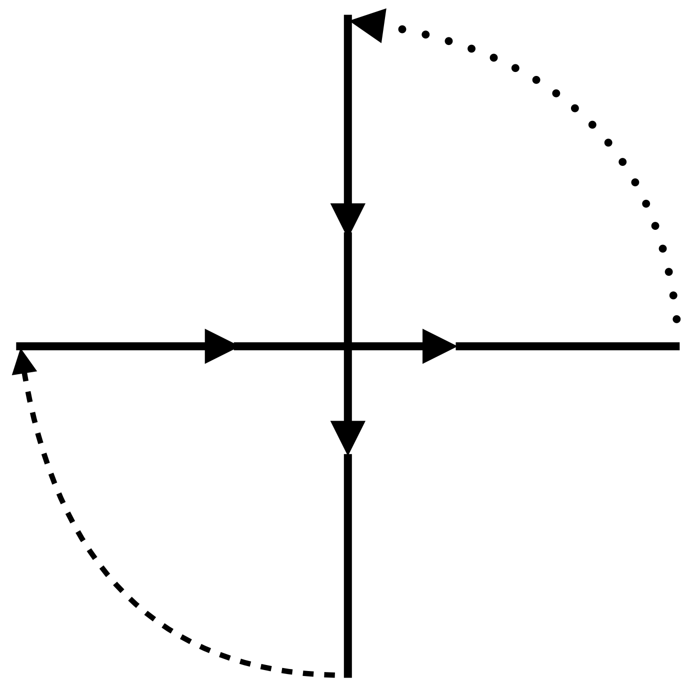

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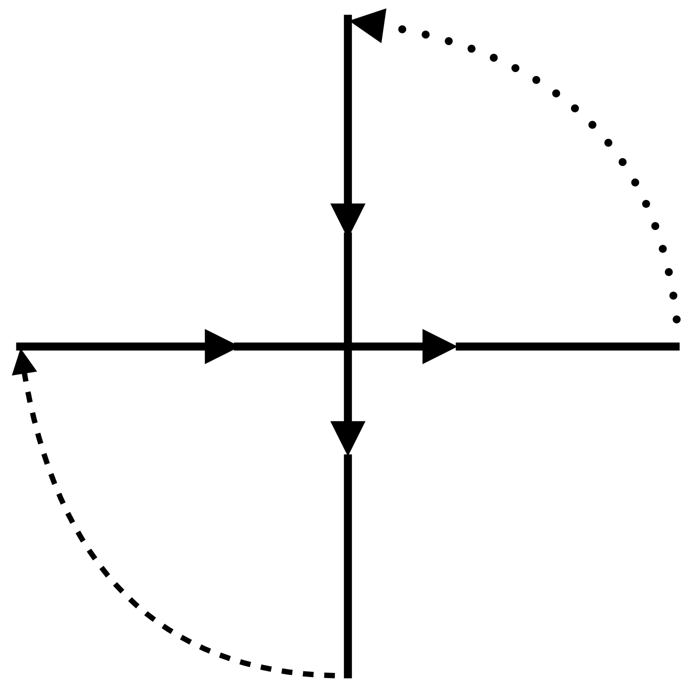

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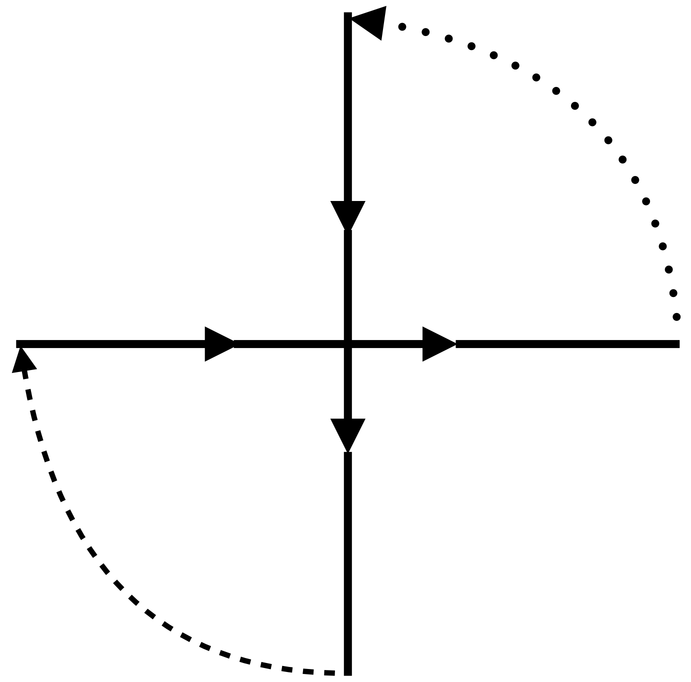

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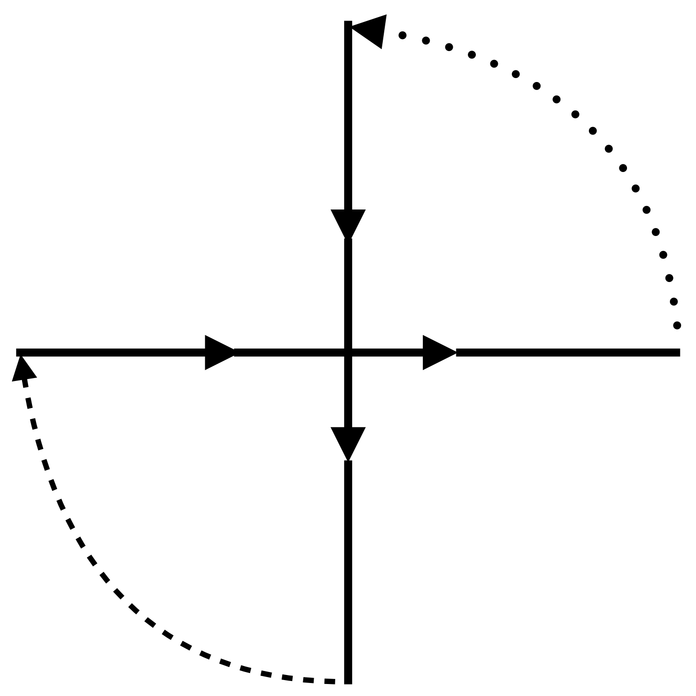

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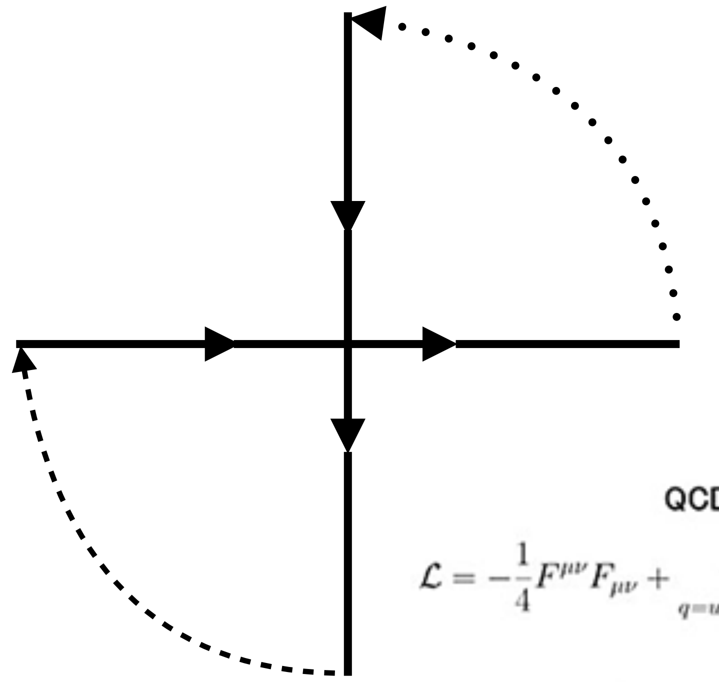

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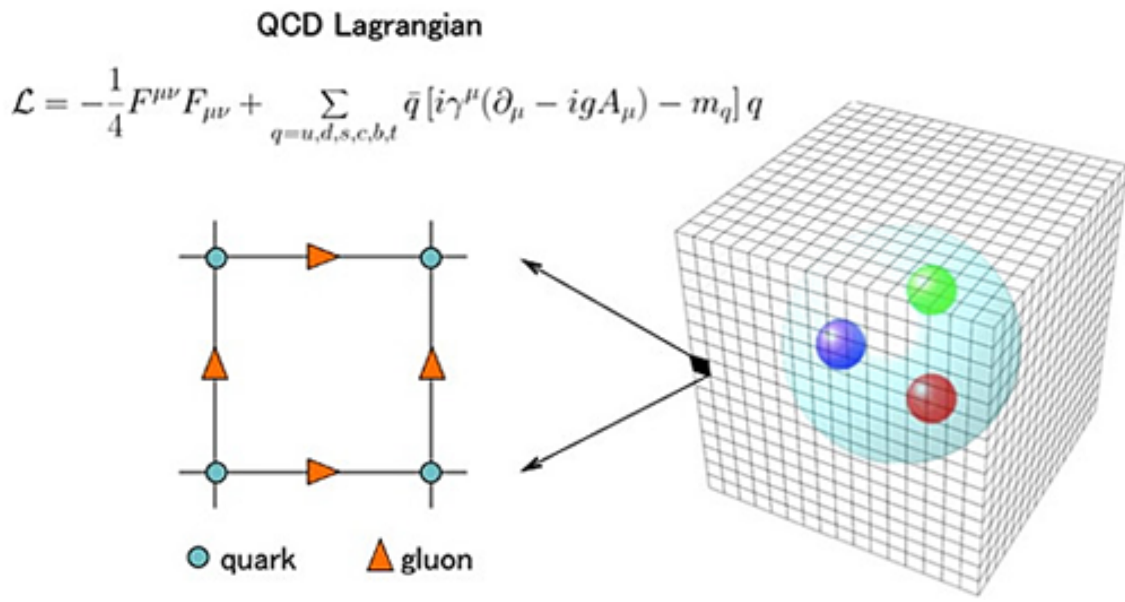

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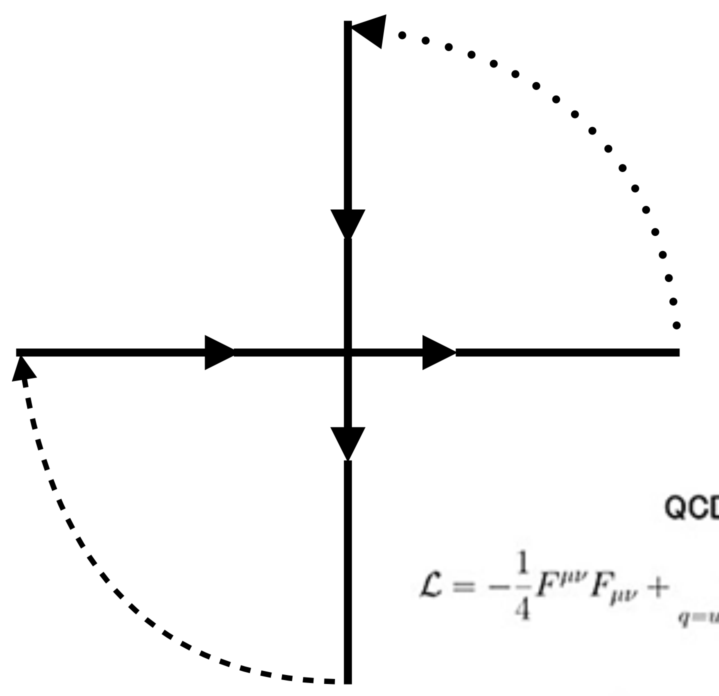

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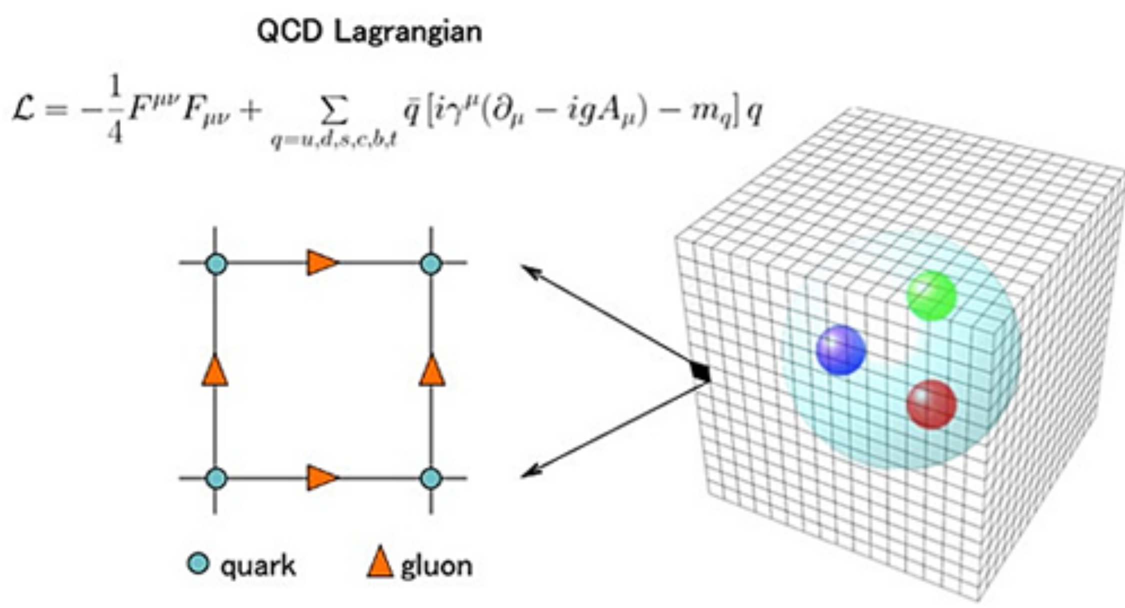

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Non-perturbative

Input parameters
$a, m_q, \ldots, \alpha_s, \ldots$

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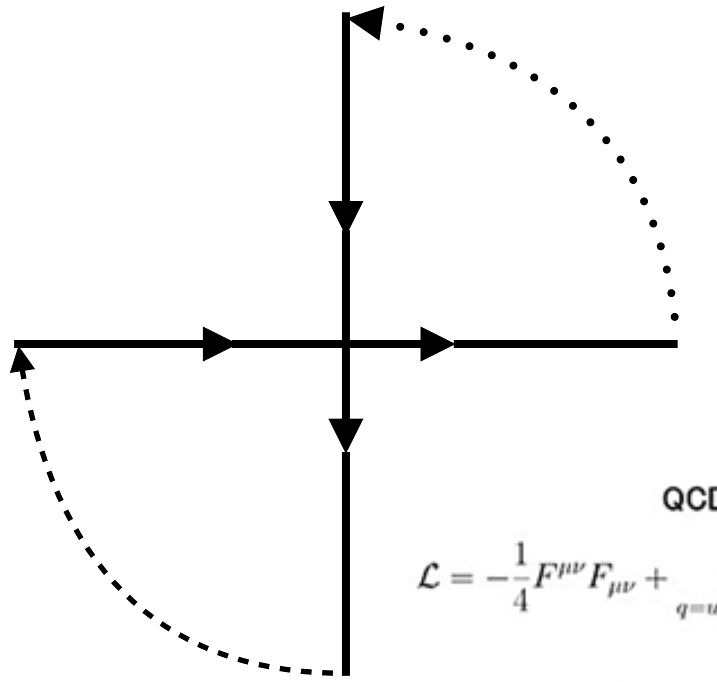

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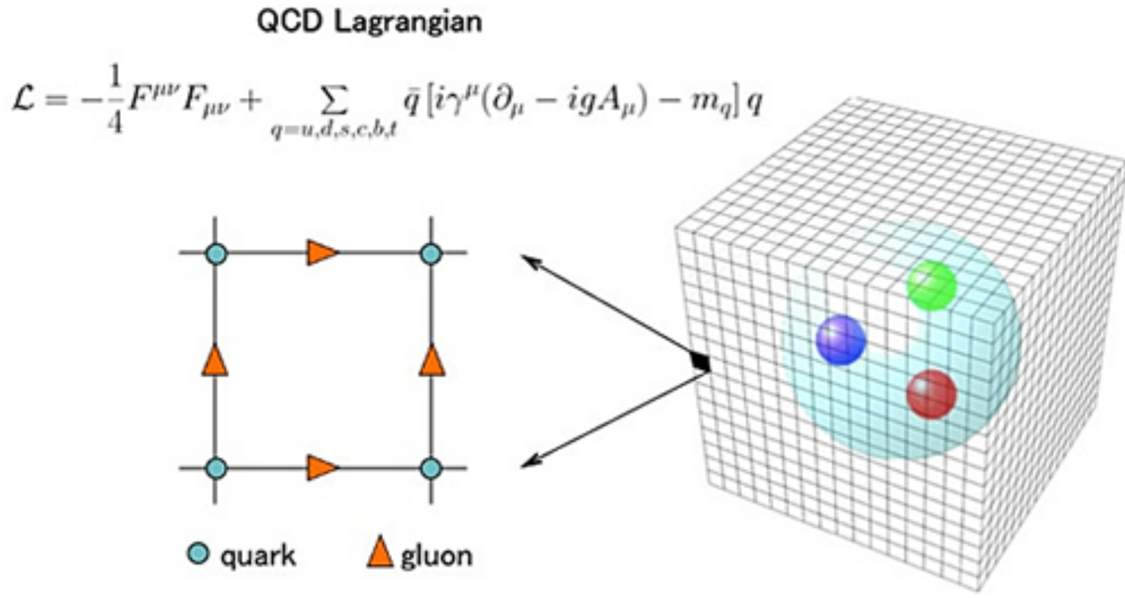

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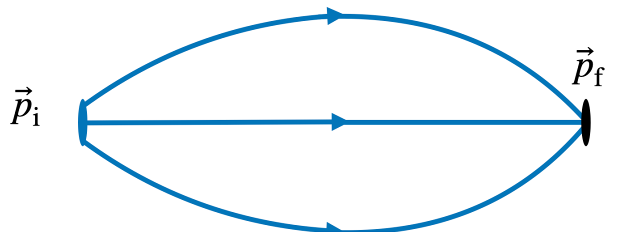
E.g. n-point correlation functions

$$\langle \Omega | \mathrm{T}\phi(x_1)\ldots\phi(x_n) | \Omega \rangle$$

Wick's theorem



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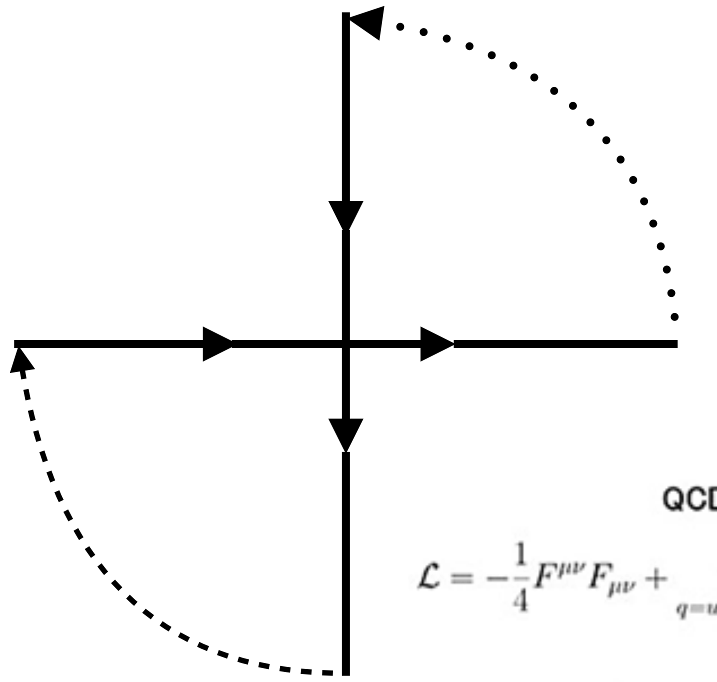

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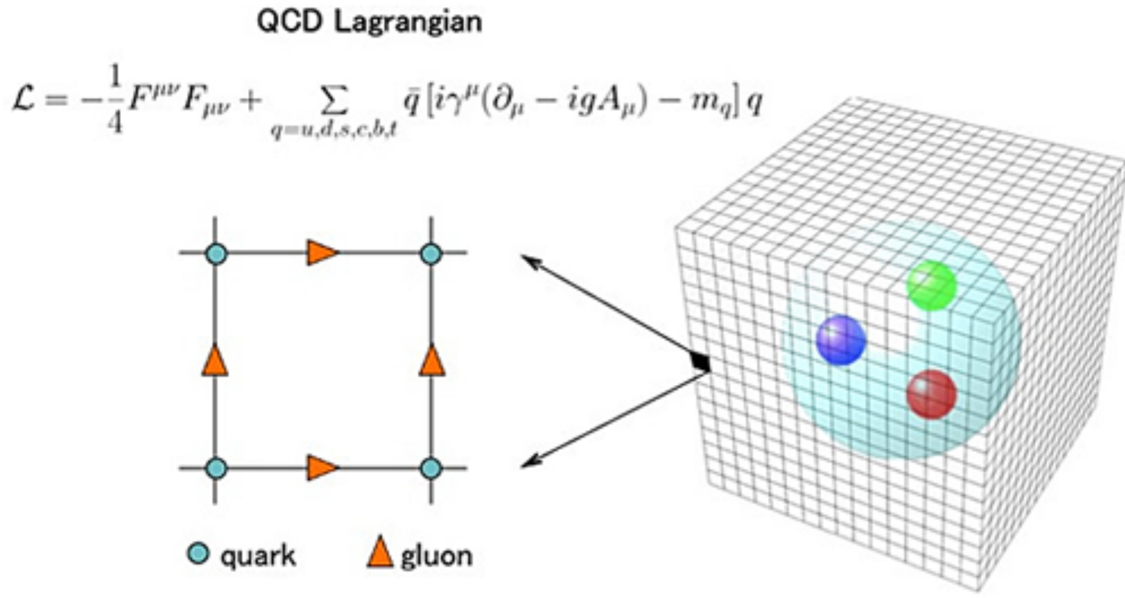

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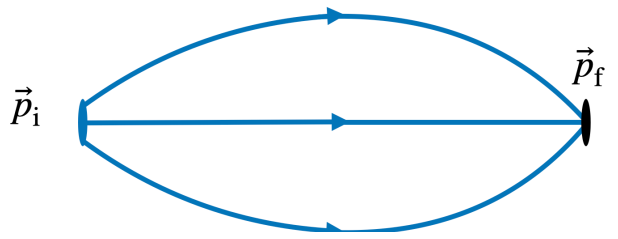

→ Finite-volume spectrum, matrix elements …

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Non-perturbative

Input parameters
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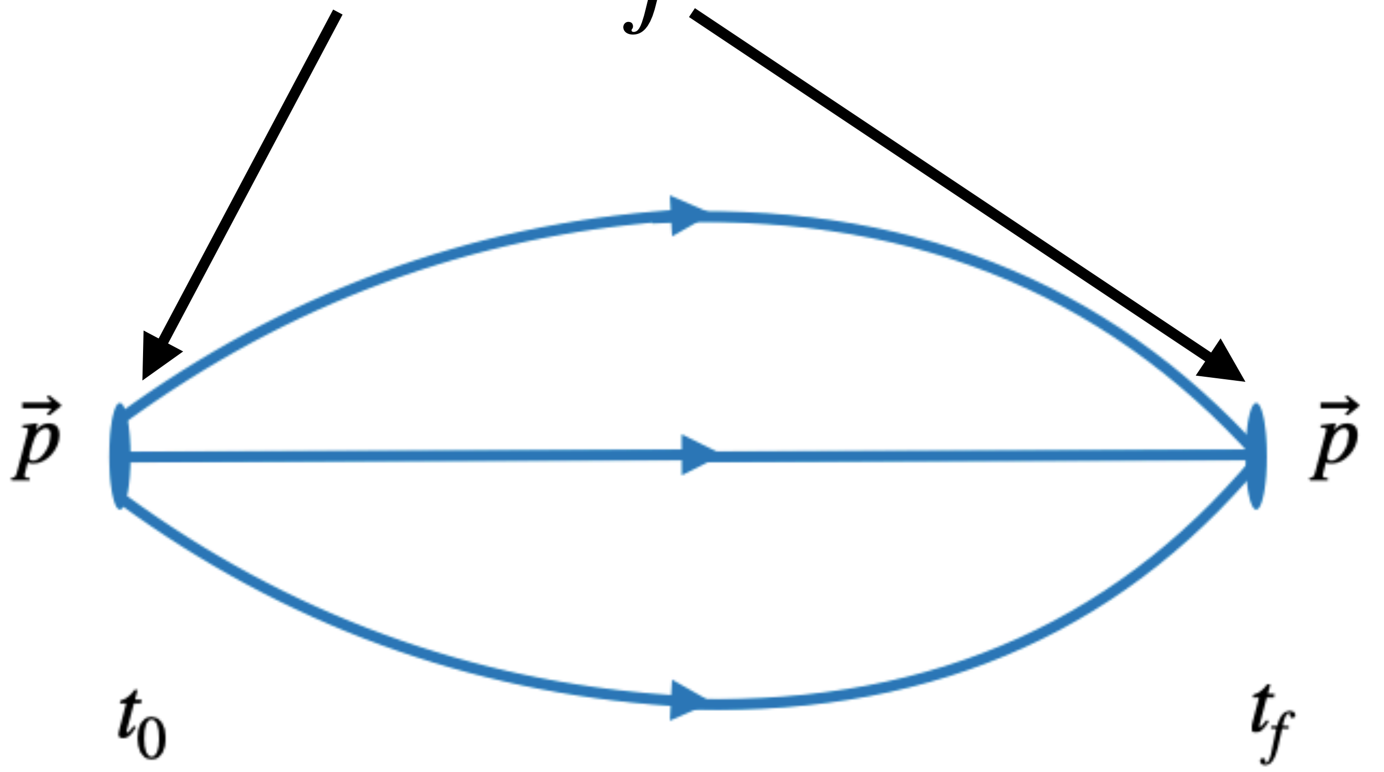
Extracting finite-volume spectrum

- What kind of observables do we measure on the lattice?
- To extract the finite-volume spectrum
  - Two-point correlation functions

$$C_{ij}(t) = \langle \chi_i(t)\chi_j^\dagger(0)\rangle$$



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interpolating operators with the same quantum numbers as the hadron state

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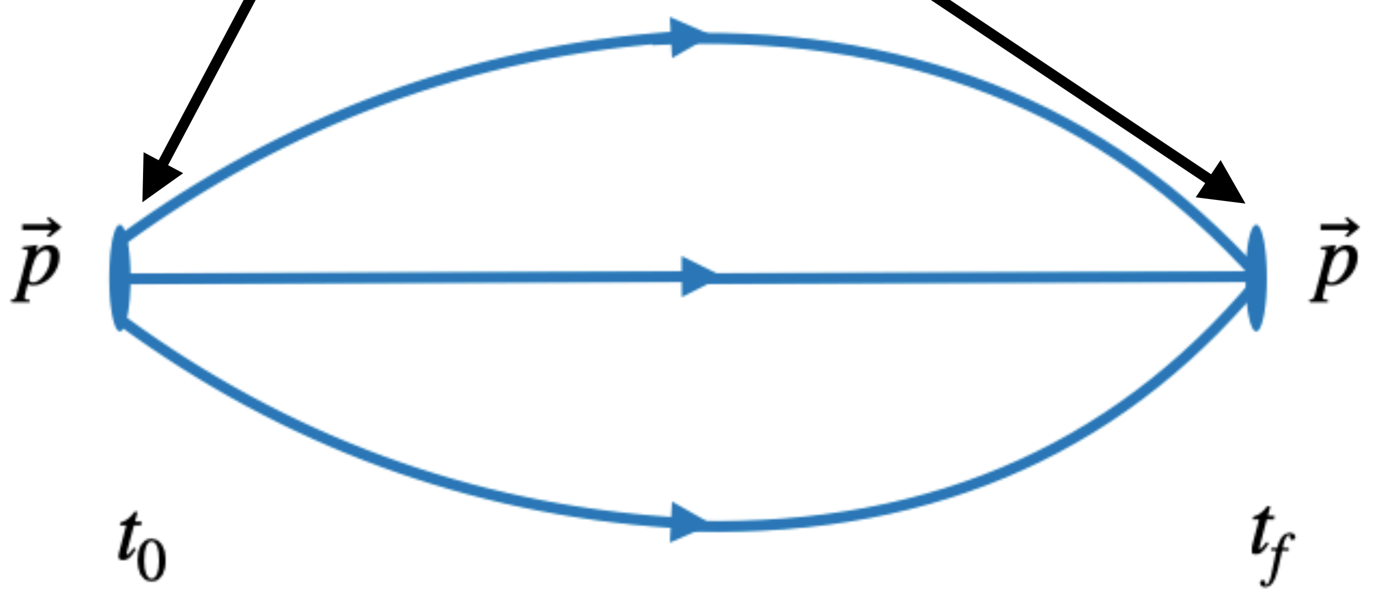
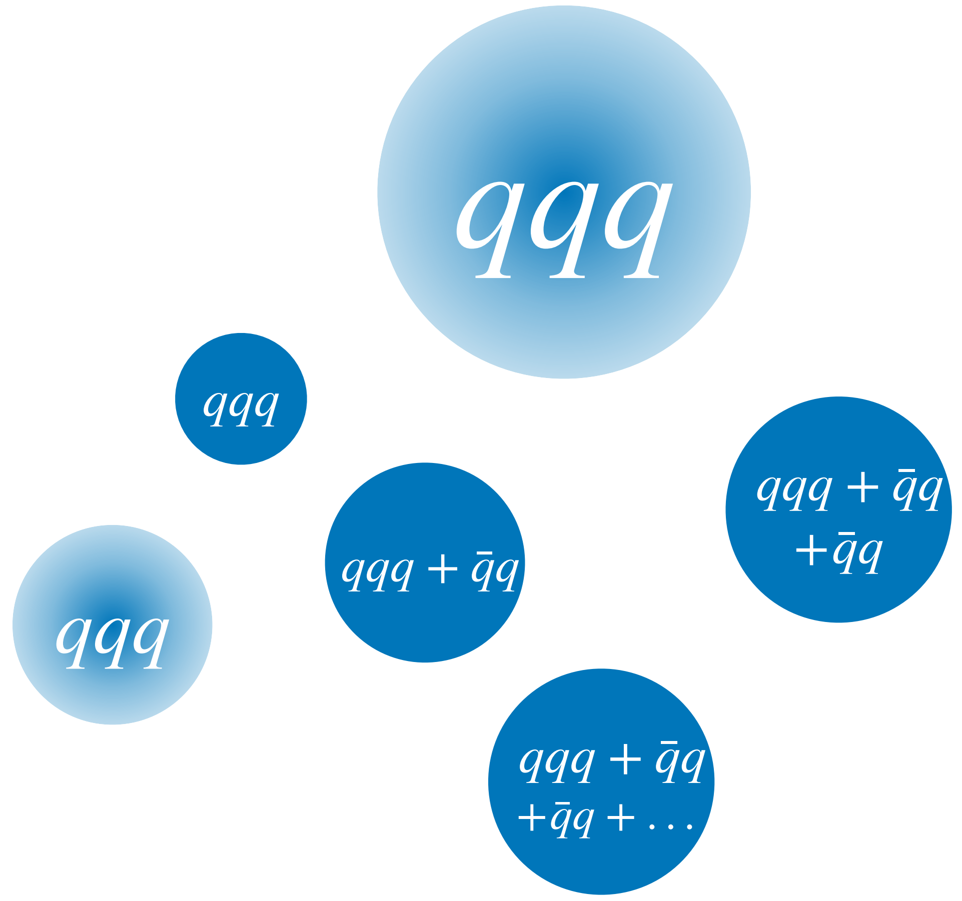


$\chi_i^\dagger \in \{$



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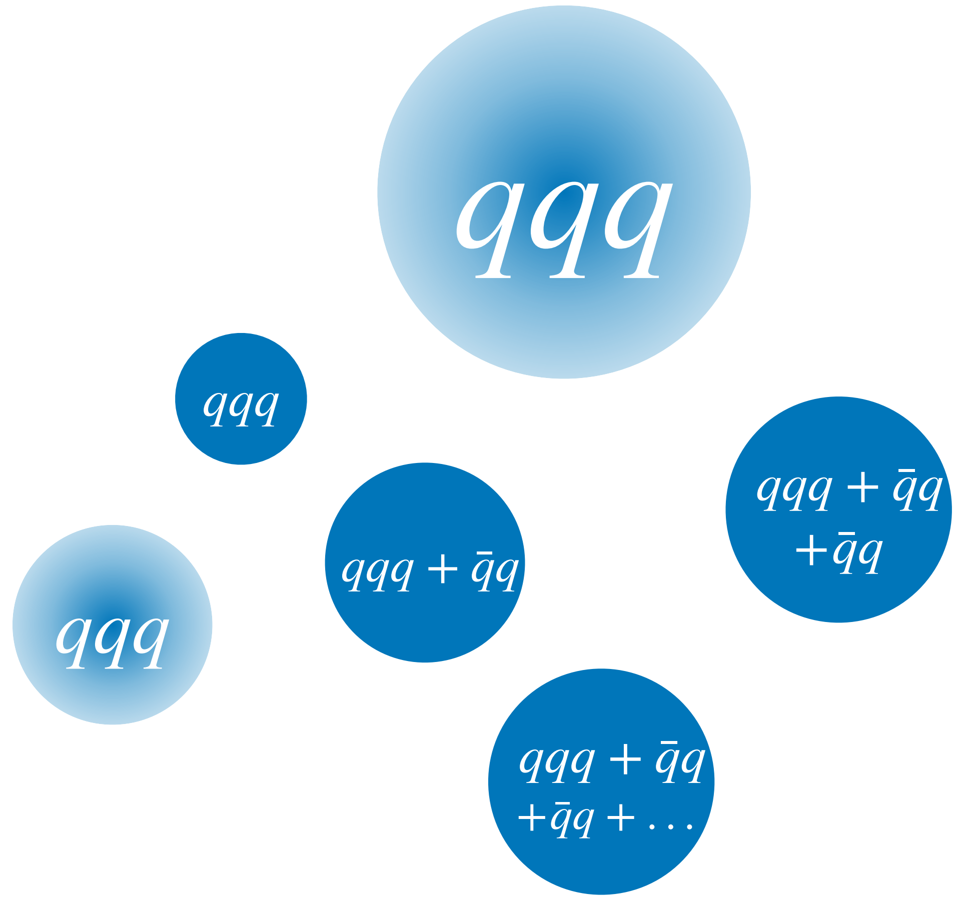
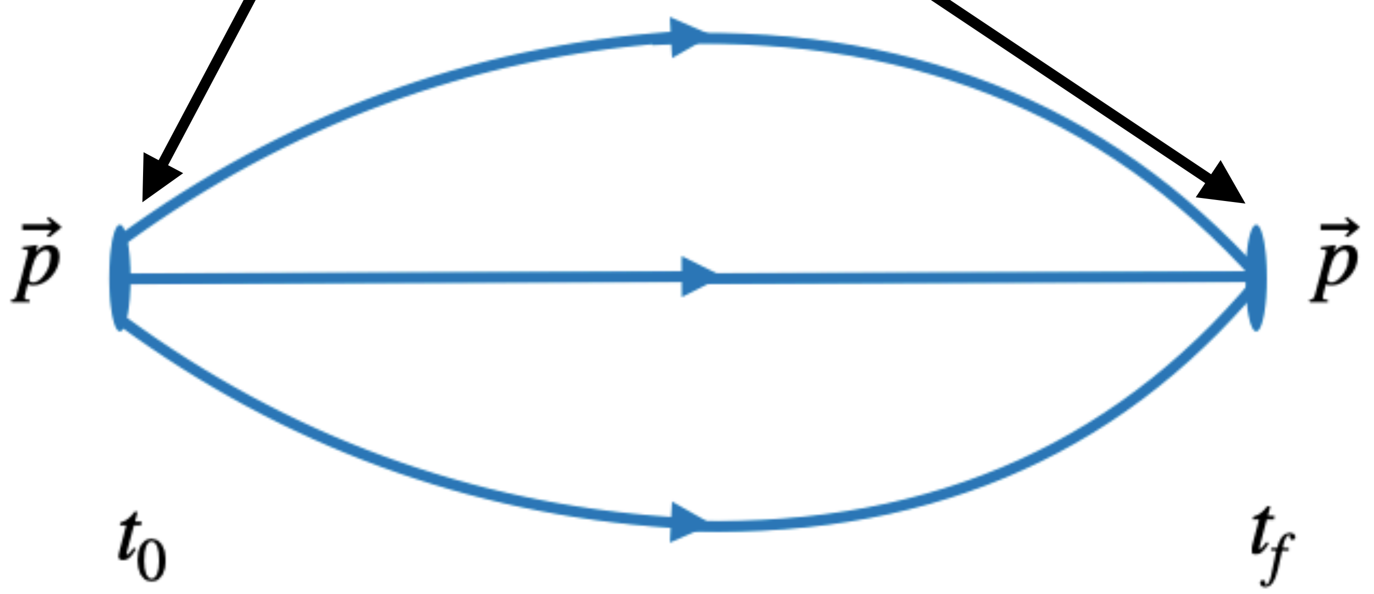


$$C_{ij}(t) = \langle \chi_i(t) \chi_j^\dagger(0) \rangle \rightarrow \sum_{n=0}^{\infty} \langle \Omega | \chi_N | n \rangle \langle n | \chi_N^\dagger | \Omega \rangle e^{-E_n t}$$

$$1 = \sum_n^{\infty} | n \rangle \langle n |$$



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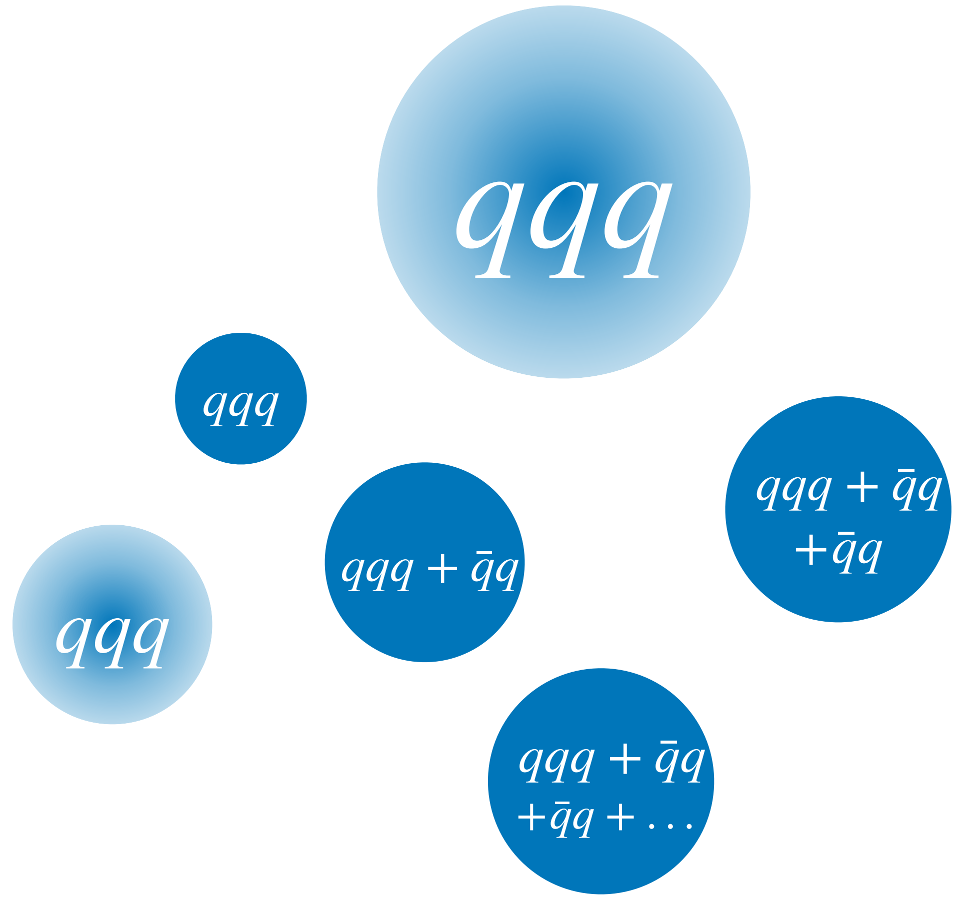

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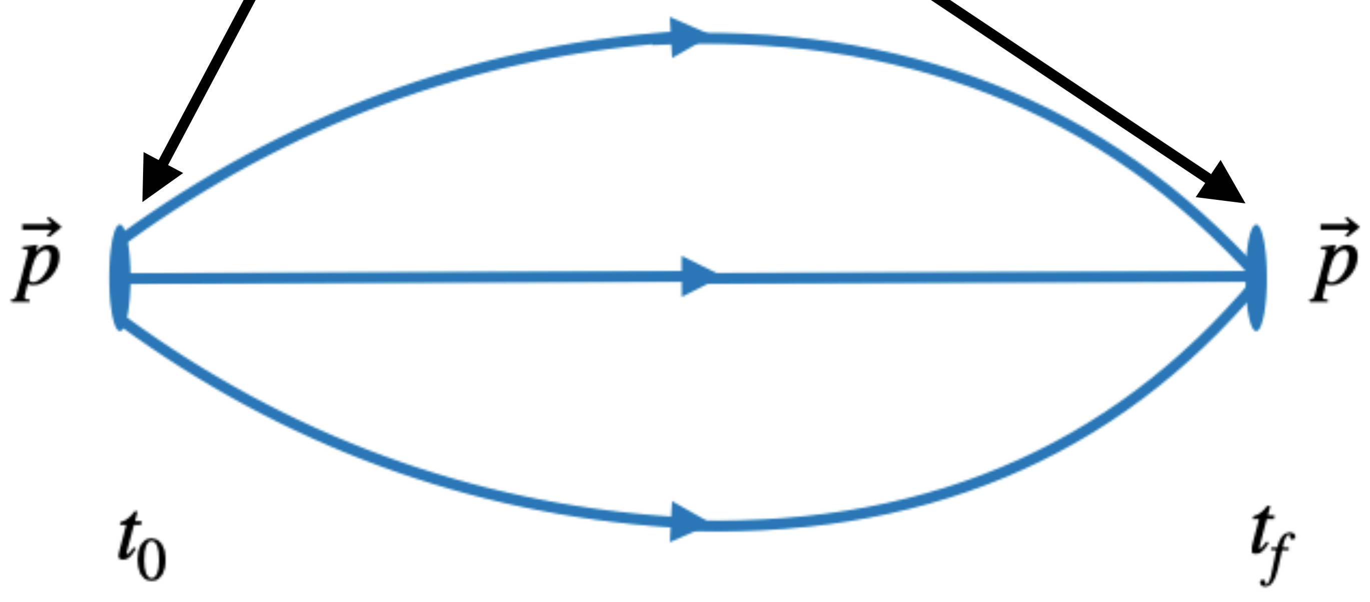

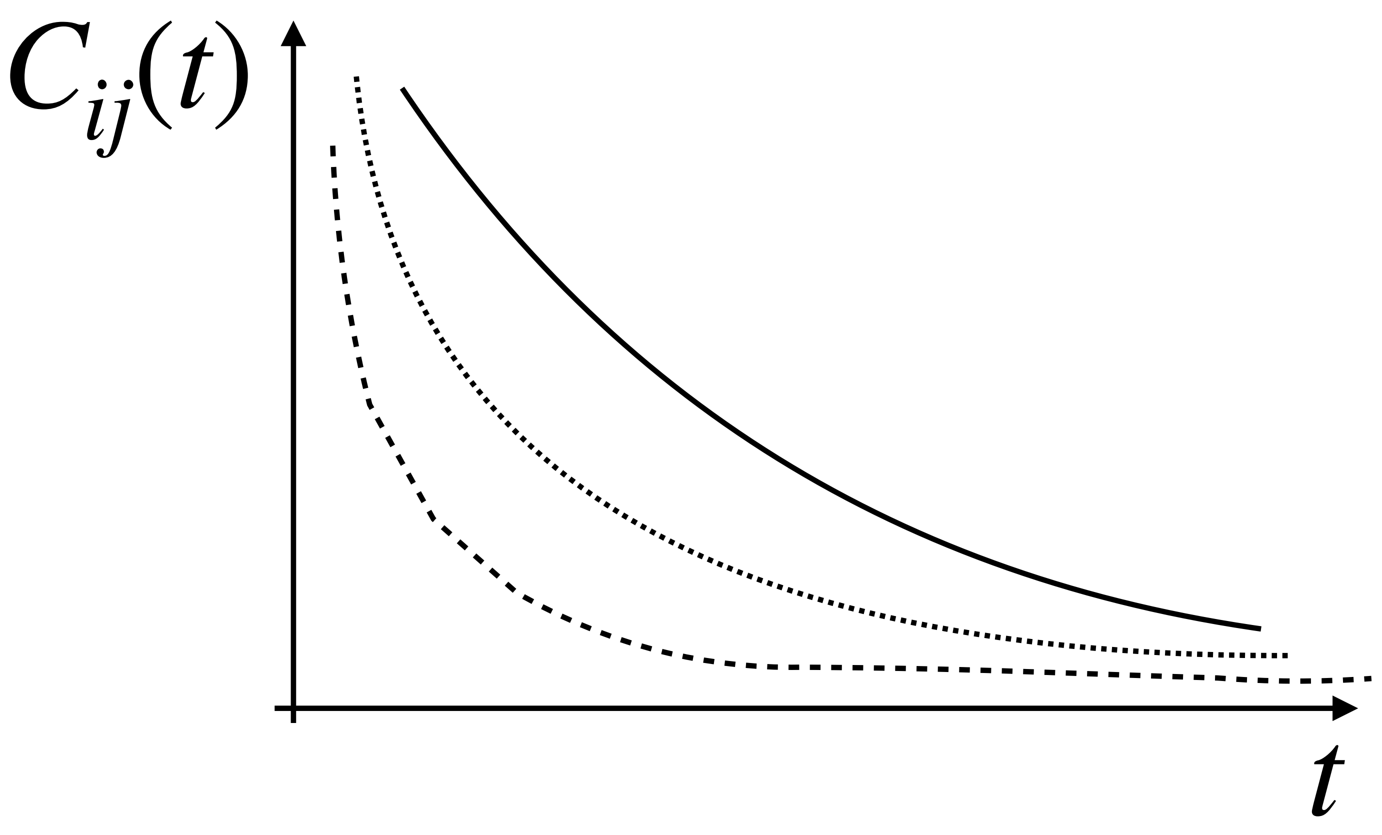

Multi-exponential fit

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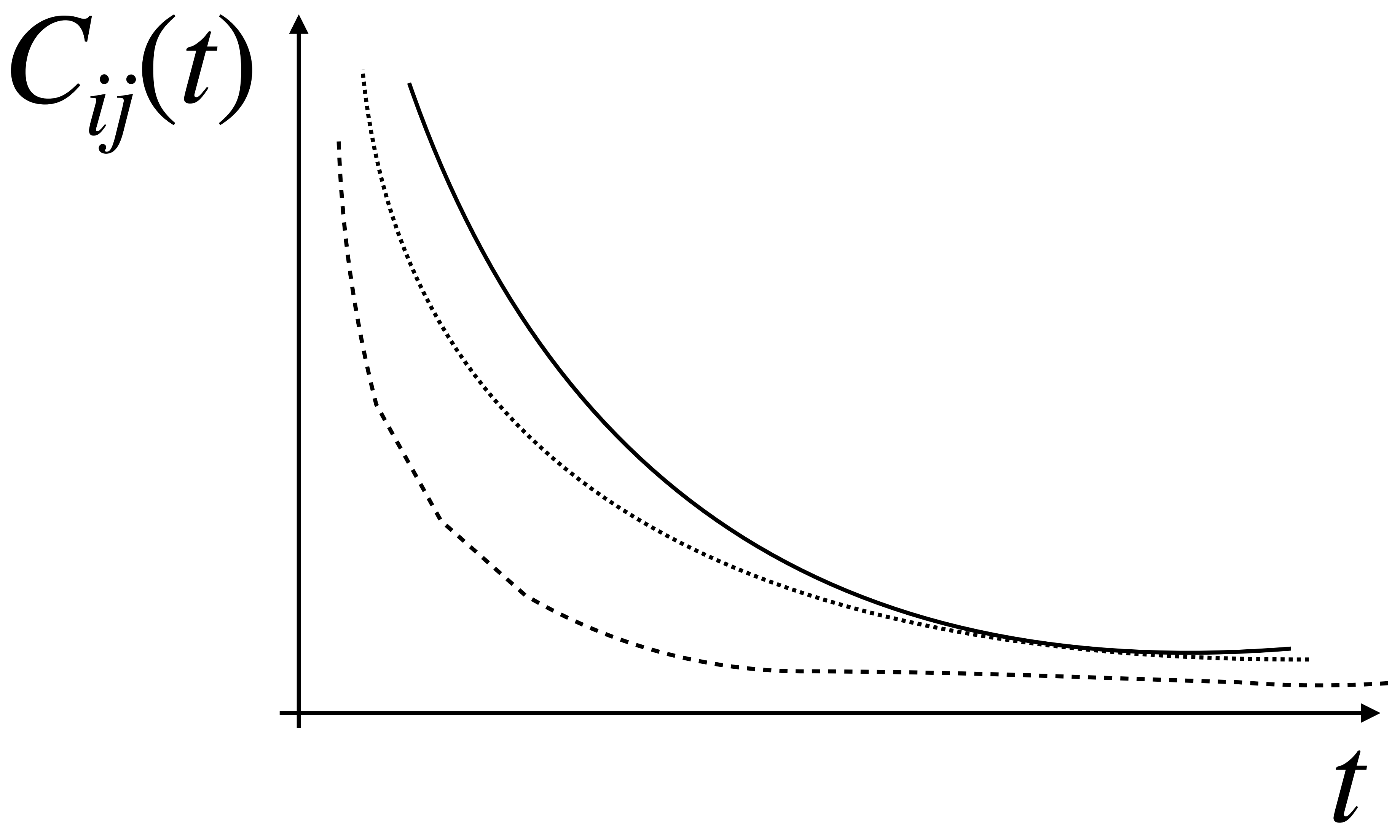
Constraint 1

$t \gg 0$

$$C_{N,2\mathrm{pt}}(t) = W_0 e^{-E_0 t}$$



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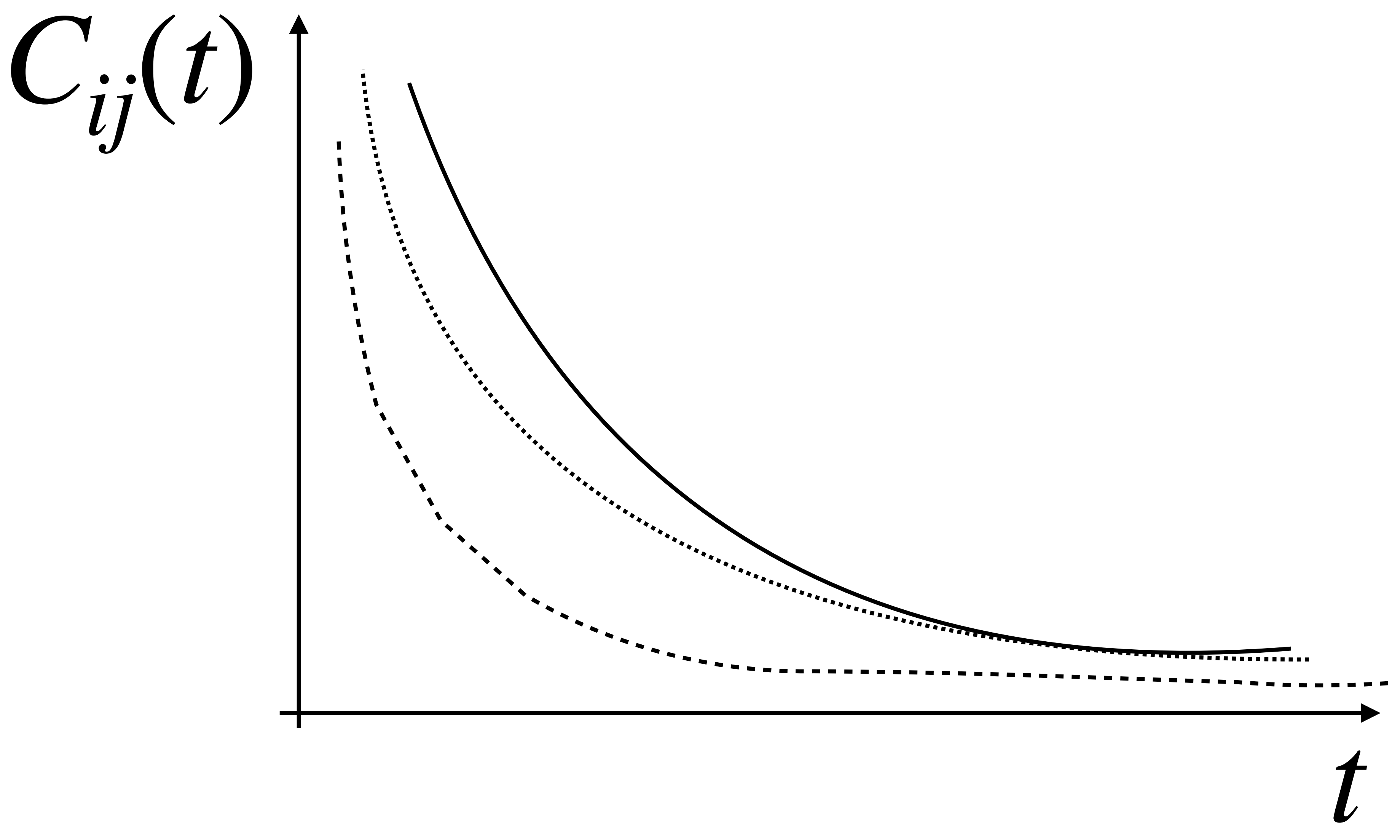

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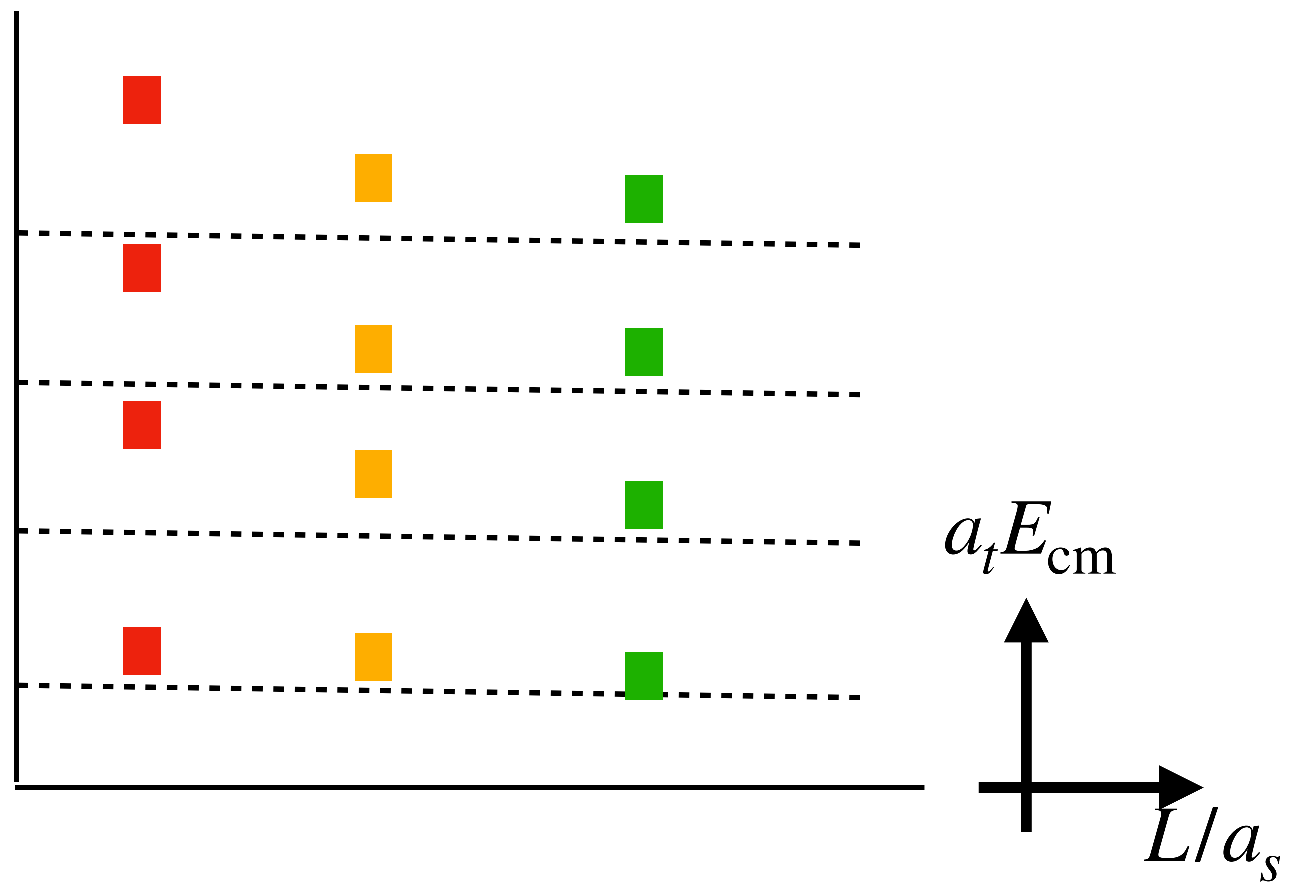
Strongly stable hadrons (bound states):
$\pi, K, D, \ldots, n, p, \ldots$
Find E_0

Finite-volume spectrum from lattice QCD

Finite-volume 2pt Correlation Functions → Finite-volume spectrum

$C_{N,\text{2pt}}(t;\vec{p})$

E_n



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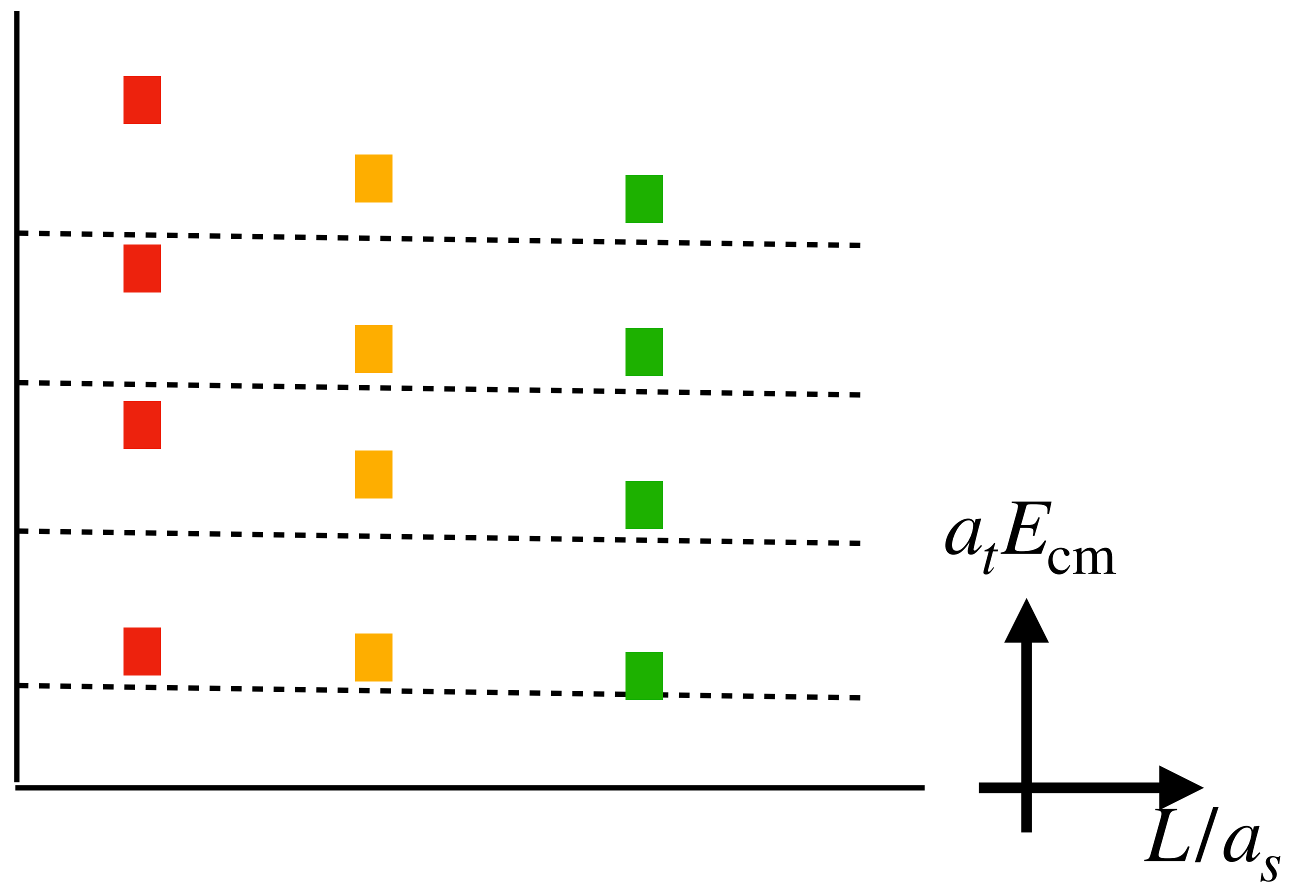

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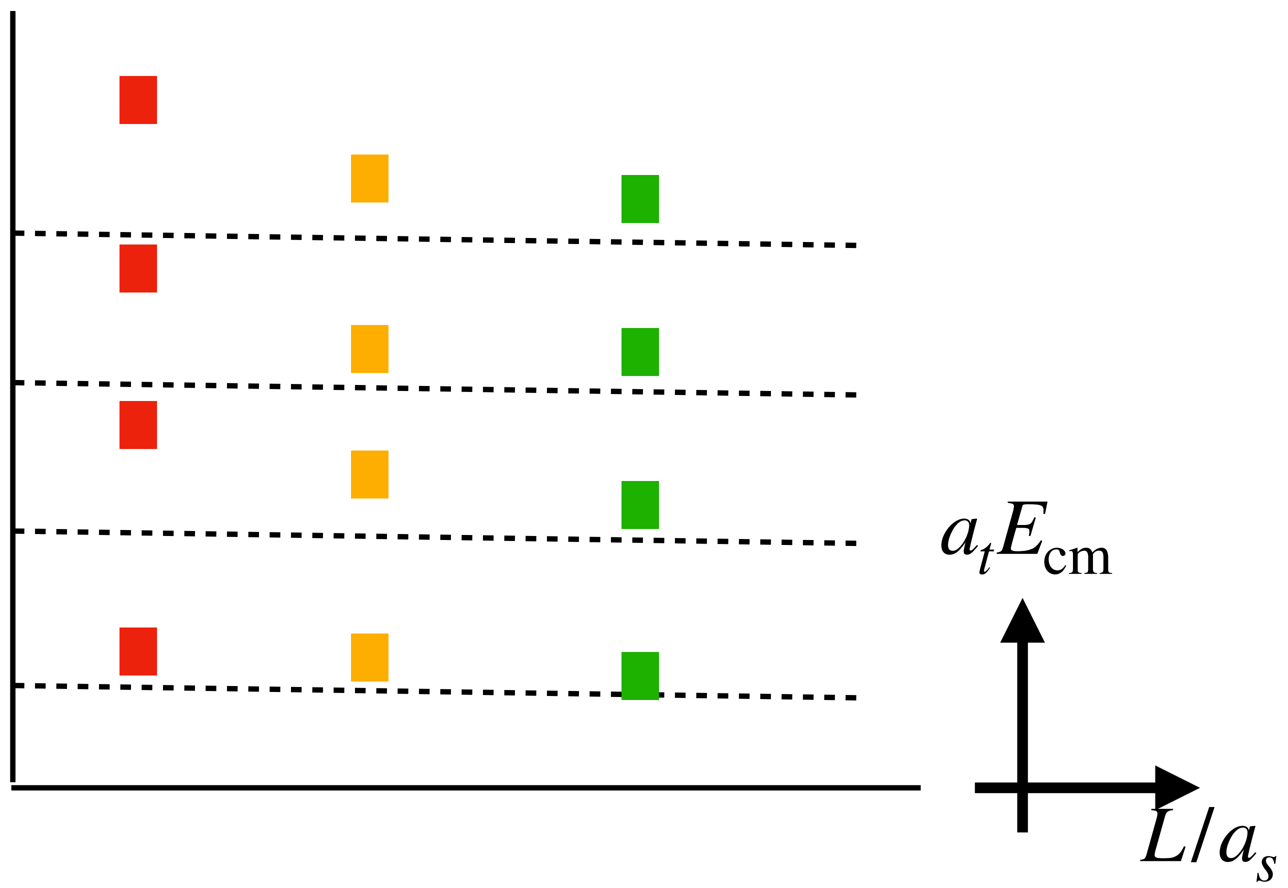
→ Near-threshold bound states & Resonances



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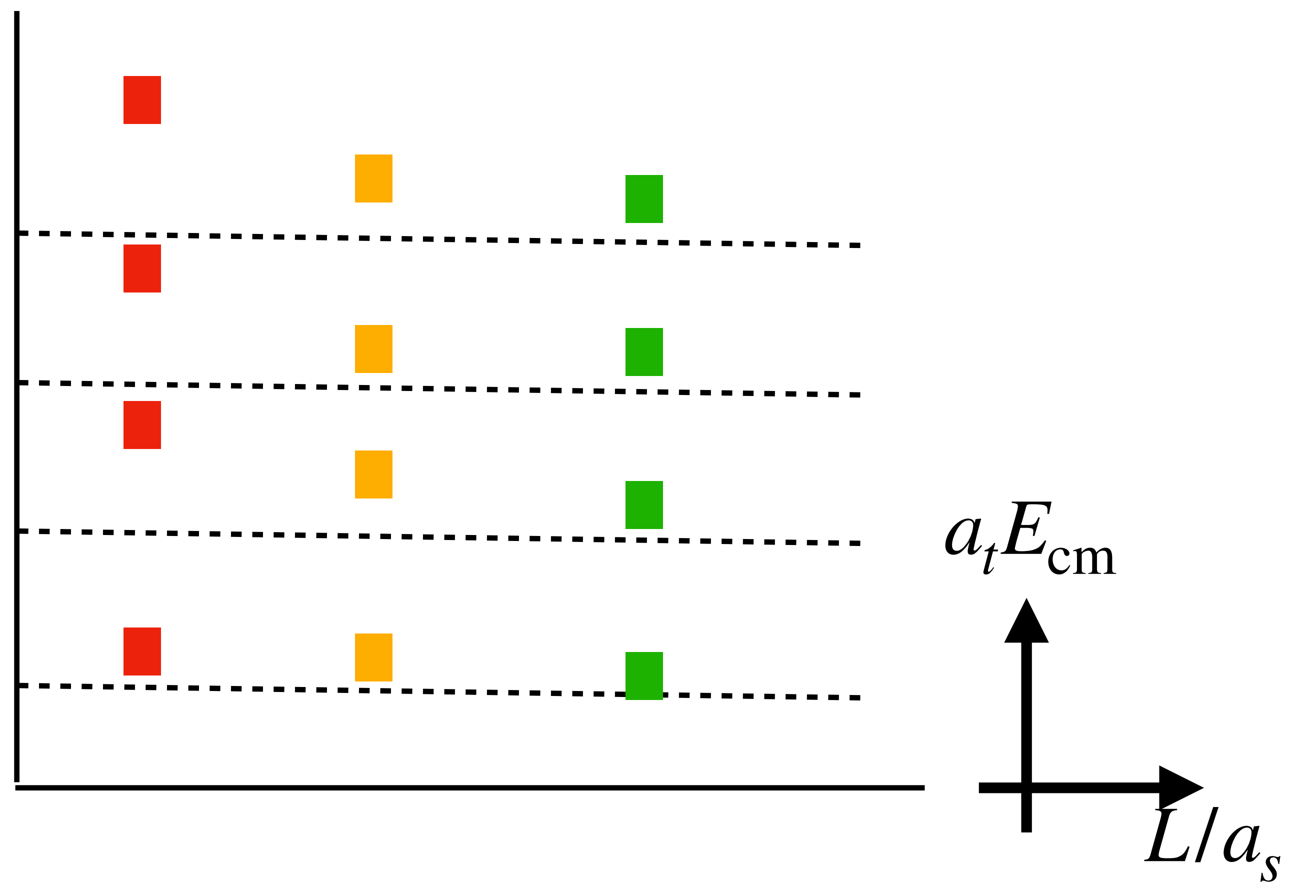
!

↓ Estimate spectrum in the infinite volume with E_n



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→ Luscher formalism
$E_n \leftrightarrow T(E_{\text{cm}})$

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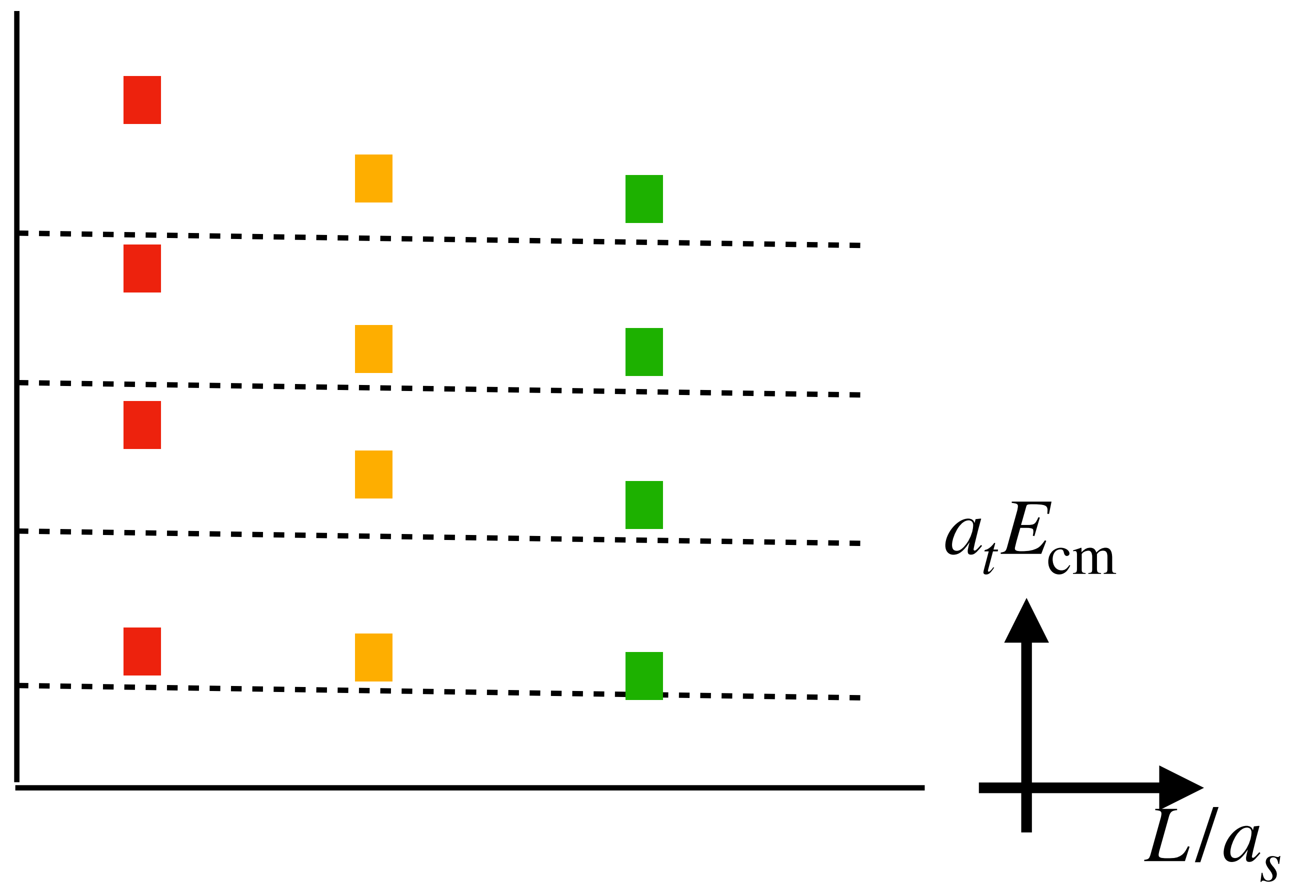

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→ Near-threshold bound states & Resonances

↓ !

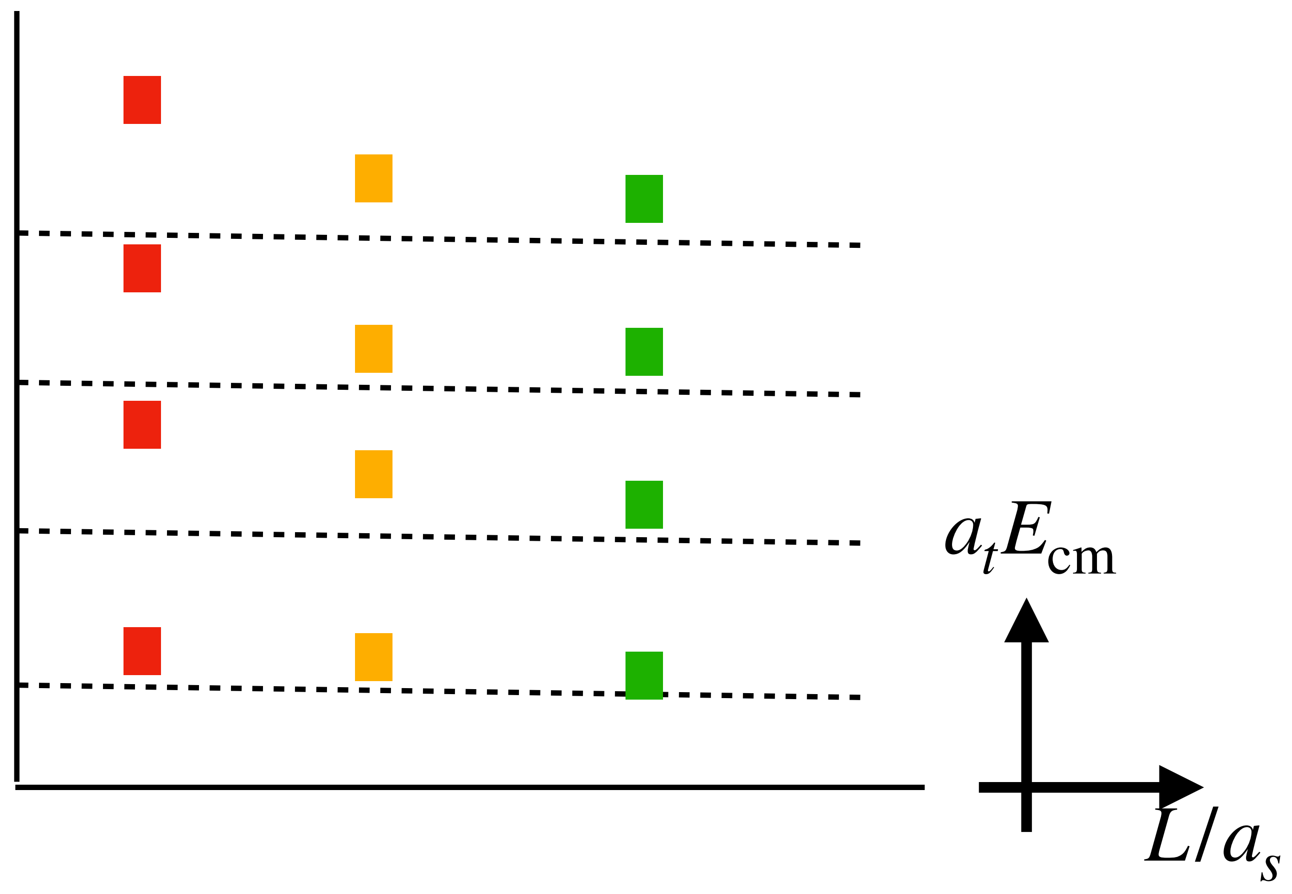
Estimate spectrum in the infinite volume with E_n

→ Luscher formalism

$E_n \leftrightarrow T(E_{\mathrm{cm}})$

↓ ✓

Scattering matrix $T(E_{\mathrm{cm}})$



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→ Near-threshold bound states & Resonances

→ Estimate spectrum in the infinite volume with E_n (!)

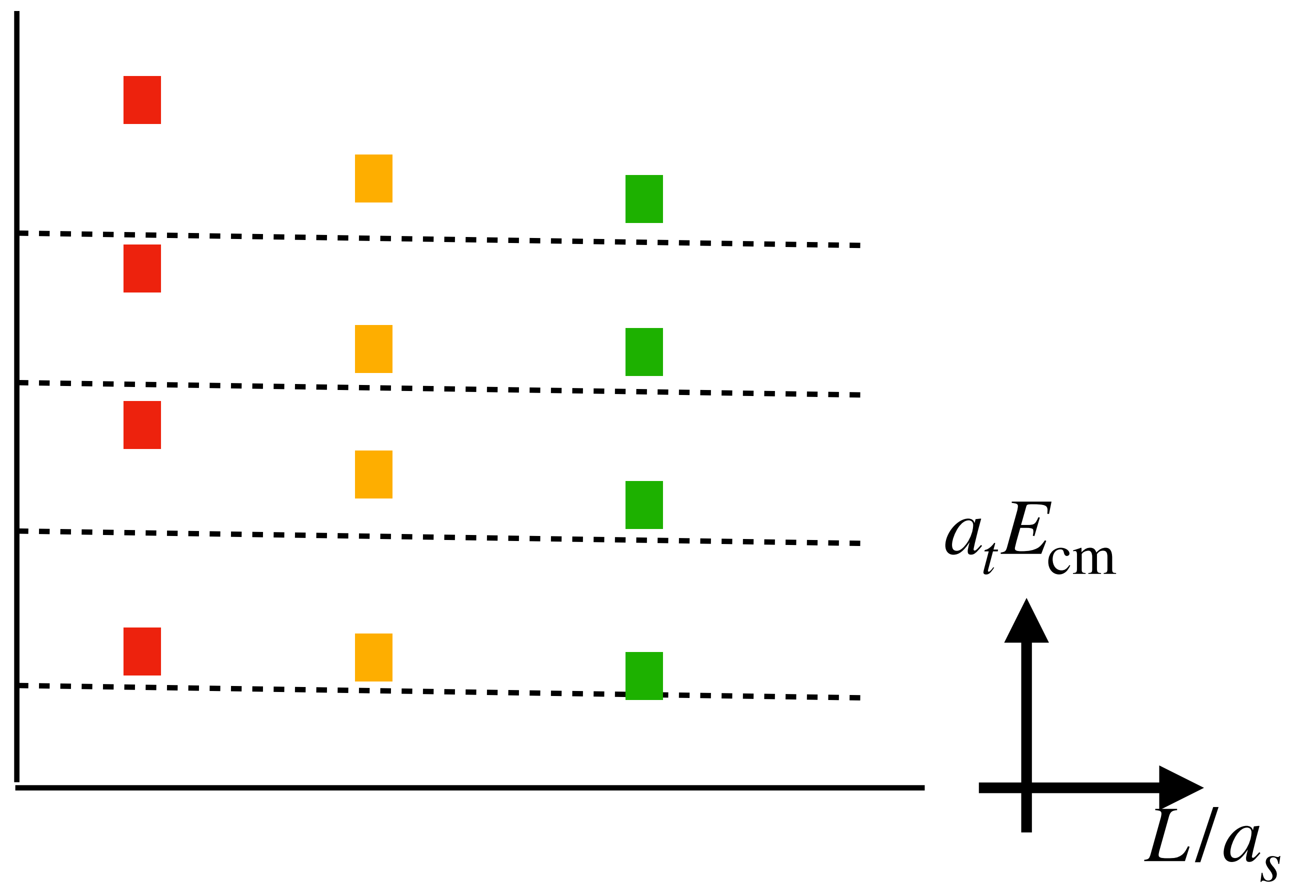
→ Luscher formalism

$E_n \leftrightarrow T(E_{\mathrm{cm}})$

→ Scattering matrix

$T(E_{\mathrm{cm}})$

→ $E_{\mathrm{cm}}^{\mathrm{p}} = m_{\mathrm{R}} + \frac{1}{2} i\Gamma$



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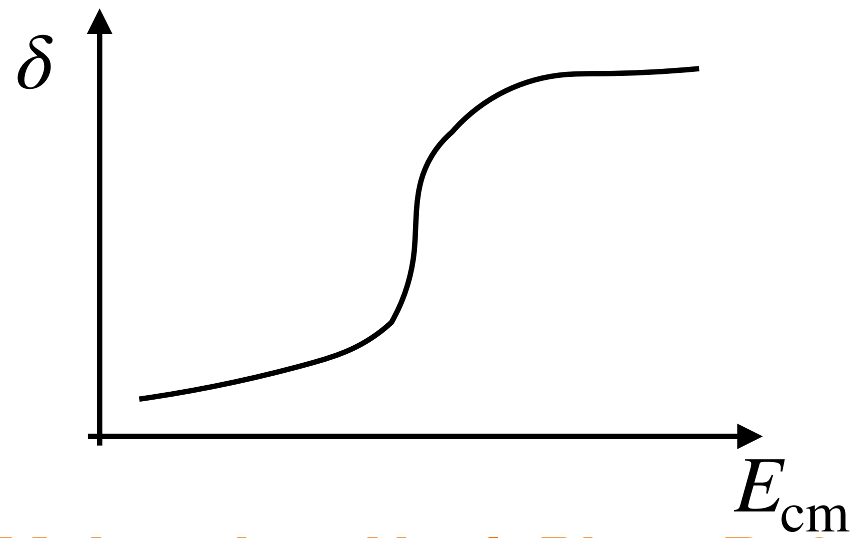
→ Near-threshold bound states & Resonances

→ Estimate spectrum in the infinite volume with E_n (!)

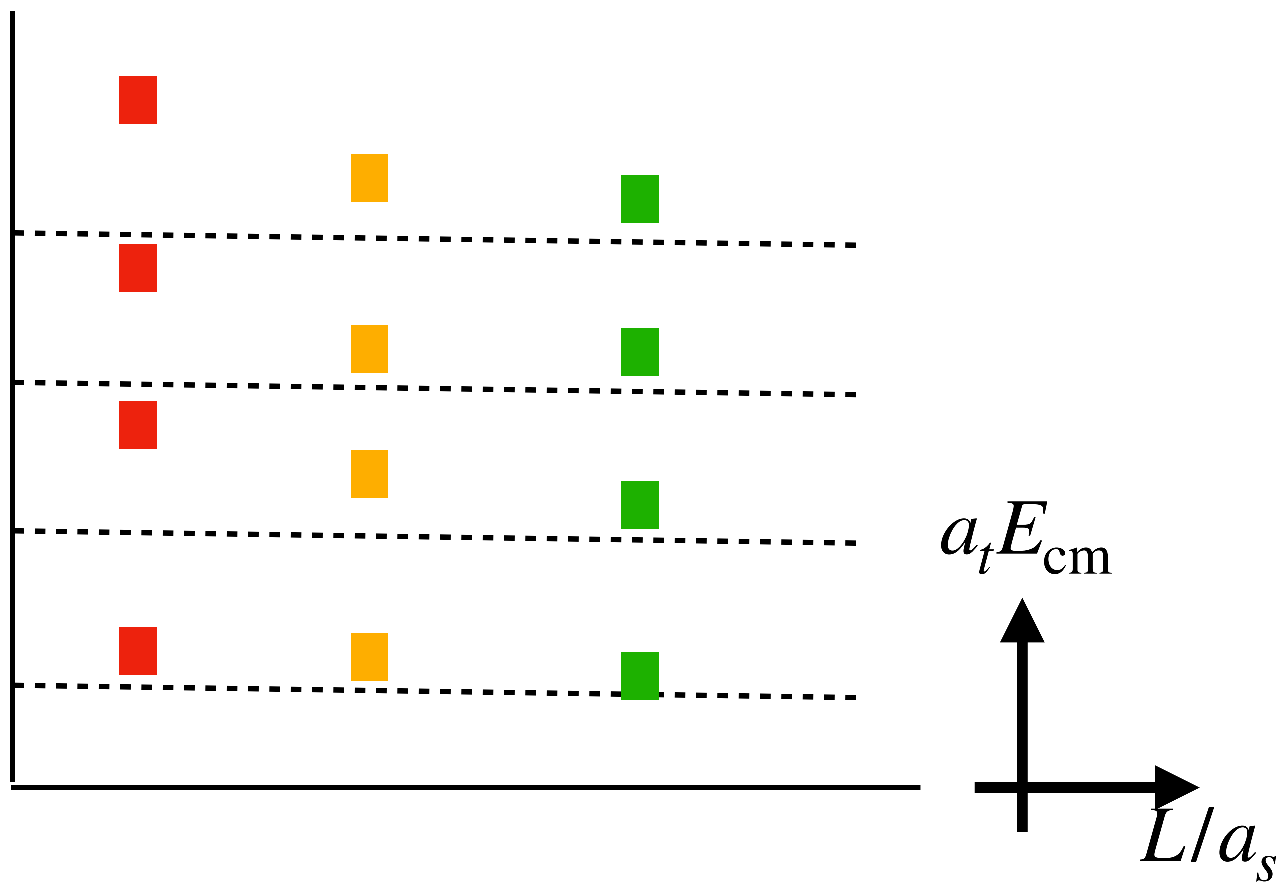
→ Luscher formalism
$E_n \leftrightarrow T(E_{\mathrm{cm}})$

→ Scattering matrix
$T(E_{\mathrm{cm}})$

→ $E^{\mathrm{p}}_{\mathrm{cm}} = m_{\mathrm{R}} + \frac{1}{2}i\Gamma$



M. Luscher, Nucl. Phys. B, 354, (1991)



Finite-volume spectrum from lattice QCD

Finite-volume 2pt Correlation Functions

$C_{N,2\text{pt}}(t;\vec{p})$

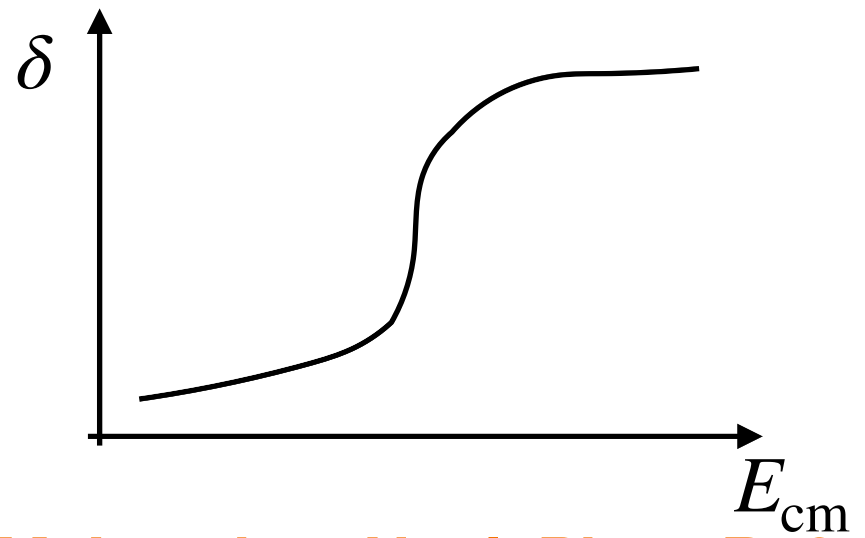
Finite-volume spectrum

E_n

Strongly stable hadrons (bound states):

$\pi, K, D, \ldots, n, p, \ldots$

Find E_0 for $t \gg 0$



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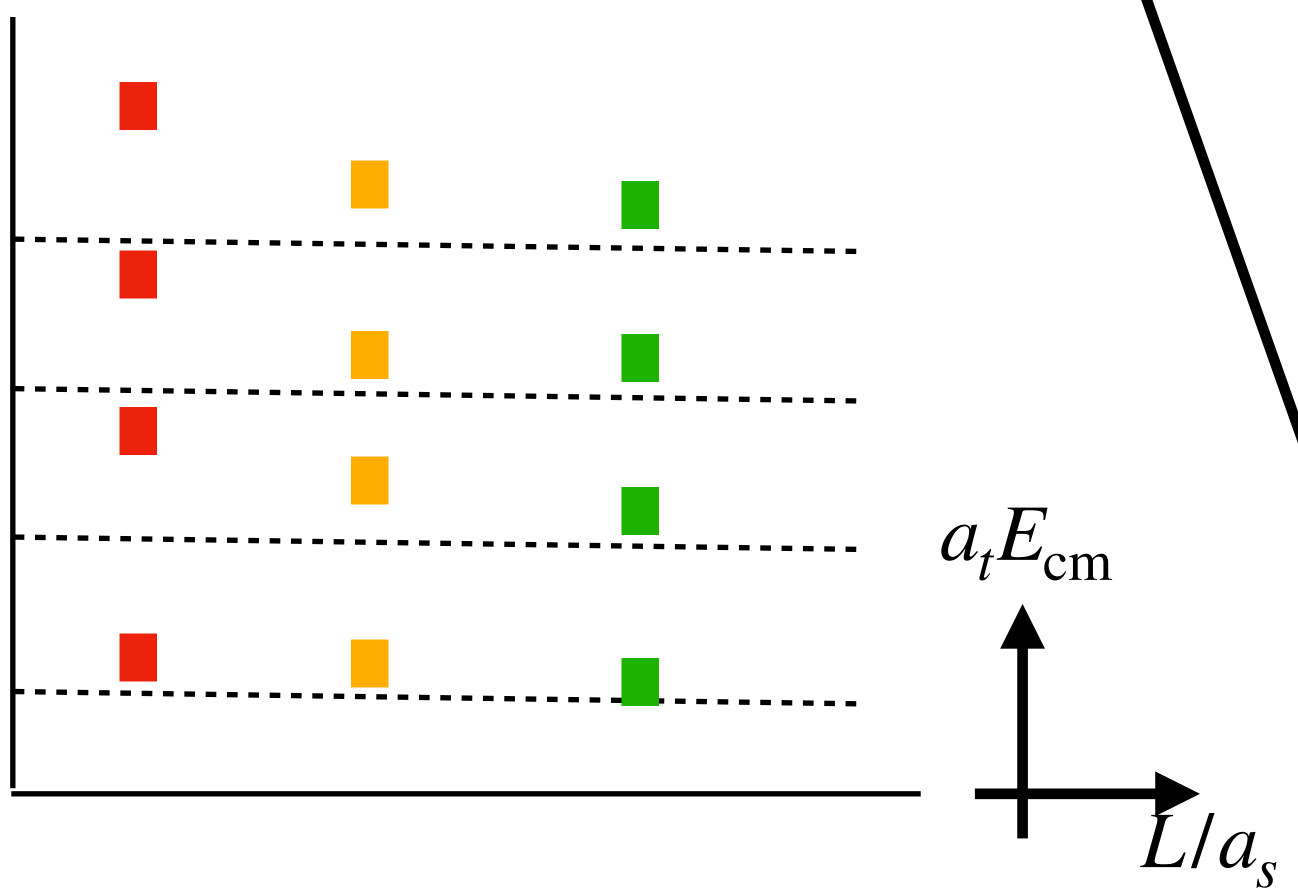
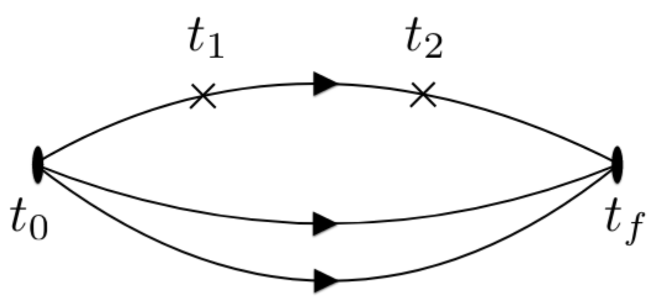


Constrain 3pt/4pt correlation functions to extract form factors, etc.



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?

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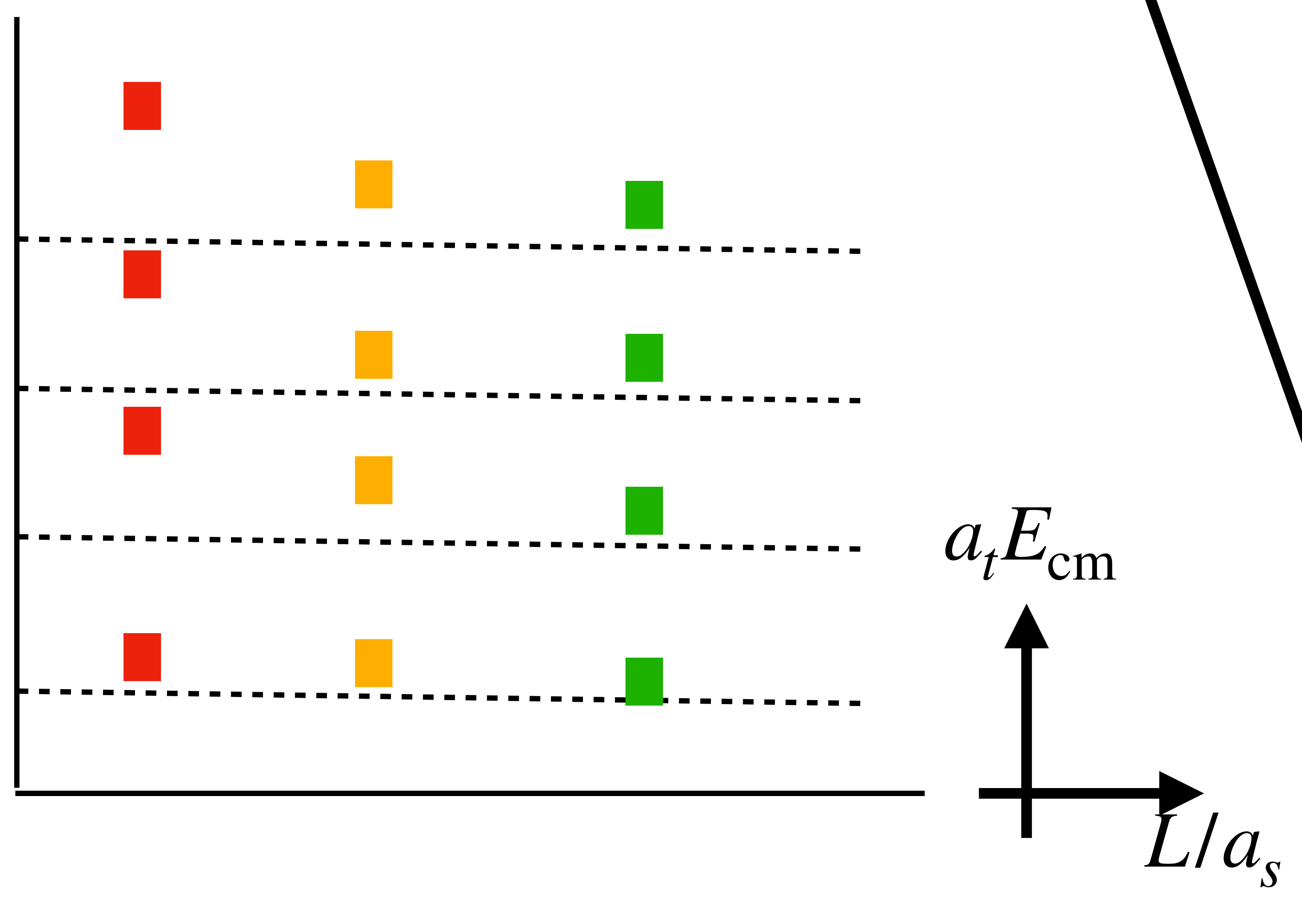

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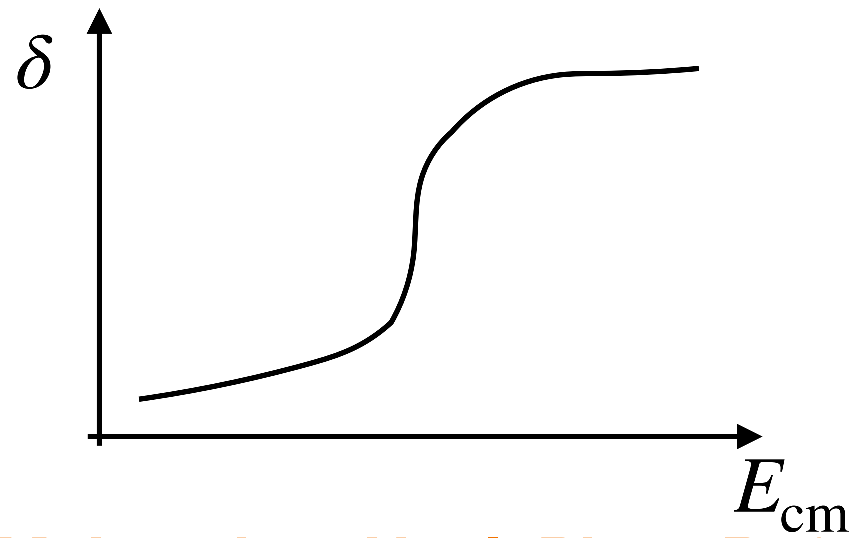

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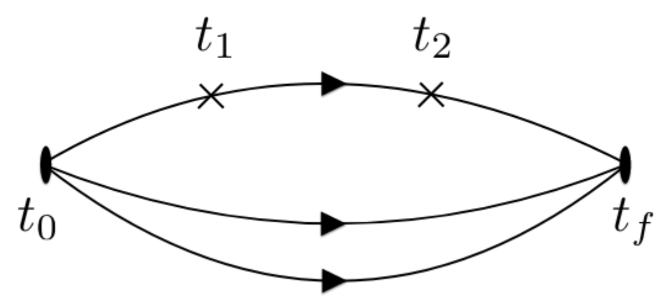


Outline

- Introduction
  - Finite-volume spectrum from lattice QCD
- How to extract finite-volume spectrum from lattice QCD
  - Multi-exponential fit
  - Variational methods (GEVP)
  - An example: Roper state from lattice QCD
- Application of the Bayesian Reconstruction
- Conclusion and outlook

Multi-exponential fit

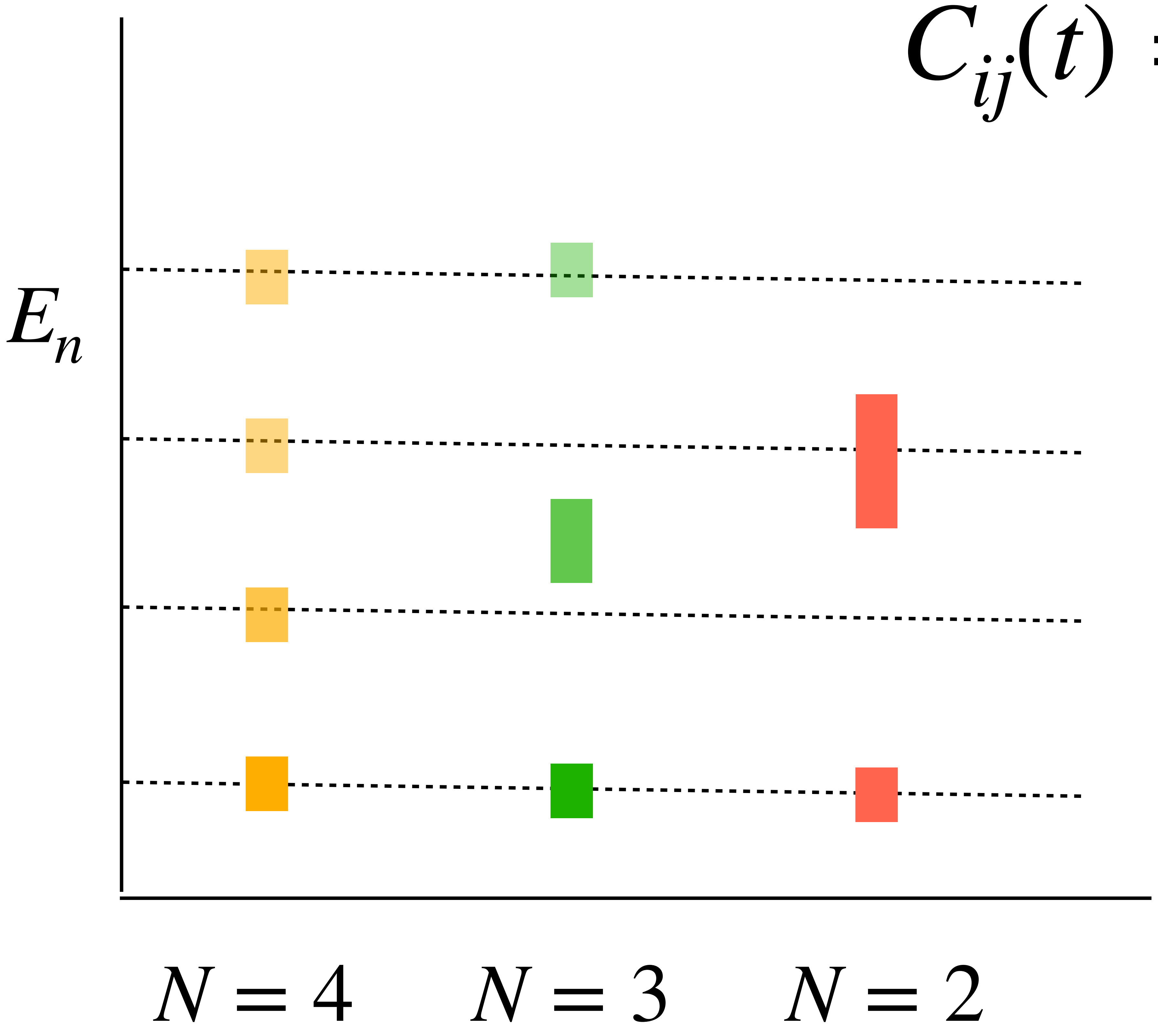
- To extract the finite-volume spectrum
  - Two-point correlation functions $\chi_i^\dagger = \chi_j^\dagger =$ qqq

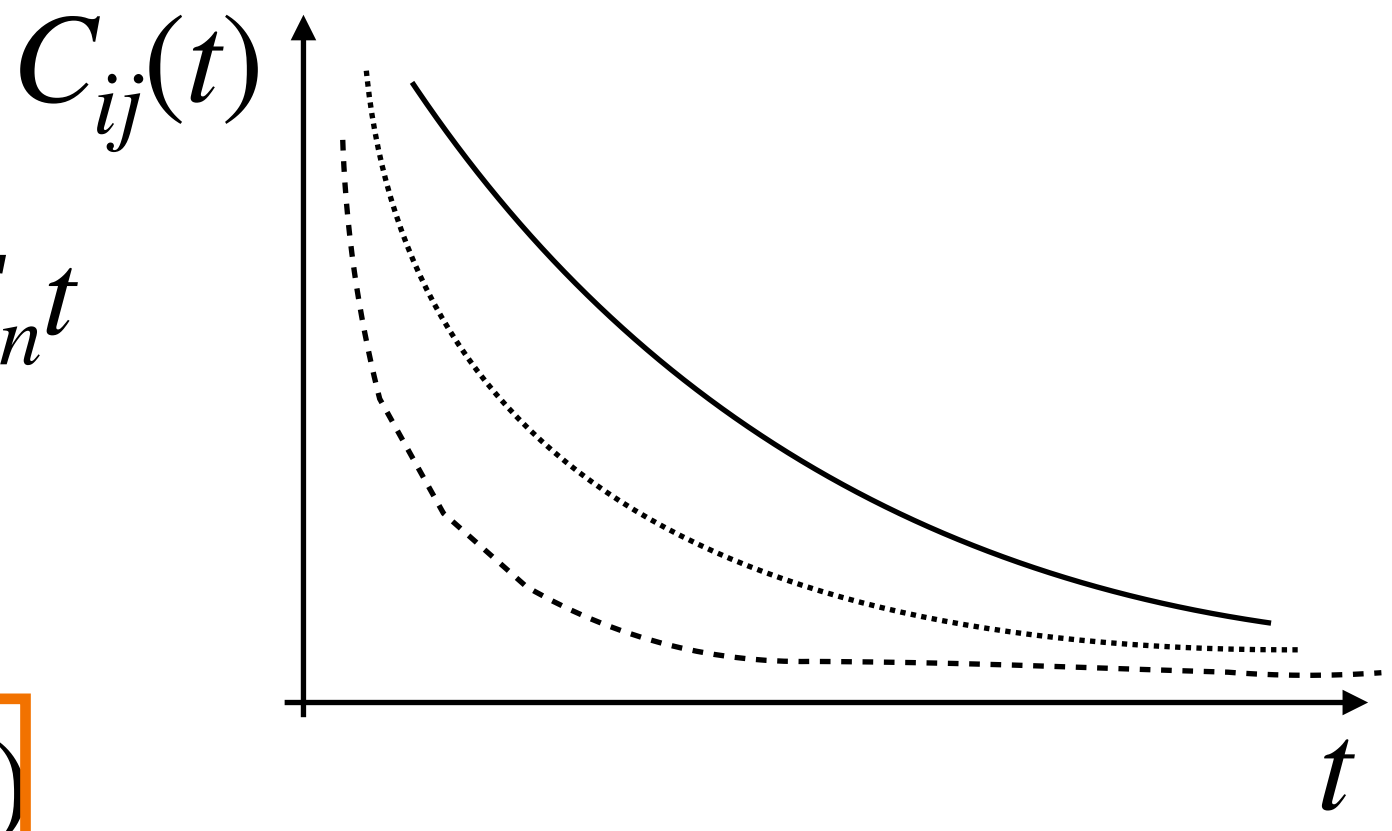
$$C_{ij}(t) = \langle \chi_i(t) \chi_j^\dagger(0) \rangle = \sum_{n=0}^{\infty} W_n e^{-E_n t}$$

Constraint 1

$$C_{N,\text{2pt}}(t) = \sum_{n=0}^{N} W_n e^{-E_n t}$$

$$\chi^2 \equiv \sum_{ij} (D_i - \tilde{D}_i) C_{ij}^{-1} (D_j - \tilde{D}_j)$$





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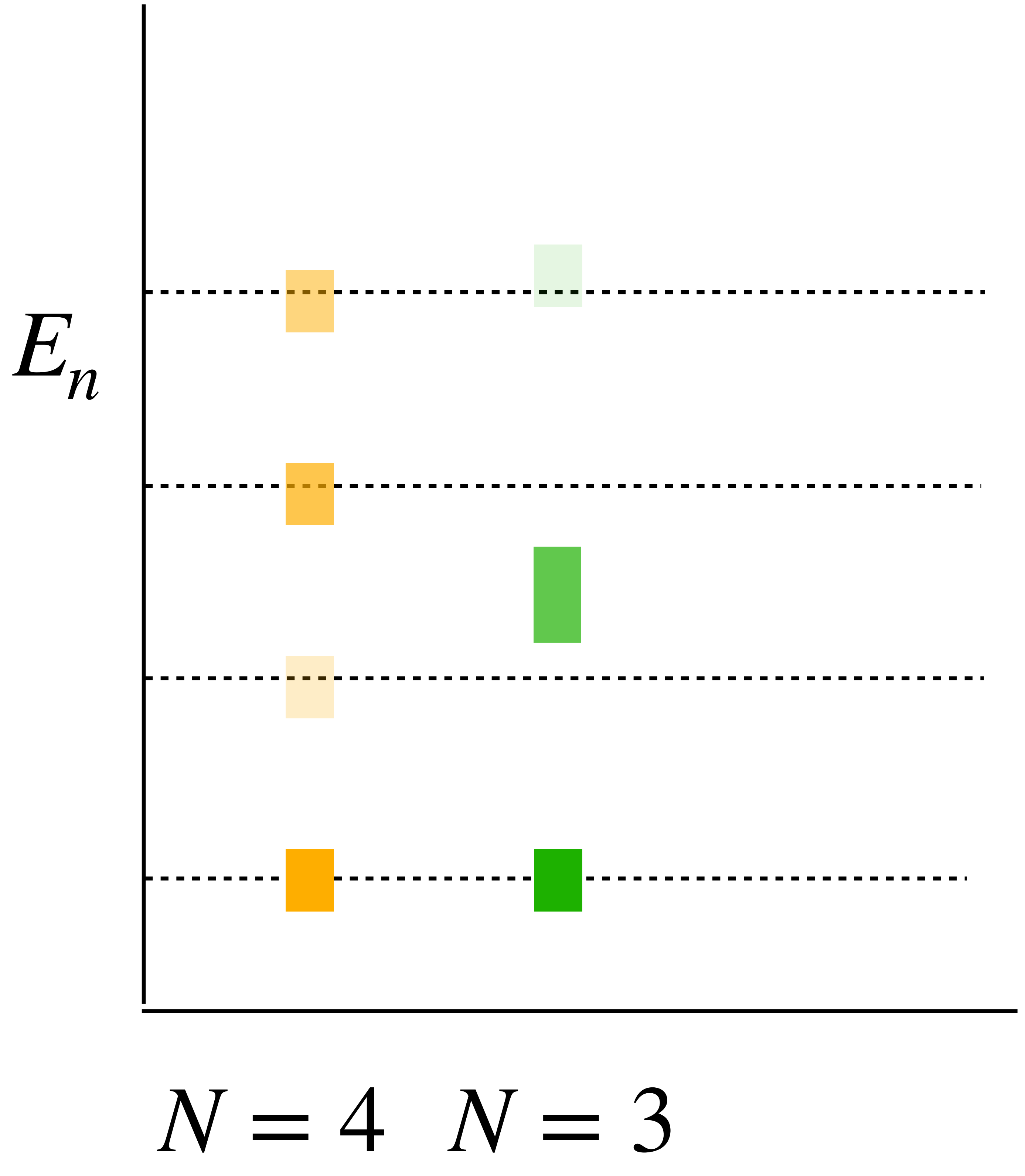

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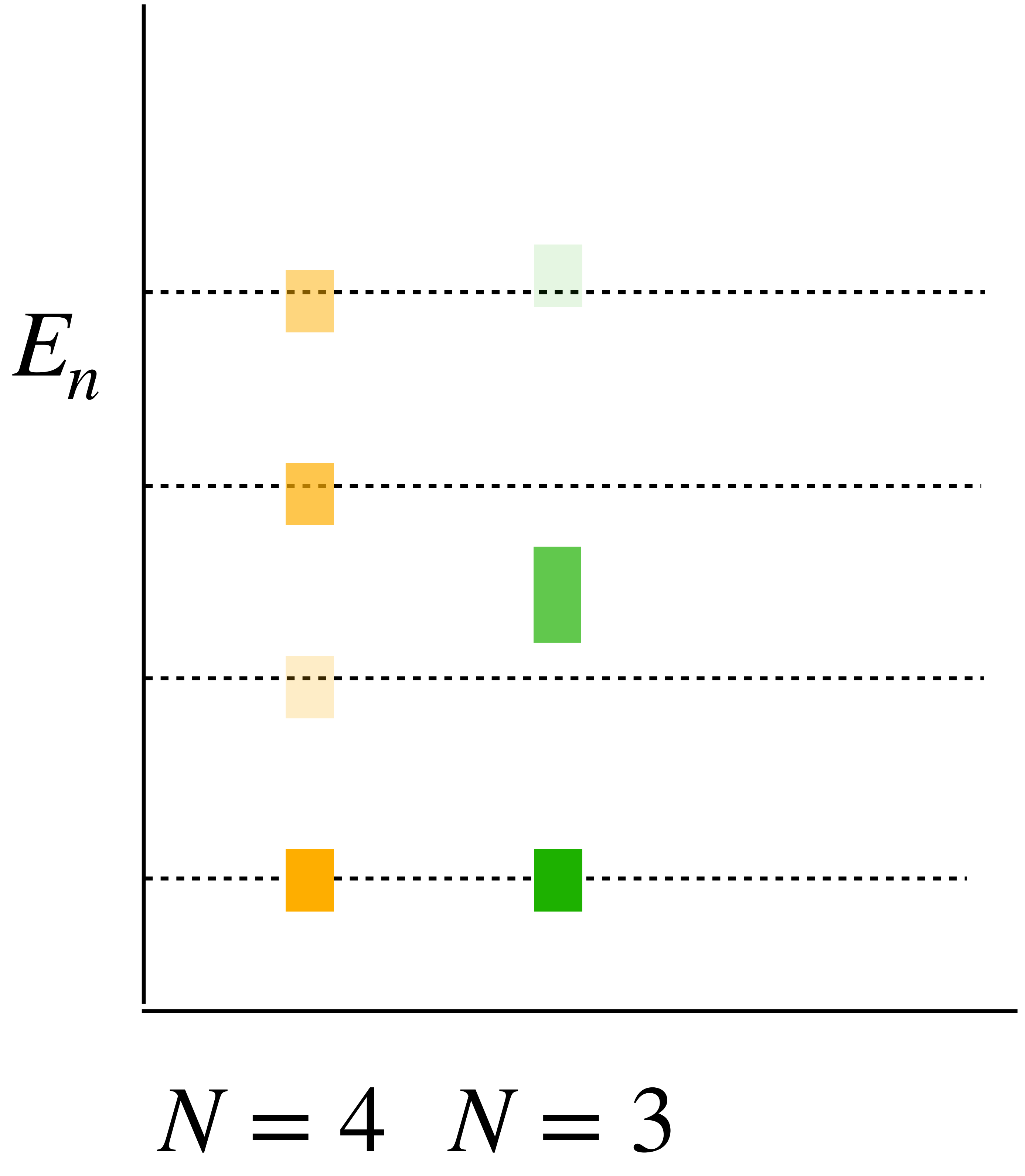

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G. P. Lepage et al., arXiv: hep-lat/0110175

$$+ \quad \chi^2_{\text{prior}} \equiv \sum_n \frac{(W_n - \tilde{W}_n)^2}{\tilde{\sigma}^2_{W_n}} + \sum_n \frac{(E_n - \tilde{E}_n)^2}{\tilde{\sigma}^2_{E_n}}$$

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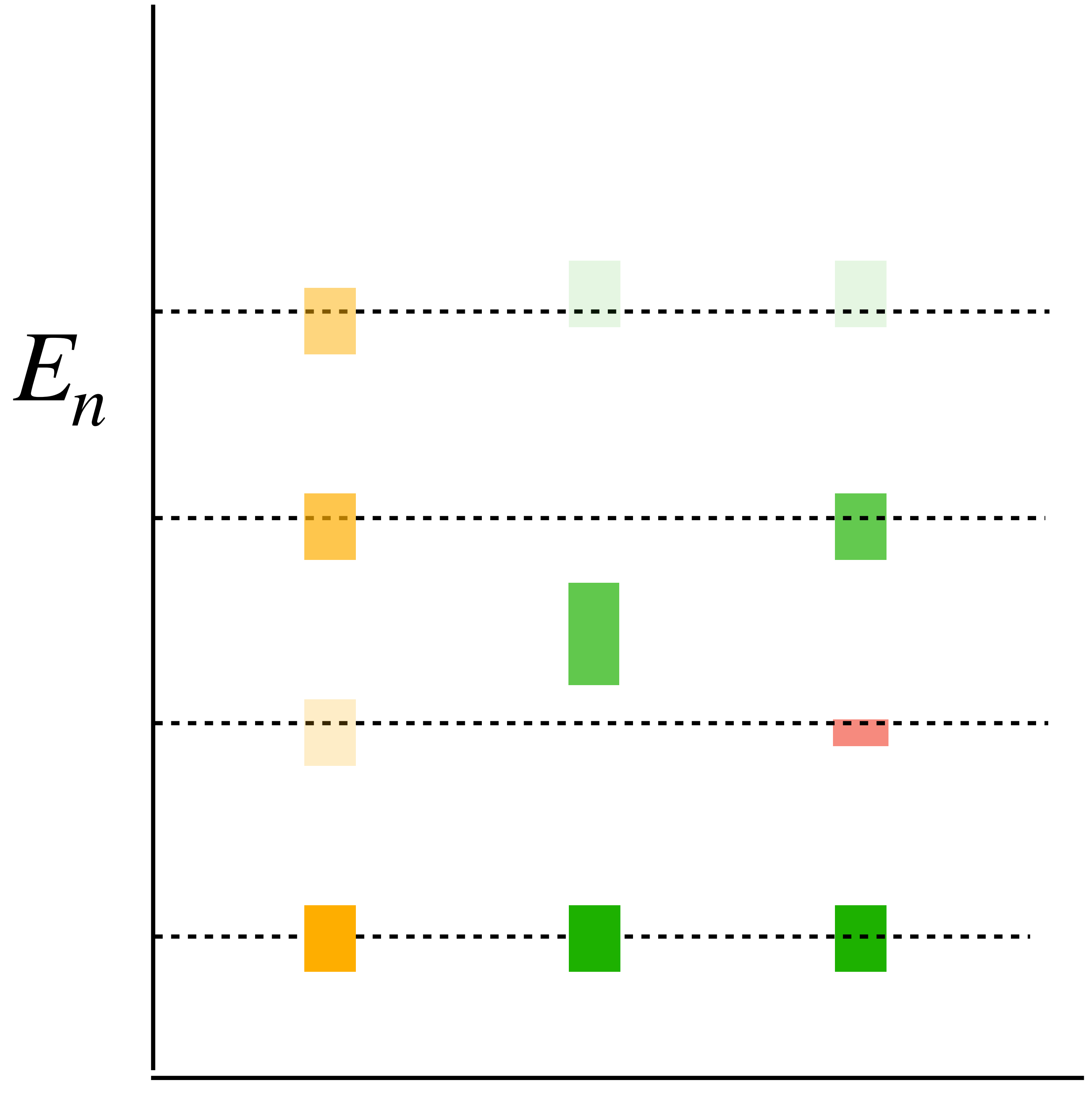

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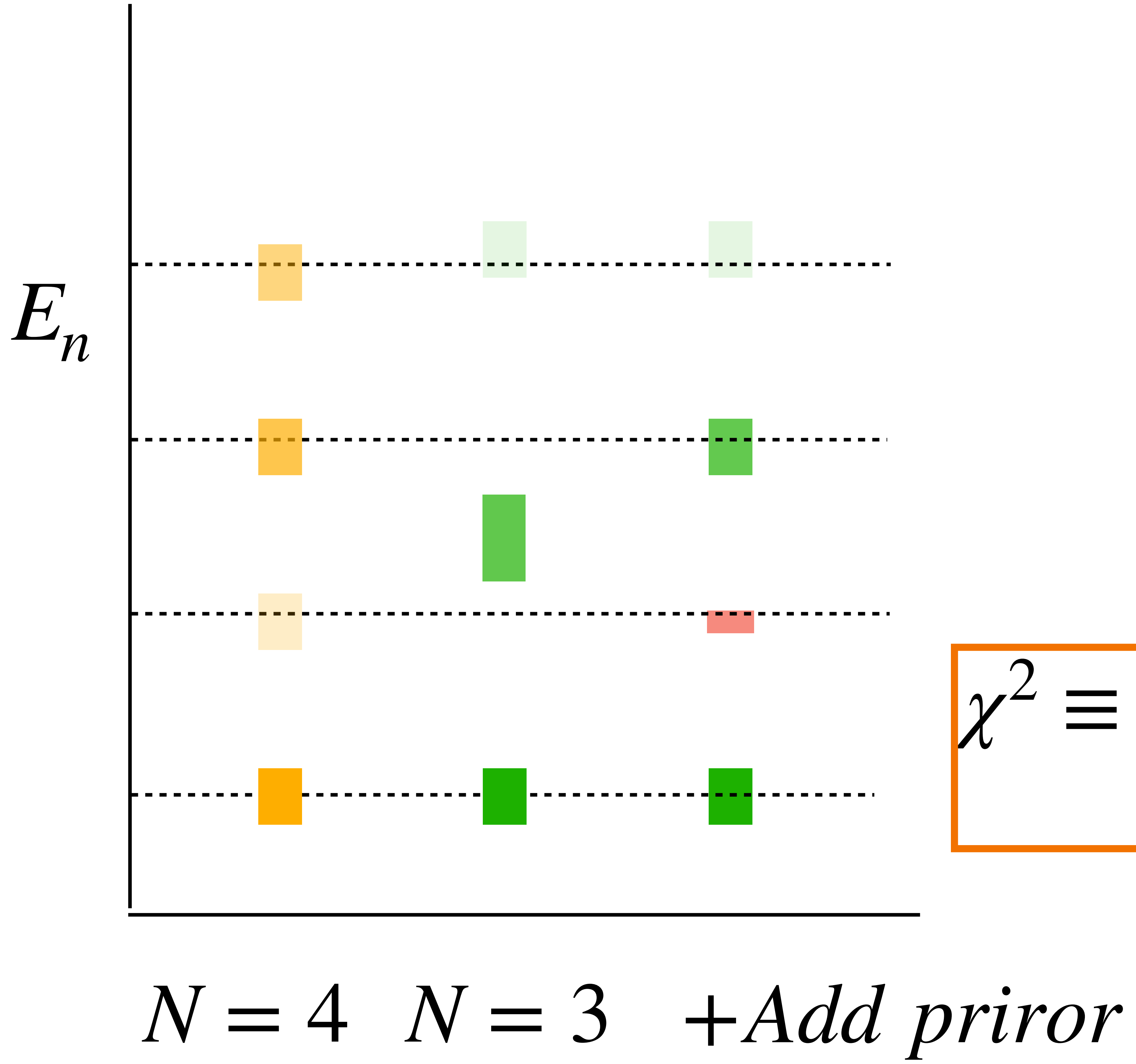

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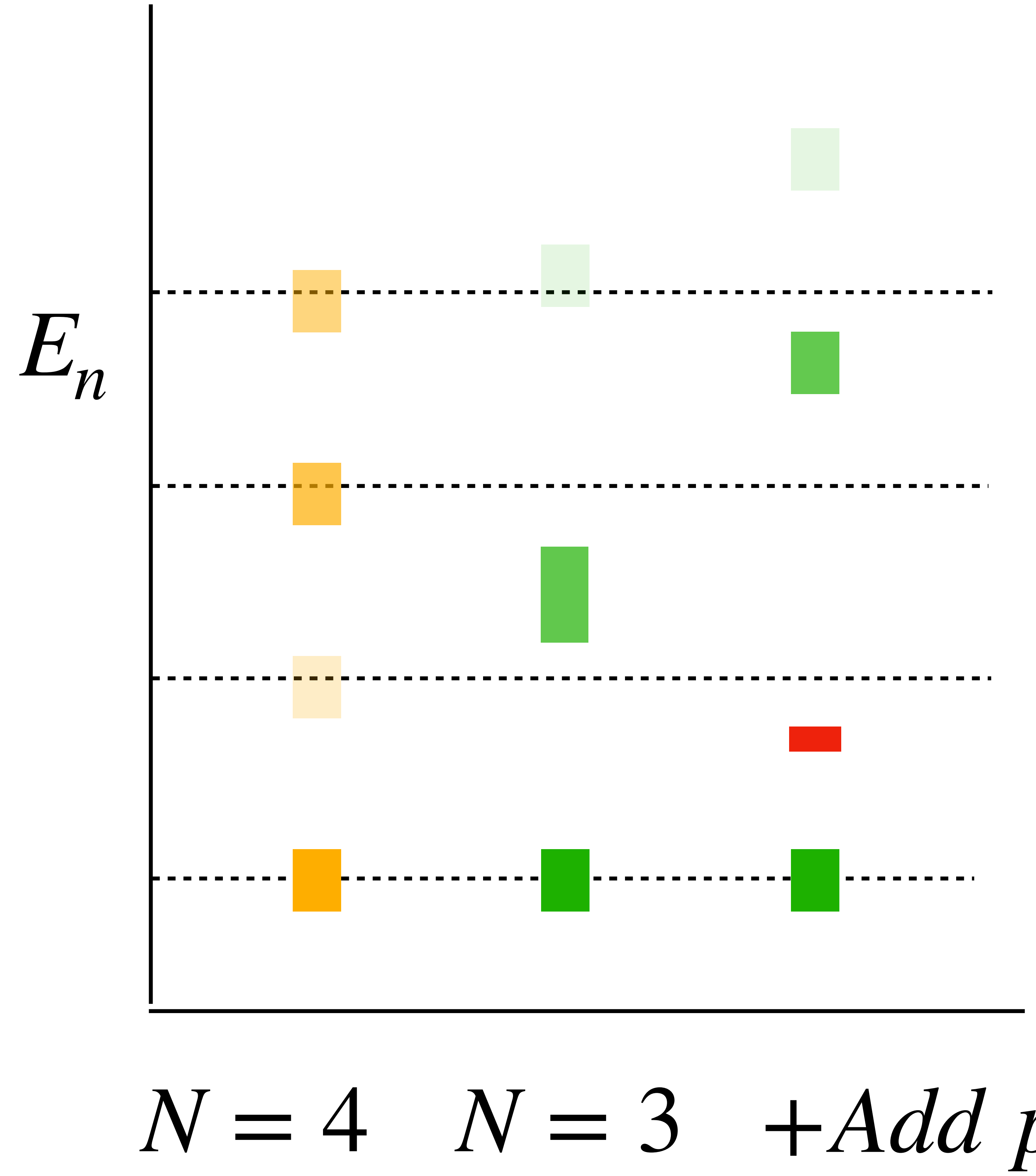

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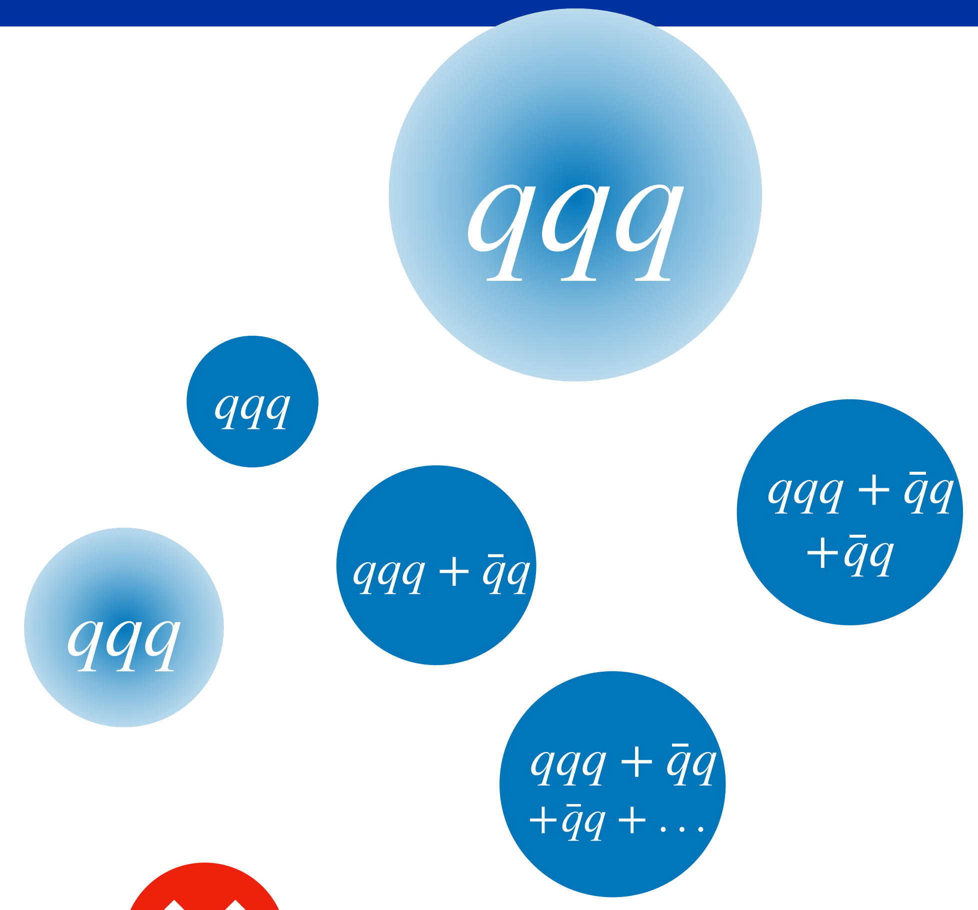
Multi-exponential fit

- To extract the finite-volume spectrum
  - Two-point correlation functions

interpolating operators with the same quantum numbers as the hadron state

$\chi_i^\dagger \in \{$  $\}$

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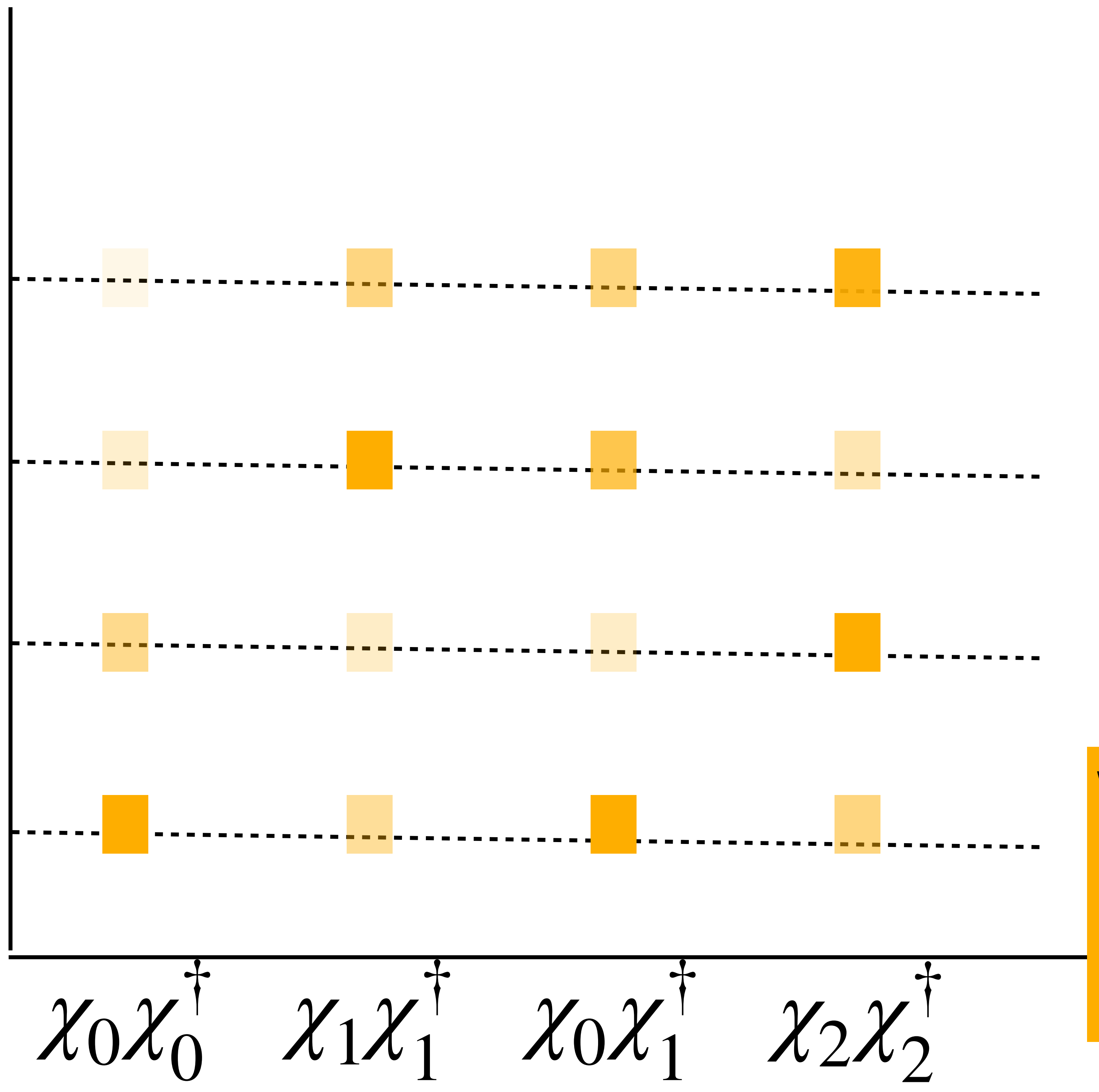

Why not find linear combinations of $\chi_i^\dagger$'s with the maximum overlap to the states n's?

Multi-exponential fit

- To extract the finite-volume spectrum
  - Two-point correlation functions

Choose N basis operators: $\chi_i^\dagger$

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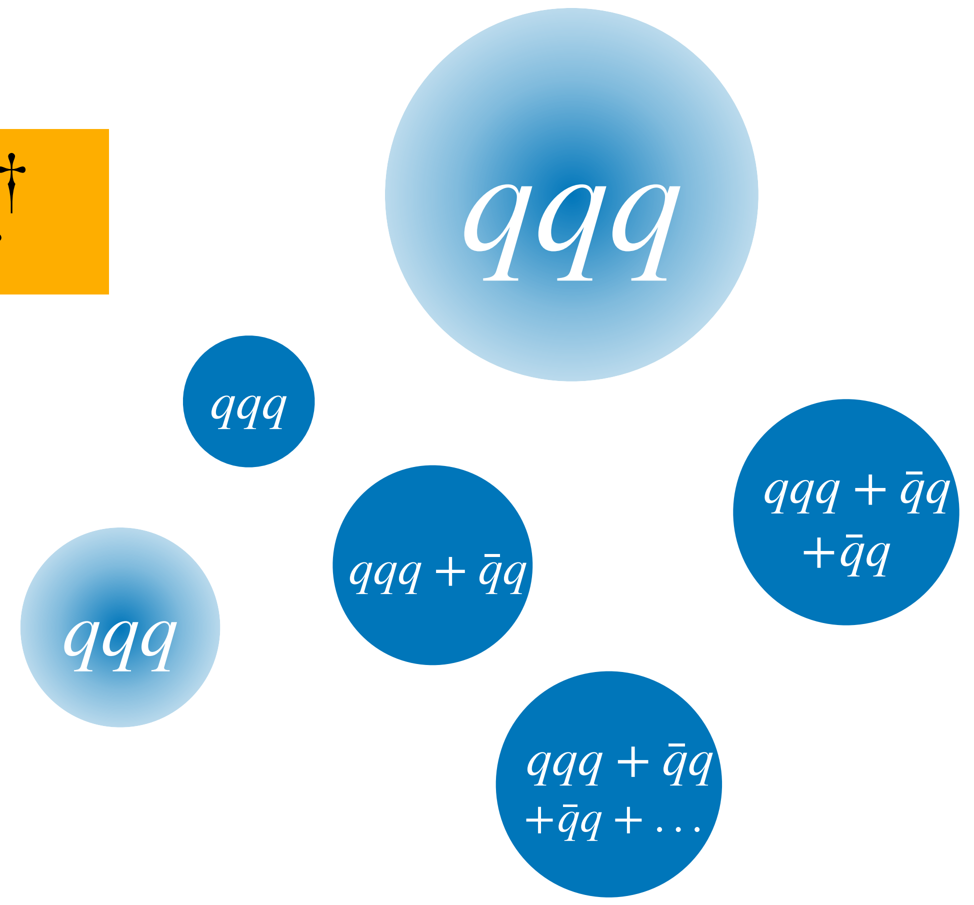

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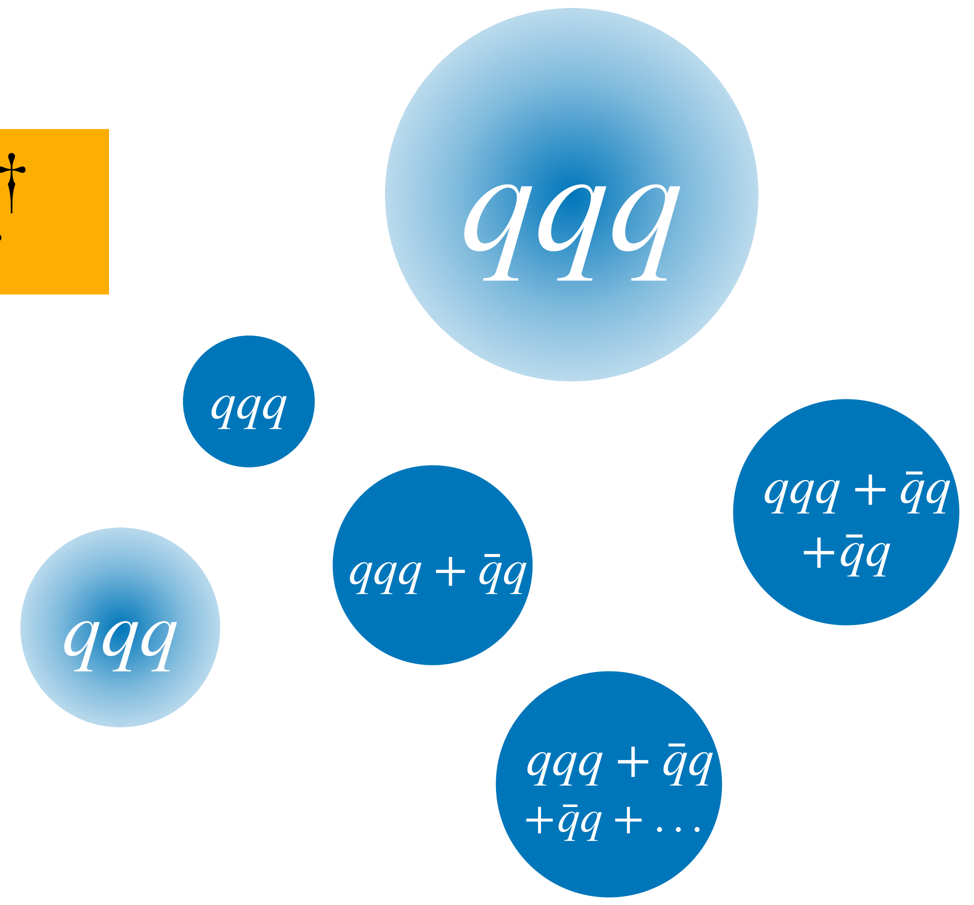

Find linear combinations of $\chi_i^\dagger$'s

$$X_n^\dagger = \sum_i v_i^{(n)} \chi_i^\dagger$$

Multi-exponential fit

- To extract the finite-volume spectrum

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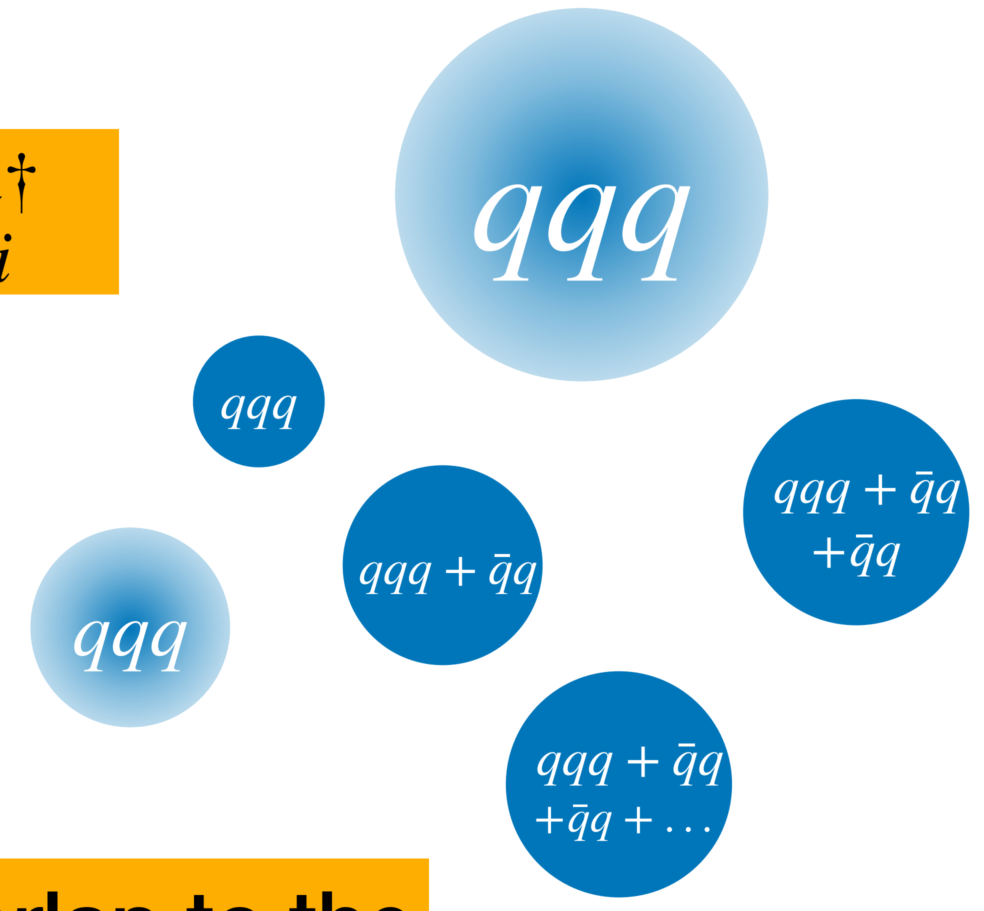

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with the maximum overlap to the states n's

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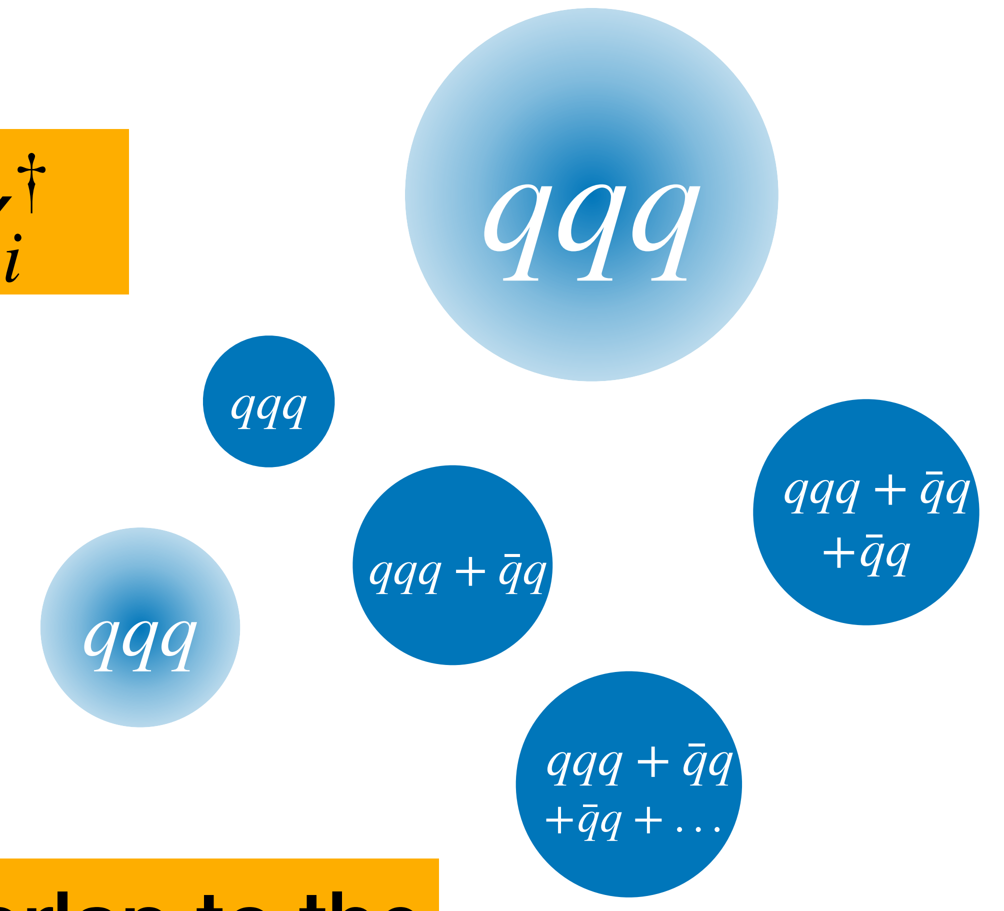

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$$\mathbf{C}(t) = \begin{pmatrix} C_{00}(t) & C_{01}(t) & \dots & C_{0N}(t) \\ C_{10}(t) & C_{11}(t) & \dots & C_{1N}(t) \\ \dots & \dots & \dots & \dots \\ C_{N0}(t) & C_{n1}(t) & \dots & C_{NN}(t) \end{pmatrix}$$

$$v_i^{(n)*} C_{ij}(t_0)\, v_j^{(n')} = \delta_{n,n'}$$

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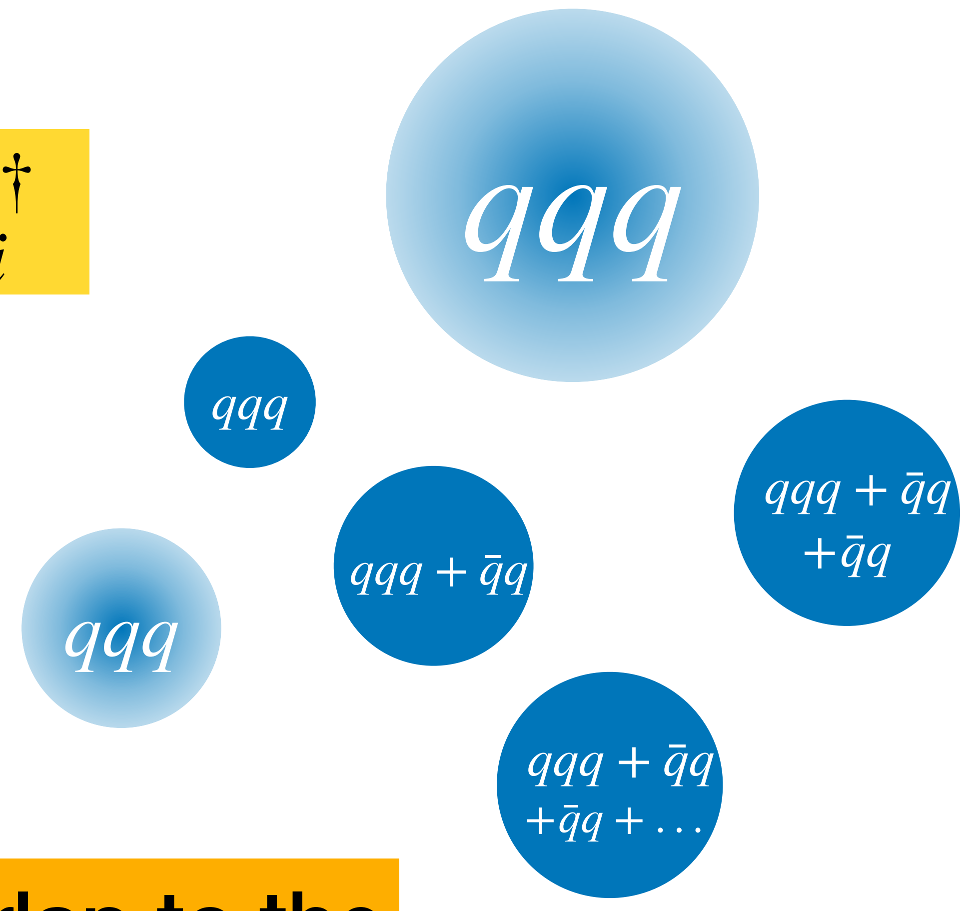

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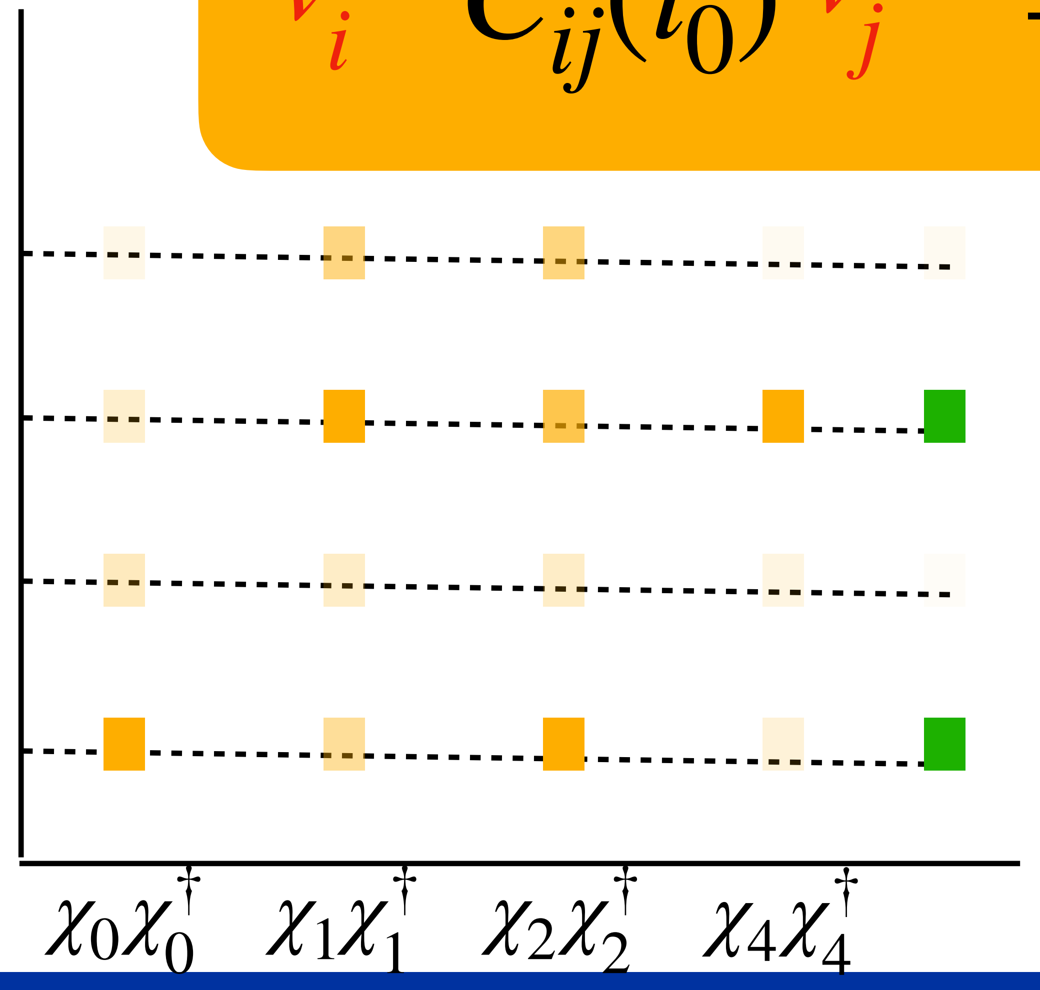
$$\lambda^{(n)}(t) = e^{-E_n t}$$

generalized eigenvalue problem (GEVP)



Q2: How to make sure we are not missing some of the $E_i's$?

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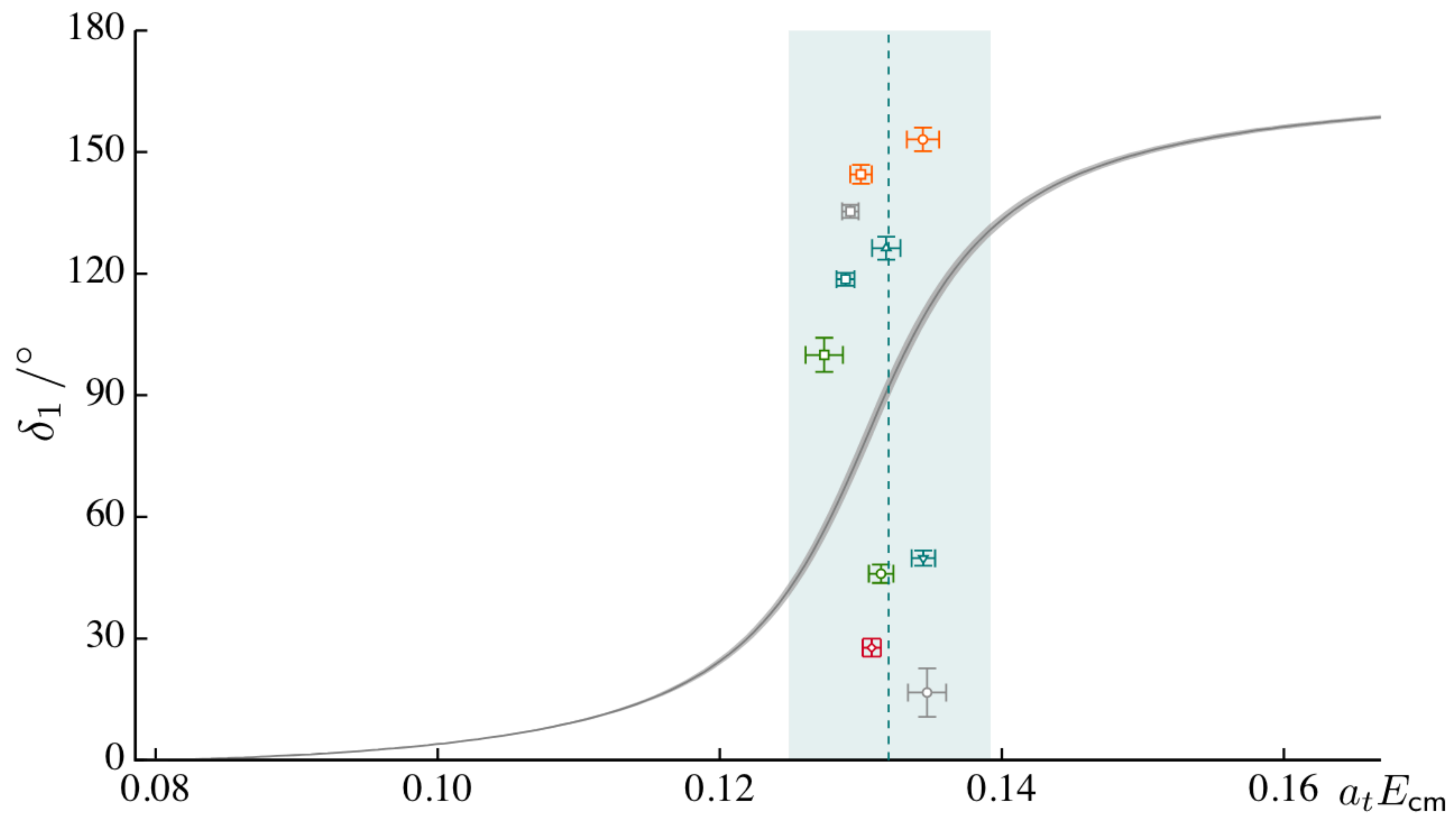

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Wilson et al.,*Phys. Rev. D*, vol. 92, no. 9, p. 094502, Nov. 2015

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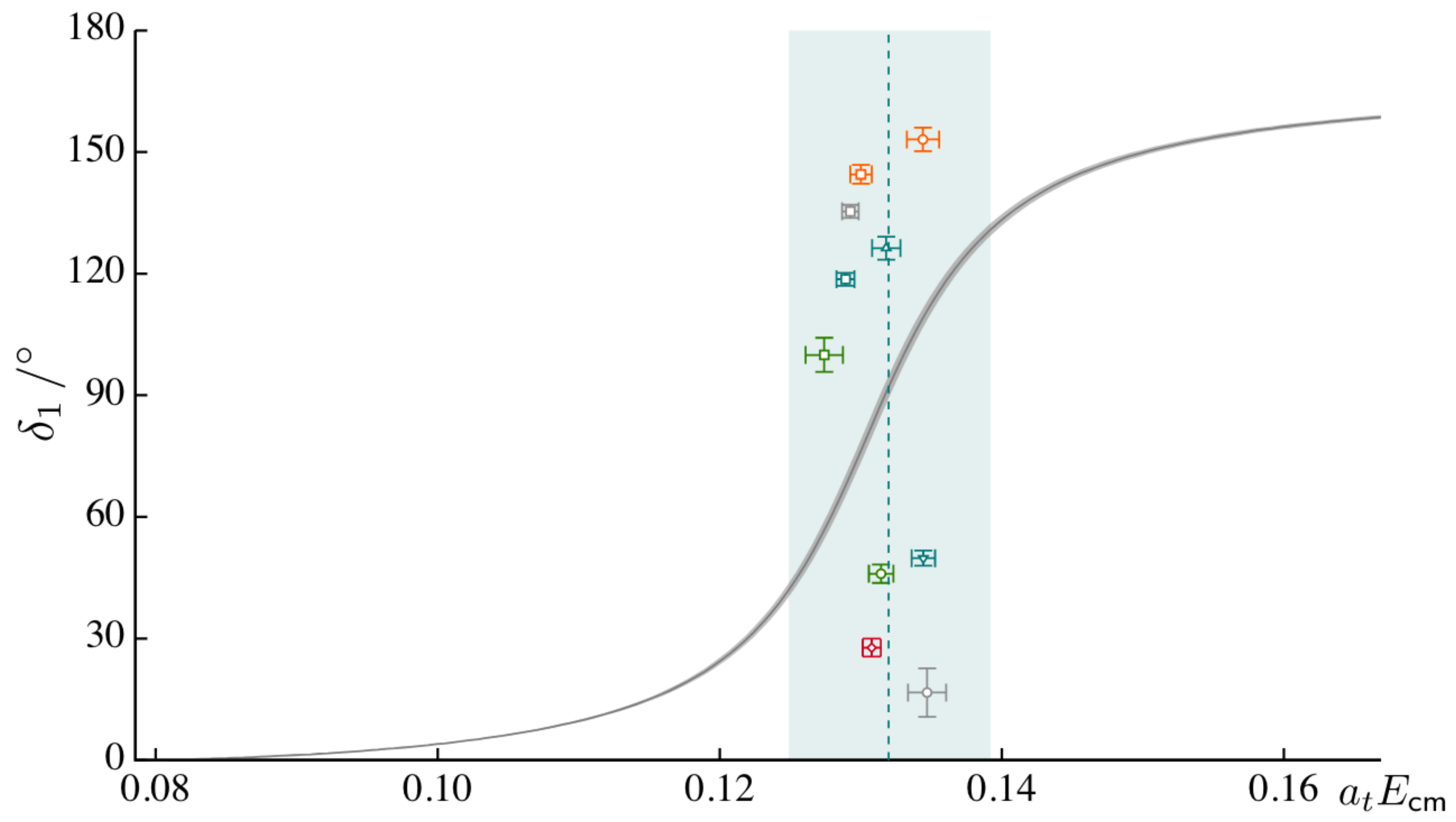

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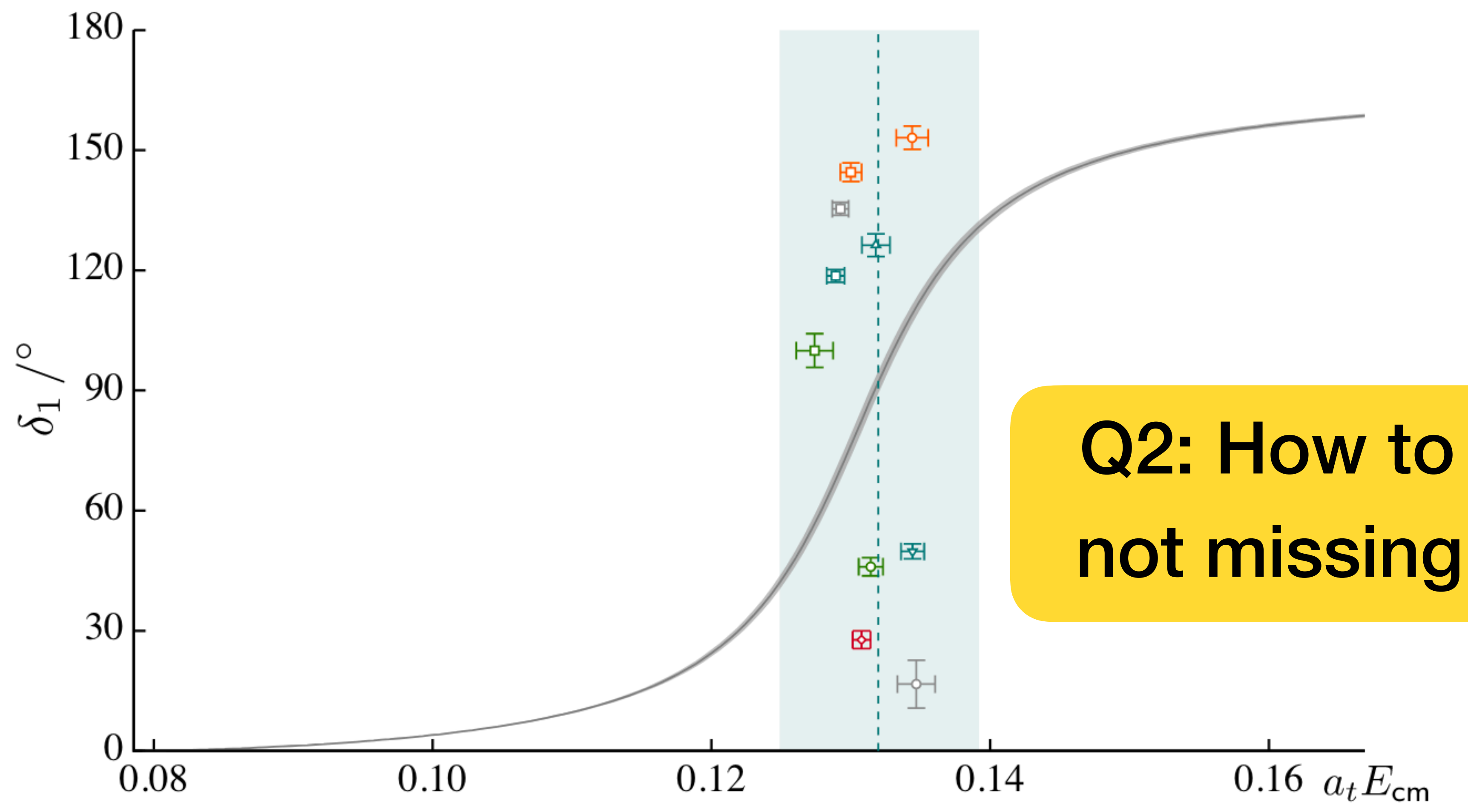

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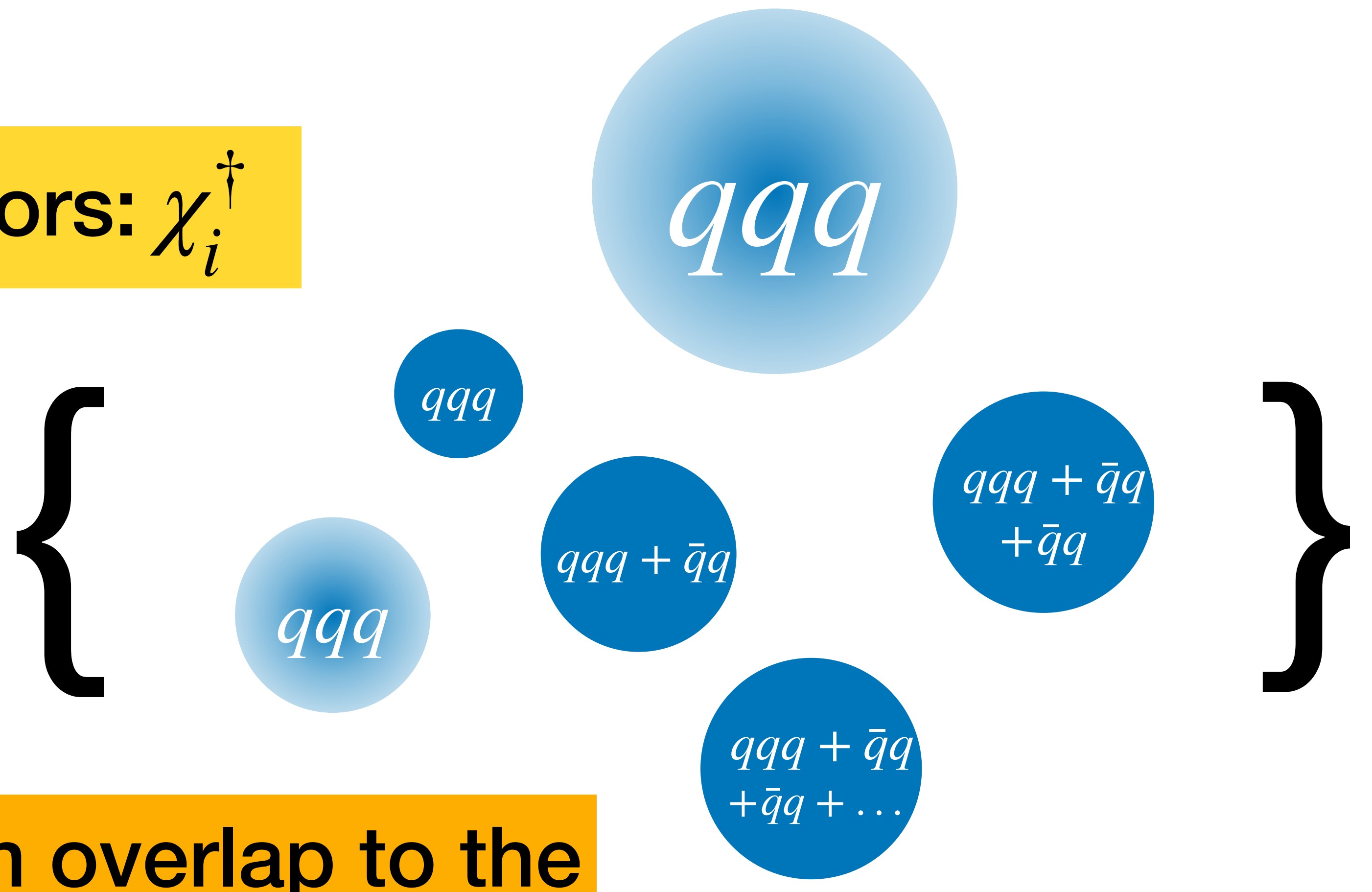

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$$\uparrow$$

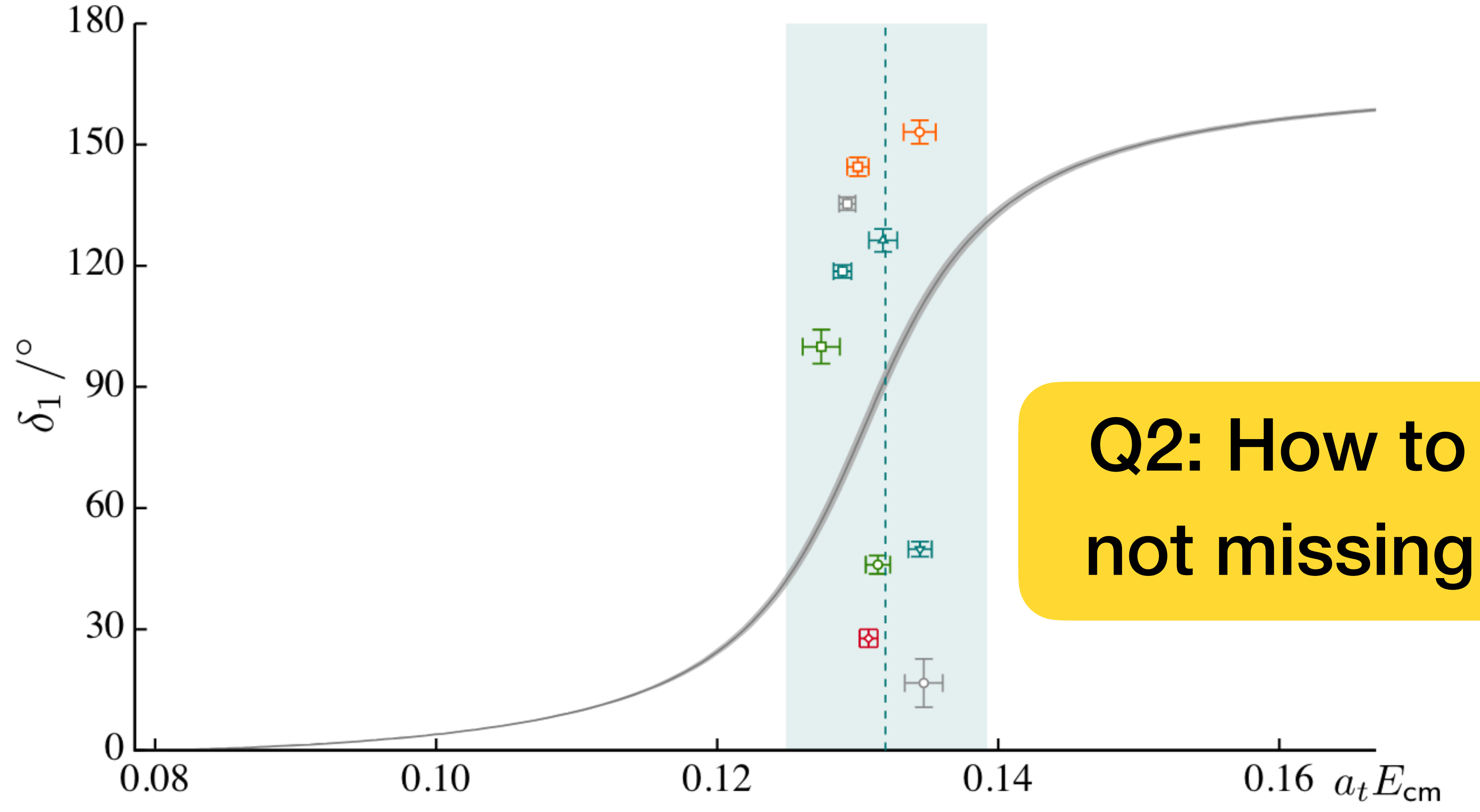
$$= e^{-E_n t}$$

generalized eigenvalue problem (GEVP)

More operators! “distillation”

M. Peardon et al. (Hadron Spectrum), Phys.Rev. D80, 054506 (2009)

Cost for GEVP is high!!!



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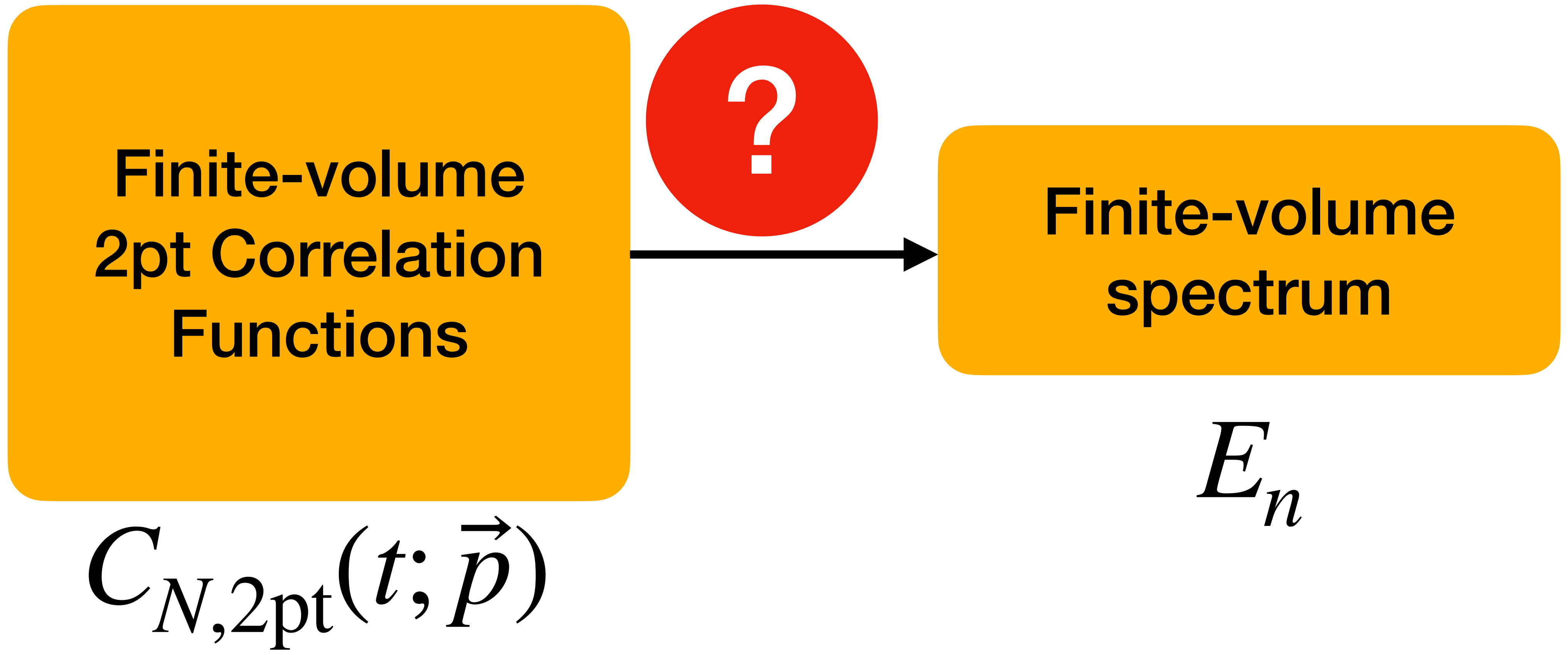
Finite-volume spectrum from lattice QCD

Q2*: Do we really need to make sure we are not missing (resolve) some of the $E_i's$?



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Strongly stable hadrons (bound states):

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Find E_0 for $t \gg 0$

Near-threshold bound states & Resonances

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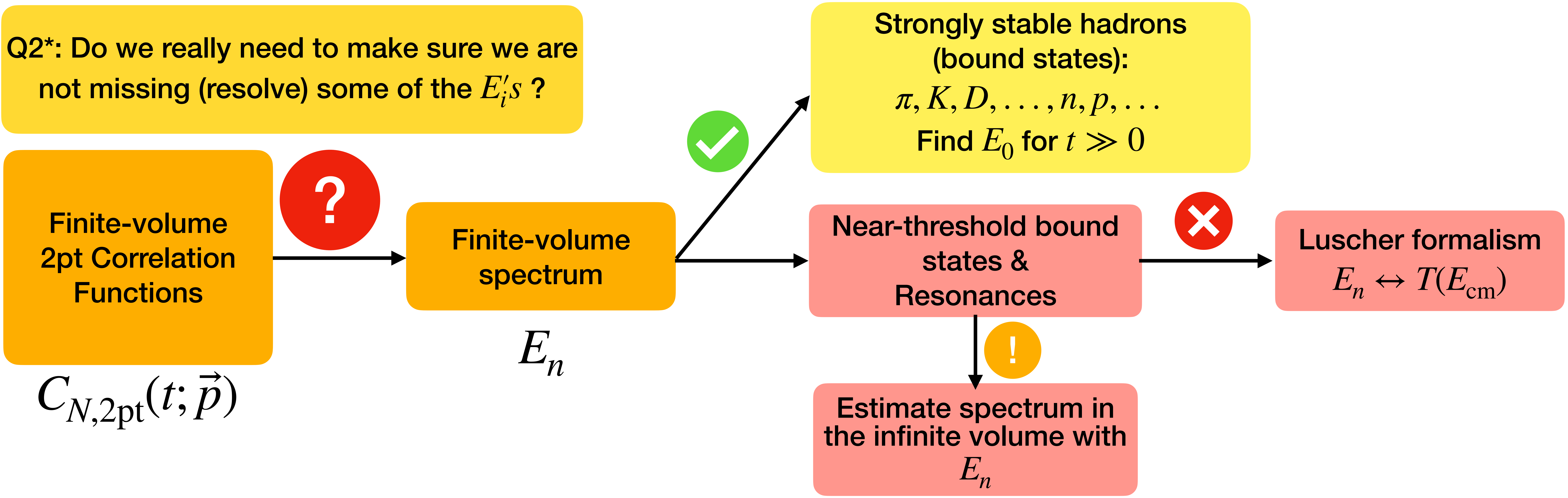

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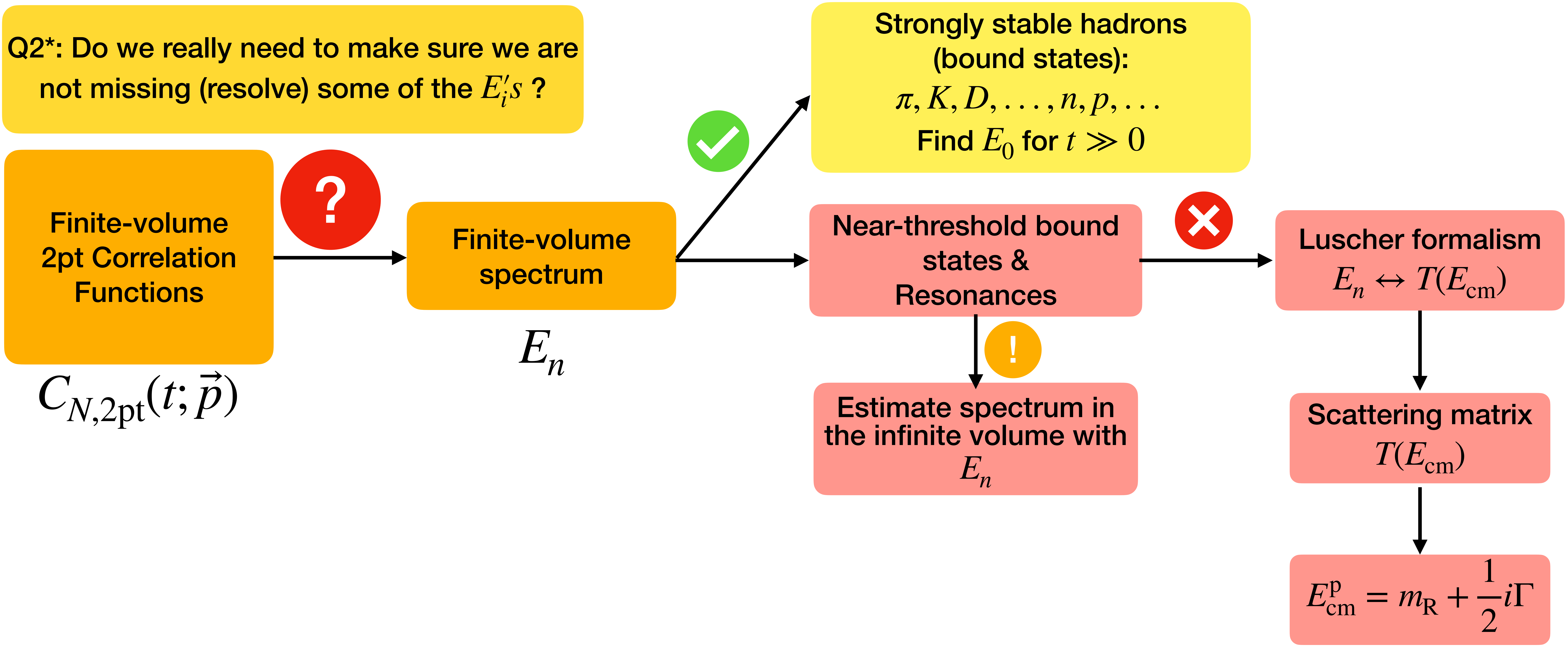

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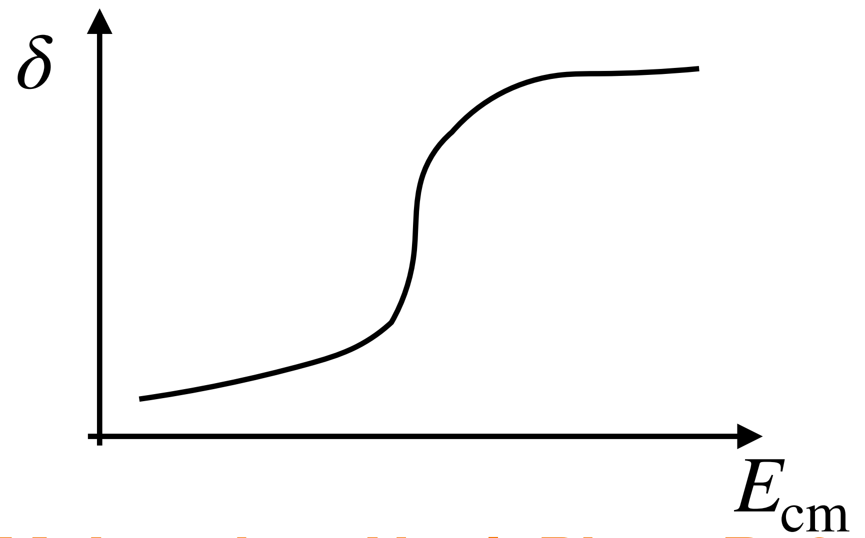

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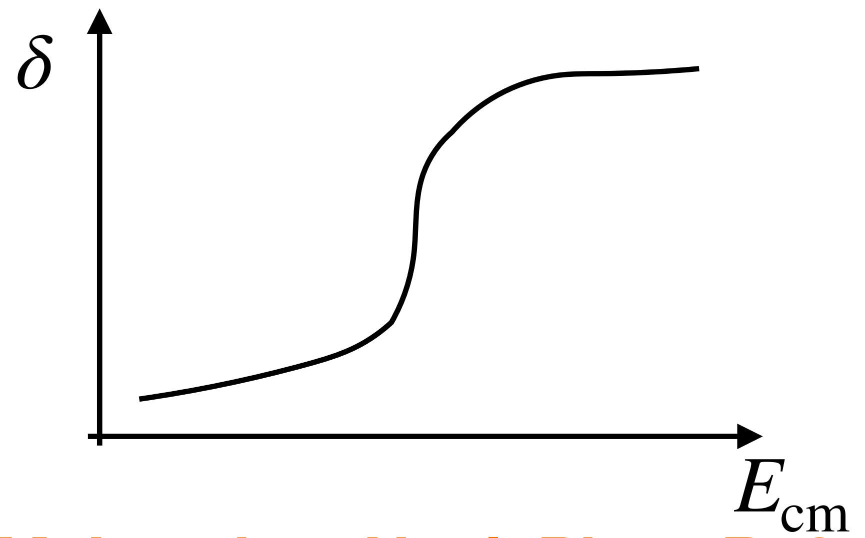

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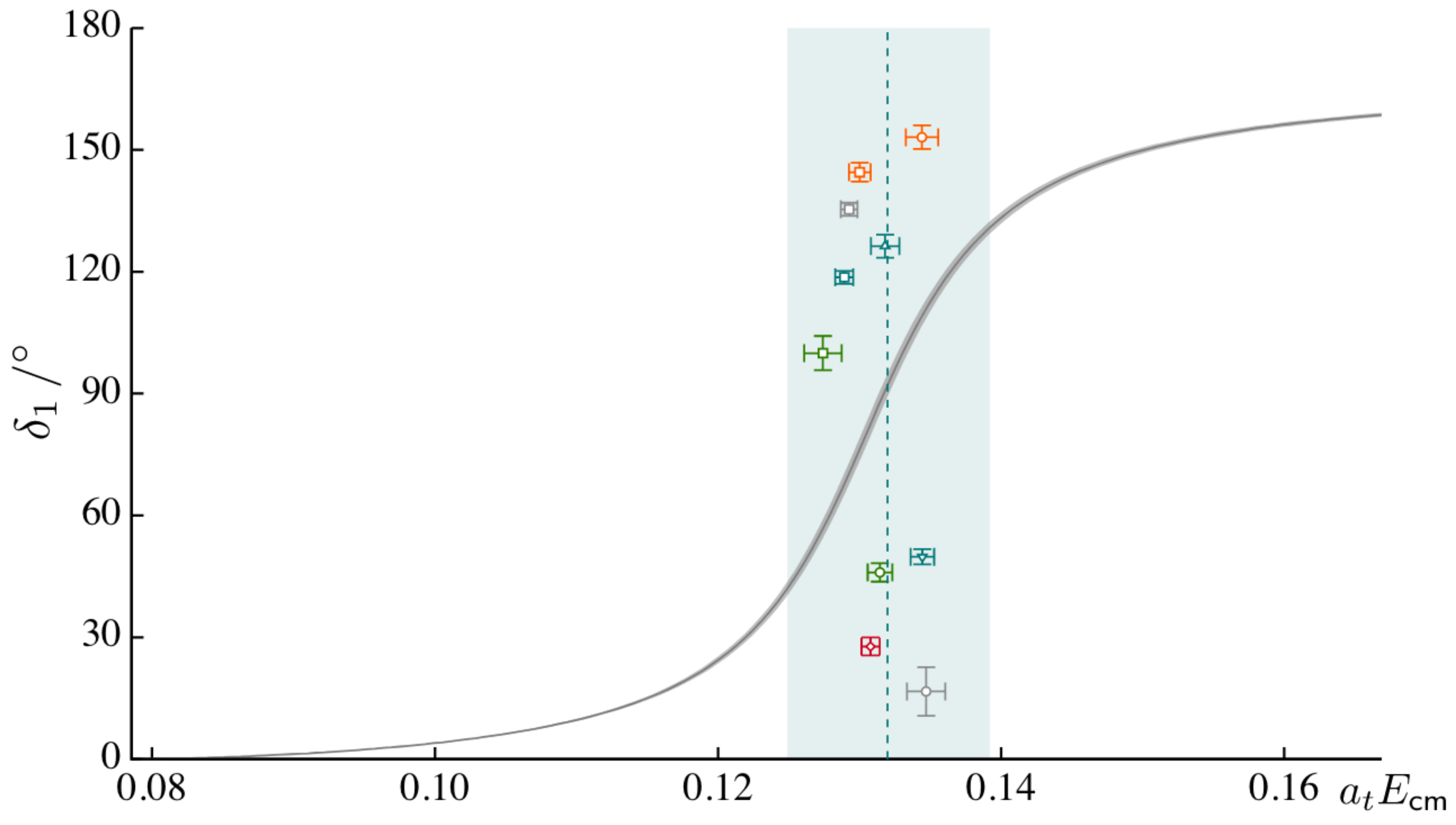

Wilson et al., *Phys. Rev. D*, vol. 92, no. 9, p. 094502, Nov. 2015

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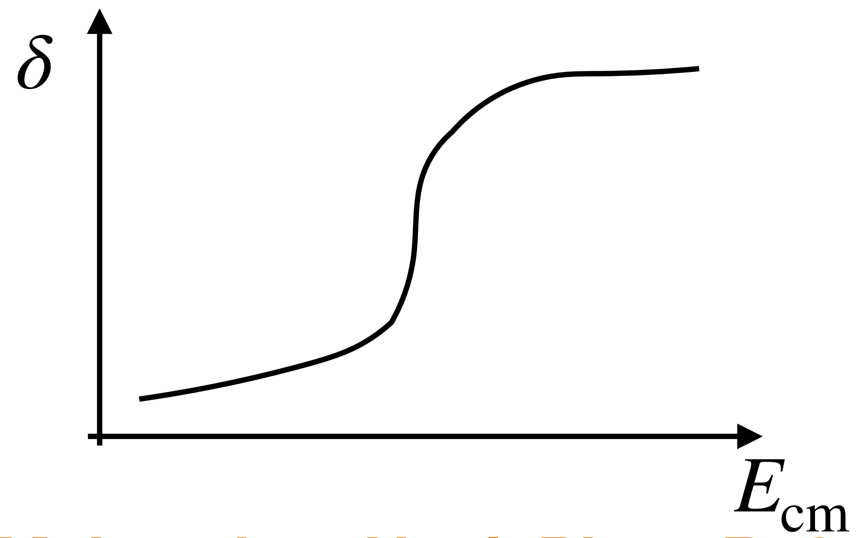

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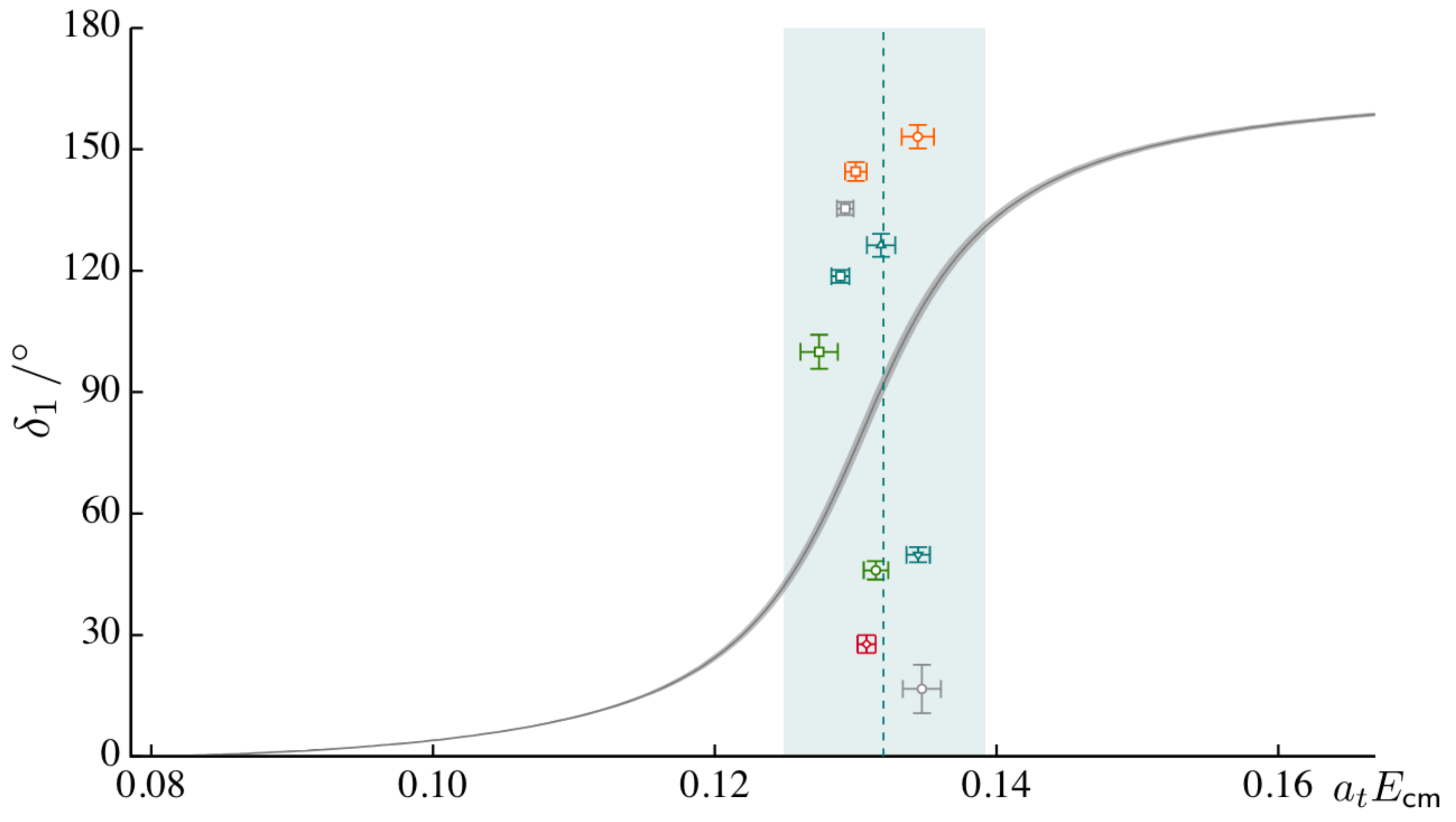

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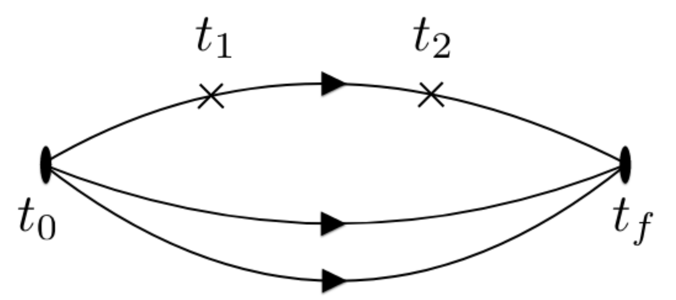

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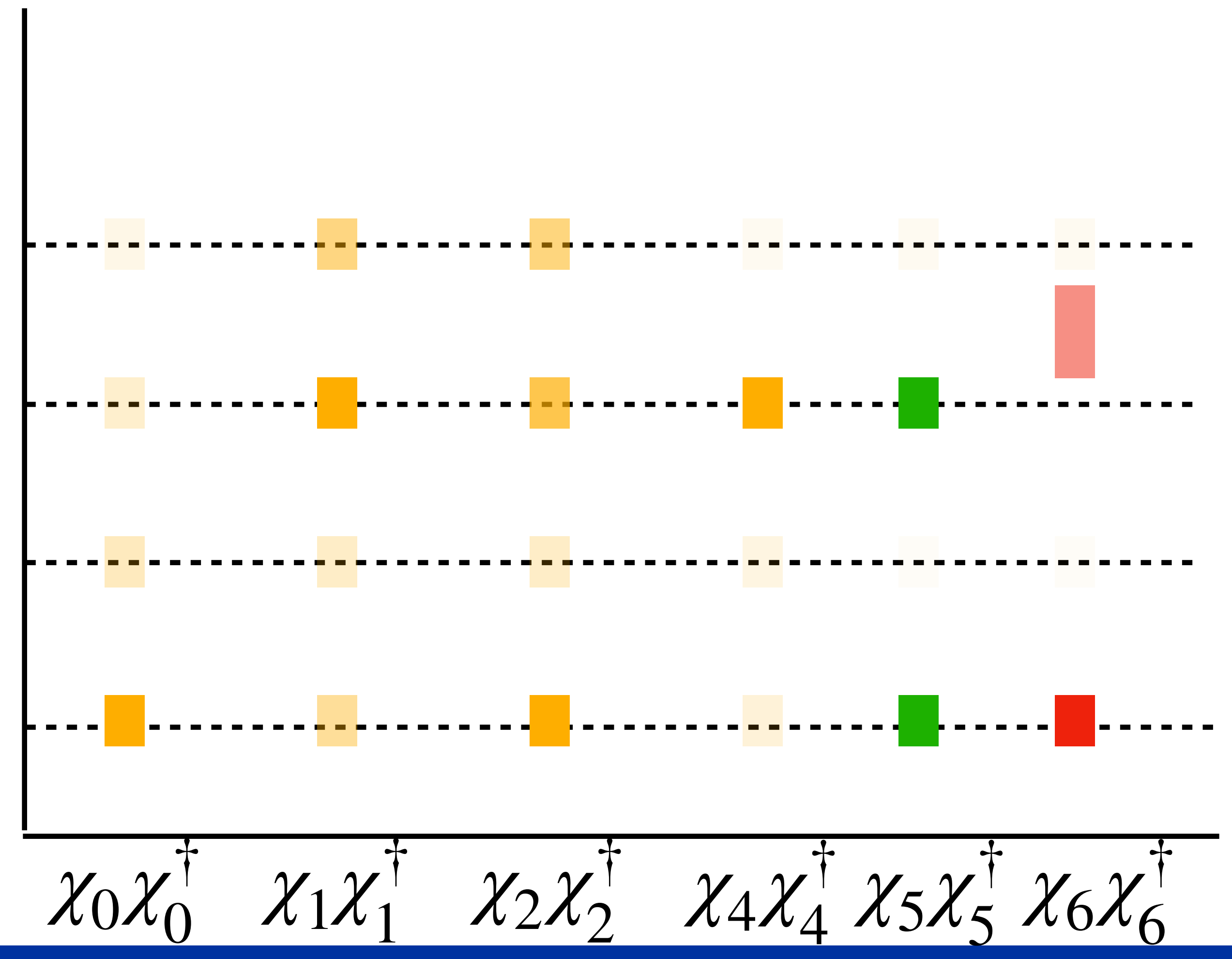

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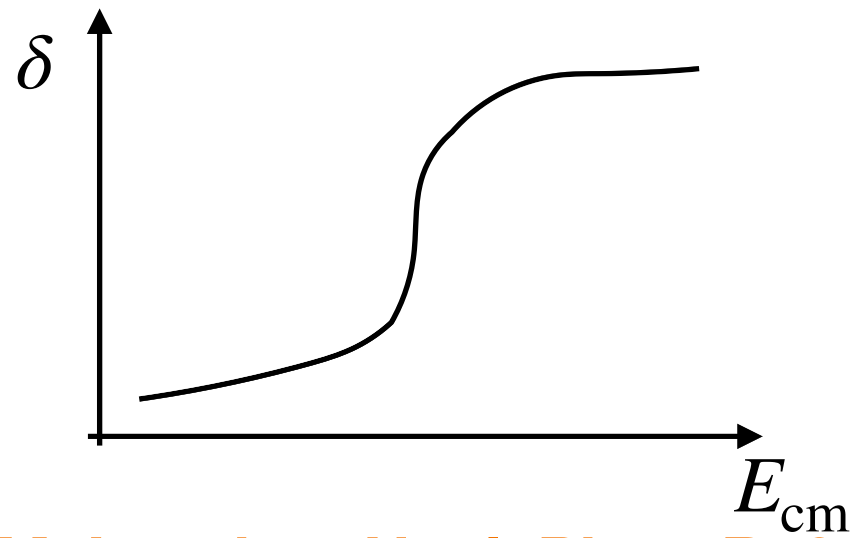

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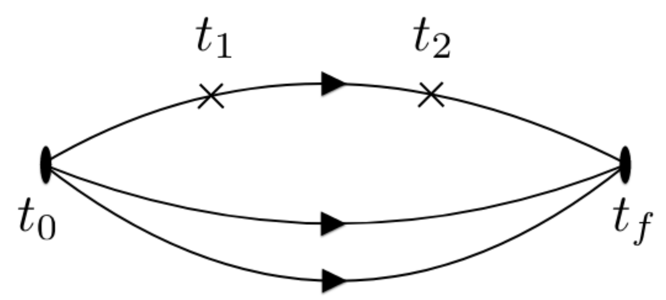

Example: Roper State from Overlap Fermions

$N(1440)$ $1/2^+$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Re(pole position) = 1360 to 1380 (≈ 1370) MeV
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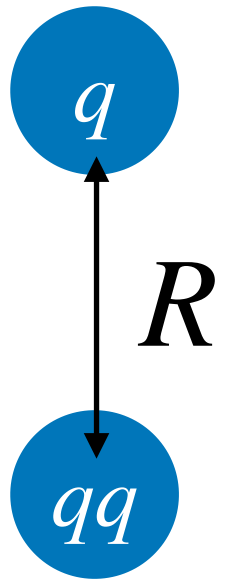

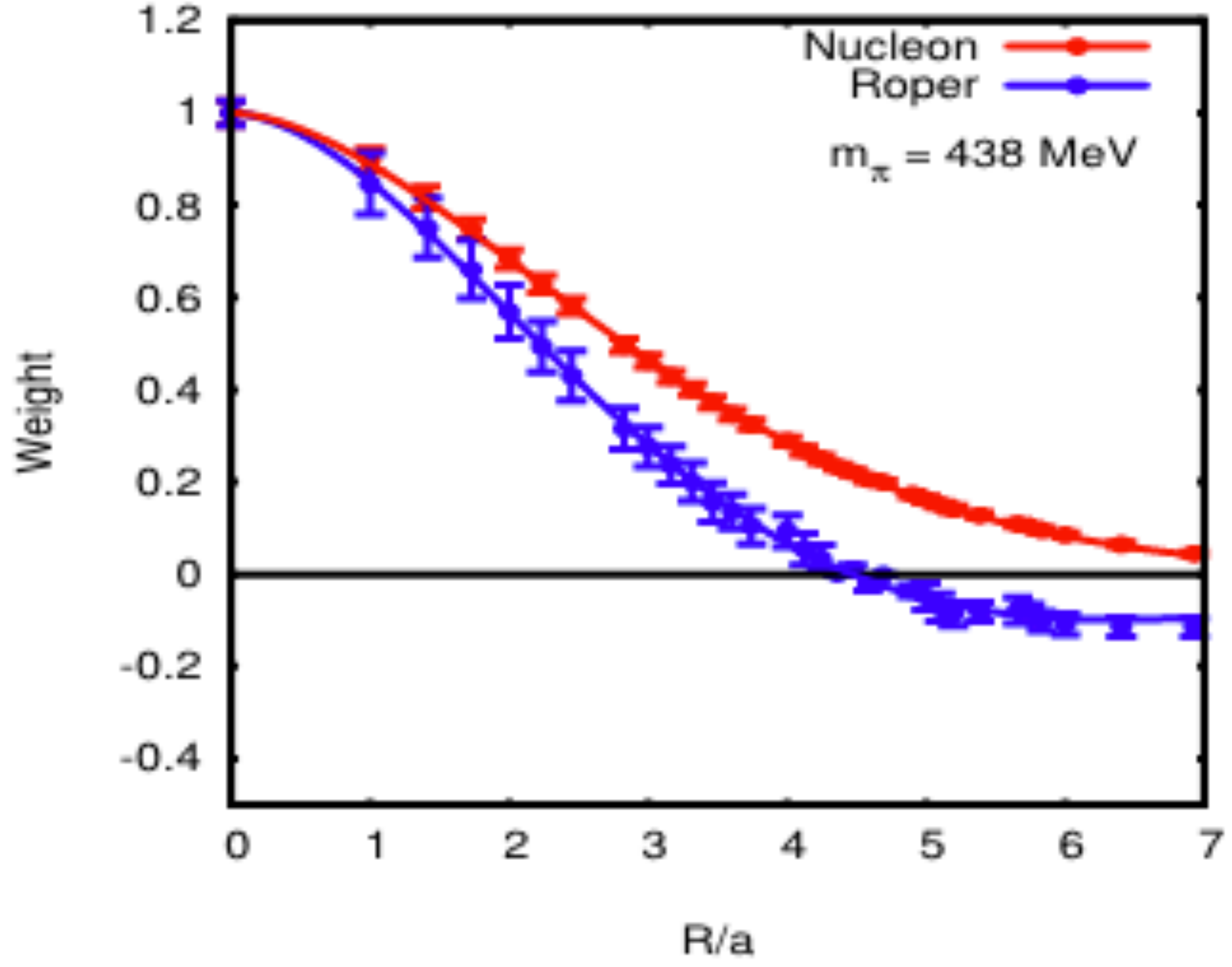

Nucleon and Roper wavefunctions in the Coulomb gauge

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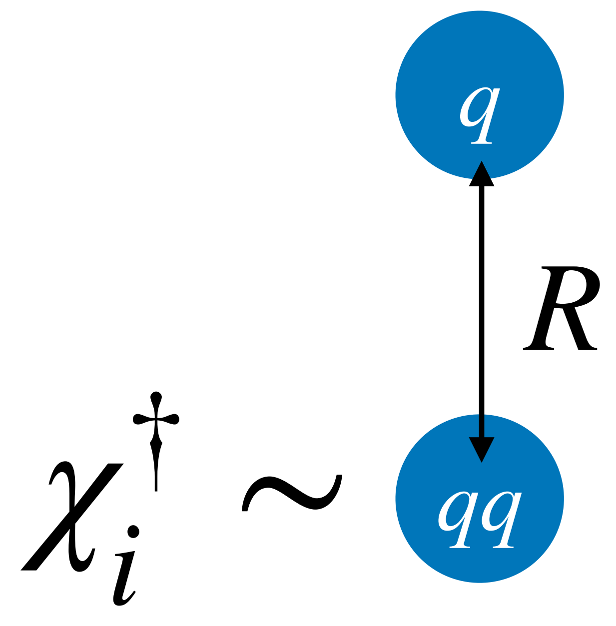

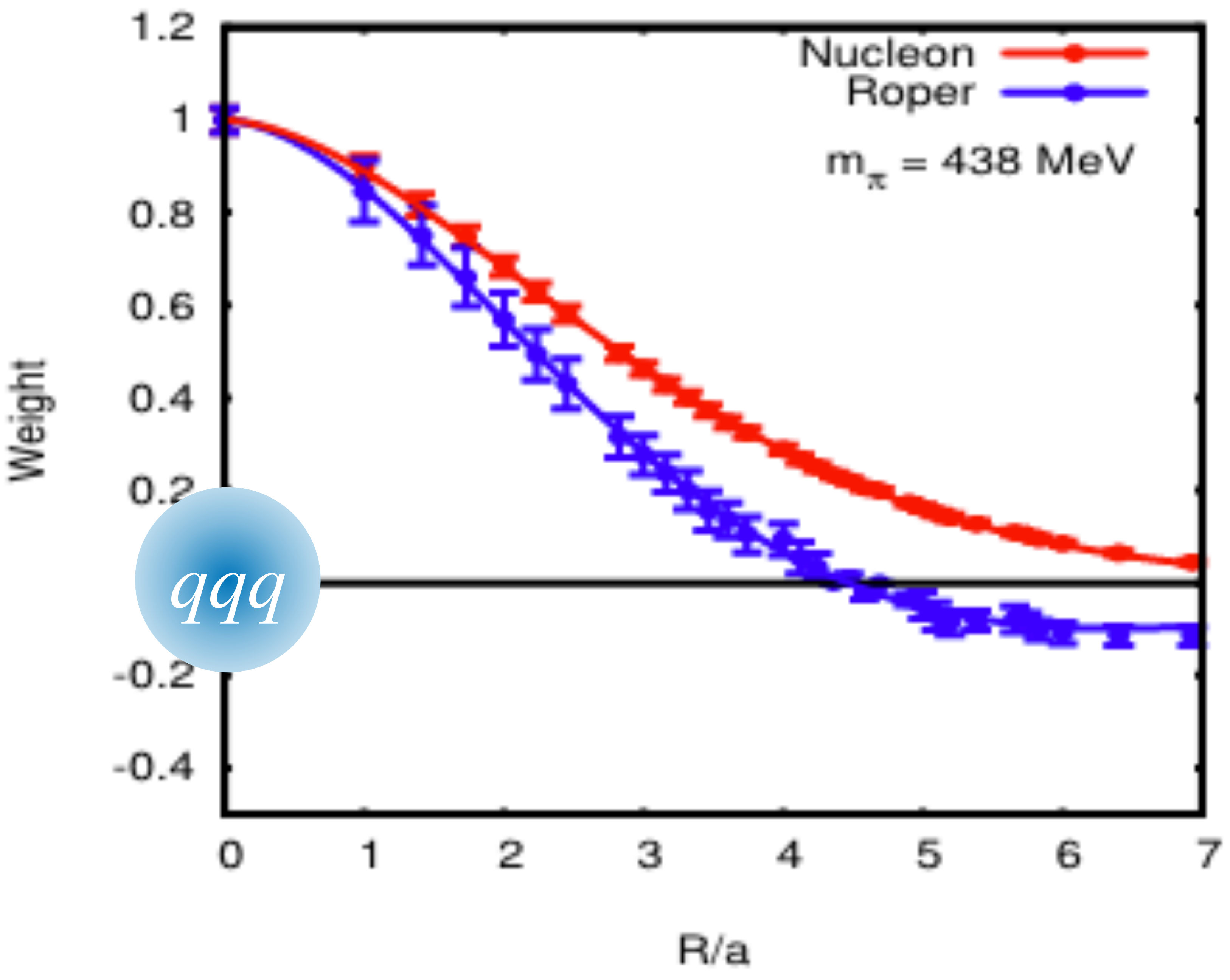

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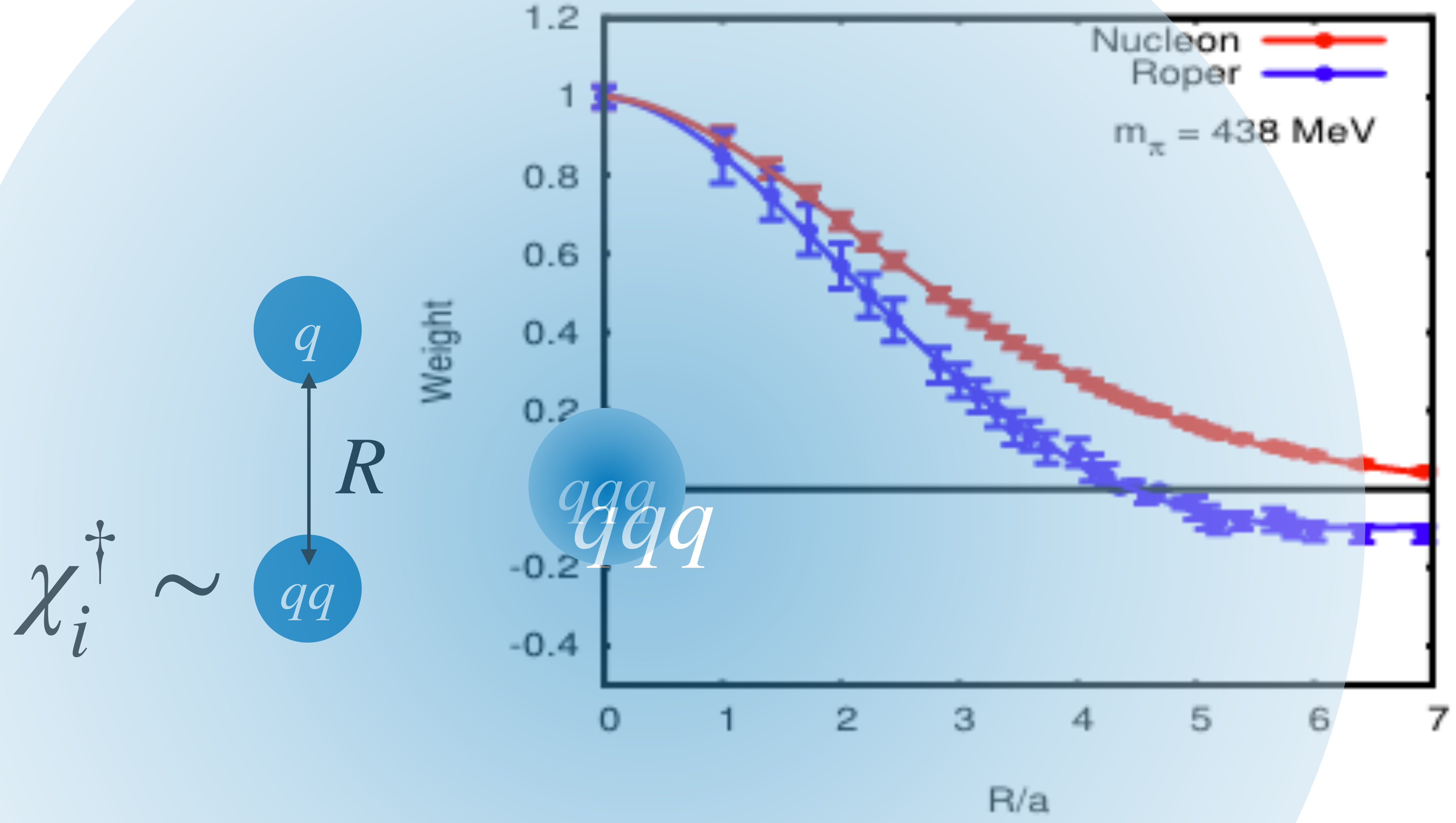

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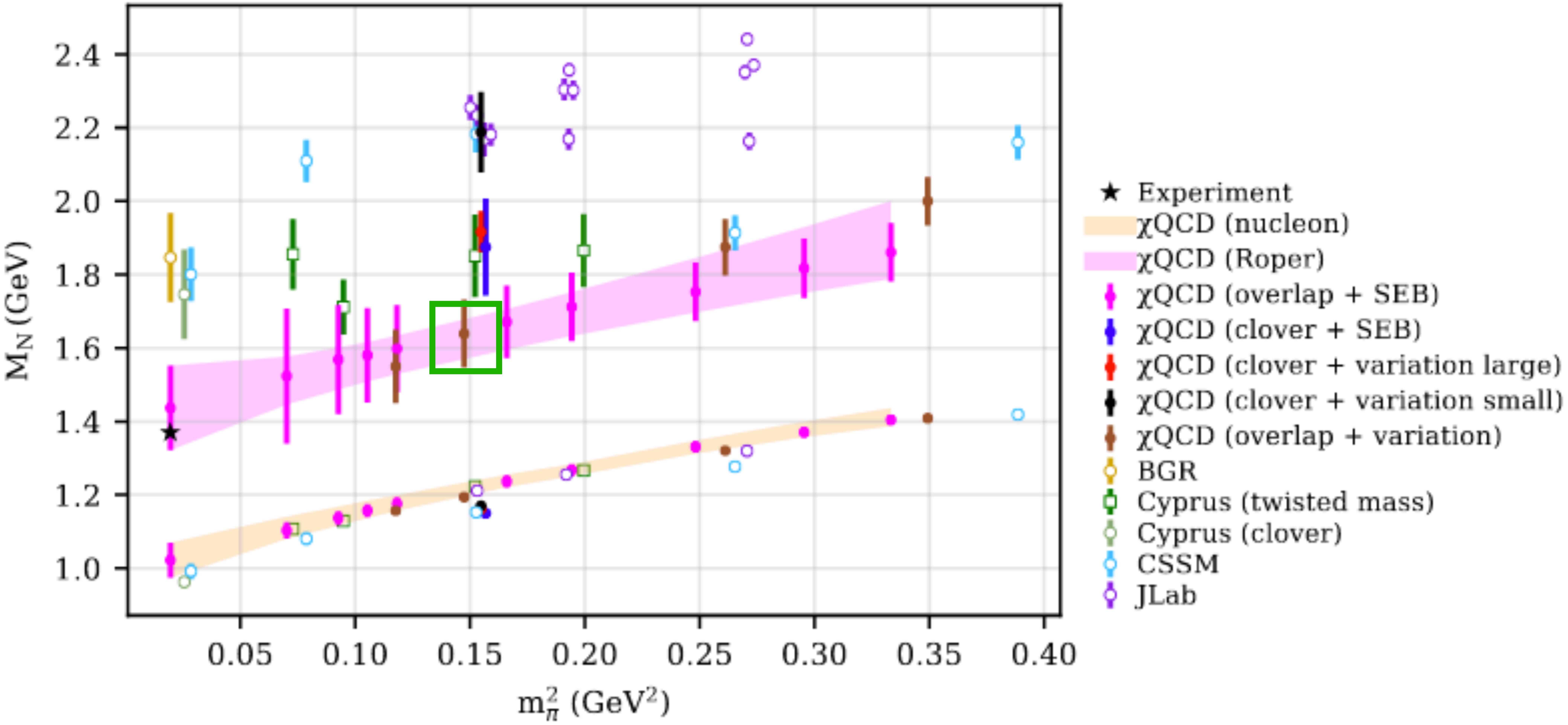

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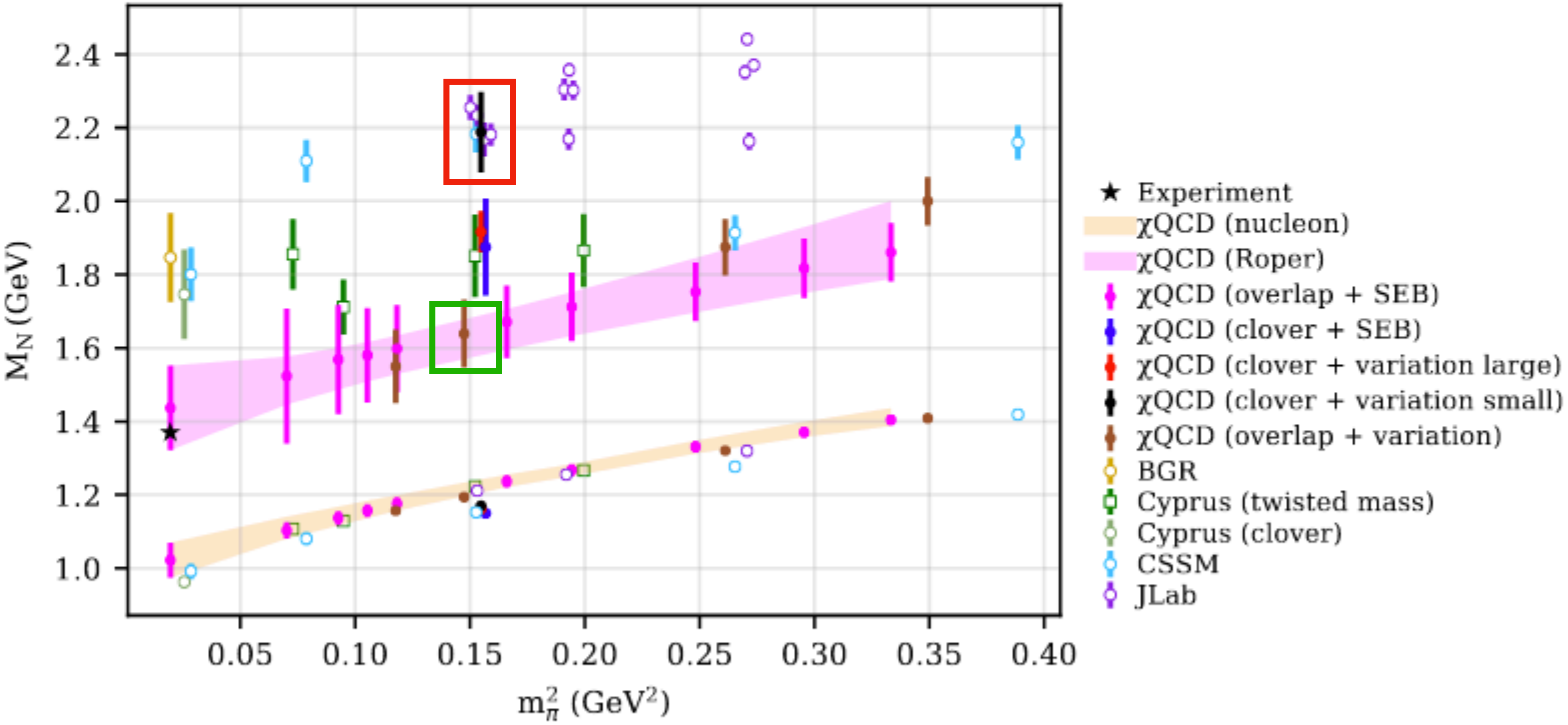

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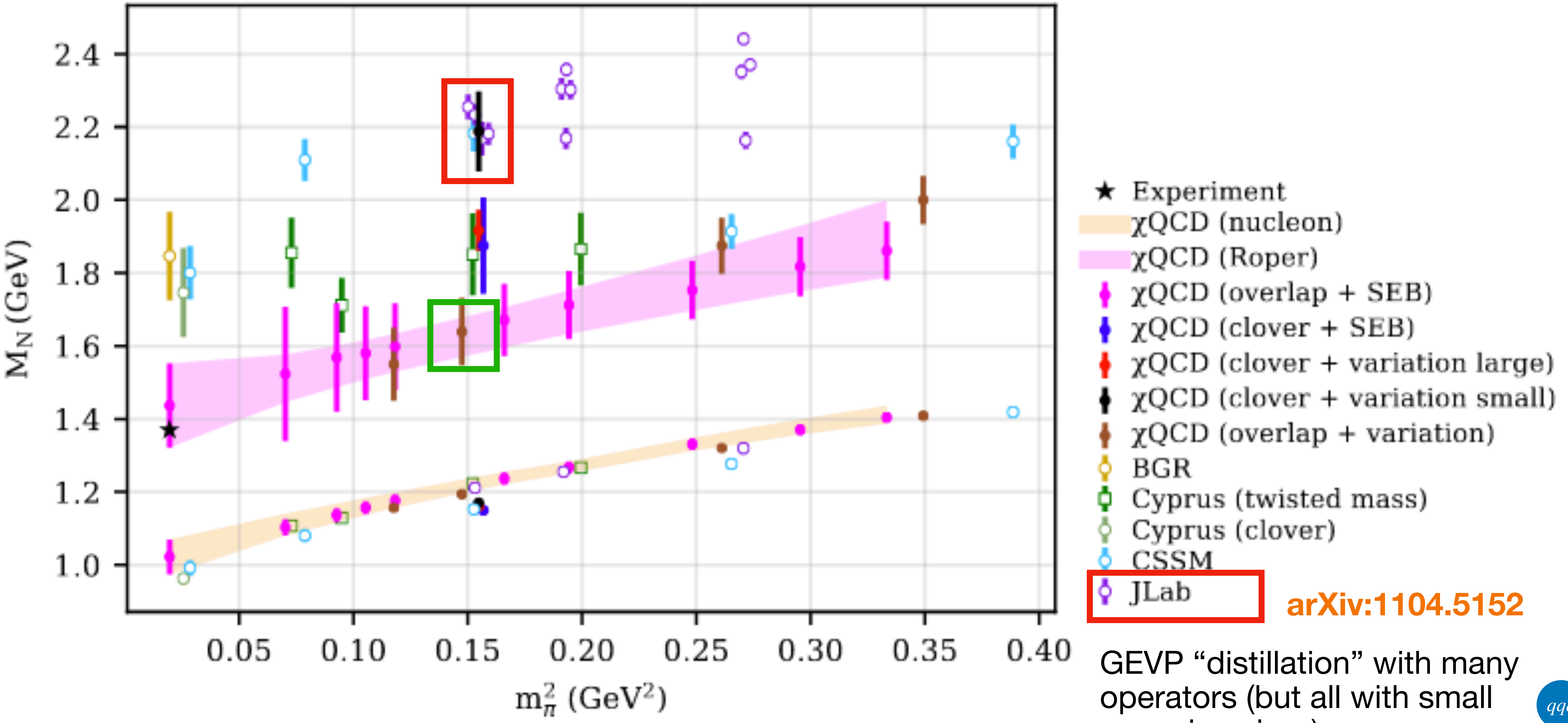
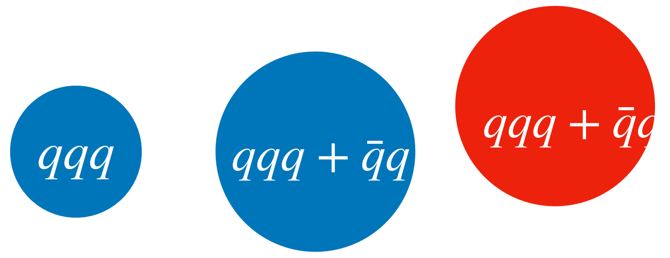


arXiv:1104.5152

GEVP "distillation" with many operators (but all with small smearing sizes)



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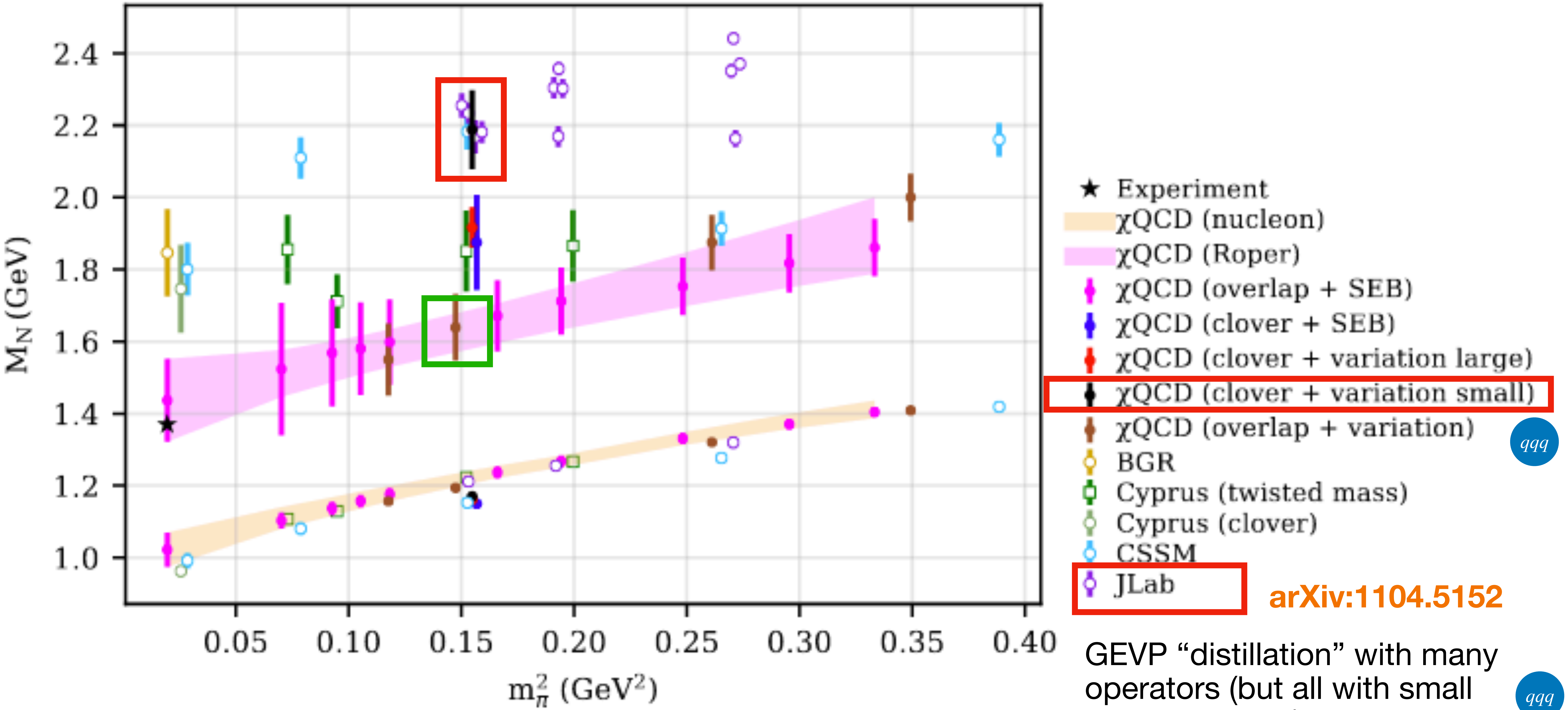

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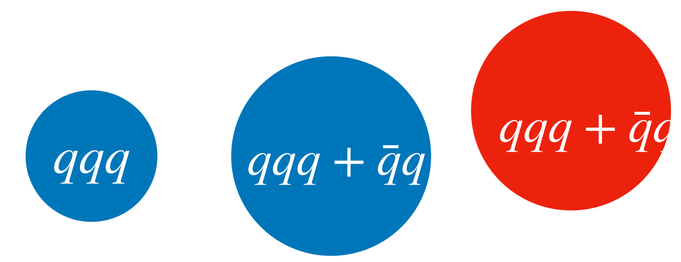

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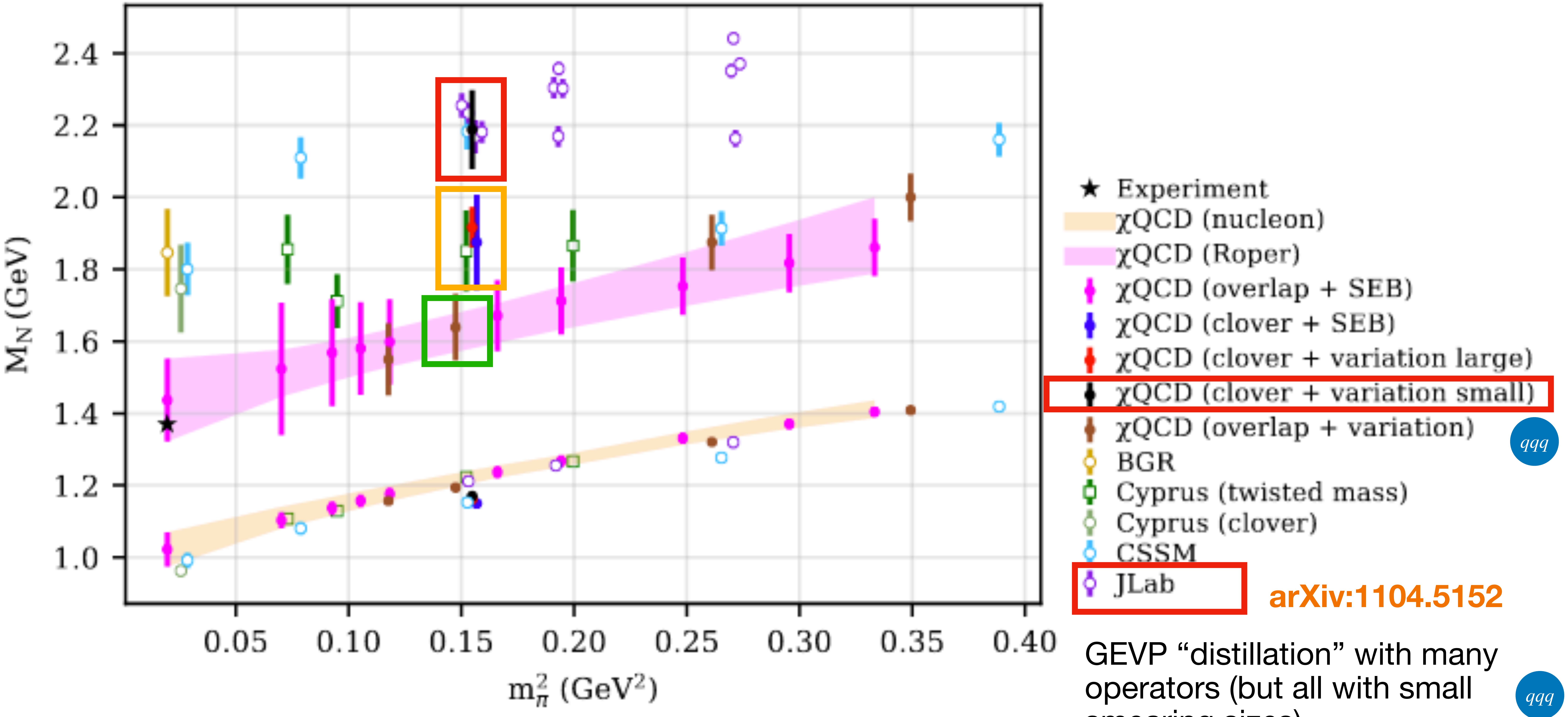

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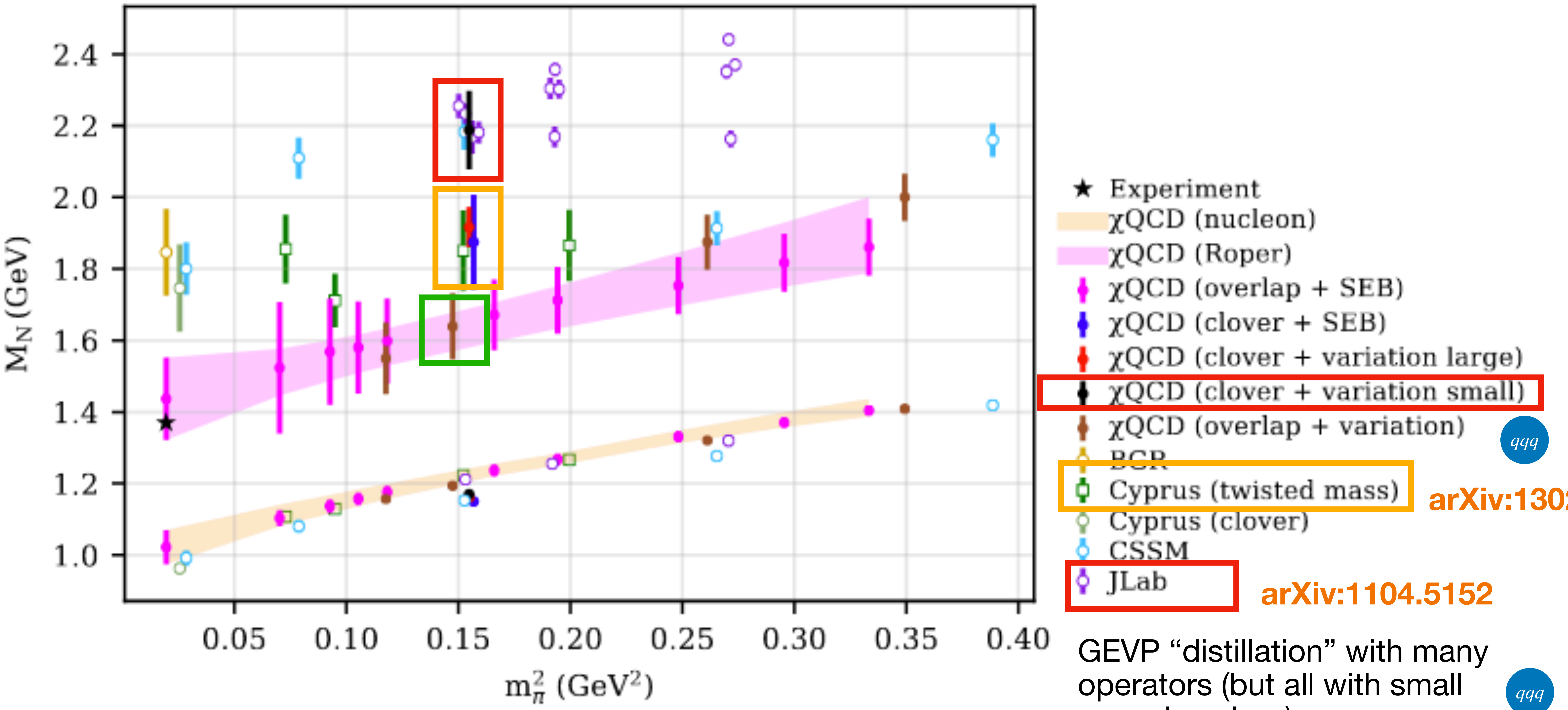

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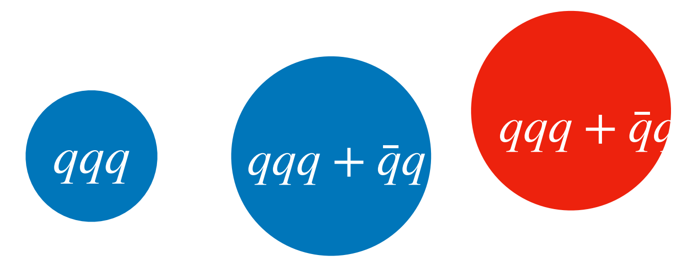

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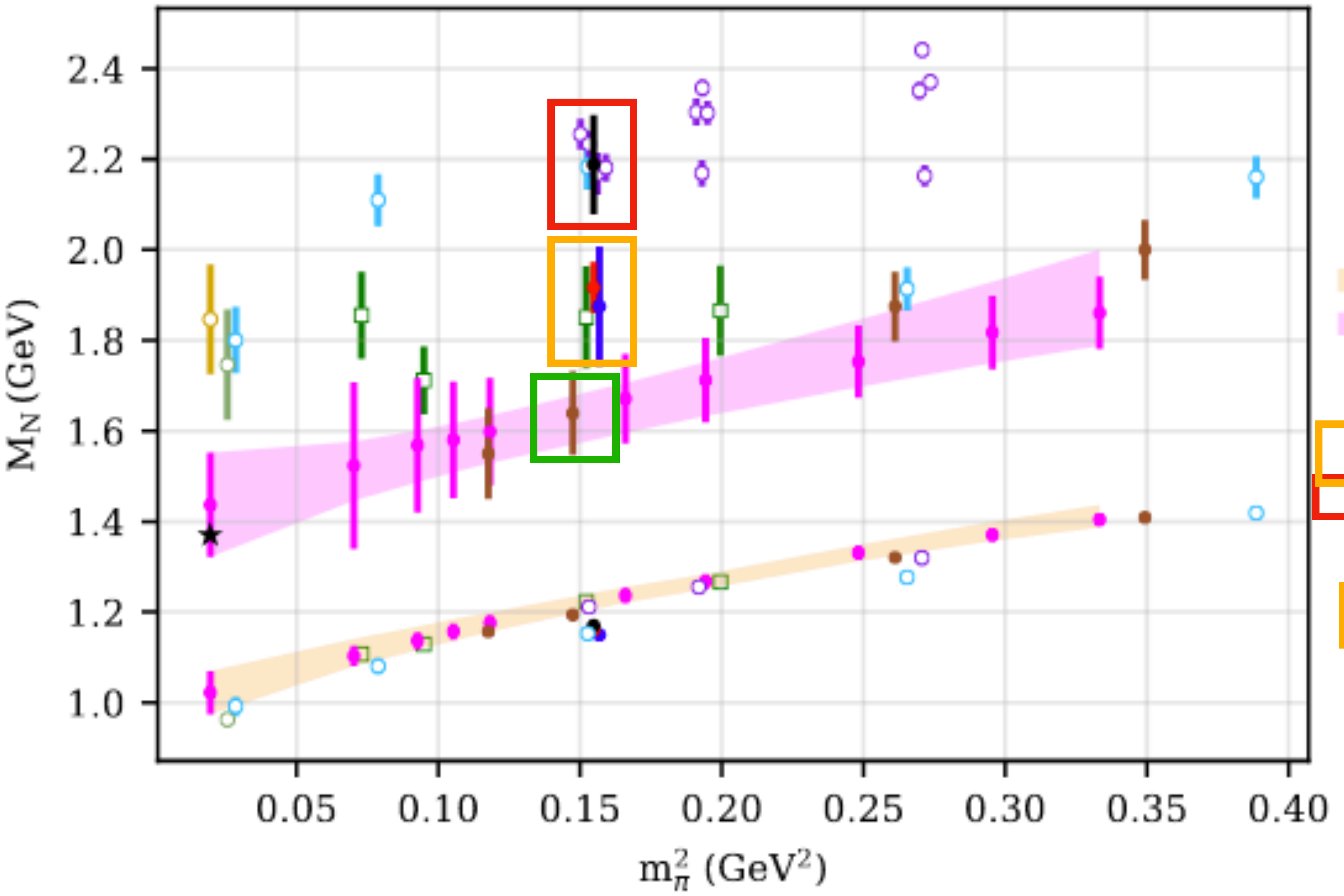

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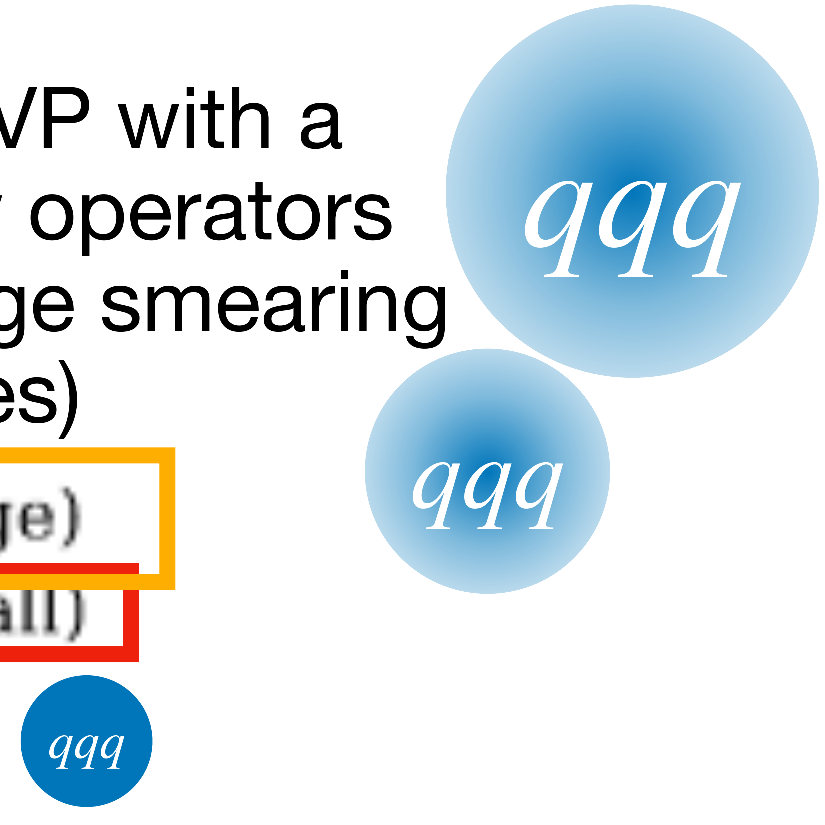

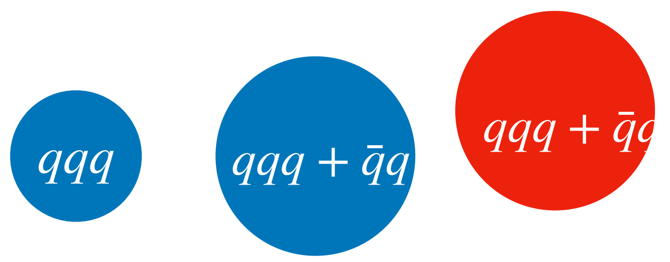

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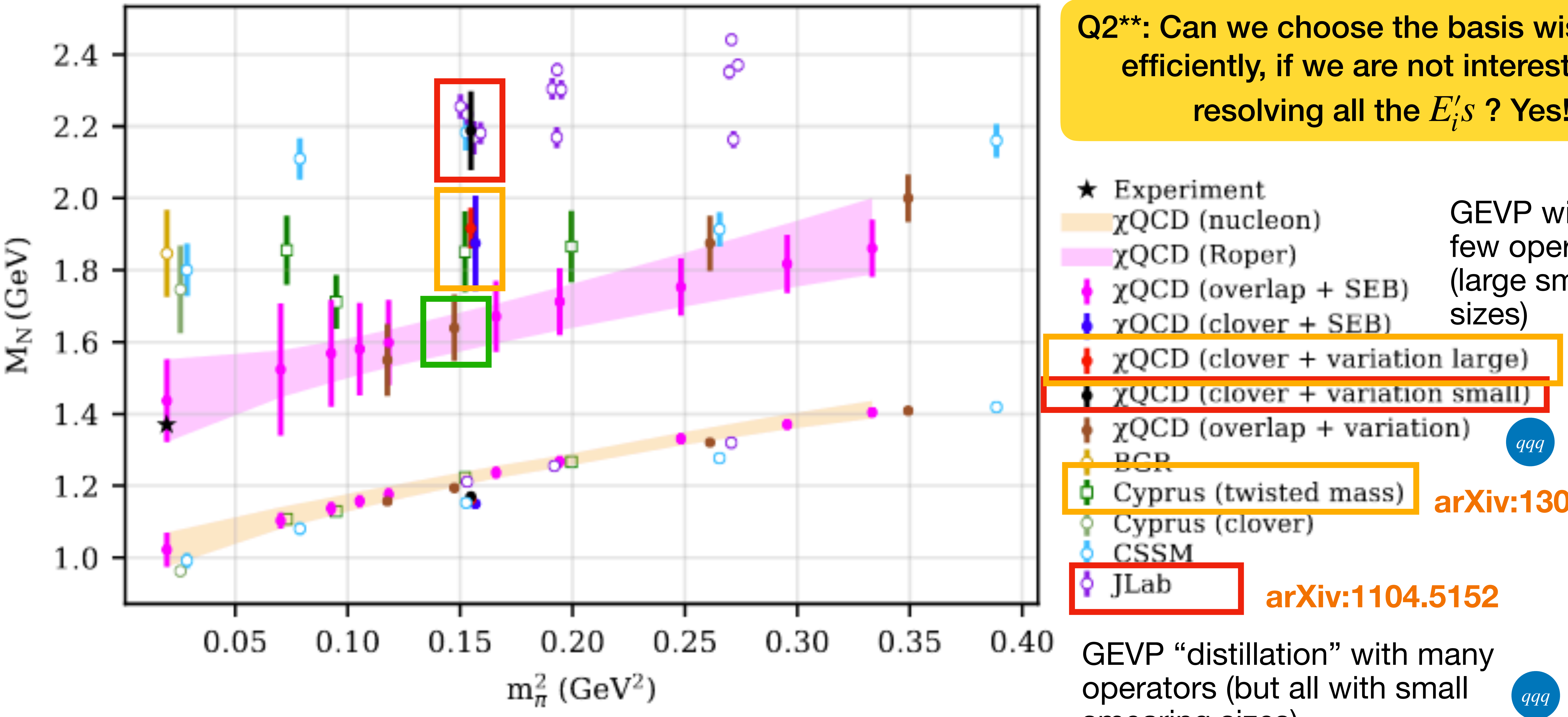

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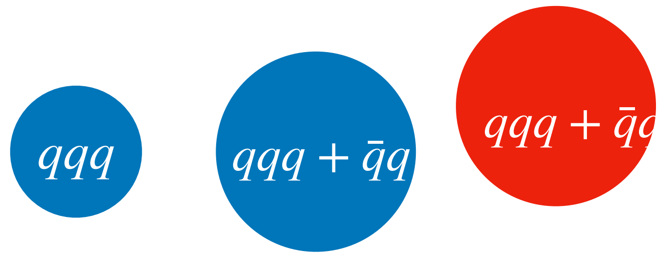

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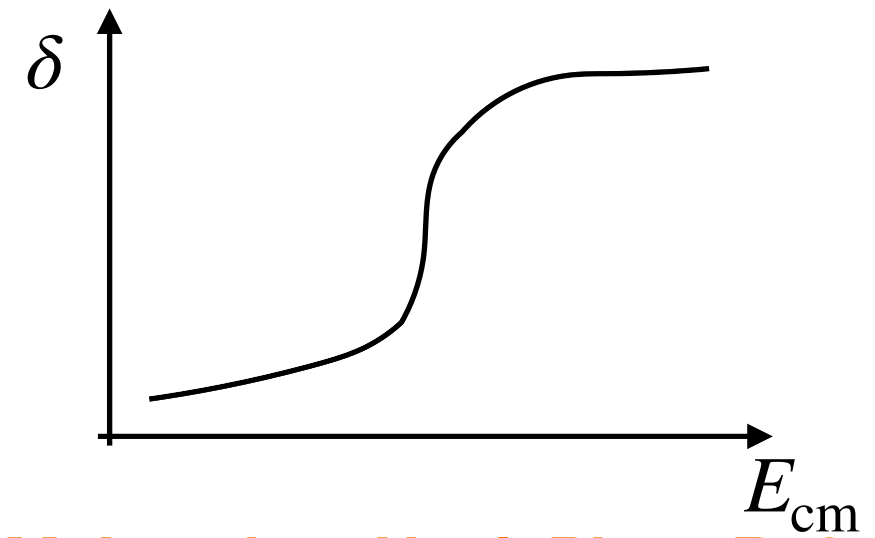

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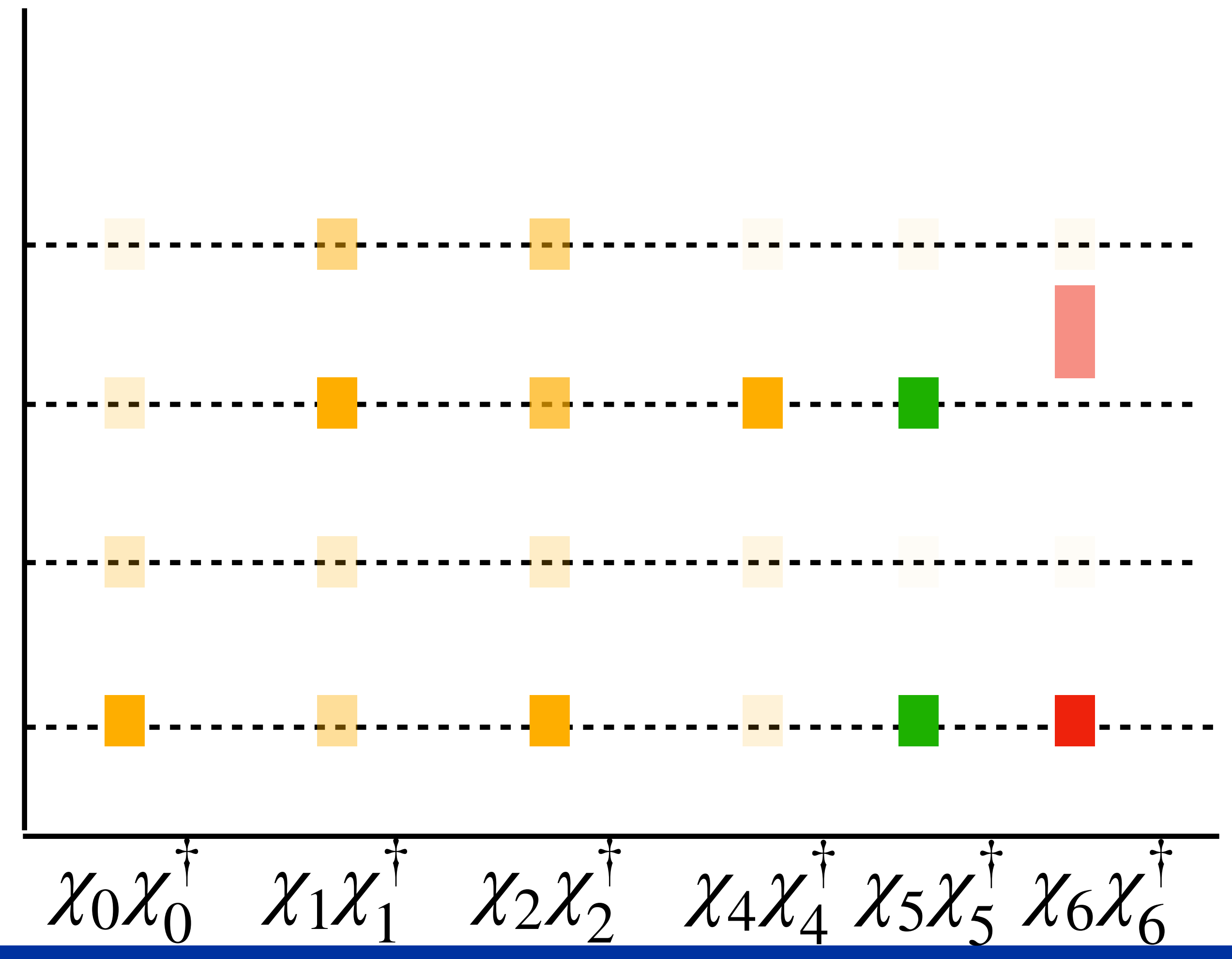

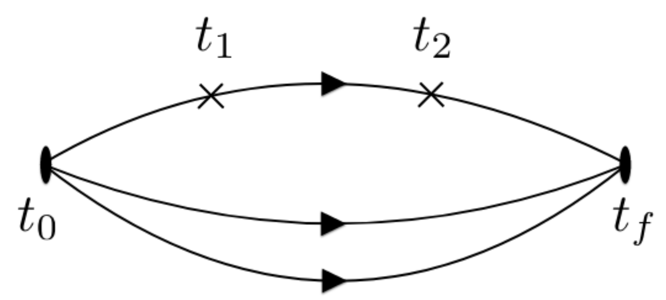

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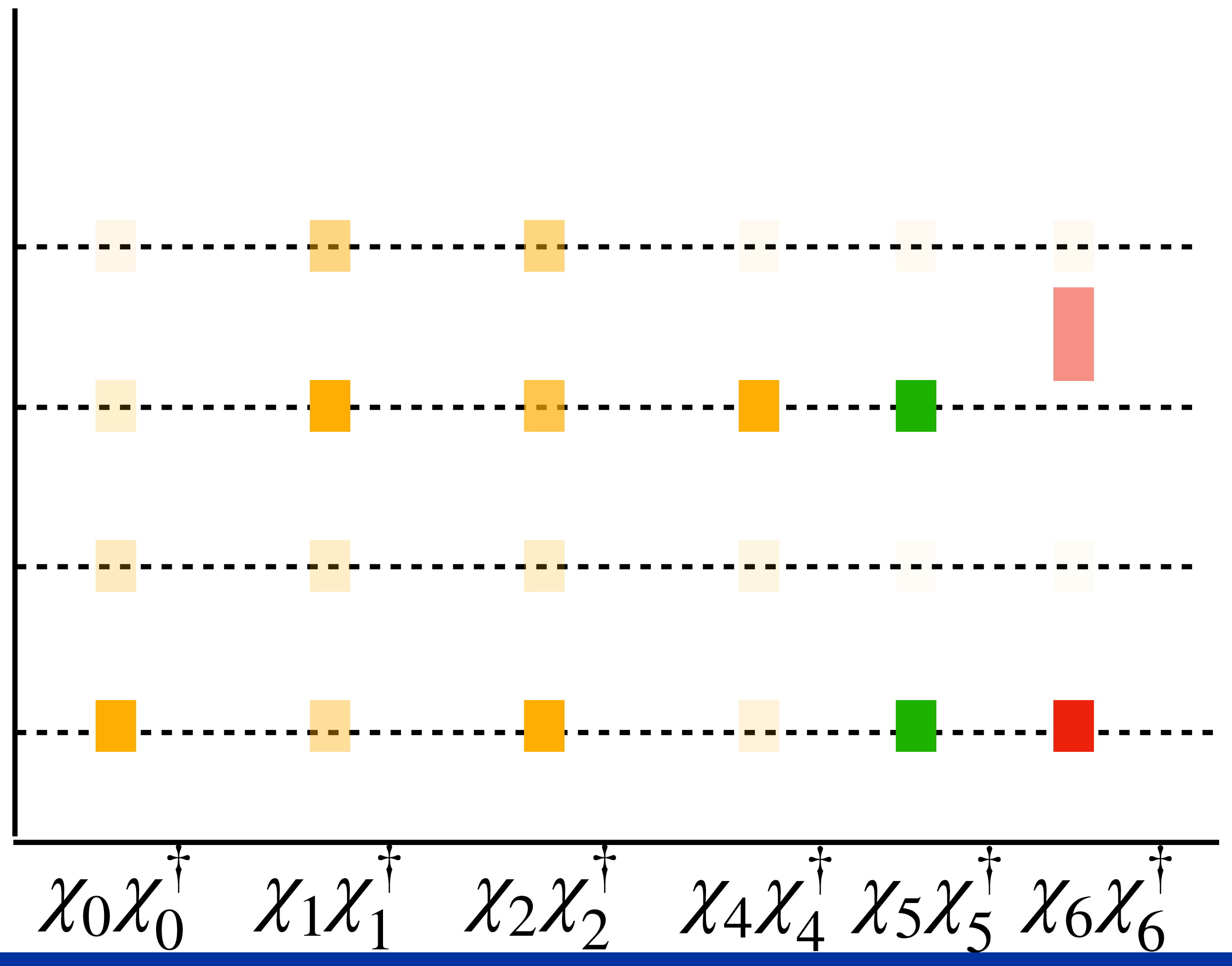

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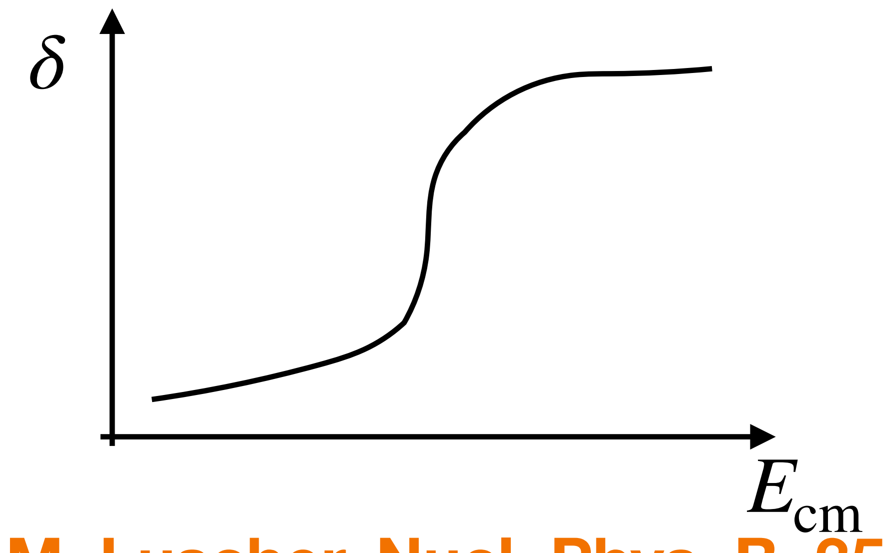

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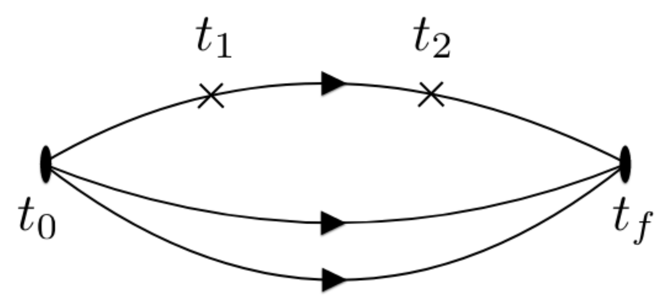

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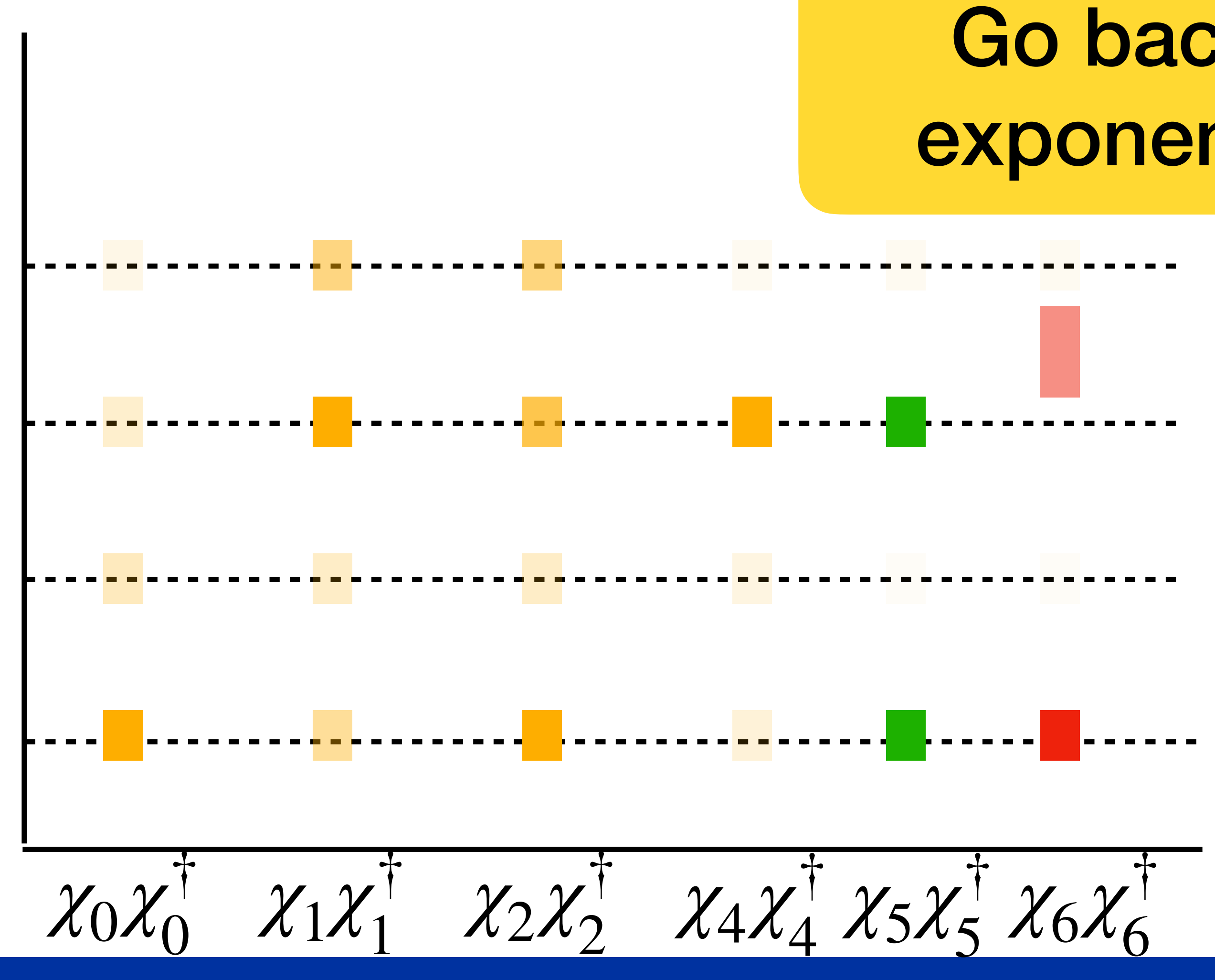

Only one operator? Go back to multi-exponential priors?

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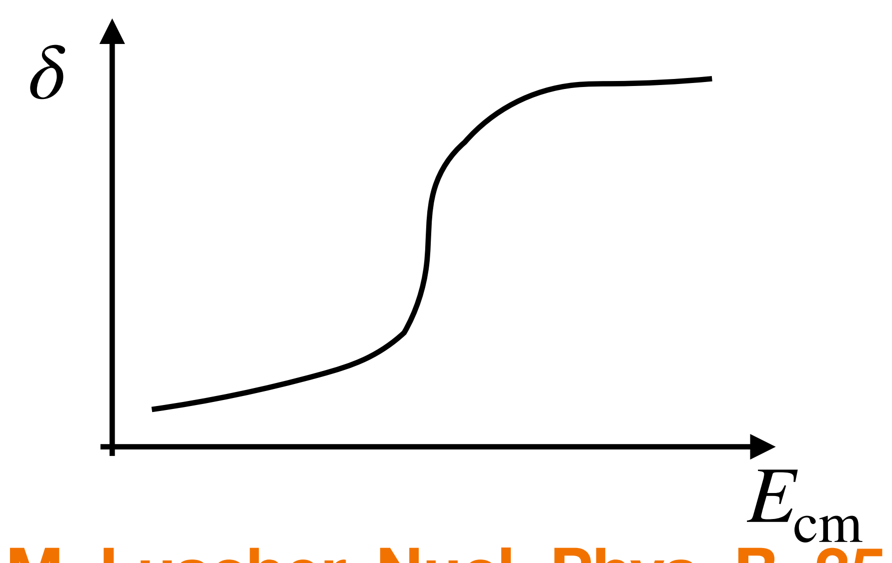

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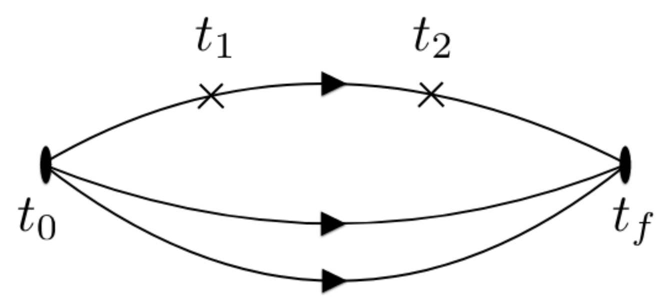


The Bayesian Reconstruction for inverse problems

A. Rothkopf, "Bayesian inference of real-time dynamics from lattice QCD," Front. Phys., arXiv:2208.13590

Yannis Burnier and Alexander Rothkopf, Phys. Rev. Lett. 111, 182003

Marginalize α_l with $P[\alpha] = 1$

$$\underbrace{P[\rho|D,m]}_{\text{posterior*}} = \underbrace{P[D|\rho,I]}_{\text{likelihood}} \times \underbrace{\prod_l \int d\alpha_l P[\rho|\alpha,m]P[\alpha]}_{prior*} / \underbrace{P[D|m]}_{\text{evidence}}$$

$$e^{-Q} \qquad e^{-L} \qquad e^{S}$$

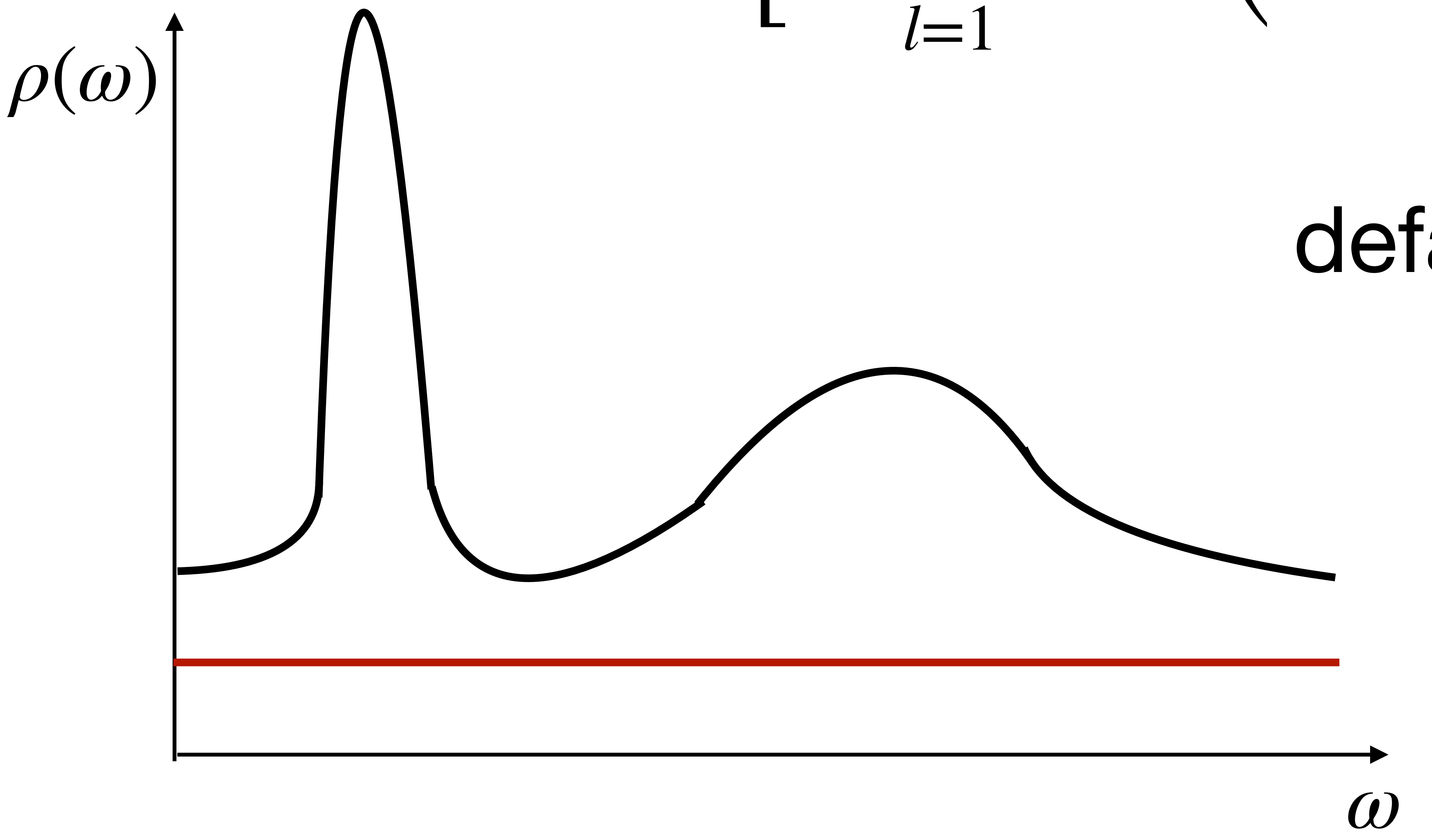
$$Q = L - S + \dots$$

$$\chi^2 \equiv \sum_{ij} (D_i - \tilde{D}_i) C^{-1}_{ij} (D_j - \tilde{D}_j)$$

$$P[\rho|\alpha,m] = \exp\left[- \sum_{l=1}^{N_\mu} \alpha_l \Delta\mu \left(1 - \frac{\rho_l}{m_l} - \log[\frac{\rho_l}{m_l}] \right) \right]$$

fitted spectrum

default model

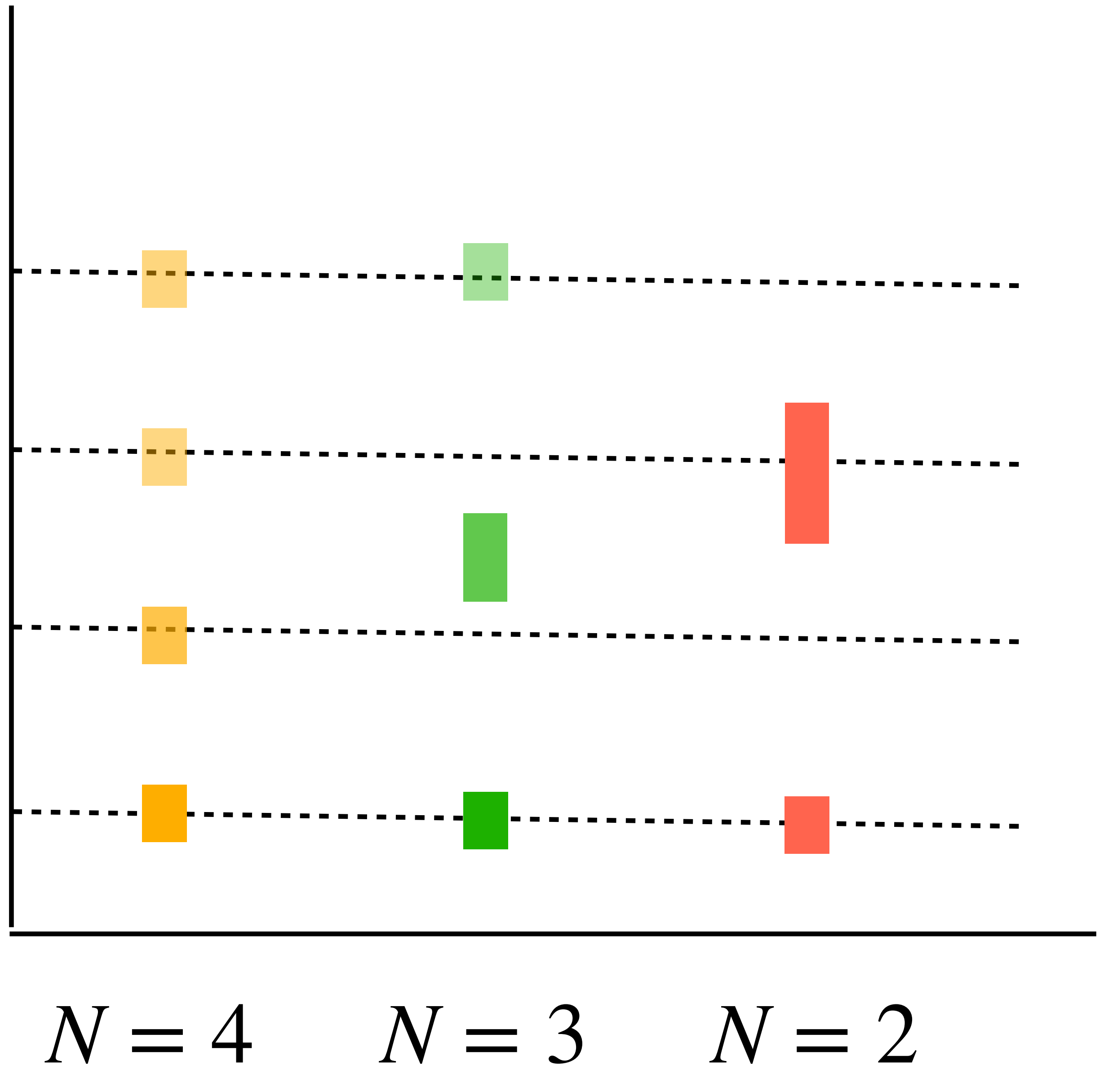


The Bayesian Reconstruction on nucleon correlators

- To extract the finite-volume spectrum
  - Two-point correlation functions

$$C_{ij}(t) = \langle \chi_i(t)\chi_j^\dagger(0)\rangle = \sum_{n=0}^{\infty} W_n e^{-E_n t}$$

$$\sum_{n=0}^{N} W_{ij,n} e^{-E_n t}$$



- Finite-volume spectrum found in terms of spectral function $\rho(\omega)$

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M. T. Hansen et al.,
Phys. Rev. D 96, 094513 (2017)

Talks by Liang, Sufian

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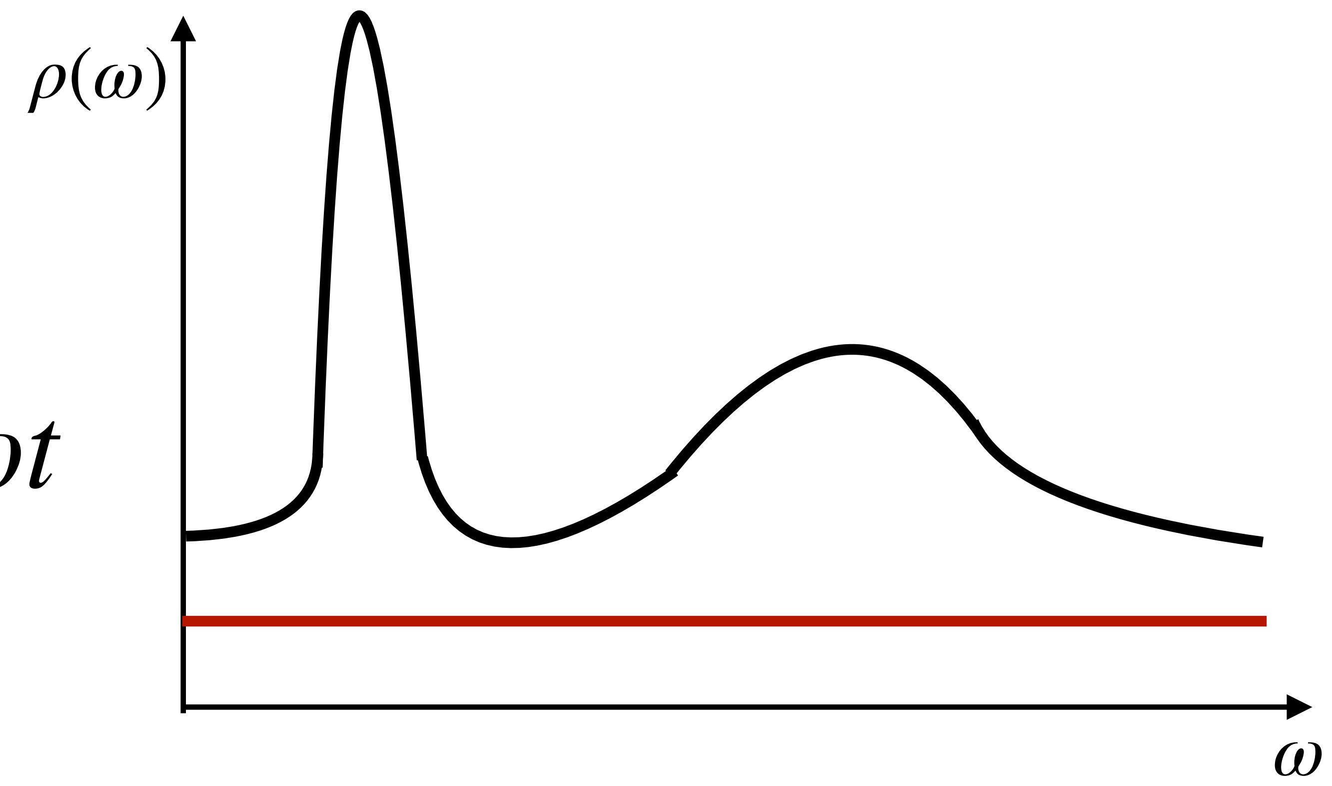

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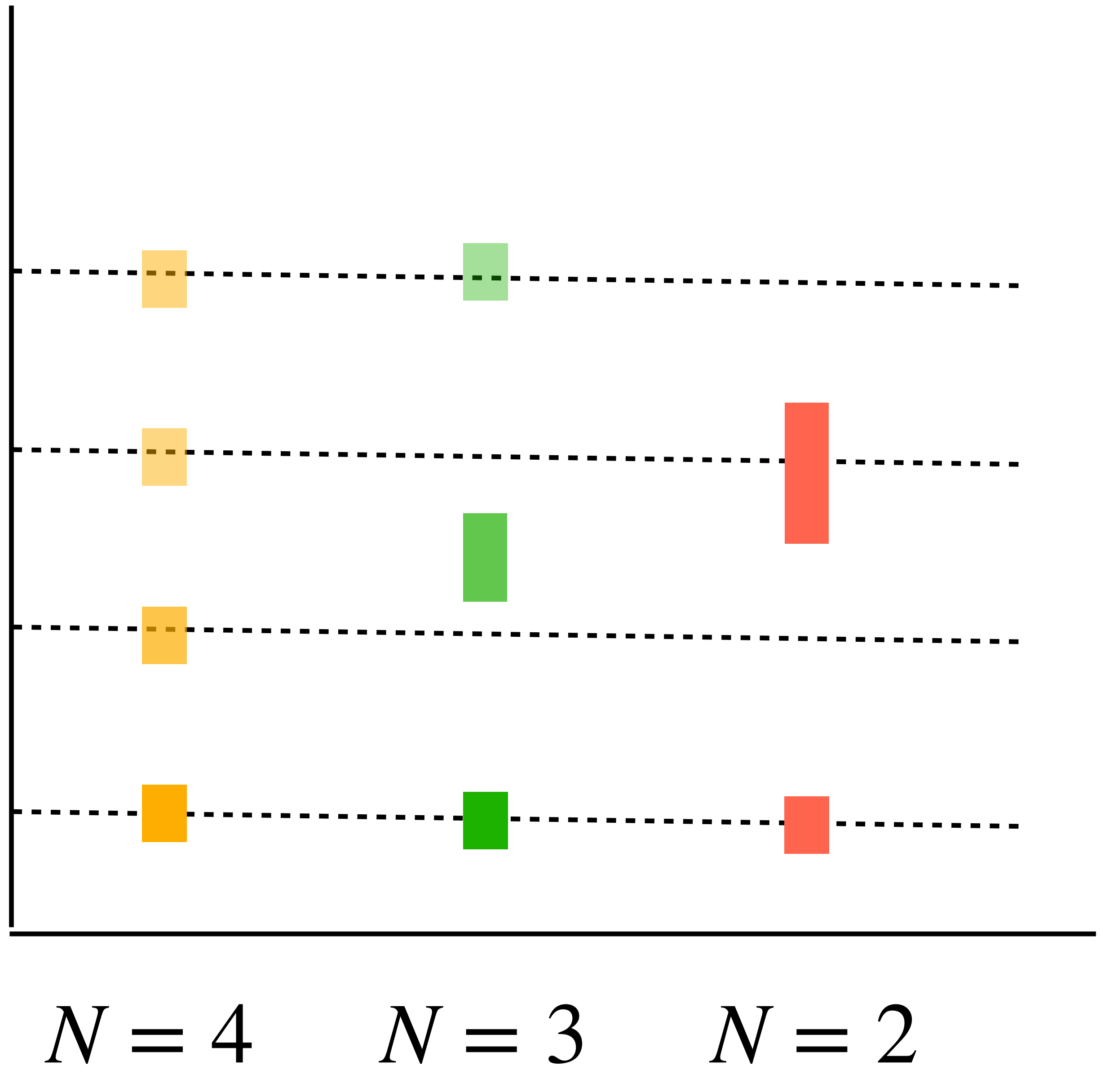

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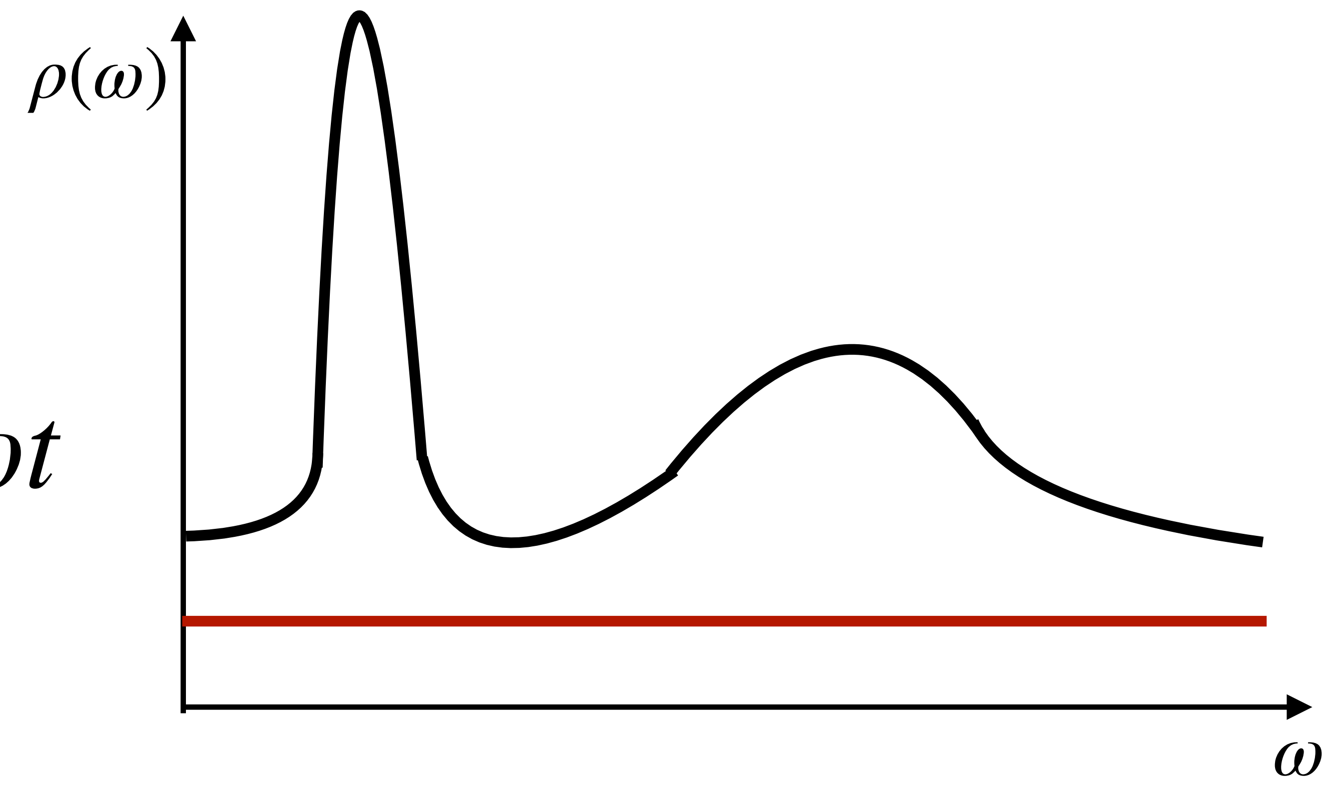

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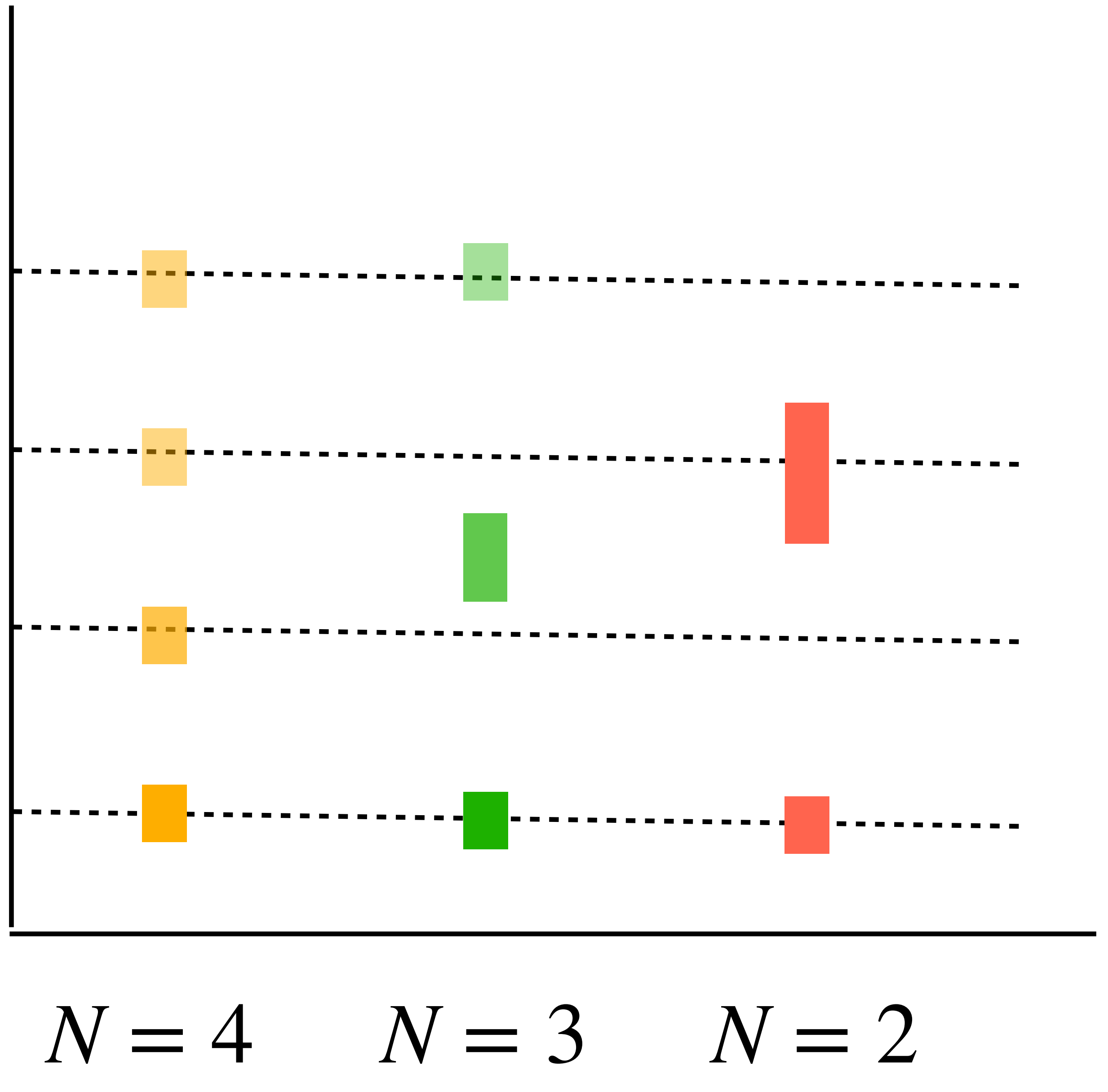

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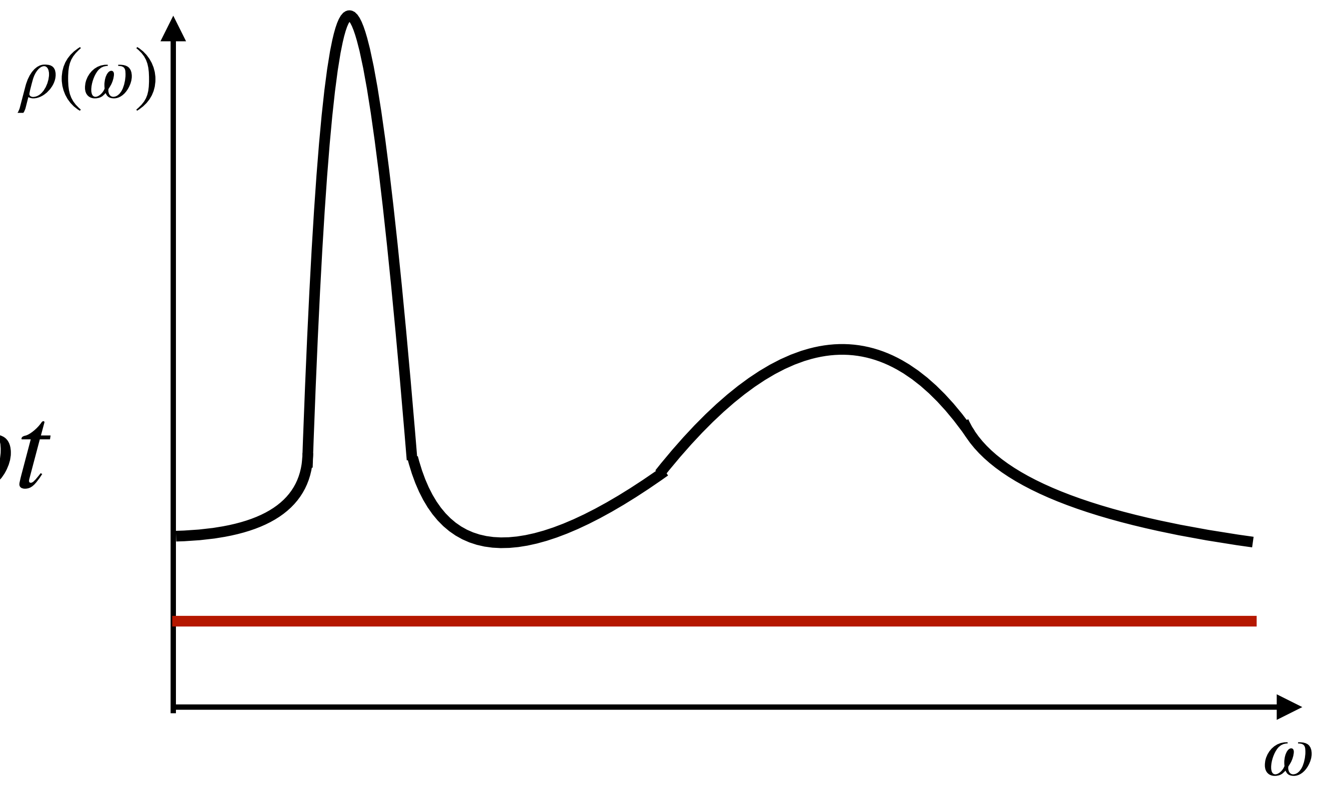

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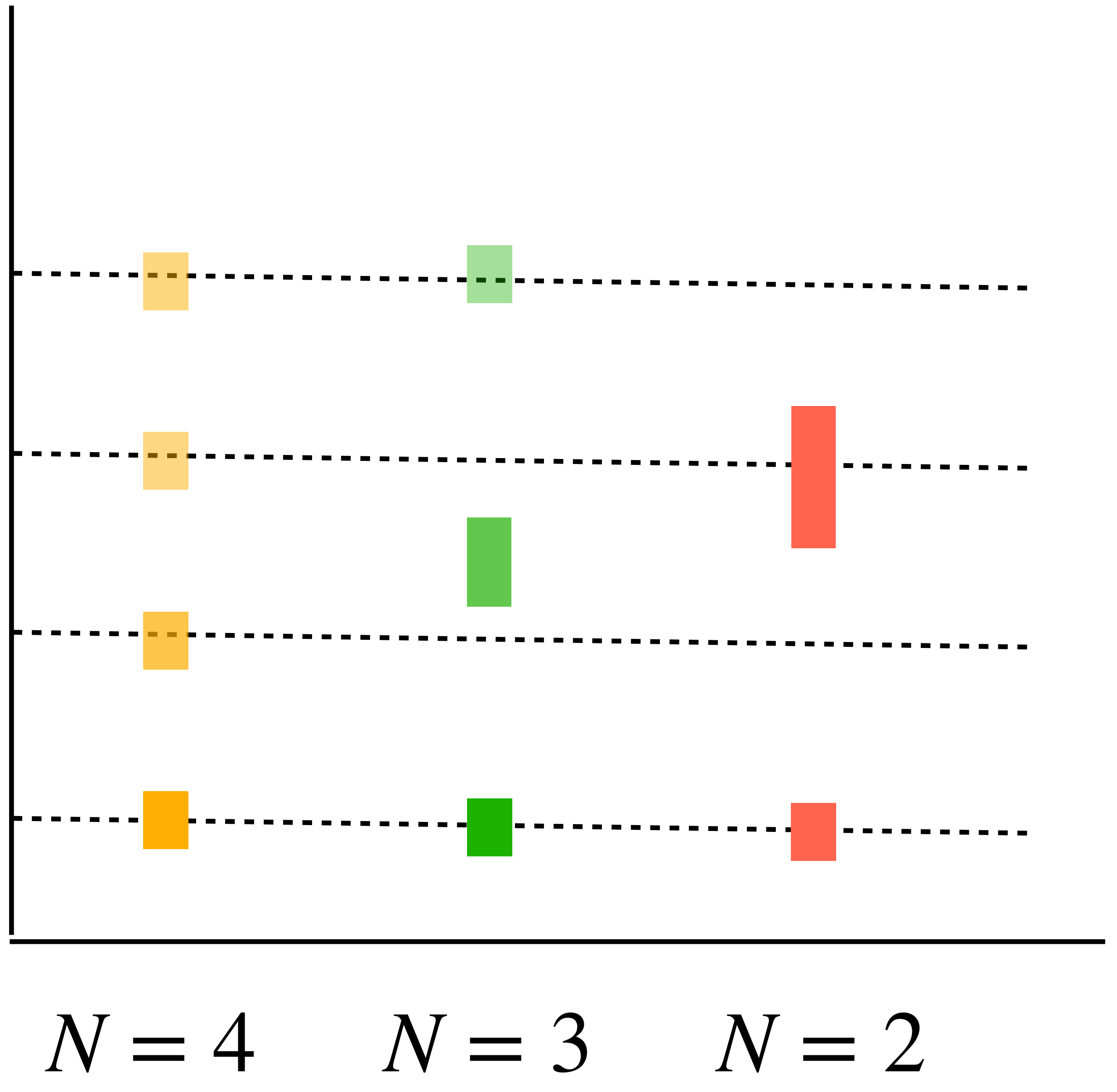

G. P. Lepage et al., arXiv: hep-lat/0110175

$$\chi^2 \equiv \sum_{ij} (D_i - \tilde{D}_i) C_{ij}^{-1} (D_j - \tilde{D}_j)$$

$$+ \quad \chi^2_{\text{prior}} \equiv \sum_n \frac{(W_n - \tilde{W}_n)^2}{\tilde{\sigma}^2_{W_n}} + \sum_n \frac{(E_n - \tilde{E}_n)^2}{\tilde{\sigma}^2_{E_n}}$$

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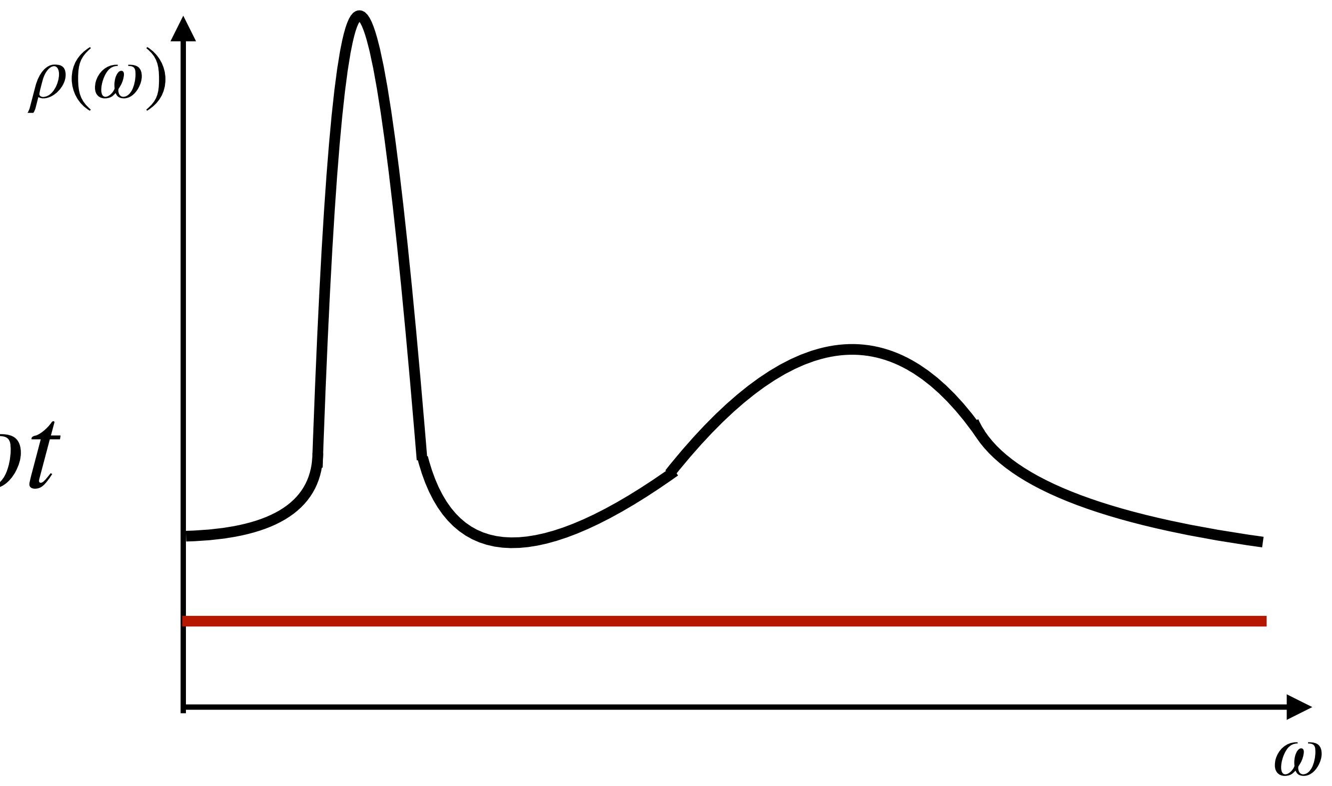

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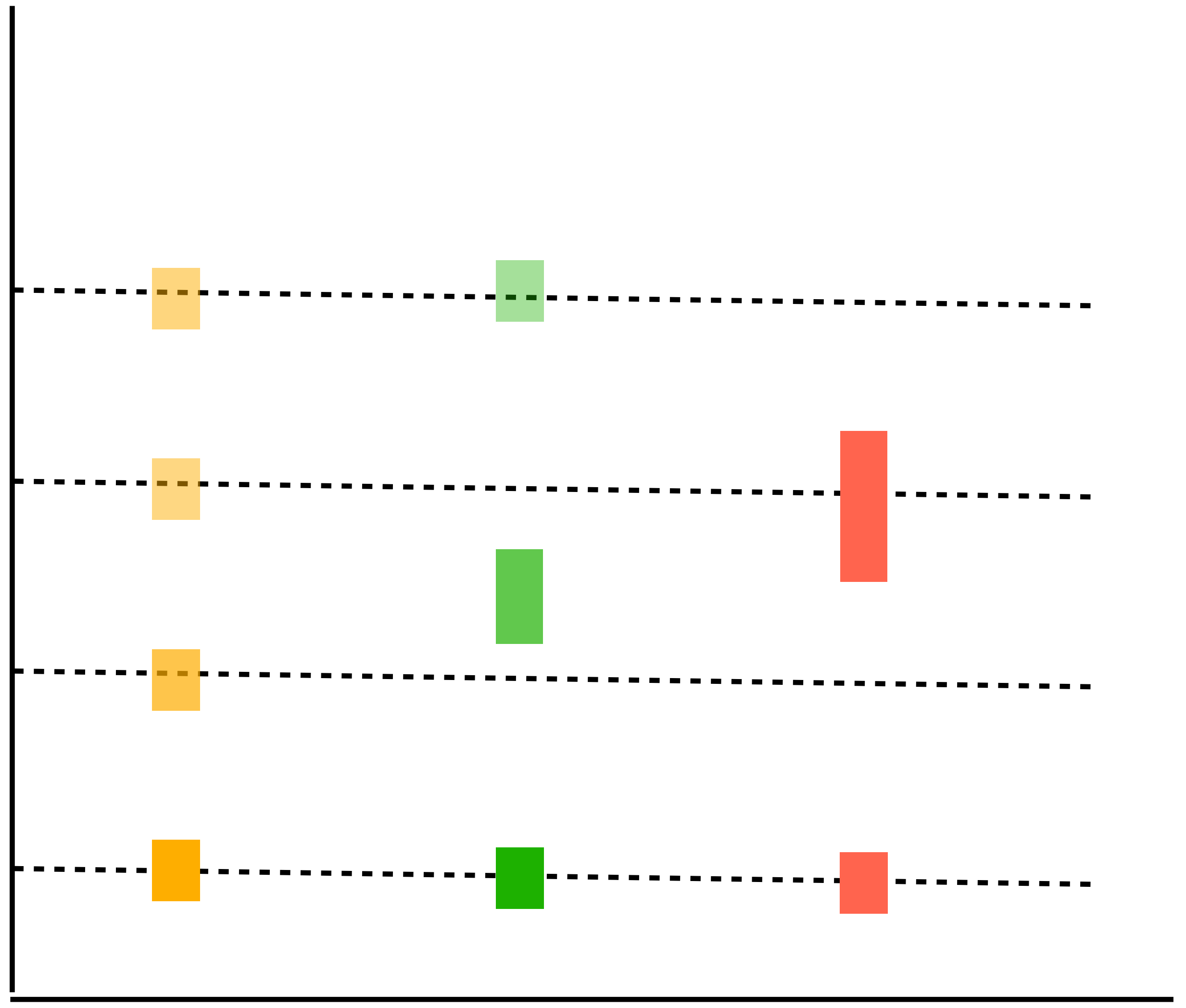

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$$Q = L - S + \ldots$$

- Finite-volume spectrum found in terms of spectral function $\rho(\omega)$

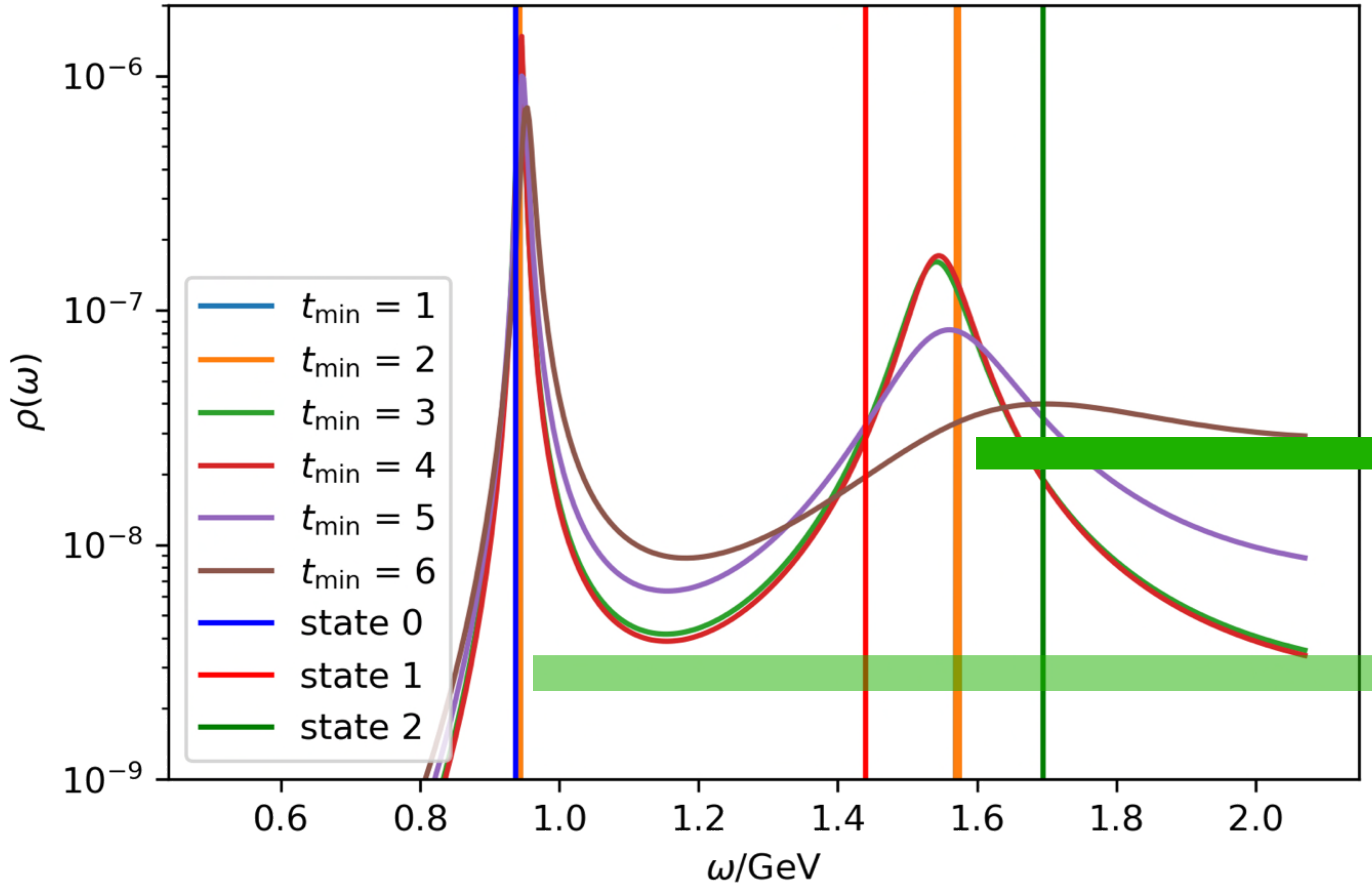
Finite-volume spectrum with BR preliminary

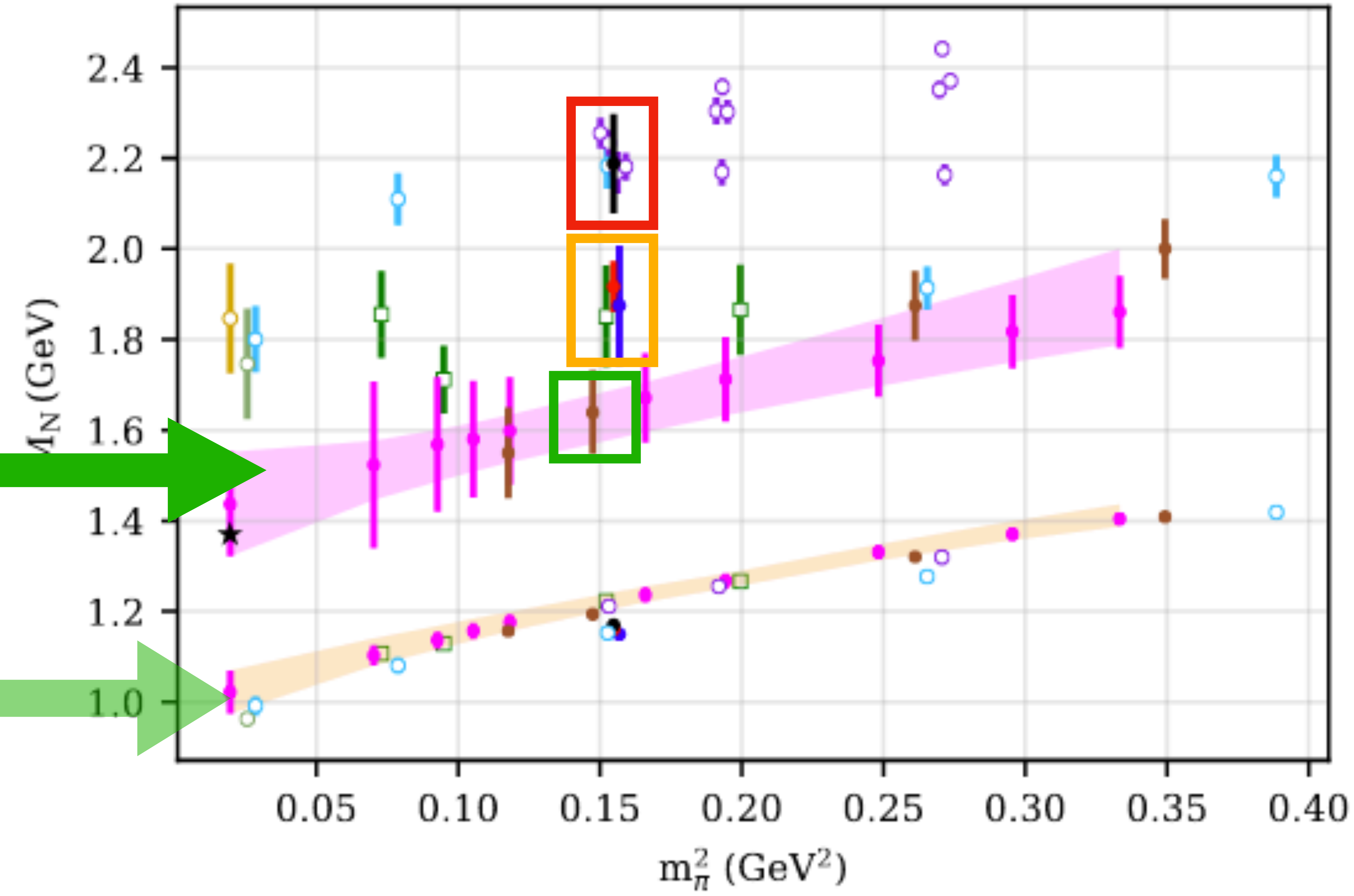
Ens.	Action (F+G)	$1/a$ (GeV)	Lattice volume	m_l	m_s	m_{res}	m_π (MeV)	Size (fm)
				(in lattice units)				
10	MDWF+I	1.730(4)	$48^3\times96\times24$	0.00078	0.0362	0.000614	139	5.5

T. Blum et al., Phys. Rev. D 93, 074505

Default model

$$m(\omega) = const\,.$$





Finite-volume spectrum from lattice QCD

Finite-volume 2pt Correlation Functions

$C_{N,2\mathrm{pt}}(t;\vec{p})$

?

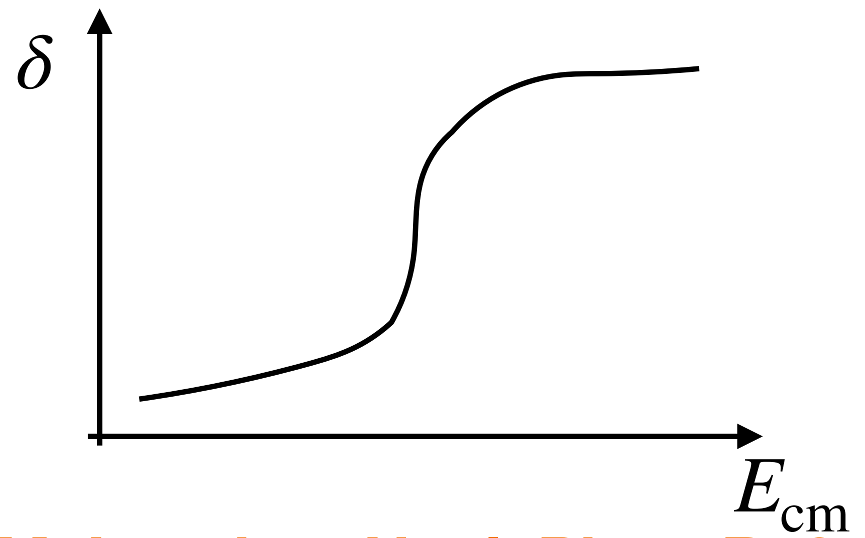
Finite-volume spectrum

E_n

Strongly stable hadrons (bound states):

$\pi, K, D, \ldots, n, p, \ldots$

Find E_0 for $t \gg 0$



M. Luscher, Nucl. Phys. B, 354, (1991)

Near-threshold bound states & Resonances

Luscher formalism

$E_n \leftrightarrow T(E_{\mathrm{cm}})$

!

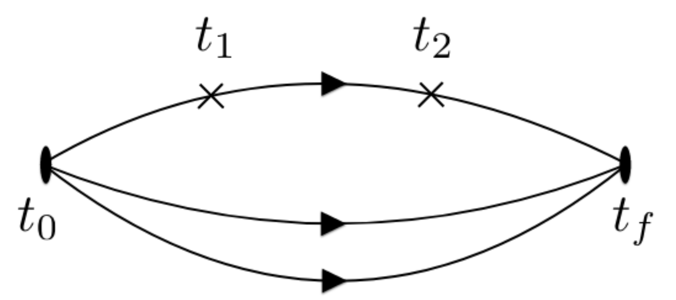
Estimate spectrum in the infinite volume with E_n

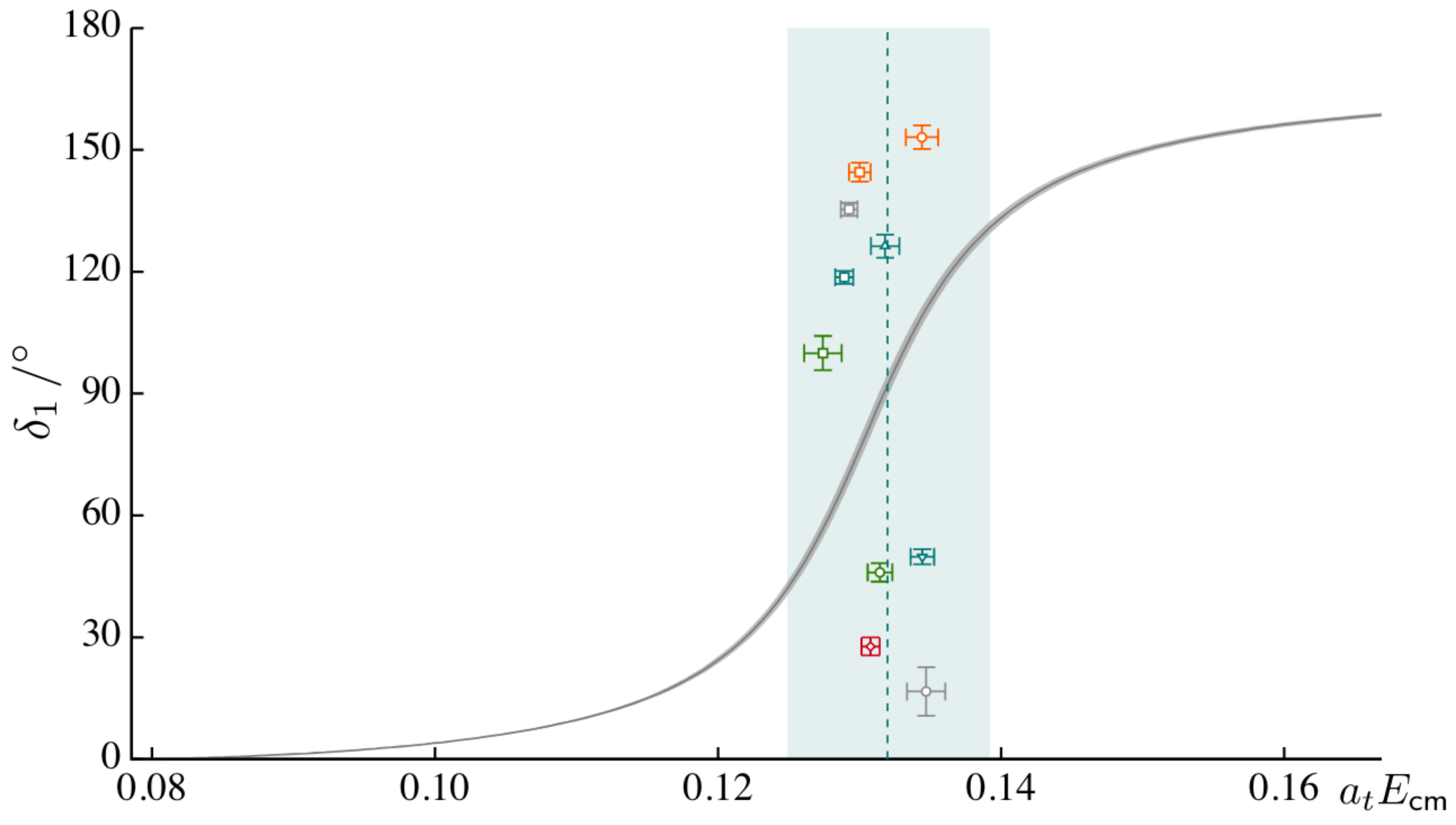
Scattering matrix

$T(E_{\mathrm{cm}})$

$$E^{\mathrm{p}}_{\mathrm{cm}} = m_{\mathrm{R}} + \frac{1}{2} i\Gamma$$

Constrain 3pt/4pt correlation functions to extract form factors, etc.





Wilson et al., *Phys. Rev. D*, vol. 92, no. 9, p. 094502, Nov. 2015

Finite-volume spectrum from lattice QCD with BR

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Near-threshold bound states & Resonances

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Estimate spectrum in the infinite volume with E_n

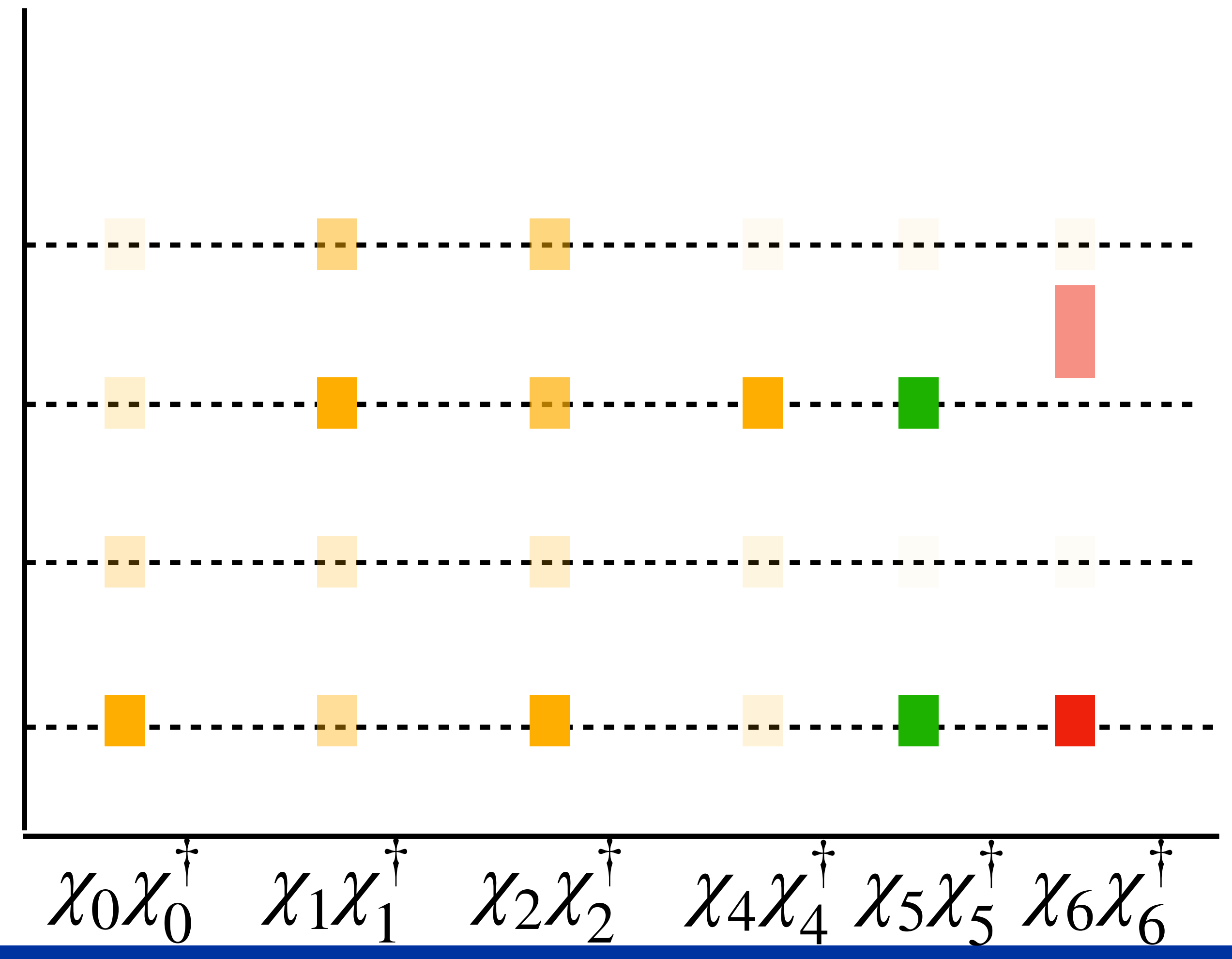
M. T. Hansen et al., Phys. Rev. D 96, 094513 (2017)

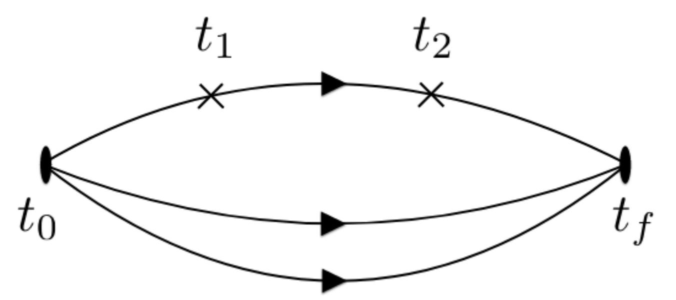
Smeared spectral functions from BR

Talk by Liang

Talk by Liu

Constrain 3pt/4pt correlation functions to extract form factors, etc.



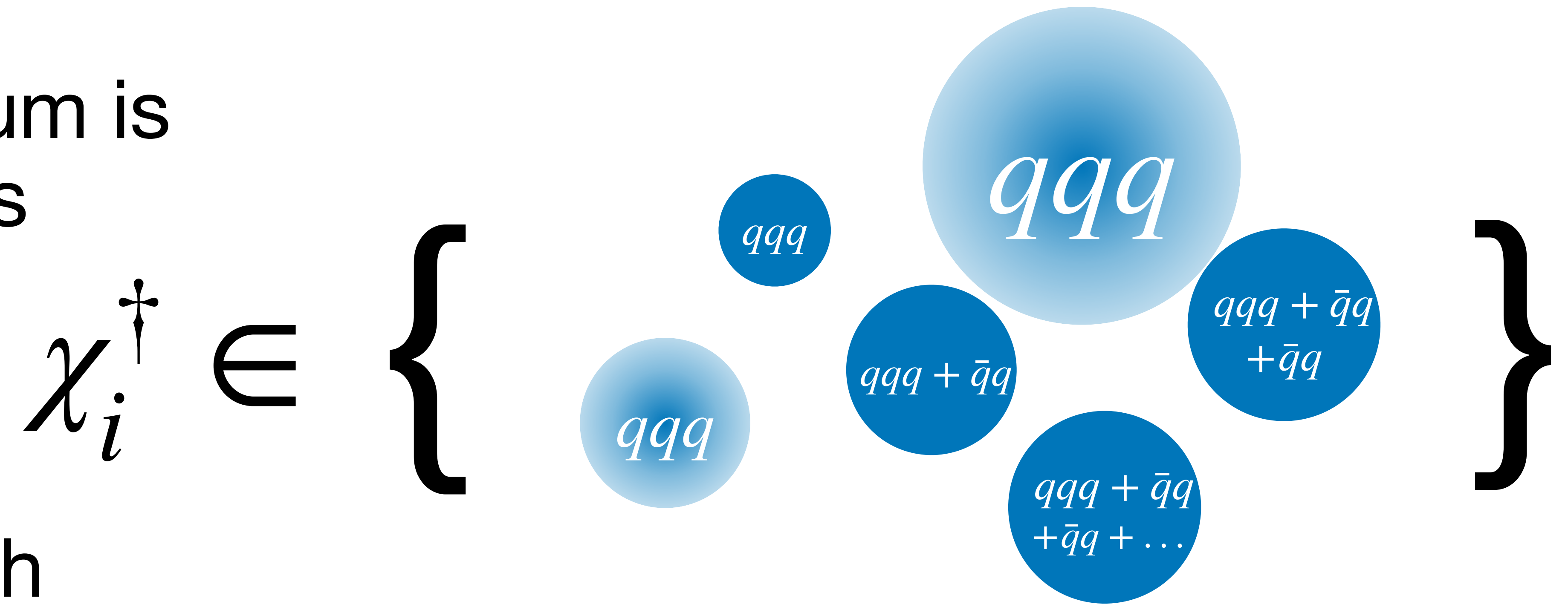


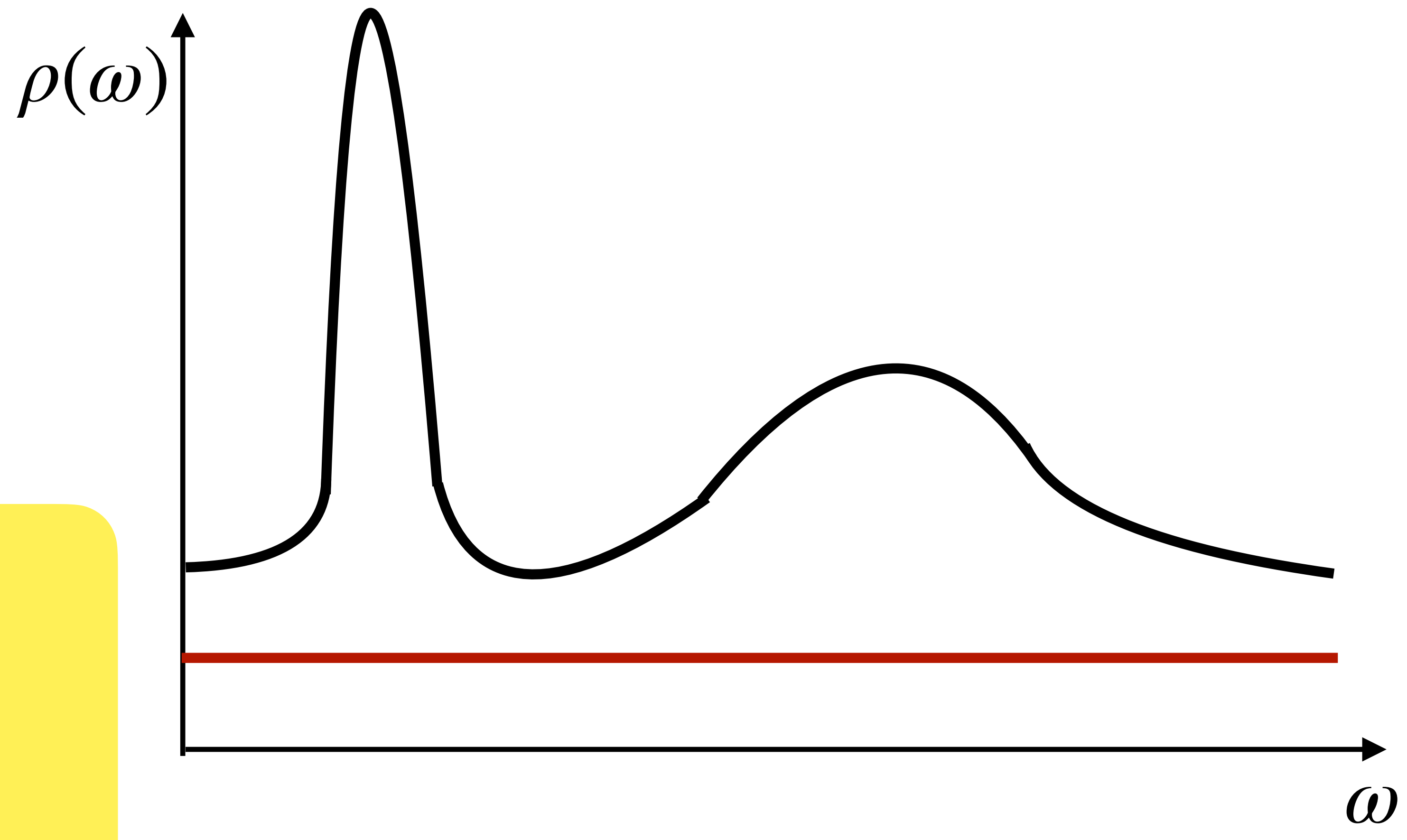
Conclusion and outlook

- Extraction of finite-volume spectrum is important in lattice QCD for various physics topics.

- We don't **always** have to use **many interpolating operators** with **high computational cost.**
  - Theories can help us **choose the basis of interpolating operators wisely/ efficiently.**

- **The Bayesian Reconstruction (BR) method can be used** to extract the finite-volume spectrum $\rho(\omega)$.
  - More on-going tests of BR
  - To be continued …





M. T. Hansen et al., Phys. Rev. D 96, 094513

Strongly stable hadrons (bound states): $\pi, K, D, \dots, n, p, \dots$ Find E_0 for $t \gg 0$

Near-threshold bound states & Resonances → Smeared spectral functions from BR

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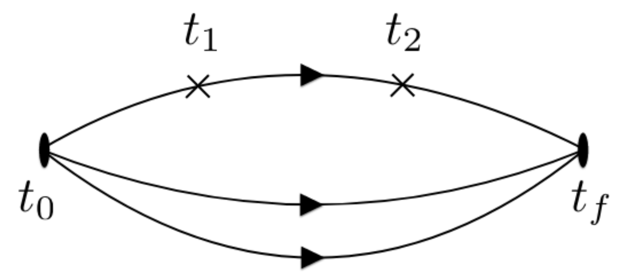

Talk by Liu

Thanks for your attention!