Los Alamos NATIONAL LABORATORY

Proton decay matrix elements on the lattice

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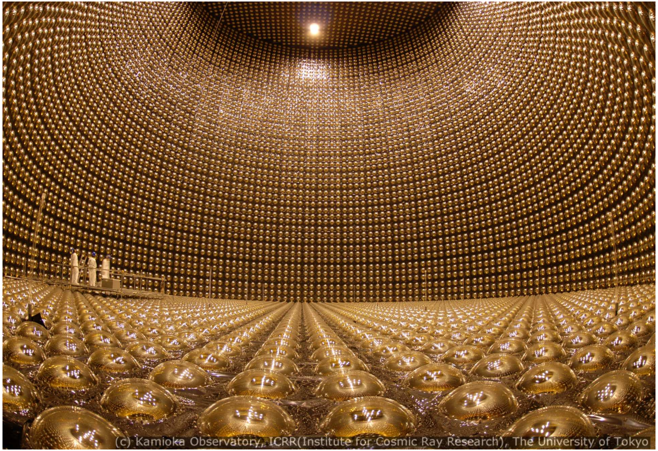
Motivation

- Proton lifetime tests baryon number conservation
- Probes scales inaccessible to colliders: Limits on GUT, extra dim., etc
- Limits on stability of nuclear matter



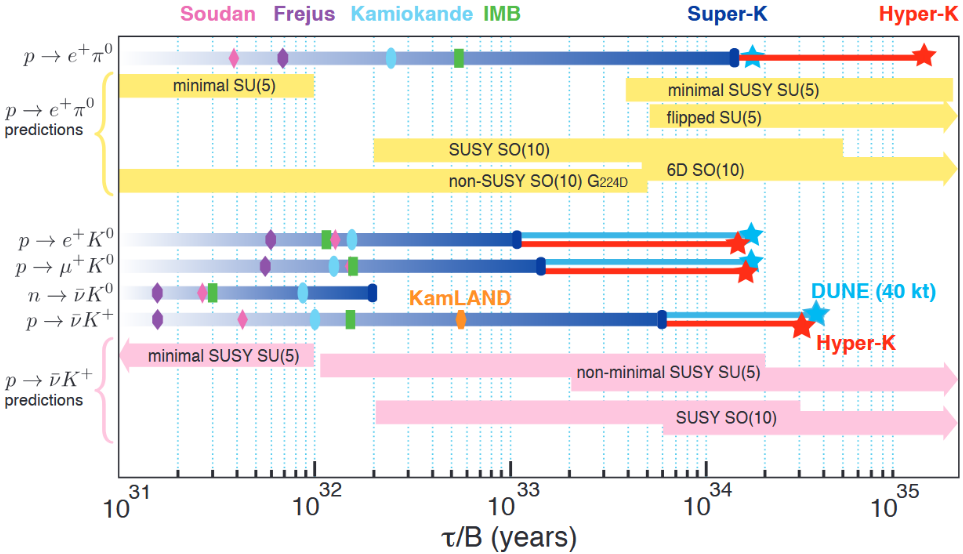
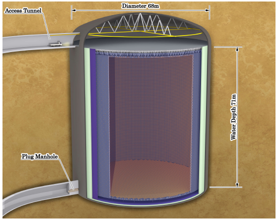
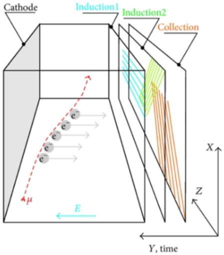


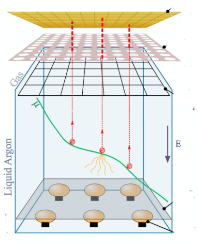
Figure: Proton lifetime bound

Experimental sensitivity to proton lifetime

- Hyper Kamiokande
  (Water Cherenkov detector)
  $p \rightarrow e^+\pi^0$: 7.8×10^{34}years
  $p \rightarrow \bar{\nu}K^+$: 3.2×10^{34} years
  Hyper K design report 2018
- DUNE
  (LArTPC)
  $p \rightarrow \bar{\nu}K^+$: 1.3×10^{34} years
  Assuming 30% eff [DUNE TDR]
- JUNO
  (Liquid Scintillator)
  $p \rightarrow \bar{\nu}K^+$: 9.6×10^{33} years
  Earlier talk by C. Jiang







BNV operators at low scale

- (SUSY-)GUT provides the effective operator of lowest dimension 6.

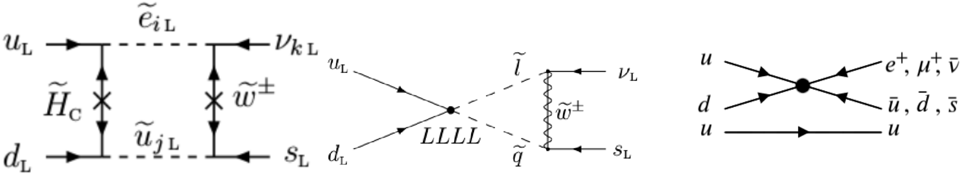


Figure: Proton decay operator at different scales (a) $\sim \Lambda_{GUT}$ (b) $\sim \Lambda_{SUSY}$ (c) $\sim \Lambda_{EW}$

Model parameters come into Wilson coefficients

- $Y_{qq}, Y_{ql}, Y_{ud}, Y_{ue}$
- M_{H_C}
- $m_{\tilde{l}}$, $m_{\tilde{q}}$, triangle loop integrals, ...

whereas the model-independent content remains in the effective operator.
Each model provides a different list of operators and lifetime estimates.

Hadronic matrix element of BNV op

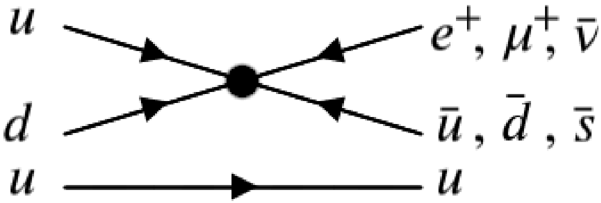
Dim=6 effective operator

$$\langle \Pi \bar{\ell} | p \rangle_{GUT} \sim C^{\Gamma\Gamma'} \langle \Pi \bar{\ell} | p \rangle_{SM} = C^{\Gamma\Gamma'} \bar{v}_\ell \langle \Pi | \mathcal{O}_{qqq} | p \rangle \tag{1}$$

where $C^{\Gamma\Gamma'}$ is a wilson coefficient, Π is a pseudoscalar meson, p is a proton, and $(XY)_\Gamma = (XP_\Gamma Y)$. Our calculations include:

$$\mathcal{O}_I = \epsilon^{abc} (\bar{q}_1^{aC} P_{\chi^I} q_2^b)(\bar{\ell}^C P_{\chi'_I} q_3^c) \tag{2}$$

- $\langle \pi^0 | (\bar{u}^C d)_\chi u_L | p \rangle$
- $\langle \pi^+ | (\bar{u}^C d)_\chi d_L | p \rangle$
- $\langle K^0 | (\bar{u}^C s)_\chi u_L | p \rangle$
- $\langle K^+ | (\bar{u}^C s)_\chi d_L | p \rangle$
- $\langle K^+ | (\bar{u}^C d)_\chi s_L | p \rangle$
- $\langle K^+ | (\bar{d}^C s)_\chi u_L | p \rangle$



Decay rate

The decay rate $\Gamma(N \to \Pi + \bar{\ell})$ is calculated from the hadronic matrix element,

$$\langle \Pi(p')\bar{\ell}(q)|O^{\Gamma\Gamma'}|N(p,s)\rangle \tag{3}$$

$$= \bar{v}_\ell(q)P_{\Gamma'}\left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{M_N}W_1^{\Gamma\Gamma'}(q^2)\right]u_N(p,s) \tag{4}$$

$$= \bar{v}_\ell(q)P_{\Gamma'}W_0^{\Gamma\Gamma'}(q^2)u_N(p,s) + O(m_\ell/M_N)\bar{v}_\ell(q)u_N(p,s) \tag{5}$$

, where Π is a PS meson, N a nucleon, and $W_{0,1}^{\Gamma\Gamma'}$ decay form factor [S.Aoki et al, PRD62:014506 (200)]. Then the decay rate is

$$\Gamma(N \to \Pi + \bar{\ell}) = \frac{(M_N - M_\Pi)^2}{32\pi M_N^3}\left|\sum_I C_I W_0^I(N \to \Pi + \bar{\ell})\right| \tag{6}$$

Lattice QCD in a nutshell

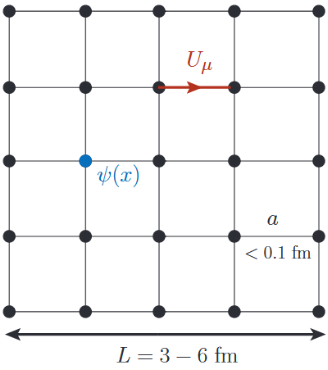
Vacuum expectation of an observable on lattice is given by

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \int \mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\mathcal{O}[\bar{\psi}, \psi, U] e^{-S_E[U,\bar{\psi},\psi]} \tag{7}$$

- 4D Spacetime Lattice with size $L^3 \times T$, lattice spacing a between the nearest point
- $\psi(x)$ fermion field on lattice site x
- $U_\mu(x)$ Gauge field on link between lattice site
- Statistical evaluation using Markov Chain Monte Carlo method

$$\overline{\mathcal{O}} = \sum_{i=1}^{n} \langle \mathcal{O} \rangle_F \left[U_\mu^{(i)} \right] = \langle \mathcal{O} \rangle + \delta \mathcal{O} \tag{8}$$

- Interpolating operator is used to create hadrons (ex. $\chi_\pi(x) = \bar{\psi}(x)\gamma_5\psi(x)$)



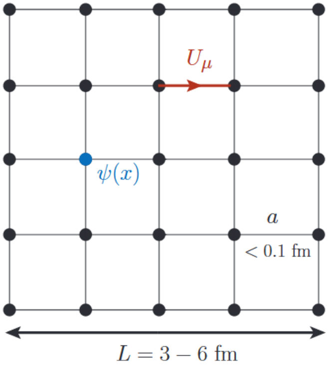
Lattice QCD in a nutshell

Statistical Uncertainties

- Using Monte Carlo method introduces uncertainty $\sim 1/\sqrt{N}$

Systematic Uncertainties

- Finite lattice spacing
- Unphysical quark mass
- Finite Volume
- Excited states contamination
- Renormalization



Lattice QCD in a nutshell

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \sum_{f=1}^{N_f} \bar{\psi}_f(x)\left(i\not{D} - m_f\right)\psi_f(x) \quad , \quad \not{D} = \gamma^\mu\left[\partial_\mu - igA_\mu(x)\right] \tag{9}$$

Vacuum expectation of an observable is given by

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int \mathcal{D}[U] \int \mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\mathcal{O}[\bar{\psi},\psi,U]e^{iS[U,\bar{\psi},\psi]} \tag{10}$$

In EW, $S[U,\bar{\psi},\psi] \sim S_0[U,\bar{\psi},\psi] + \alpha_e S_{\mathrm{int}}[U,\bar{\psi},\psi]$

In QCD, Non-perturbative due to strong coupling

Solution: Rotate to Euclidean 4D spacetime and discretize to solve numerically!

History of Proton decay matrix element calc

Ref.	JLQCD (2000)	CP-PACS & JLQCD (2004)	RBC (2007)	QCDSF (2008)	RBC/UKQCD (2008, 2014)	RBC/UKQCD (2017)	RBC/UKQCD (2022)
fermion	Wilson	Wilson	DW	Wilson	DW	DW	DW
N_f	0	0	0, 2	2	3	3	3
Volume (fm^3)	$(2.4)^2 \times 4.1$	$(3.3)^3$	Q $(1.6)^3$ 2 $(1.9)^3$	$(1.68)^3$	$(2.65)^3$	$(2.65)^3$	$(4.8)^3$ $(4.5)^3$
a (fm)	0.09	0	0.1 0.12	0.07	0.11	0.11	0.2 0.14
M_π (GeV)	0.45-0.73	0.6 - 1.2	0.39-0.58 0.48 - 0.67	0.42-1.8	0.34-0.69	0.34-0.69	0.14 (physical)
Renorm.	One-loop $1/a, \pi/a$	One-loop 2GeV	NPR 2GeV	NPR 2GeV	NPR 2GeV	NPR 2GeV	NPR 2GeV
α (GeV^3)	-0.015(1)	-0.0090	-0.0100(19) -0.0118(21)	-0.0091(4)	-0.0119(26)	-0.0144(15)	-0.01257(111)
β (GeV^3)	0.014(1)	0.0096	0.0108(21) 0.0118(21)	0.0091(4)	0.0128(28)	0.0144(15)	0.01269(107)

Table: Prior Studies on proton decay matrix elements

Lattice Setup

- Iwasaki gauge action +DSDR
- Domain wall fermion ($L_5 = 12$)
- AMA 32 sloppy sample

ensID	latt. size	a^{-1}(GeV)	aM_π	$M_\pi L$	# configs.	N_{sample}
24ID	$24^3 \times 64$	1.0230(20)	0.1378(7)	3.3	140	4480
32ID	$32^3 \times 64$	1.3787(48)	0.1008(5)	3.4	112	3584

Table: Ensembles used for RC to the charged pion β decay

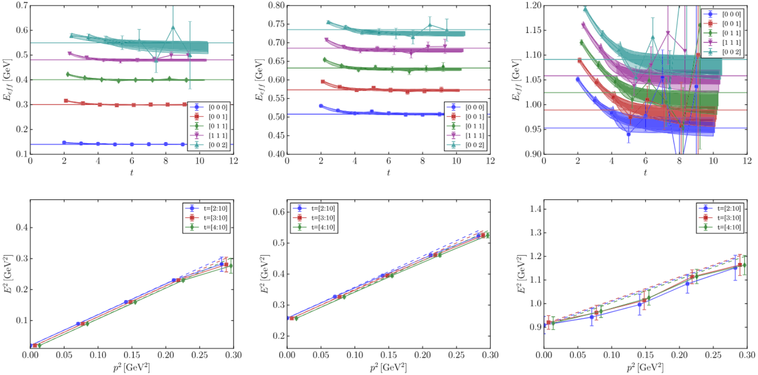
Lattice Calculation

We define the hadron interpolating operators,

$$J_N = \epsilon_{abc}(u^{a,T} C \gamma_5 d^b) u^c \tag{11}$$

$$J_{\pi^+} = \bar{d}^a \gamma_5 u^a \tag{12}$$

$$J_{K^+} = \bar{s}^a \gamma_5 u^a \tag{13}$$



Lattice Calculation

Three kinematic points for each ensemble

Π	$\vec{n}_\Pi$	$\vec{n}_N$	q^2(GeV^2) 24ID, 32ID
π	$[1\ 1\ 1]$	$[0\ 0\ 0]$	$0.010, -0.012$
	$[0\ 1\ 0]$	$[1\ 0\ 0]$	$0.113, 0.095$
	$[0\ 0\ 2]$	$[0\ 0\ 0]$	$-0.116, -0.140$
$\mathcal{K}$	$[0\ 1\ 1]$	$[0\ 0\ 0]$	$-0.034, -0.042$
	$[0\ 1\ 1]$	$[1\ 0\ 0]$	$0.058, 0.056$
	$[0\ 0\ 1]$	$[0\ 0\ 0]$	$0.075, 0.074$

Table: Table displaying values for π and $\mathcal{K}$ with corresponding vectors and q^2 values.

Nonperturbative Renormalization

$\mathcal{O}^{(ud)s}_{\Gamma\Gamma'} = \epsilon^{abc}(\bar{u}^{Ta}\Gamma d^b)\Gamma' s^c_\delta$

- Landau gauge
- RI-SMOM scheme
- Two-loop Matching (GRACEY, 2012)
- symmMOM scheme $p+q+r=0, p^2=q^2=r^2=\mu^2$
  $Z^{3q}_{IK} Proj_J[\langle \bar{p}_1\bar{q}_2\bar{r}_3\mathcal{O}^{3q}_K\rangle] = \delta_{IJ}$

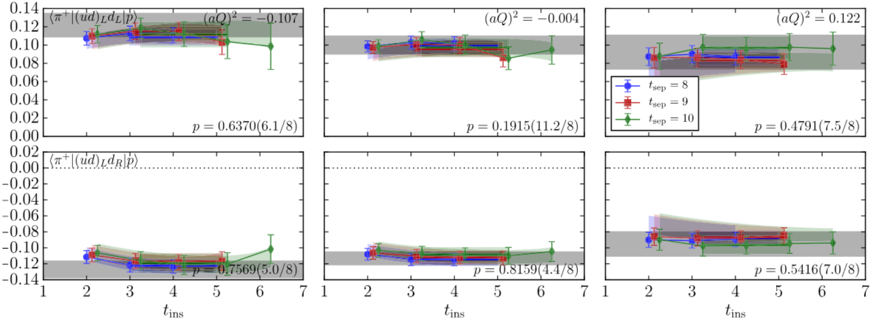
ensID	$m_q^{(1)}$	$N_{cfg}^{(1)}$	$m_q^{(2)}$	$N_{cfg}^{(2)}$	$m_q^{(3)}$	$N_{cfg}^{(3)}$
24ID	0.00107	16	0.04	18	0.085	27
32ID	0.0001	22	0.045	21		

Table: Mass choice for renormalization

	$Z^{\mathrm{(ud)s}}_{++}$	Z^{uds}_{--}	$Z^{\mathrm{(ud)d}}_{++}$	$Z^{\mathrm{(ud)d}}_{--}$
24ID	0.6671(7)(60)(87)	0.6674(7)(51)(87)	0.6671(7)(60)(87)	0.6670(7)(49)(87)
32ID	0.5895(11)(32)(77)	0.5896(9)(29)(77)	0.5893(11)(33)(77)	0.5897(9)(36)(77)

Lattice Calculation

24ID matrix element
2-state fits with energies fixed from spectrum fits



Momentum and Continuum extrapolation

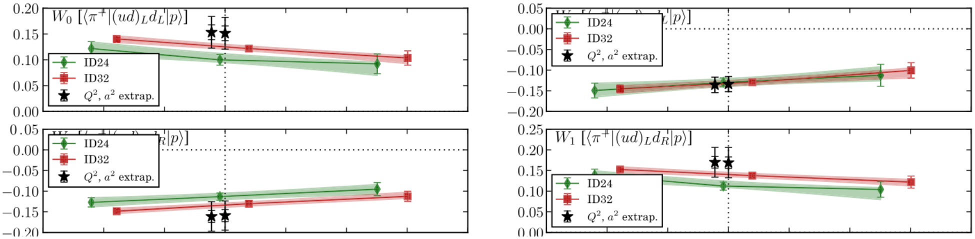
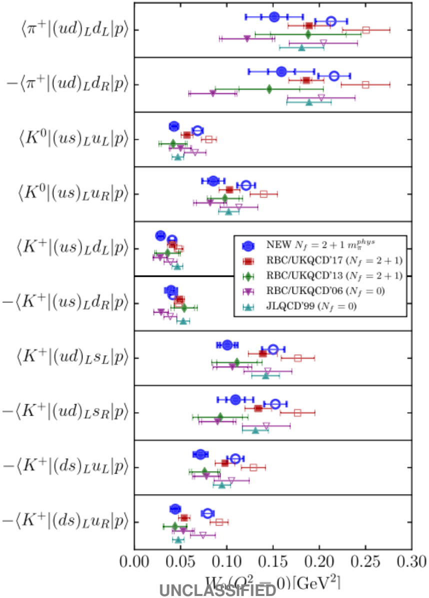


Figure: (Left) W_0 and (Right) W_1 continuum extrapolation and interpolation to kinematic point

Momentum and Continuum extrapolation



Future extensions

Vector channel decay

- No previous calculation
- $n \to K^{*+}\ell^{-} \to K^{+}\pi^{-}\nu_{\ell}$.
- K^{*} state is sharp and below the threshold on some lattices.
- For Parity conserving case, the form factor decomposition should be:

$$\langle K^{*i}\ell|(ds)(d\ell)|p\rangle = \epsilon^{i}_{\mu}(p')\bar{u}_{\ell}(q)[F_1(q^2)\gamma_{\mu} + F_2(q^2)\sigma_{\mu\nu}q^{\nu} + F_3(q^2)q_{\mu}]u_p(p) \tag{14}$$

Example: on Clover E5 lattice

- Wilson Clover lattice ensemble
- lowest threshhold for $\pi\pi$ P-wave state is 894.3(5.8) MeV.
- shows lighter vector meson mass due to discretization effect

ID	a(fm)	$m_\pi(\mathrm{MeV})$	latt. size	$m_\pi L$	# configs.
E5	0.07280(75)	271.8(2.9)	$48^3 \times 128$	4.18	
	aM_N	aM_ρ	aM_{K^*}	aM_π	aM_K	ap_{min}
	0.3736(51)	0.2858(39)	0.3388(17)	0.10037	0.21199	0.131

Table: example of ensemble with stable vector meson state

Summary

- Computing BNV hadronic matrix element was done at the physical point
- two-loop non-perturbative renormalization was done
- Future computation can contain more channels