Nuclear Matter Equation of State from In-Medium Similarity Renormalization Group

INT/N3AS Workshop EOS Measurements with Next-Generation Gravitational-Wave Detectors

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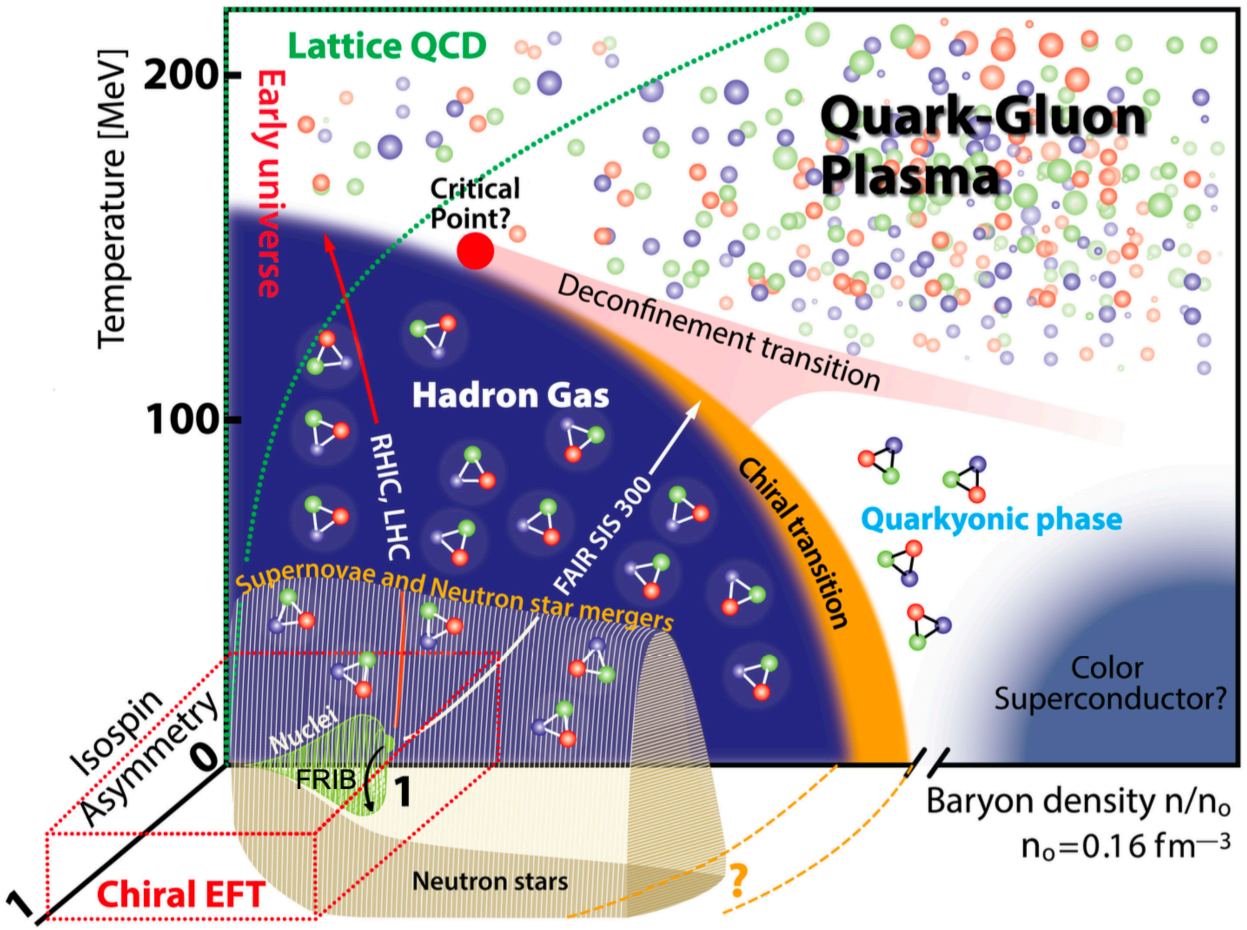
Outline

- Introduction
- IMSRG-EOS Framework
- Preliminary EOS Results
- Summary and Outlook

Introduction - Nuclear Matter and Equation of State

- Nuclear Matter: an idealized system of interacting nucleons in the thermodynamical limit



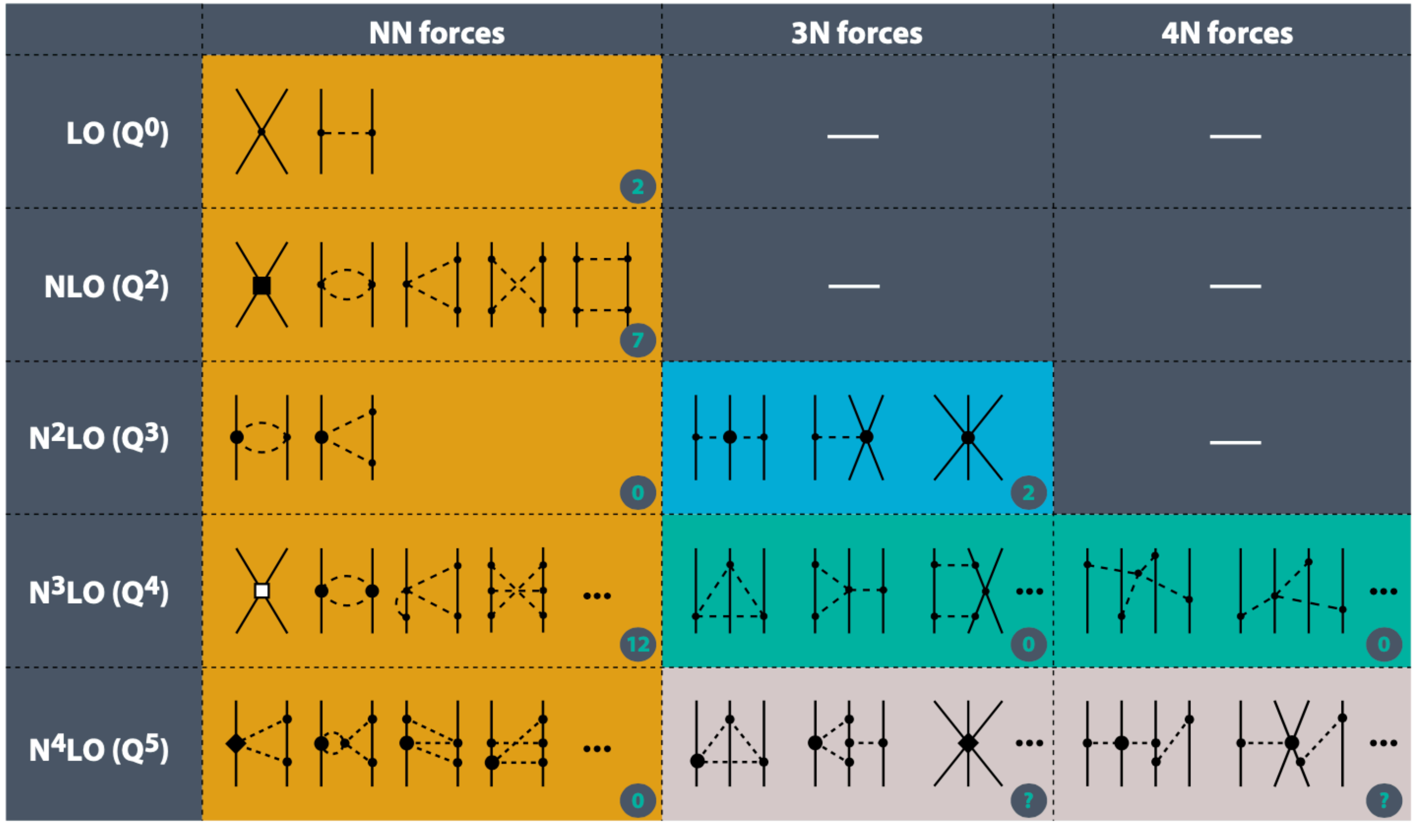
Why is it interesting to us:

- Testing ground for many body methods
- Strongly related to dense astronomical objects like neutron stars, offers a link between nuclear physics and astrophysical observables

C. Drischler et al, *Chiral Effective Field Theory and the High-Density Nuclear Equation of State*

Introduction - Chiral EFT & Ab Initio Nuclear Theory

- Chiral effective field theory:
  - Consistent NN, NNN, … interactions
  - Systematic low-momentum expansion
  - Link with underlying QCD



- Ab initio nuclear theory ≡

systematically improvable approach for quantitatively describing nuclei and nuclear matter using the finest resolution scale possible while maximizing its predictive capabilities

A Ekström et al, *What is ab initio in nuclear theory?*
C. Drischler et al, *Chiral Effective Field Theory and the High-Density Nuclear Equation of State*

Introduction - Similarity Renormalization Group

Basic Idea

continuous unitary transformation of the Hamiltonian to band-diagonal form w.r.t. a given “uncorrelated” many-body basis

Introduction - Similarity Renormalization Group

- flow equation for Hamiltonian $H(s) = U(s)HU^\dagger(s)$:

$$\frac{d}{ds}H(s) = \left[\eta(s), H(s)\right], \quad \eta(s) = \frac{dU(s)}{ds}U^\dagger(s) = -\eta^\dagger(s)$$

- choose $\eta(s)$ to achieve desired behavior, e.g.,

$$\eta(s) = \left[H_d(s), H_{od}(s)\right]$$

to suppress (suitably defined) off-diagonal Hamiltonian

$$\lim_{s\to\infty} H_{od}(s) \longrightarrow 0$$

M Hjorth-Jensen et al, *An Advanced Course in Computational Nuclear Physics: Bridging the Scales from Quarks to Neutron Stars*

Introduction - In-Medium Similarity Renormalization Group

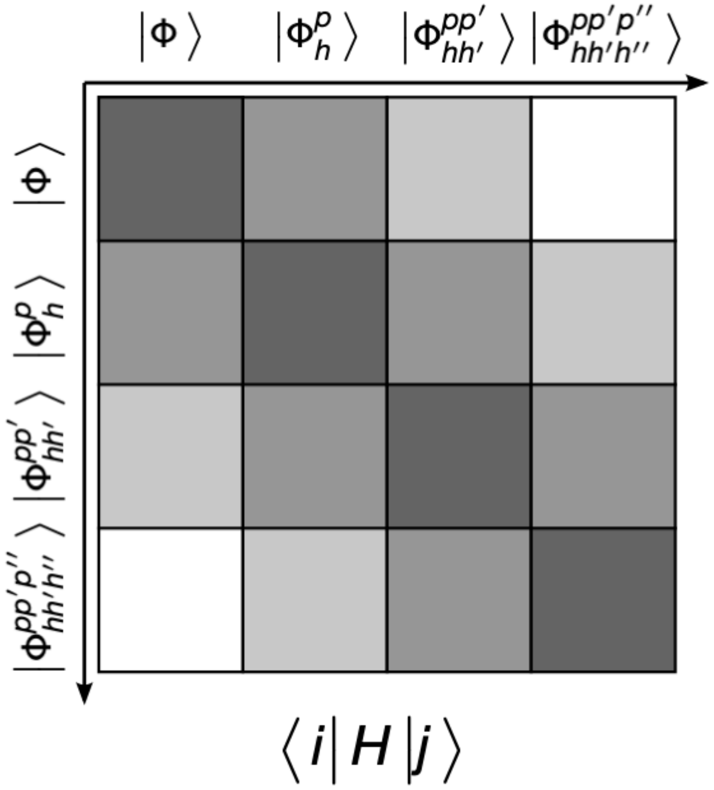
Normal-Ordered Hamiltonian

$$H = E_0 + \sum_{kl} f^k_l :A^k_l: + \frac{1}{4} \sum_{klmn} \Gamma^{kl}_{mn} :A^{kl}_{mn}: + \frac{1}{36} \sum_{ijklmn} W^{ijk}_{lmn} :A^{ijk}_{lmn}:$$

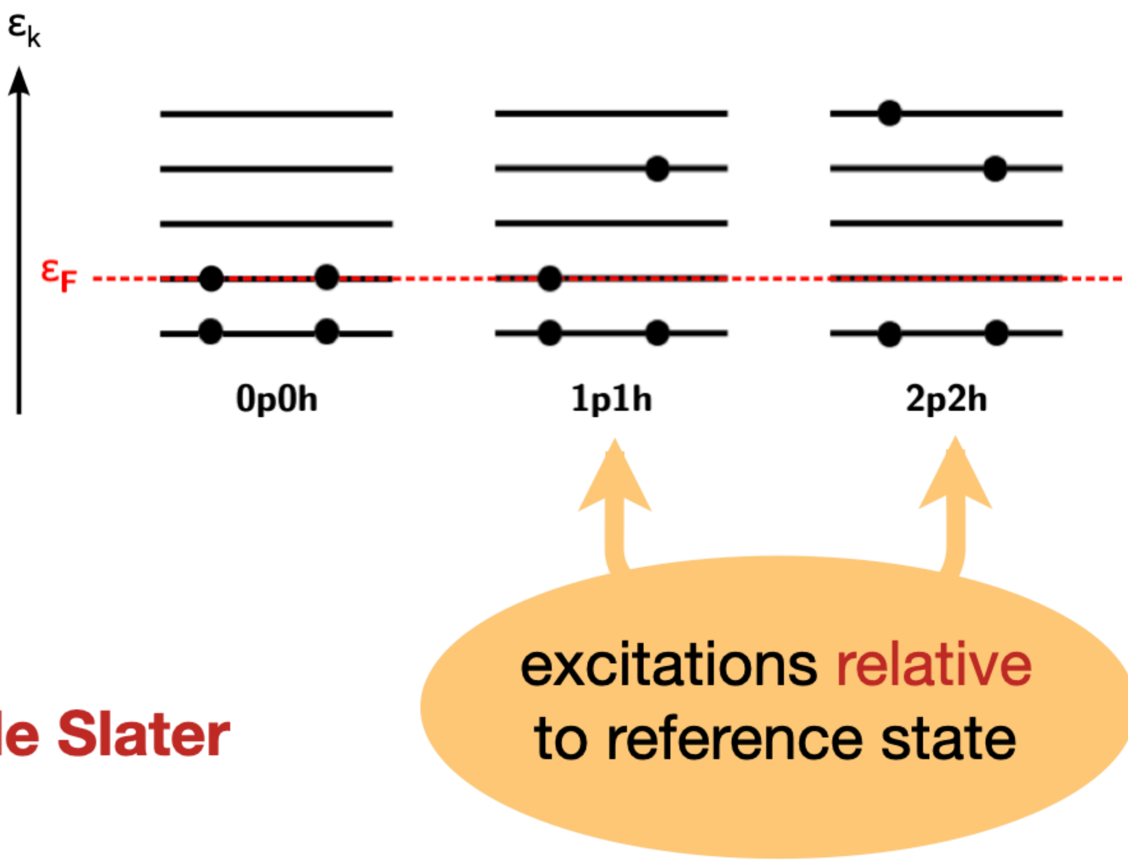
$$A^{i_1 \dots i_N}_{j_1 \dots j_N} \equiv a^\dagger_{i_1} \dots a^\dagger_{i_N} a_{j_N} \dots a_{j_1}$$

$$\langle \Phi | :A: | \Phi \rangle = 0$$

Introduction - In-Medium Similarity Renormalization Group

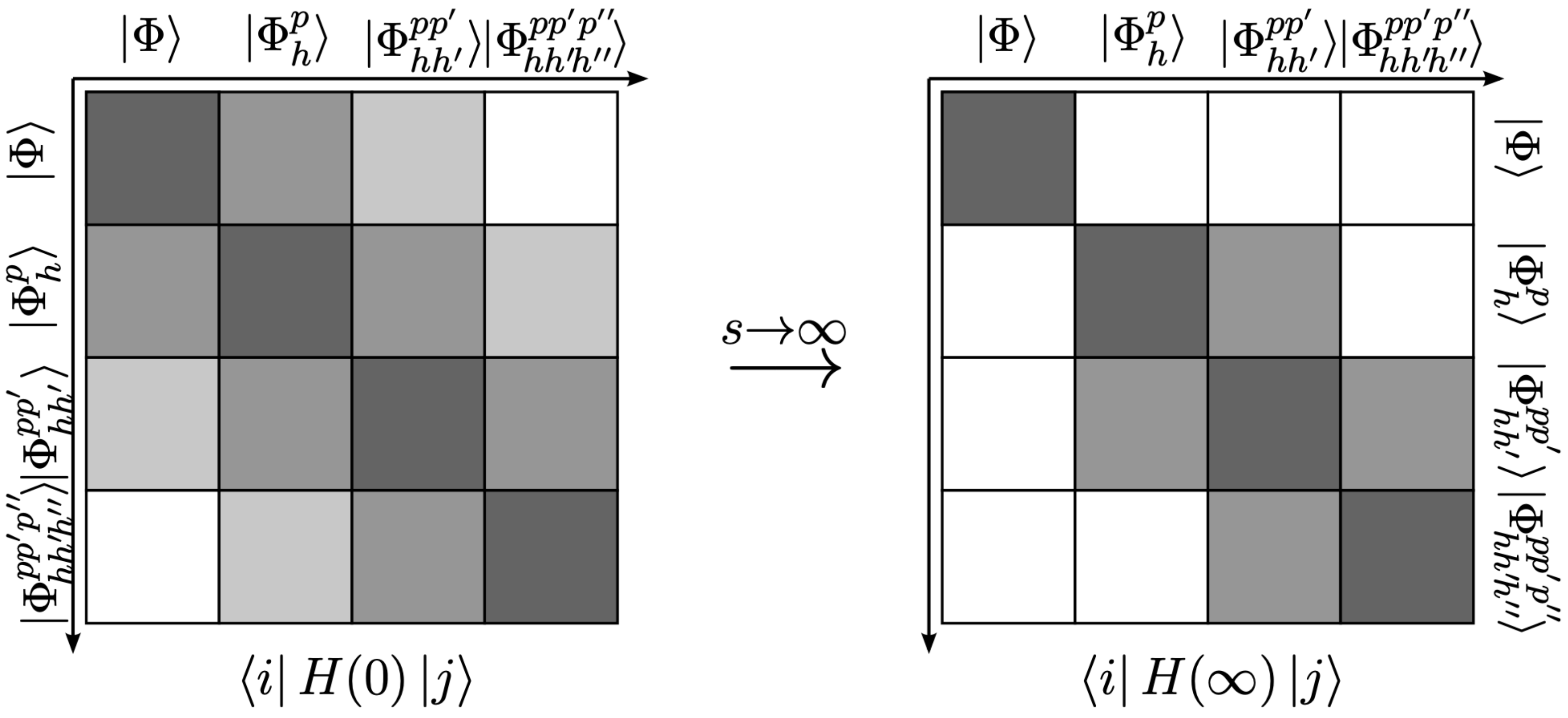


- reference state: **single Slater determinant**



Introduction - In-Medium Similarity Renormalization Group

$$\frac{d}{ds}H(s) = [\eta(s), H(s)], \quad \text{e.g.,} \quad \eta(s) \equiv [H_d(s), H_{od}(s)]$$

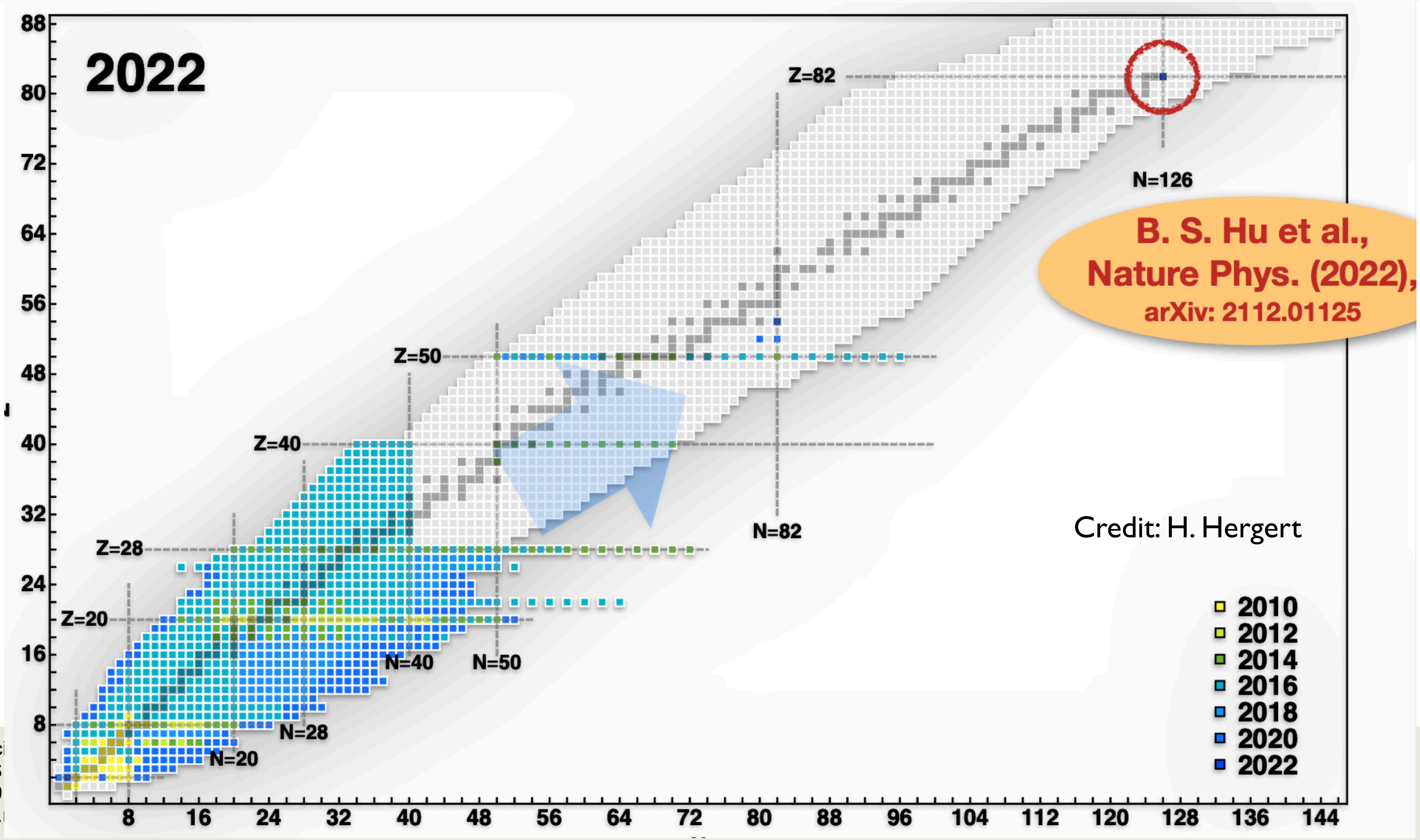


IM-SRG(2): Truncate H(s), η(s) to ***normal ordered*** 2-body terms

two-body formalism with in-medium contributions from NNN interactions

M Hjorth-Jensen et al, *An Advanced Course in Computational Nuclear Physics: Bridging the Scales from Quarks to Neutron Stars*

Introduction - Application of Ab Initio Methods in Nuclear Structure



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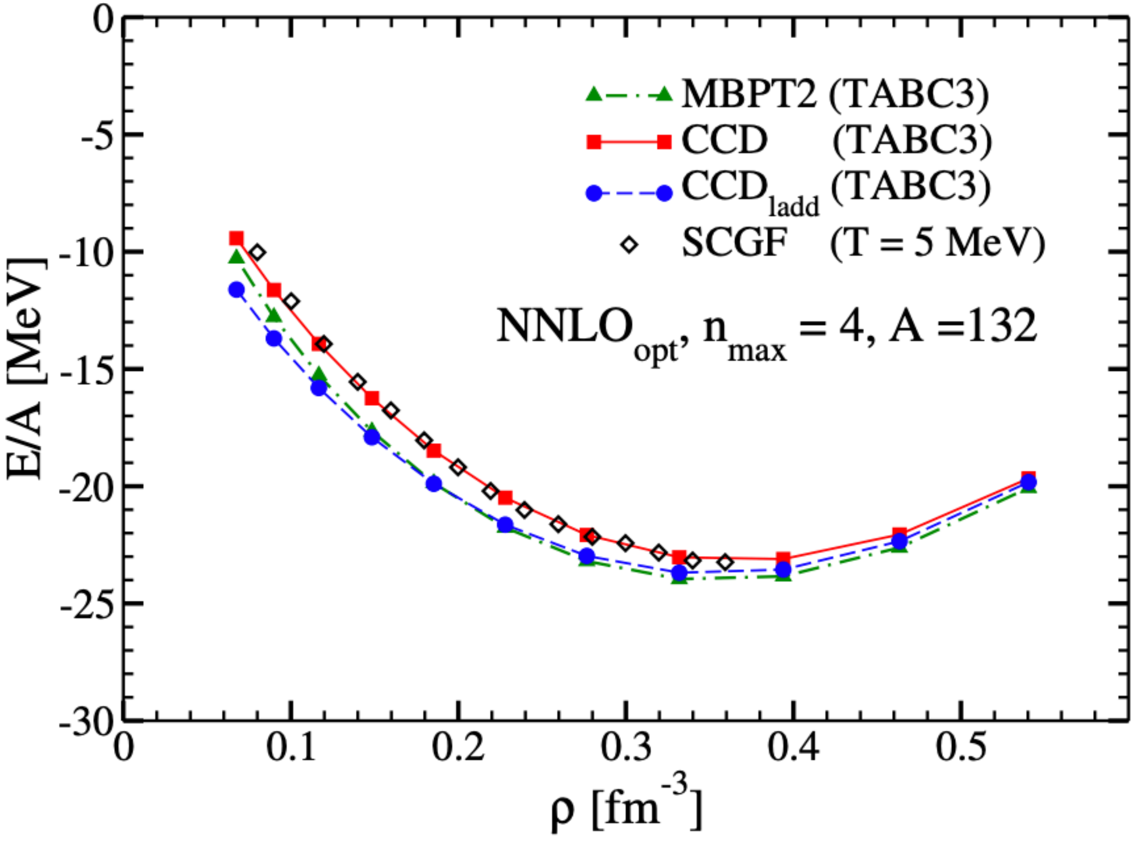
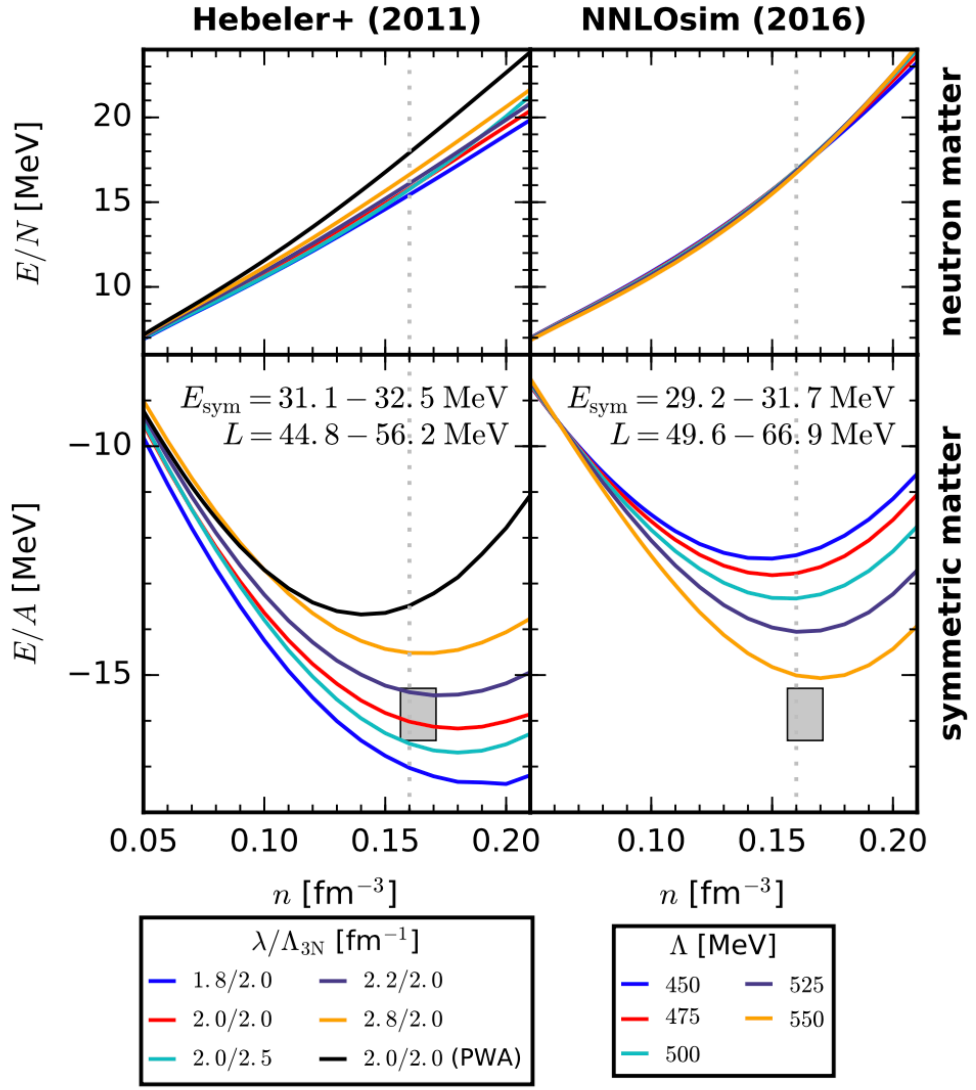
Introduction - Application of Other Ab Initio Methods in NM EOS





G. Hagen et al, *Coupled-cluster calculations of nucleonic matter*
C. Drischler et al, *Chiral Interactions up to Next-to-Next-to-Next-to-Leading Order and Nuclear Saturation*

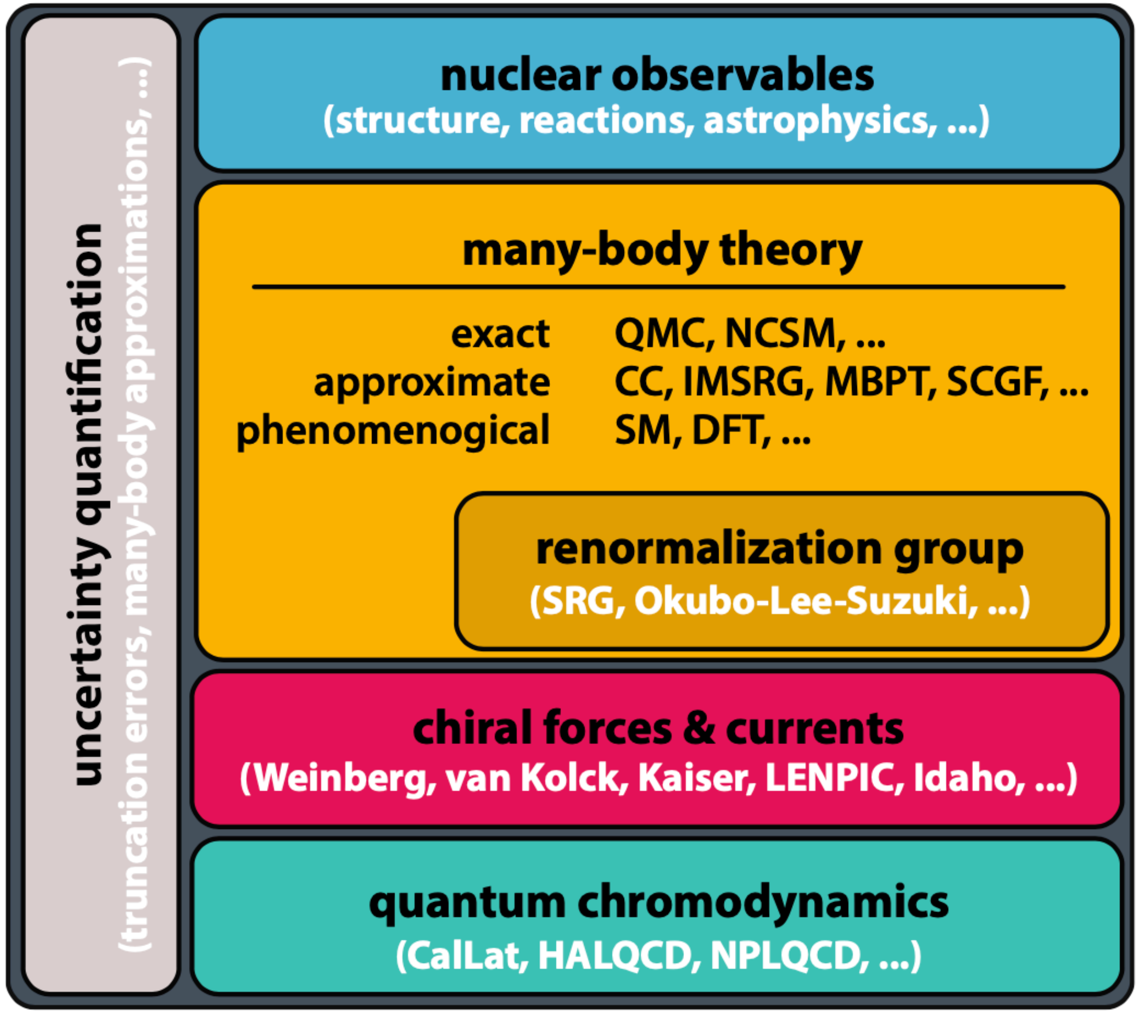
IMSRG-EOS Framework - Overview

- Physical System: Nucleons in a finite box

- Framework:

1. Single particle basis (plane waves w/Periodic boundary condition) -> Many particle basis
2. Input from chiral EFT -> Hamiltonian Matrix Elements
3. IMSRG Evolution of the Hamiltonian, NO2B level

- can be easily generalized to study other infinite systems (with arbitrary dimension) such as 2D electron gas



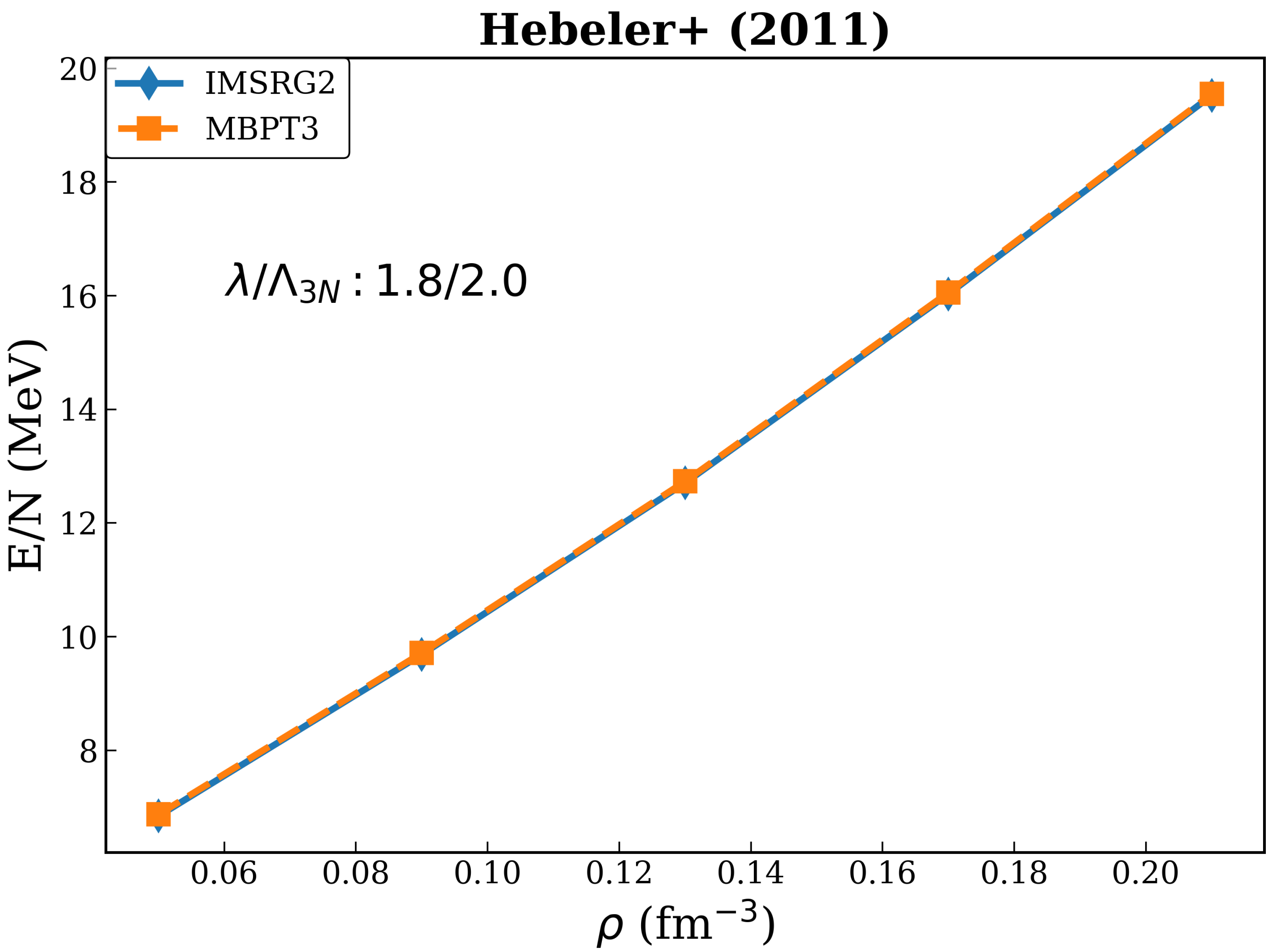
C. Drischler et al, *A Brief Account of Steven Weinberg's Legacy in ab initio Many Body Theory*

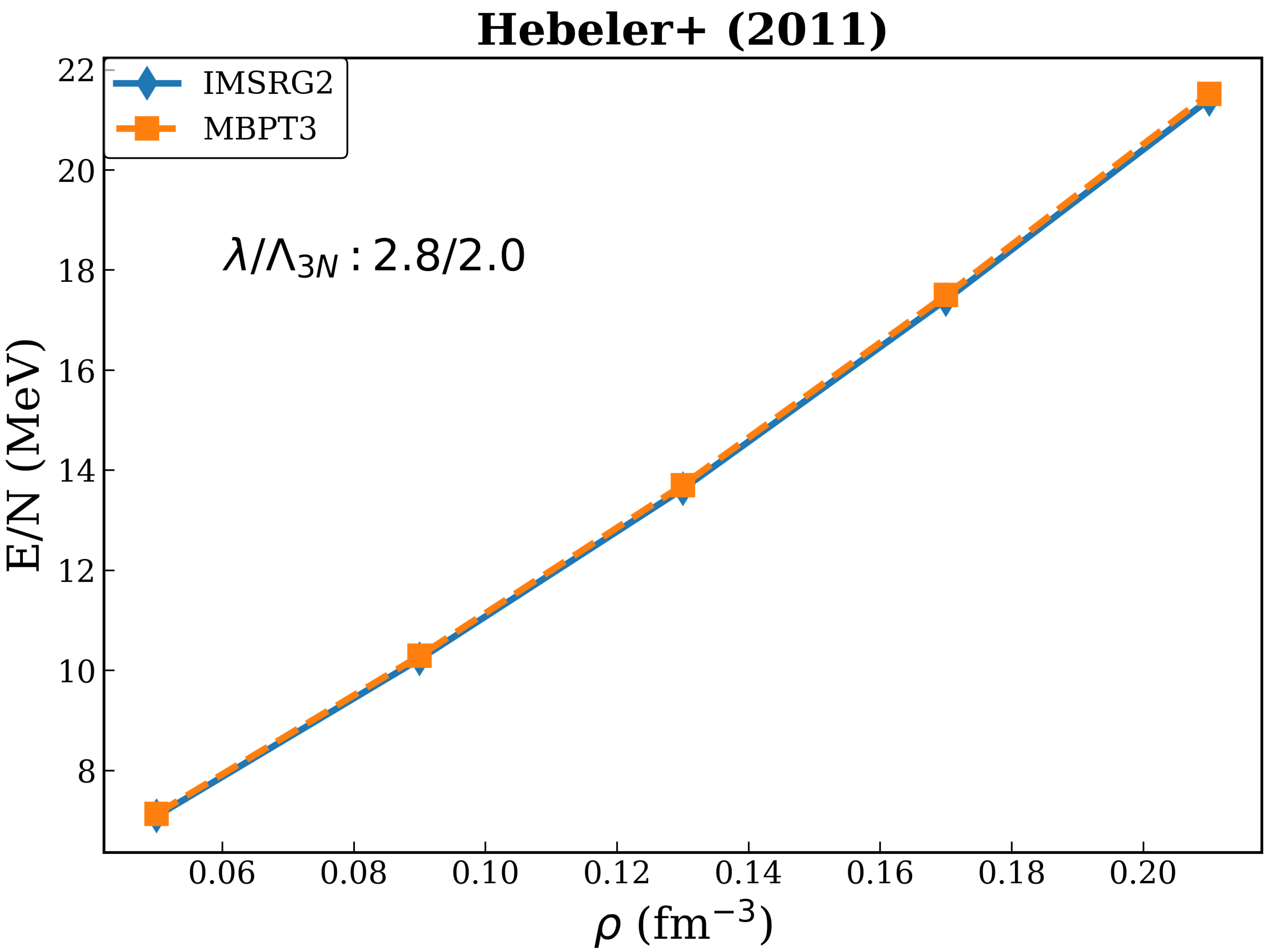
Preliminary Results - Interactions

- Hebeler Interactions:
  - Based on chiral EFT, but not fully consistent NN and 3N interactions
  - Starts from N3LO EM 500 MeV NN potential
  - NN interaction is softened by SRG evolution
  - NNLO 3N interaction adjusted to fit the triton binding energy and He charge radius
  - Denoted by λ/Λ_{3N}, where λ is the SRG flow parameter, Λ_{3N} is the 3N cutoff

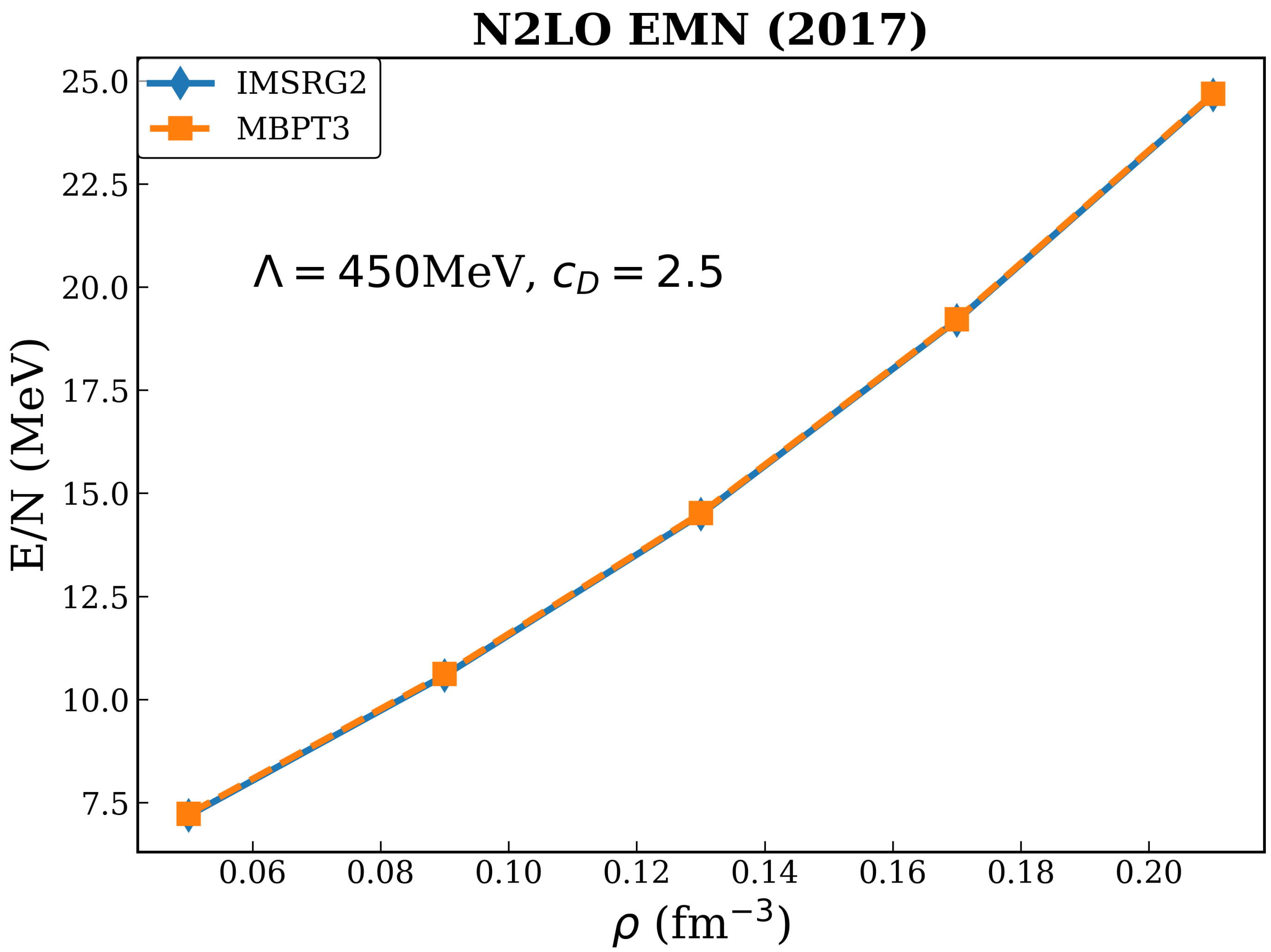
- N2LO EMN Interactions:
  - Not SRG-evolved
  - consistent NN and 3N interaction
  - c_D and c_E are fitted to the ^3H and empirical saturation properties

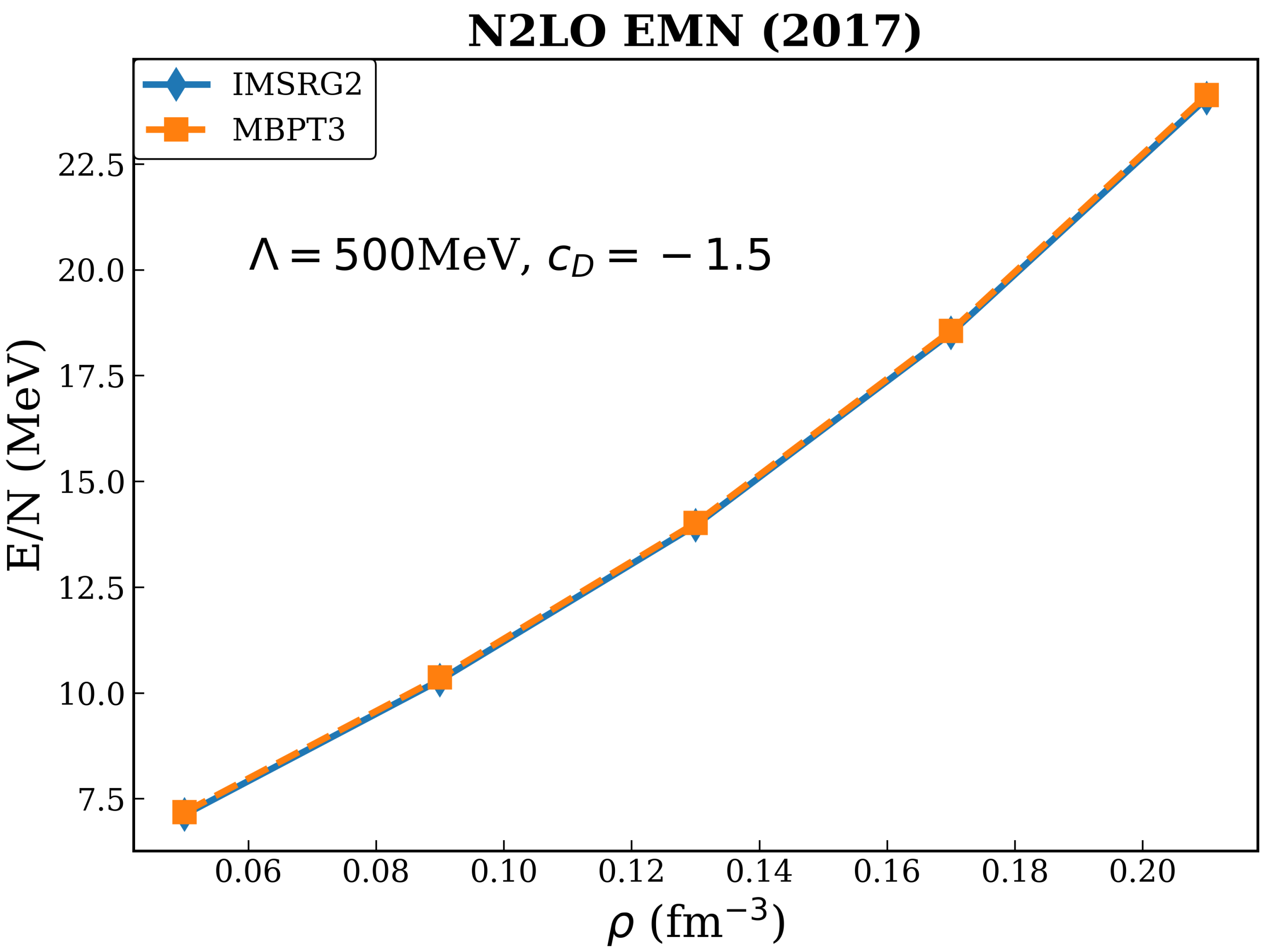
Preliminary Results - PNM EOS with Hebeler Interactions



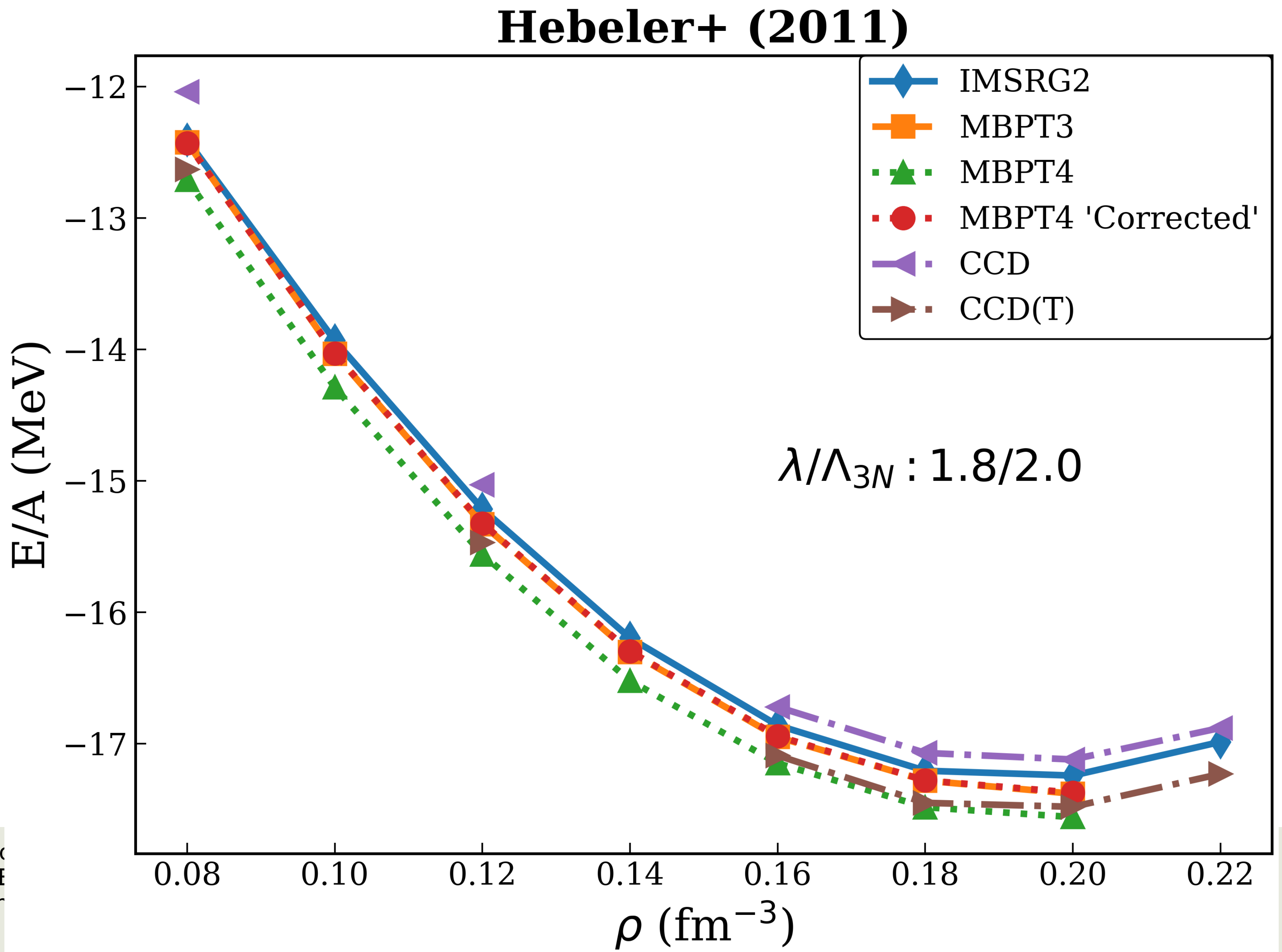


Preliminary Results - PNM EOS with N2LO EMN Interactions

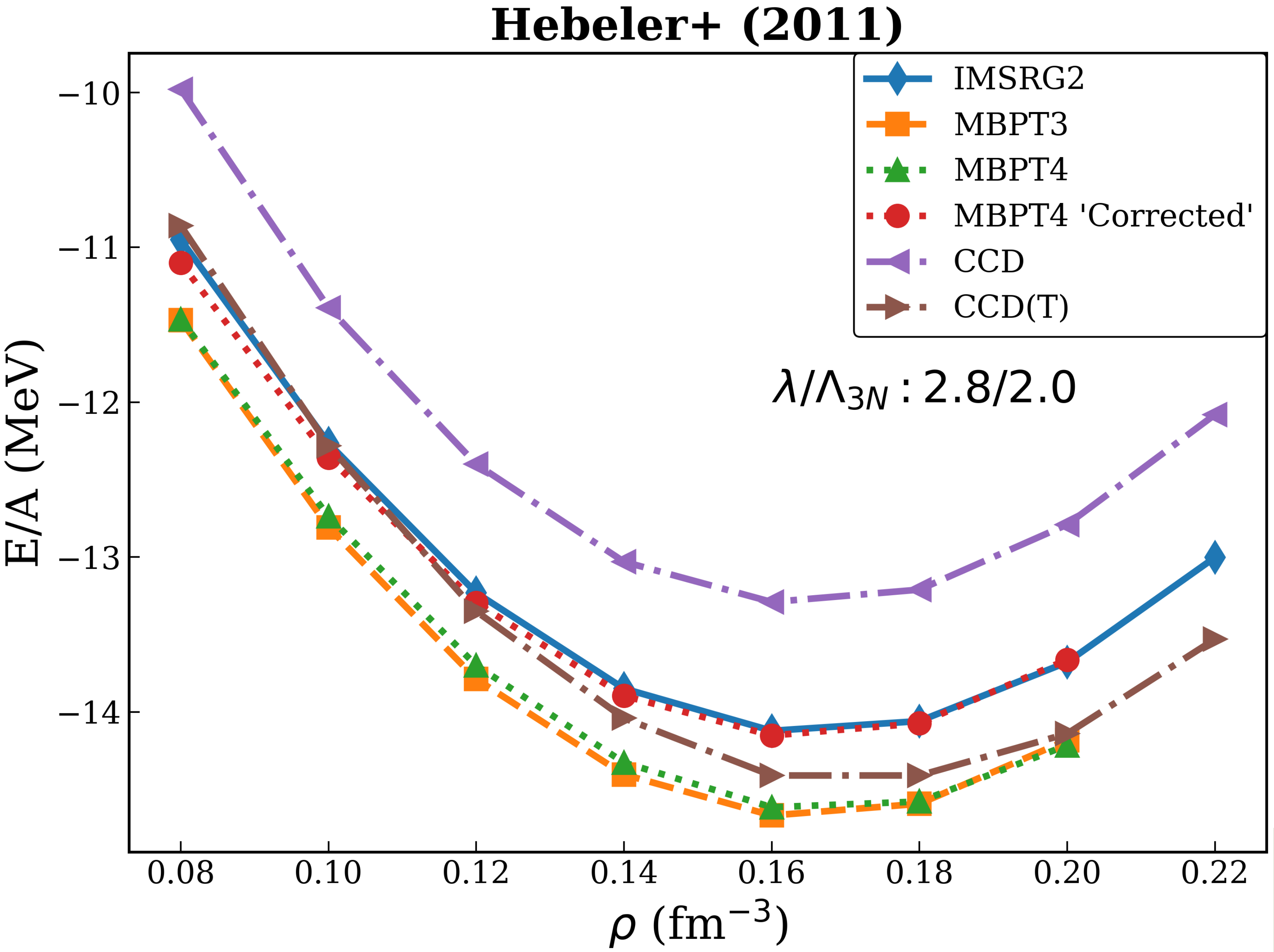




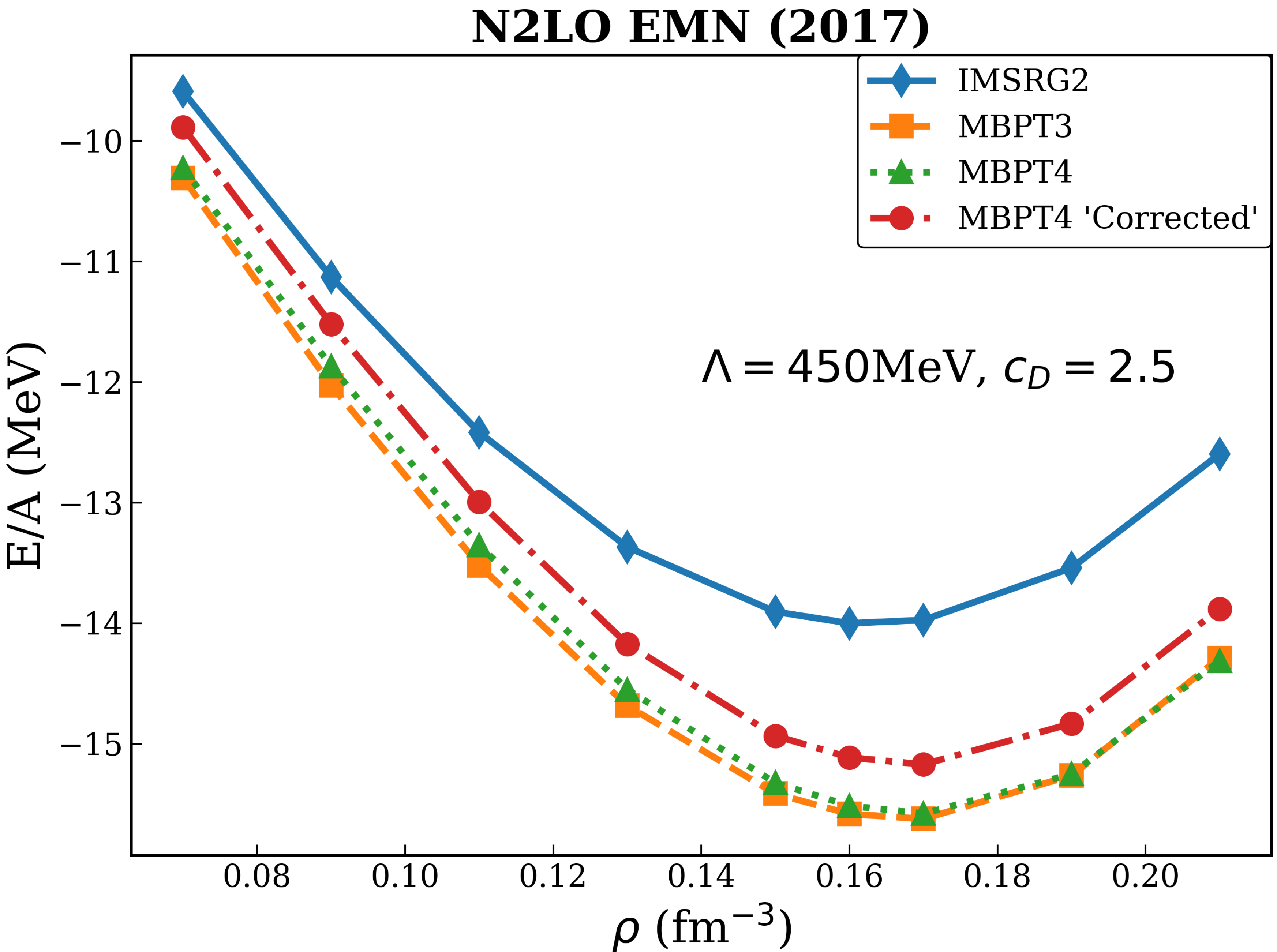
Preliminary Results - SNM EOS with Hebeler Interactions



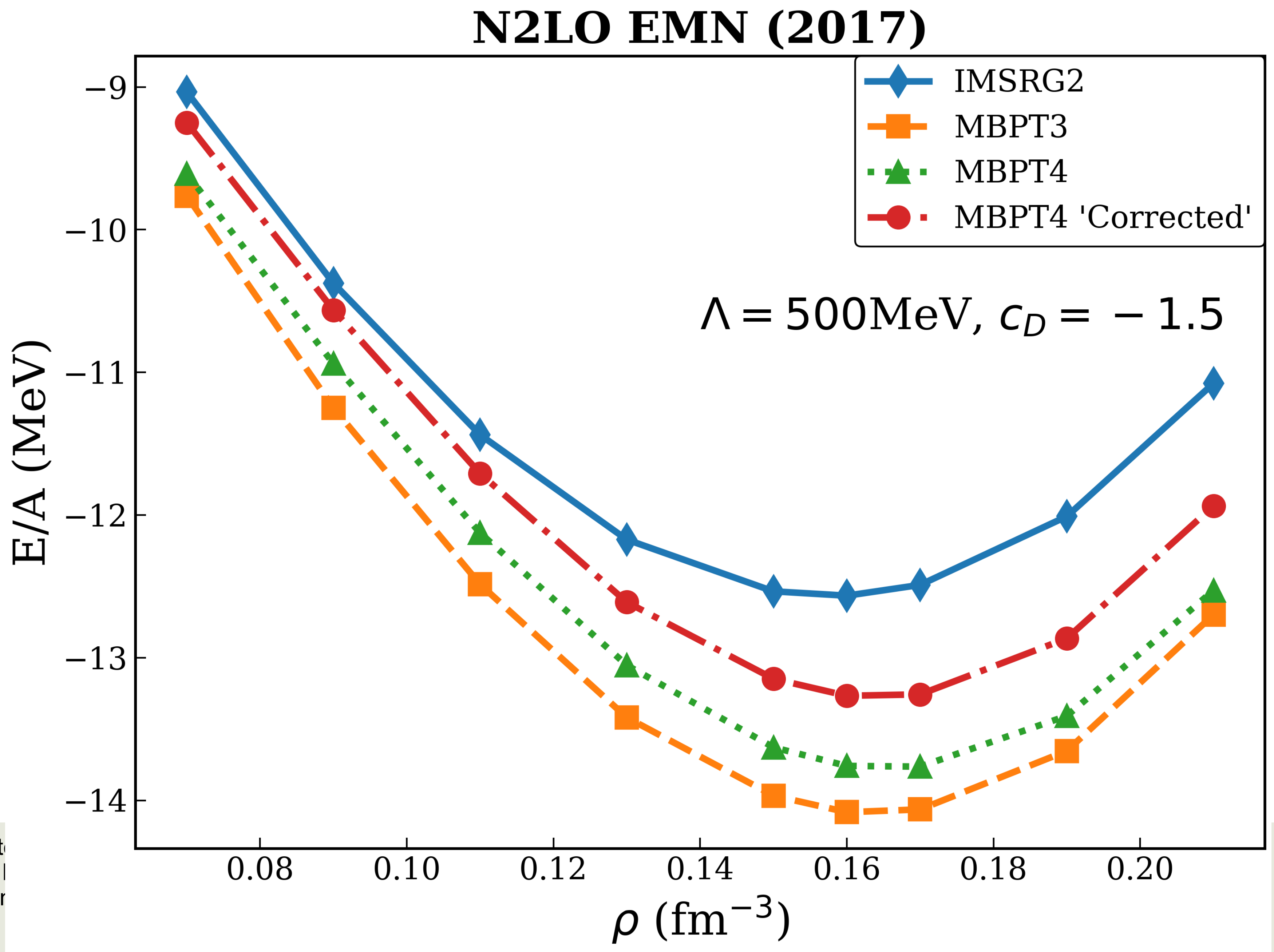
Preliminary Results - SNM EOS with Hebeler Interactions



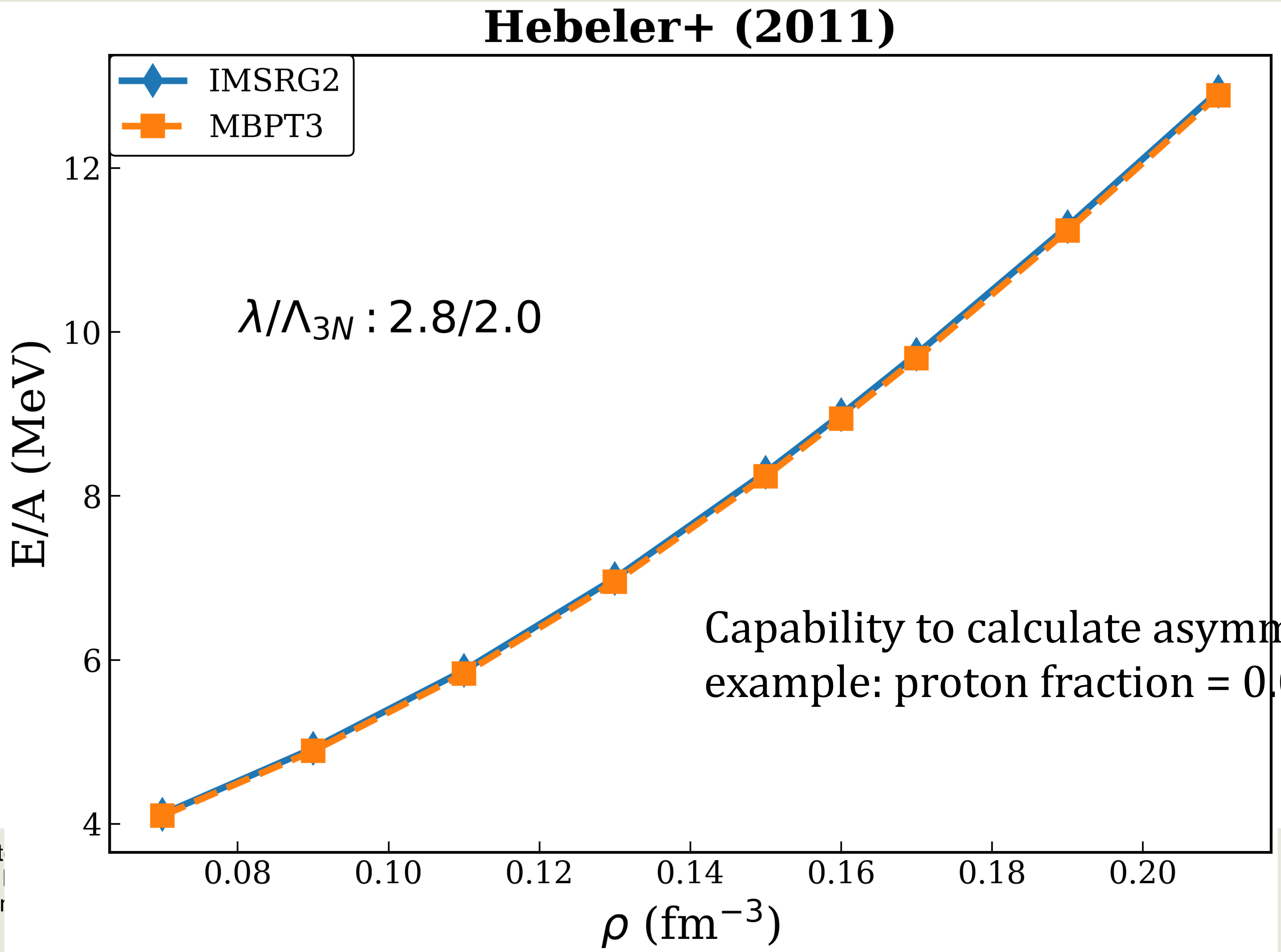
Preliminary Results - SNM EOS with N2LO EMN Interactions



Preliminary Results - SNM EOS with N2LO EMN Interactions



Preliminary Results - ANM EOS with Hebeler Interactions



Capability to calculate asymmetric matter,
example: proton fraction = 0.05

Summary

We've built a nuclear many-body modeling infrastructure based on IMSRG for EOS calculations.

- Converged calculations for a range of proton fractions
- Good agreement with other many body methods - MBPT and CC for perturbative system (PNM)
- For more correlated system (SNM), the agreement is still good for softer interactions (Hebeler).
- Noticeable discrepancies starts to occur for relatively harder interactions (N2LO EMN)
  - Still working to understand these differences (finite-size corrections, approximate IMSRG(3), etc.)

J. Davison, *Theoretical and computational improvements to the in-medium similarity renormalization group*
C. Drischler et al, Phys. Rev. C 102, 054315 (2020)
Patrick. Cook et al, arXiv:2401.11694

Outlook

- Computation of other static properties (momentum distributions, static structure factors) in progress
- Large scale EOS calculations at T = 0 for different chiral EFT interactions
  - Bayesian analysis of EFT truncation errors (BUQEYE)
  - Comparison of different many body methods
- Explore emulators for IMSRG calculations (Dynamical Mode Decomposition, Parametric Matrix Models)
- Possible extensions
  - Finite T (see Smith et al. https://arxiv.org/abs/2407.00576)
  - Approximate IMSRG(3) (see Stroberg et al., https://arxiv.org/abs/2406.13010)
  - Response via EOM (and other) techniques

C. Drischler et al, *Quantifying uncertainties and correlations in the nuclear-matter* equation of state
S. R. Stroberg et al, *IMSRG with flowing 3 body operators, and approximations thereof*
Isaac G. Smith et al, *The IMSRG at Finite Temperature*
Patrick. Cook et al, *Parametric Matrix Models*

Taste of Xian

Thank you for your attention!

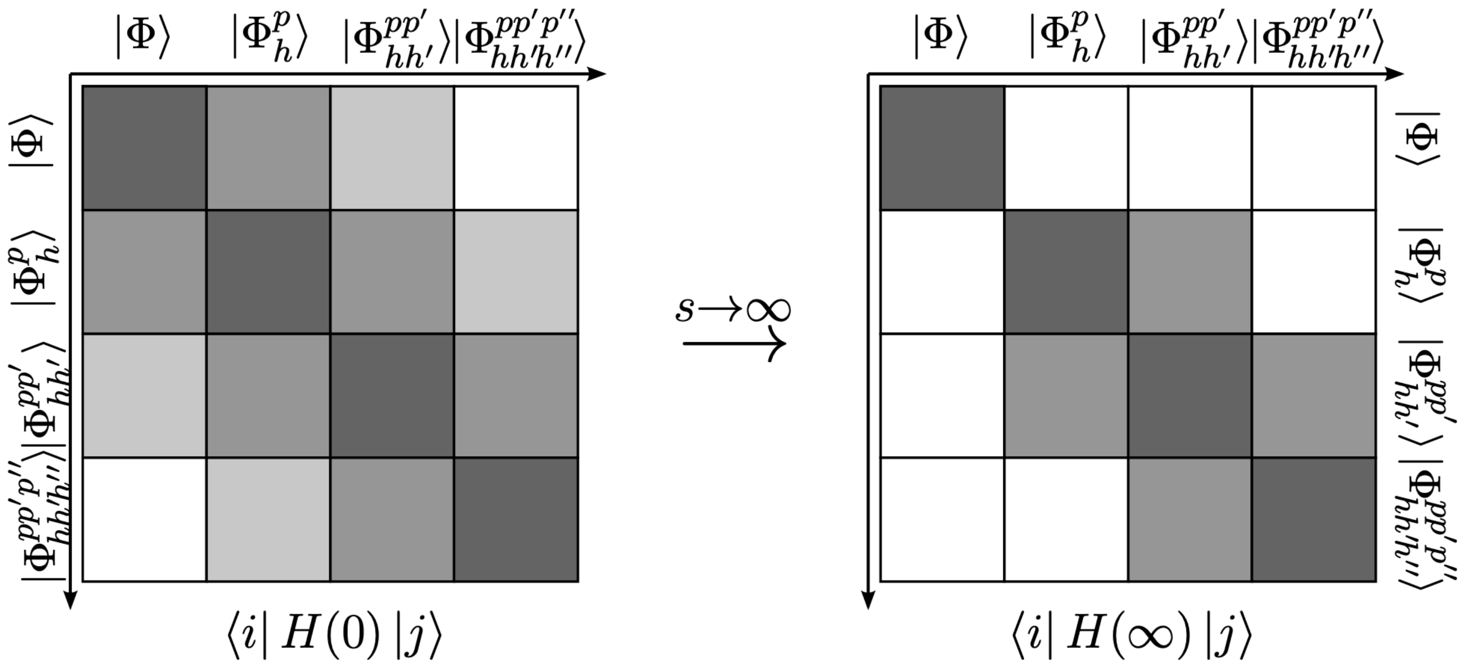
Introduction - In-Medium Similarity Renormalization Group

IMSRG(2) flow equations:

$$\frac{dE}{ds} = \sum_{ab}(n_a - n_b)\eta_{ab}f_{ba} + \frac{1}{2}\sum_{abcd}\eta_{abcd}\Gamma_{cdab}n_a n_b \bar{n}_c \bar{n}_d ,$$

$$\frac{df_{ij}}{ds} = \sum_{a}(1 + P_{ij})\eta_{ia}f_{aj} + \sum_{ab}(n_a - n_b)(\eta_{ab}\Gamma_{biaj} - f_{ab}\eta_{biaj}) + \frac{1}{2}\sum_{abc}(n_a n_b \bar{n}_c + \bar{n}_a \bar{n}_b n_c)(1 + P_{ij})\eta_{ciab}\Gamma_{abcj} ,$$

$$\frac{d\Gamma_{ijkl}}{ds} = \sum_{a}\{(1 - P_{ij})(\eta_{ia}\Gamma_{ajkl} - f_{ia}\eta_{ajkl}) - (1 - P_{kl})(\eta_{ak}\Gamma_{ijal} - f_{ak}\eta_{ijal})\} + \frac{1}{2}\sum_{ab}(1 - n_a - n_b)(\eta_{ijab}\Gamma_{abkl} - \Gamma_{ijab}\eta_{abkl}) + \sum_{ab}(n_a - n_b)(1 - P_{ij})(1 - P_{kl})\eta_{aibk}\Gamma_{bjal} .$$



M Hjorth-Jensen et al, *An Advanced Course in Computational Nuclear Physics: Bridging the Scales from Quarks to Neutron Stars*
H. Hergert et al, *Nuclear Structure from the In-Medium Similarity Renormalization Group*

Introduction - Magnus Formulation

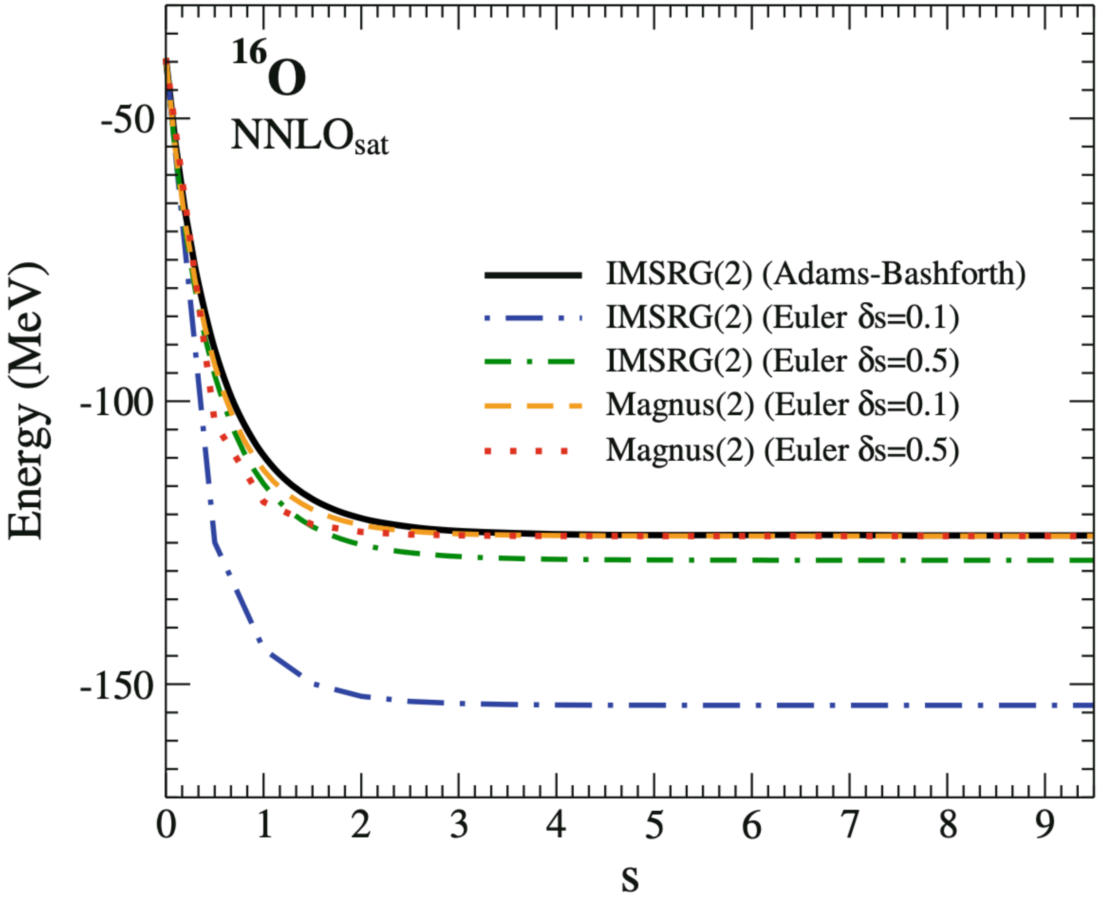
$$\hat{U}(s) \equiv e^{\hat{\Omega}(s)}$$

$$\frac{d\hat{\Omega}}{ds} = \sum_{k=0}^{\infty} \frac{B_k}{k!} \mathrm{ad}^k_{\hat{\Omega}} (\hat{\eta})$$

$$\mathrm{ad}^0_{\hat{\Omega}} (\hat{\eta}) = \hat{\eta}$$

$$\mathrm{ad}^k_{\hat{\Omega}} (\hat{\eta}) = [\hat{\Omega}, \mathrm{ad}^{k-1}_{\hat{\Omega}} (\hat{\eta})]$$

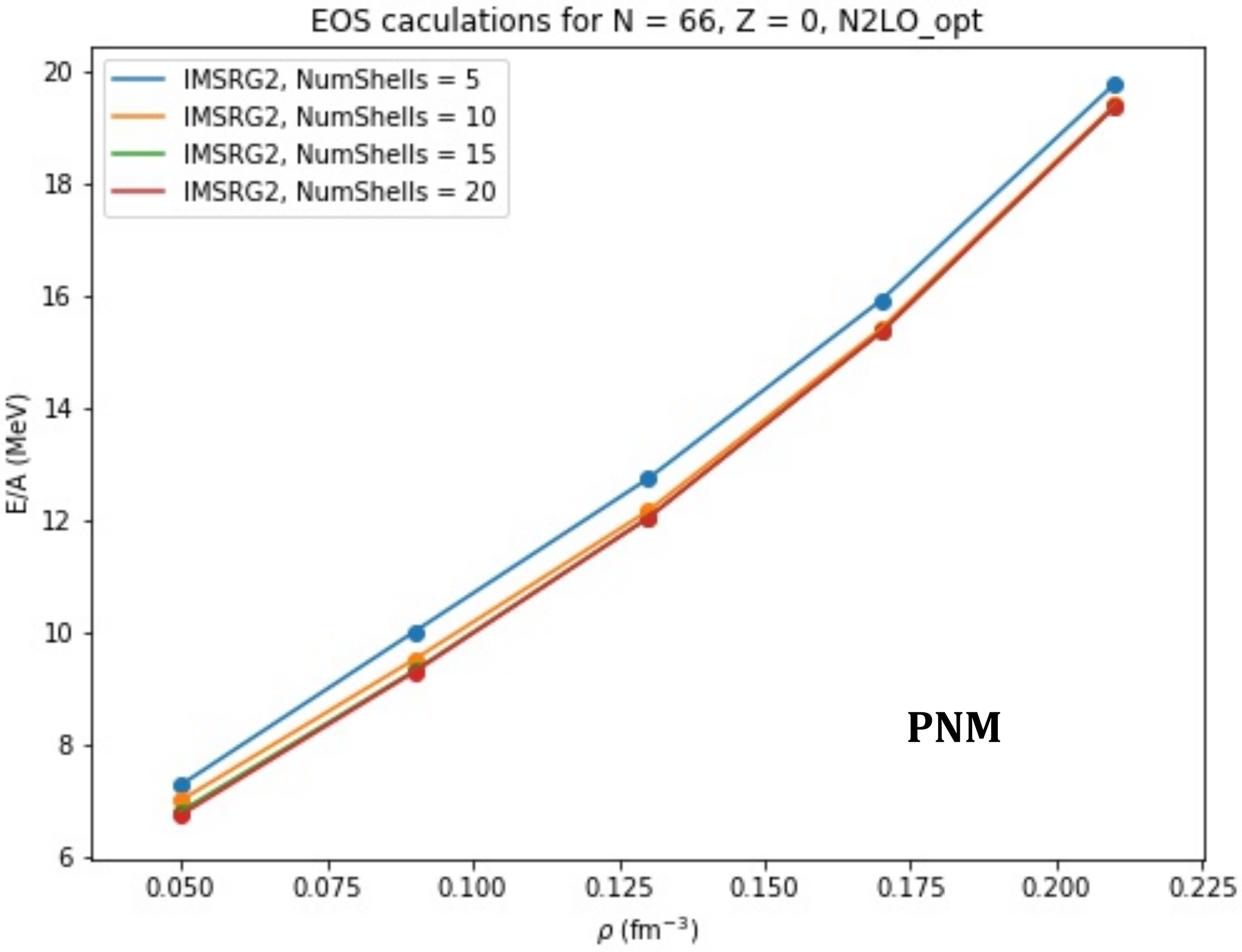
$$\hat{H}(s) \equiv e^{\hat{\Omega}(s)} \hat{H}(0) e^{-\hat{\Omega}(s)} = \sum_{k=0}^{\infty} \frac{1}{k!} \mathrm{ad}^k_{\hat{\Omega}(s)} \left(\hat{H}(0)\right)$$

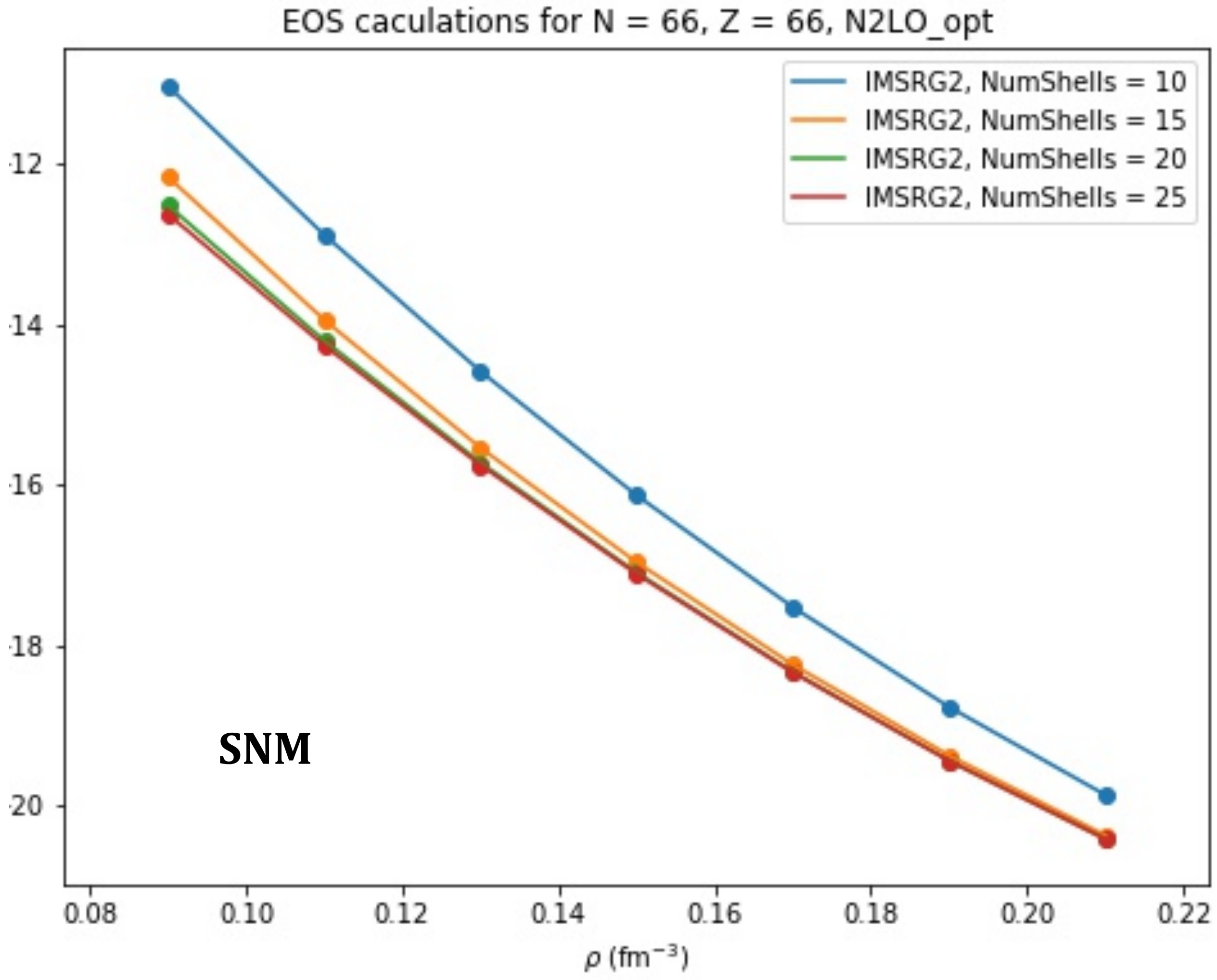


M Hjorth-Jensen et al, *An Advanced Course in Computational Nuclear Physics: Bridging the Scales from Quarks to Neutron Stars*

H. Hergert et al, *Nuclear Structure from the In-Medium Similarity Renormalization Group*

Results - Basis Convergence





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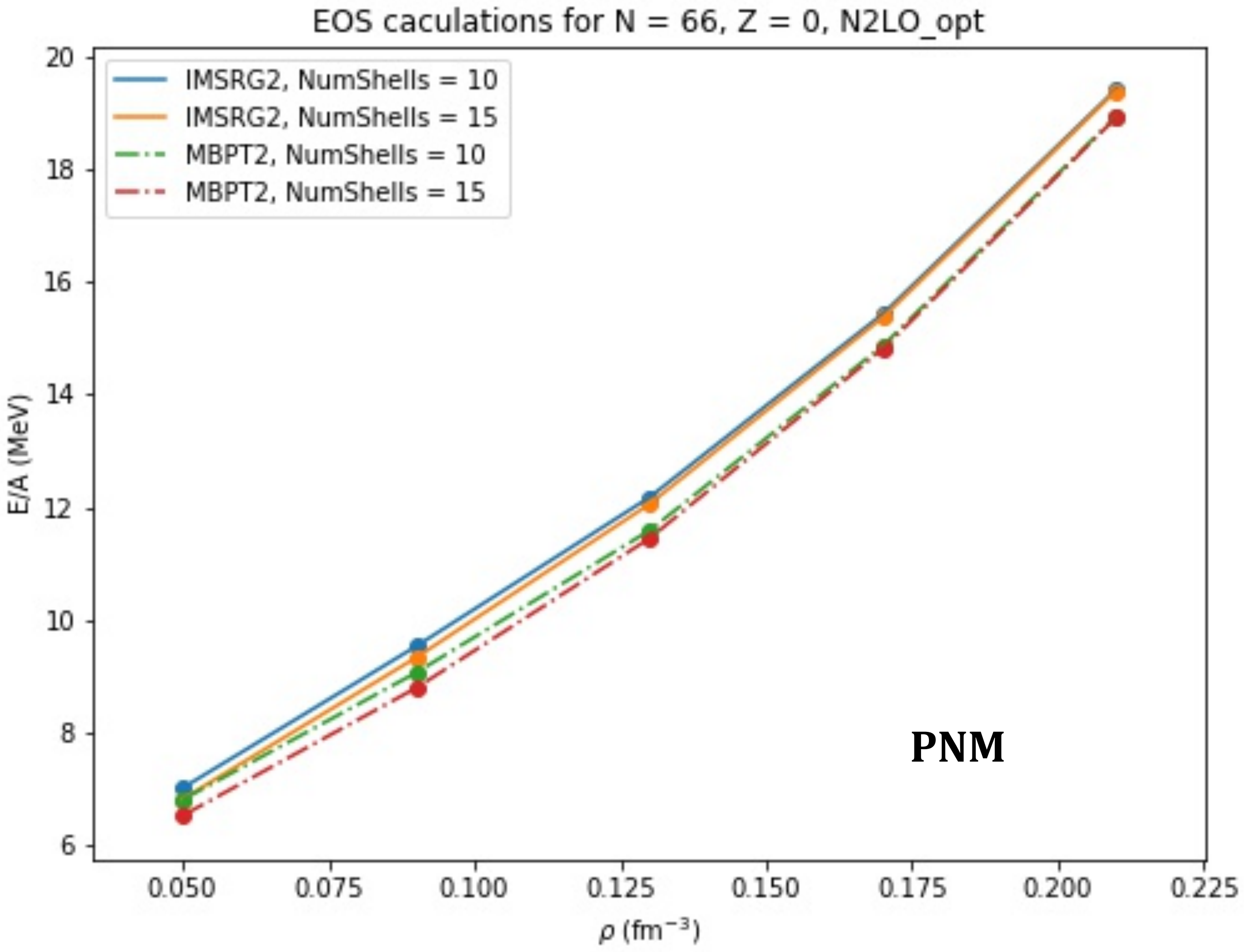
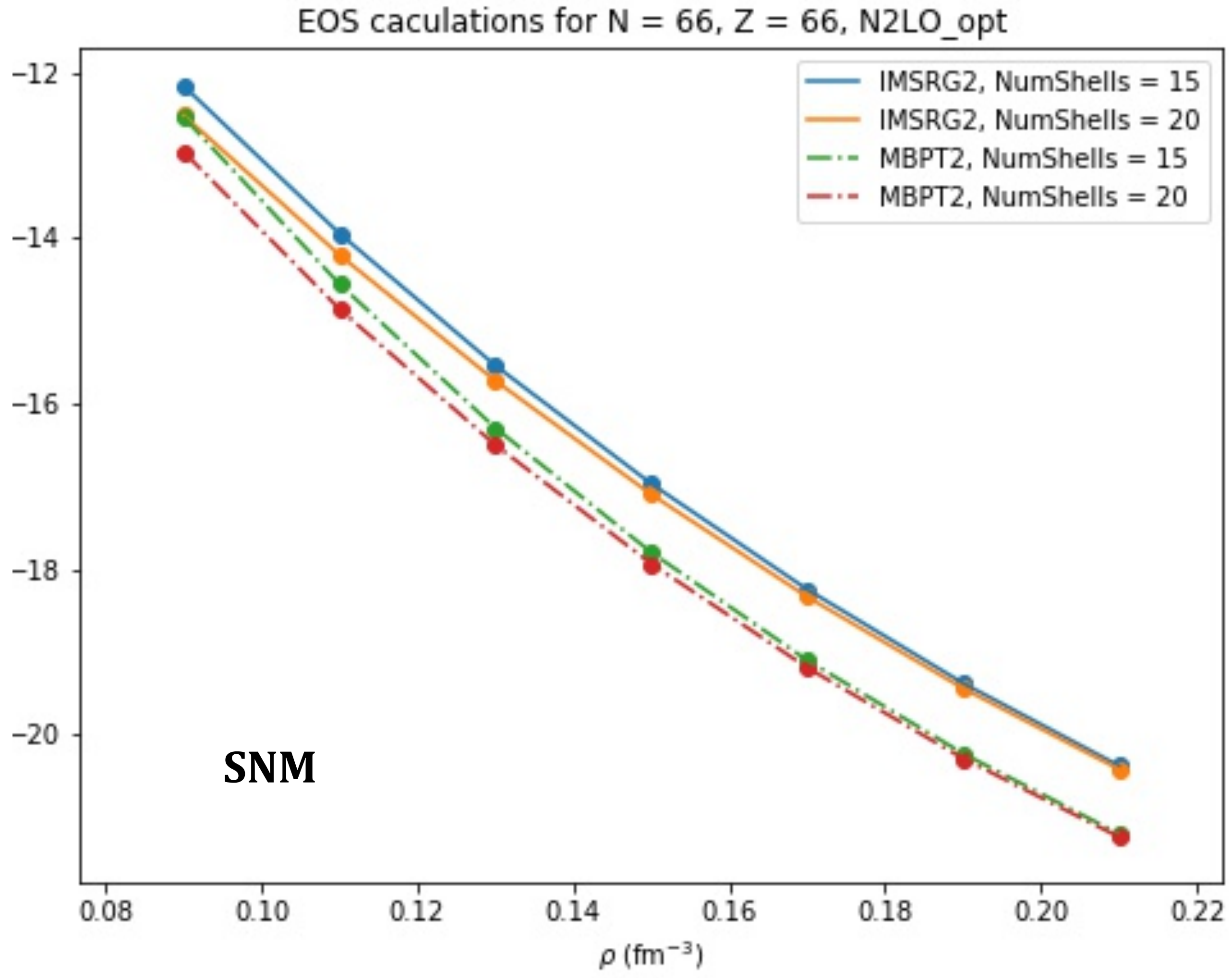
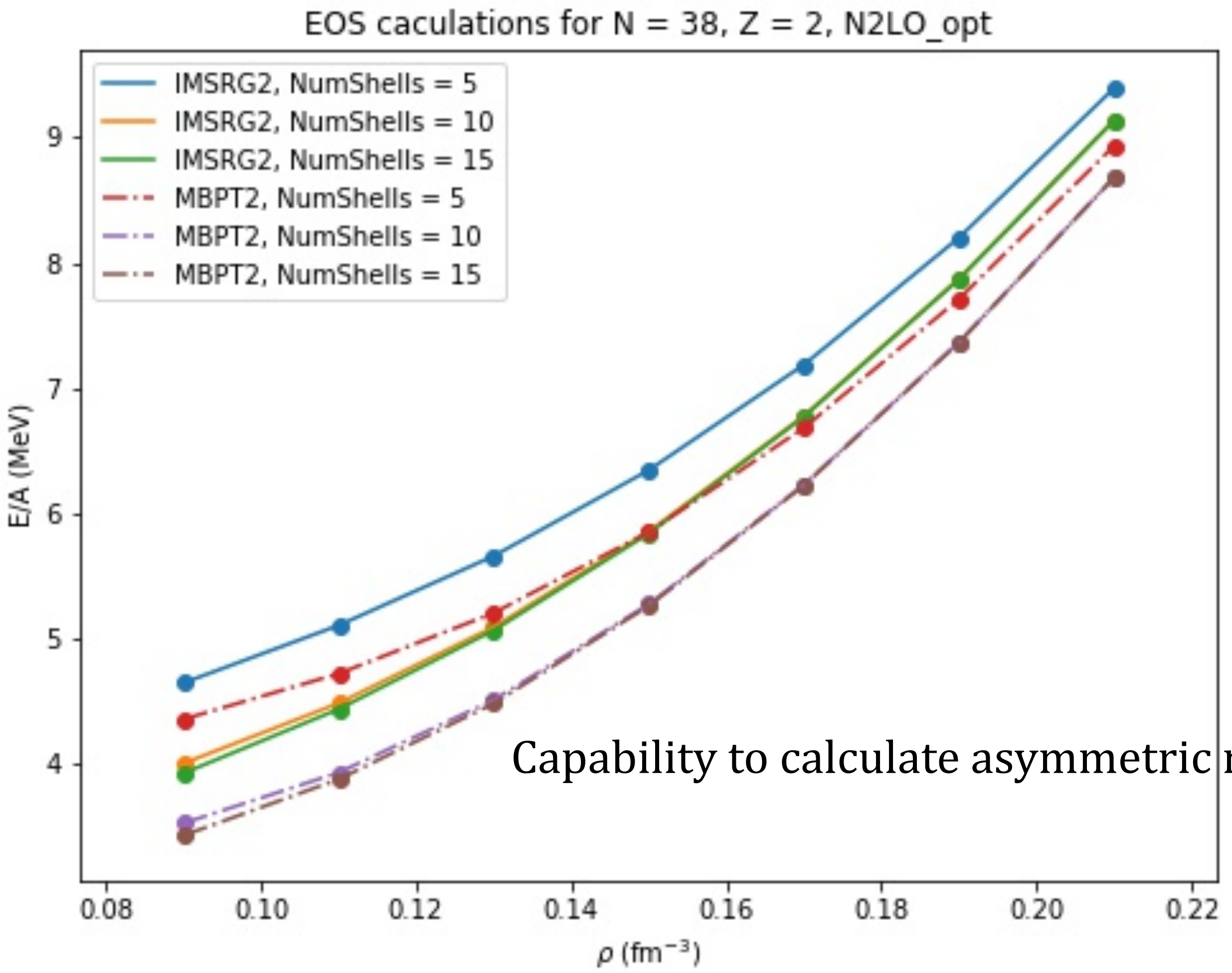


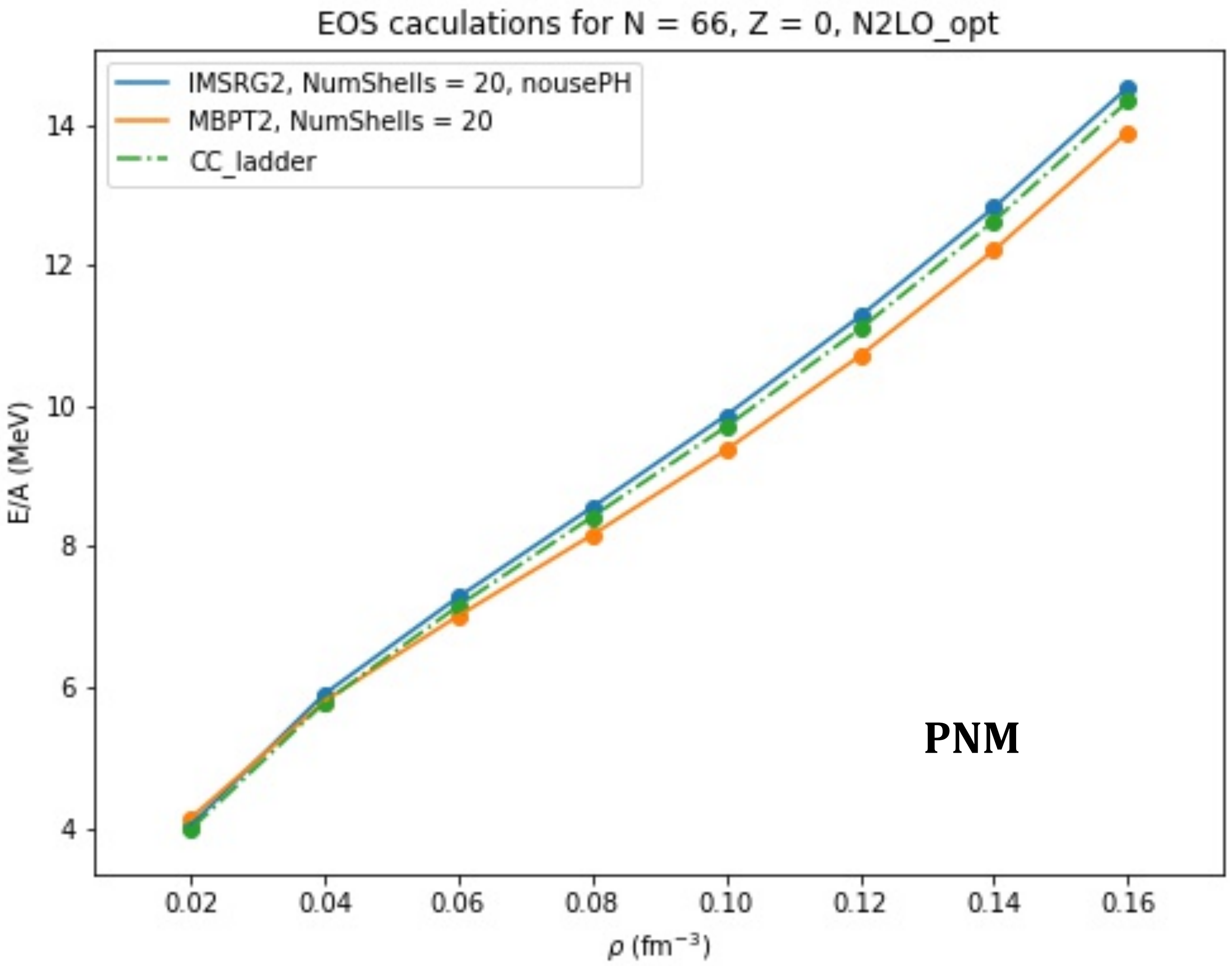


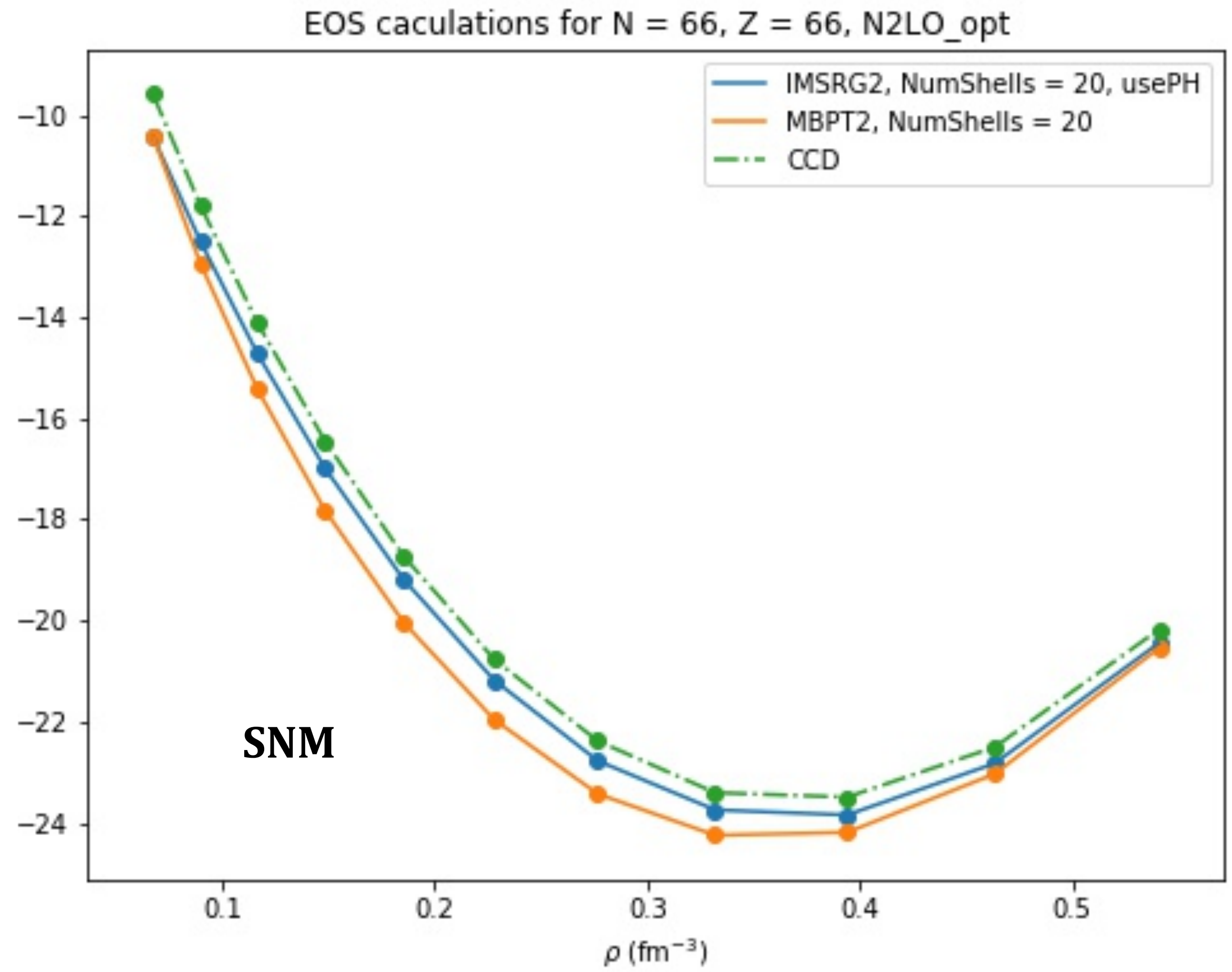
Results - Basis Convergence



Capability to calculate asymmetric matter, proton fraction = 0.05

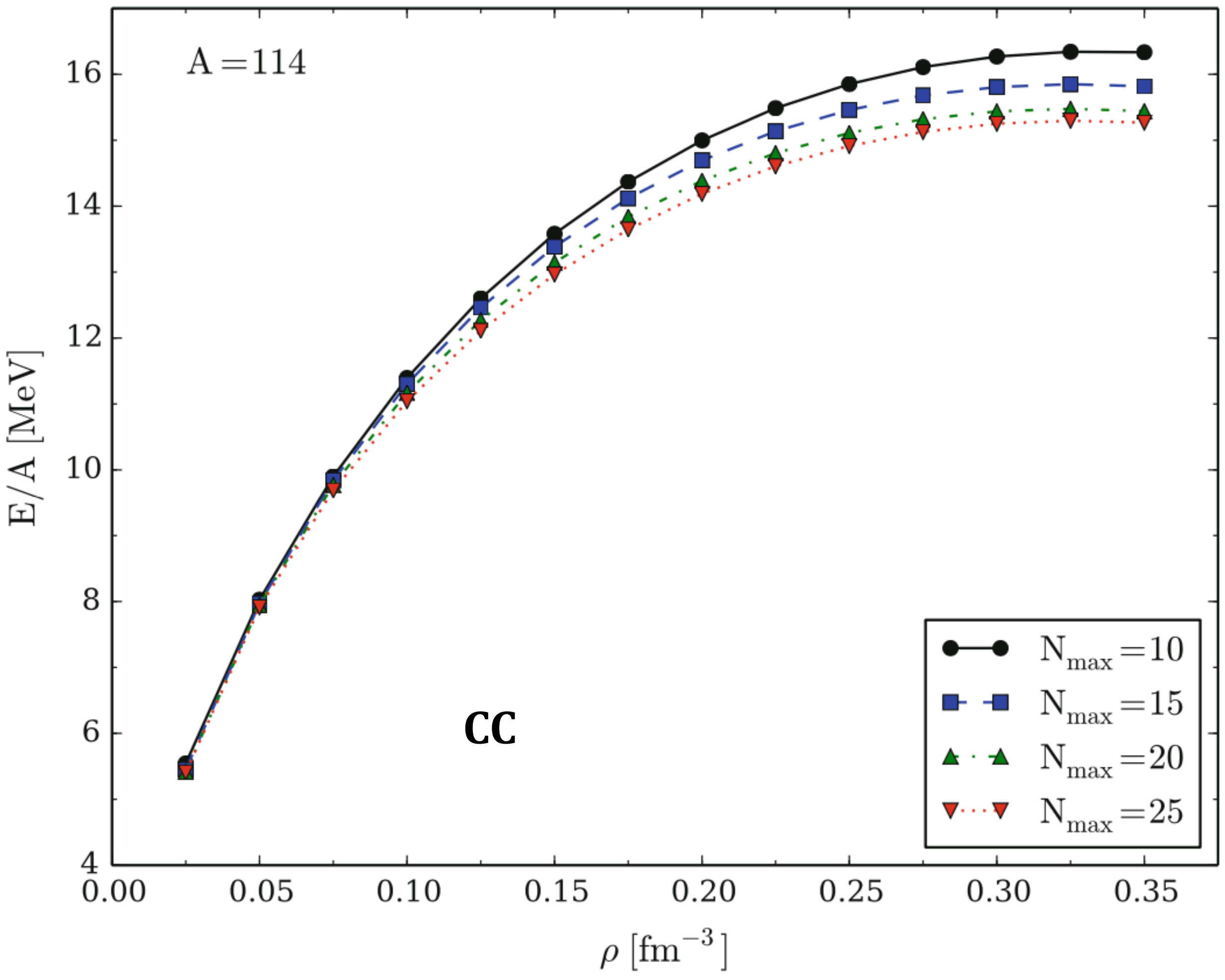
Results - Benchmark

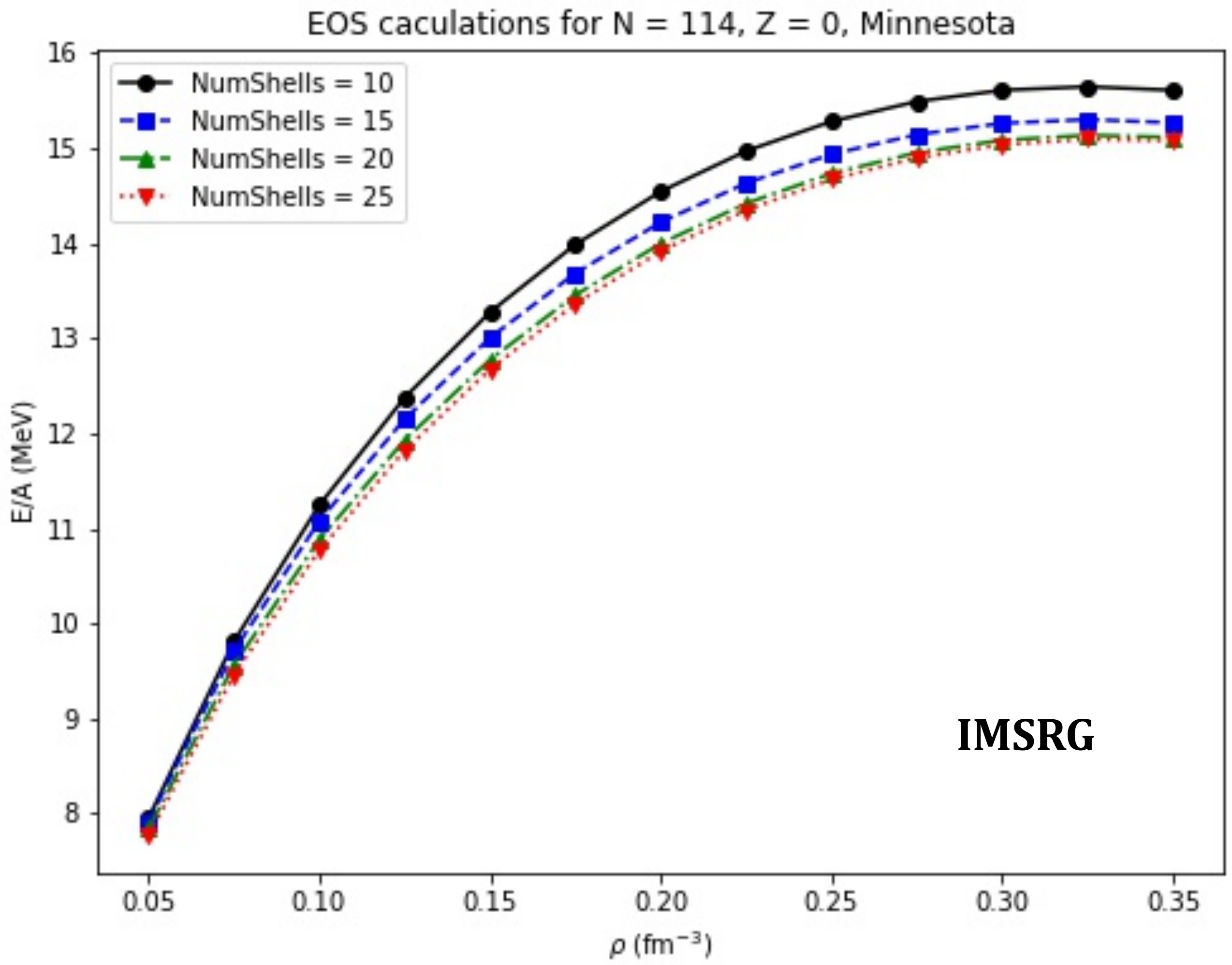




G. Hagen et al, Phys. Rev. C 89, 014319

Results - Benchmark





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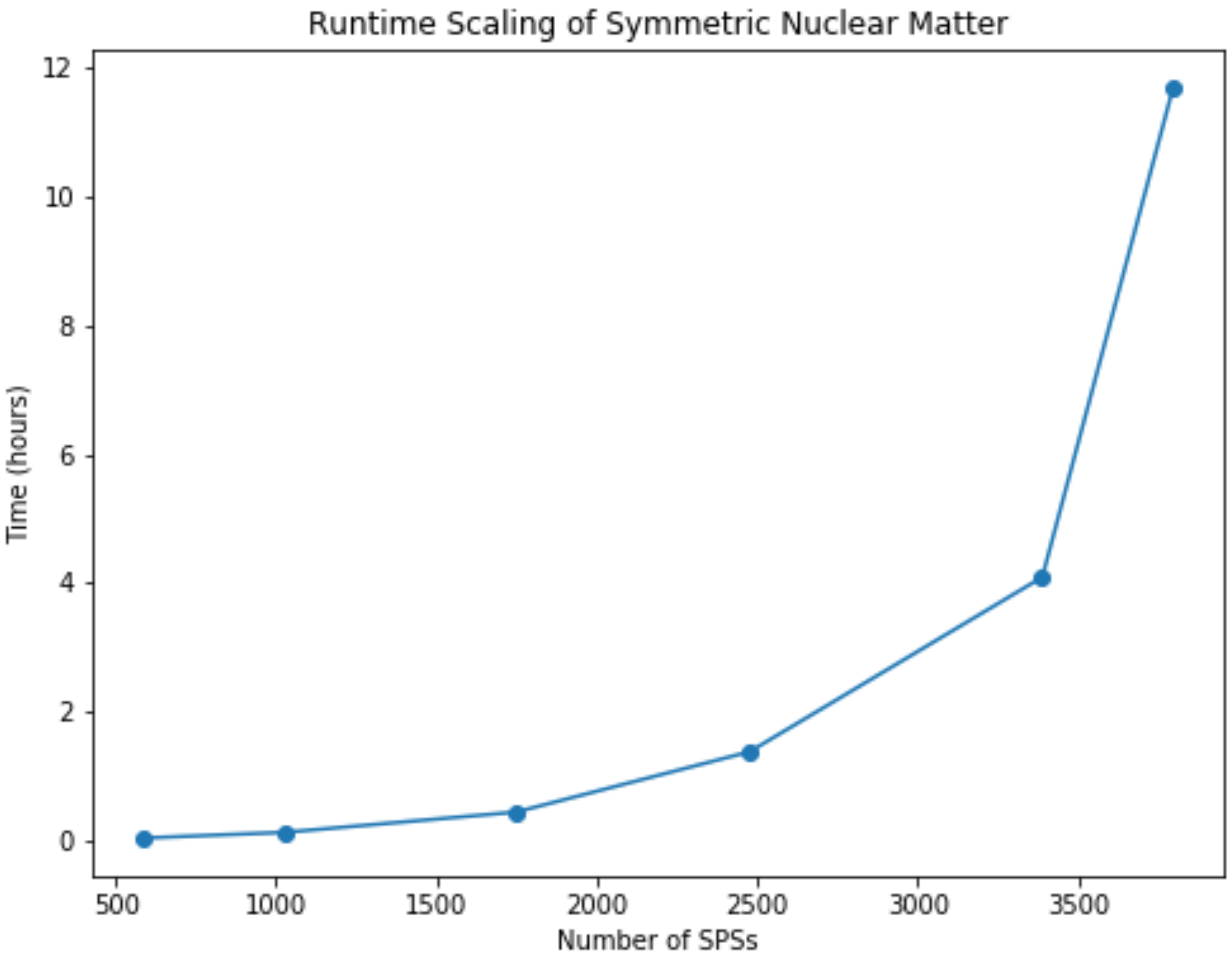
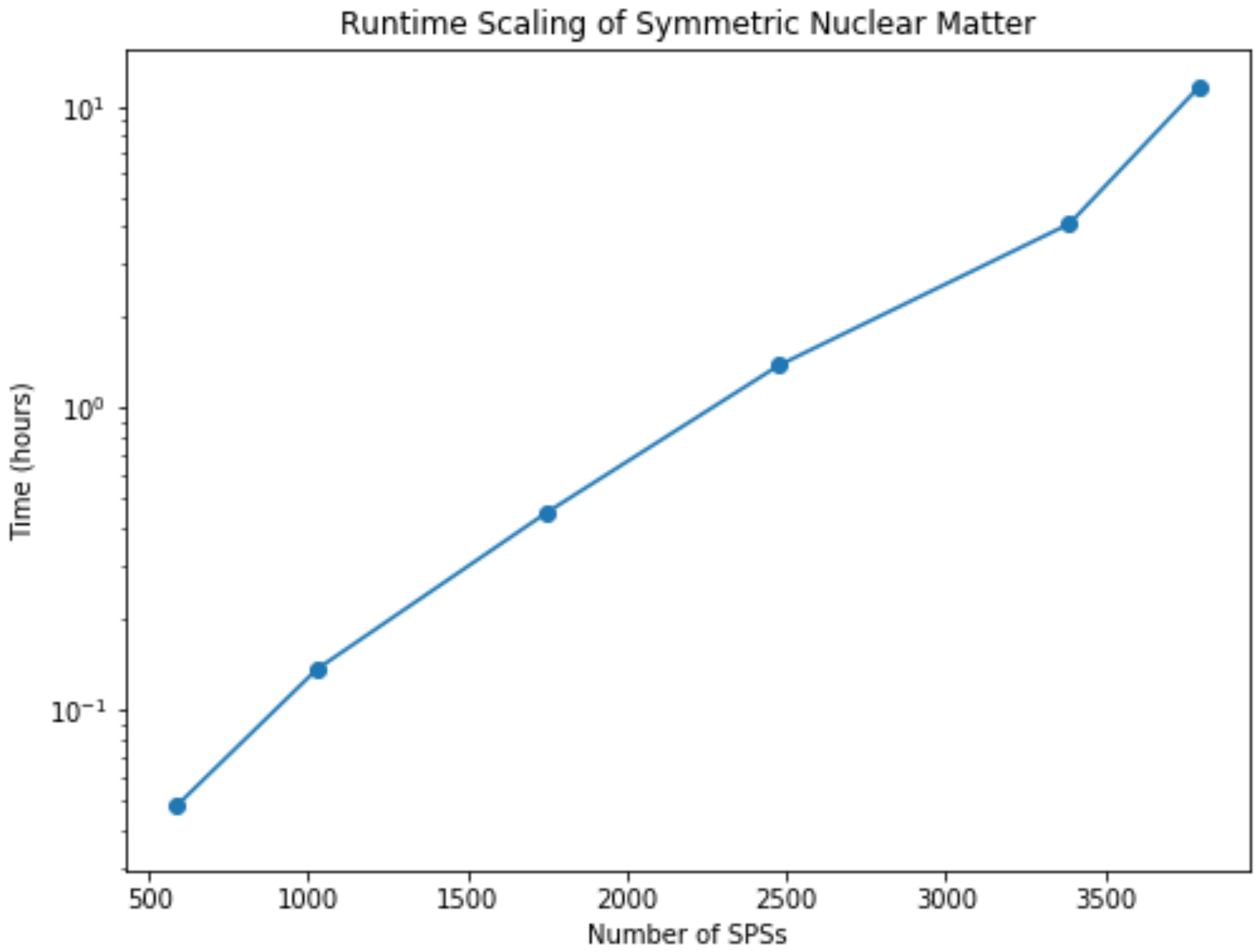
frib.msu.edu

M Hjorth-Jensen et al, *An Advanced Course in Computational Nuclear Physics: Bridging the Scales from Quarks to Neutron Stars*

Results - Scaling





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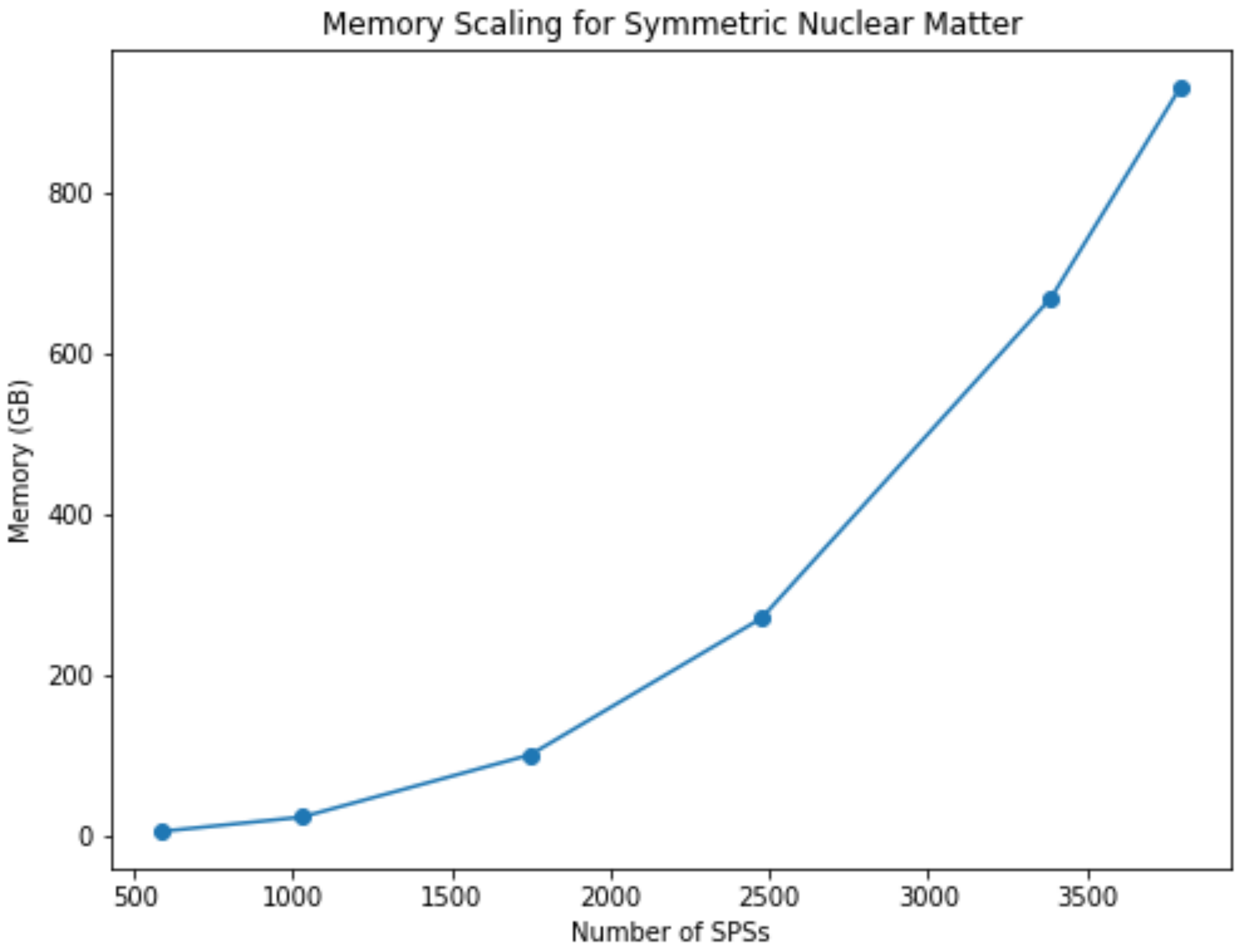
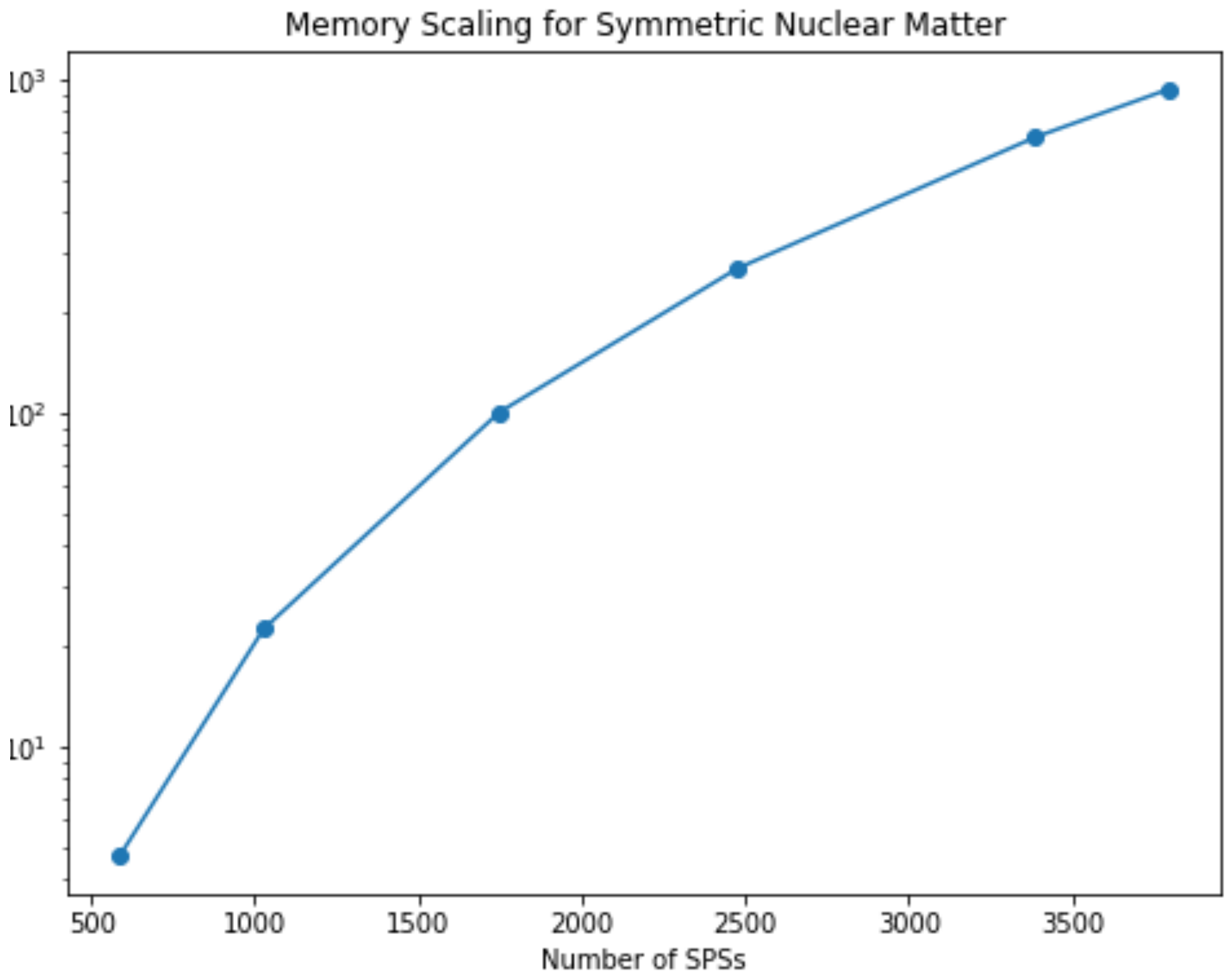
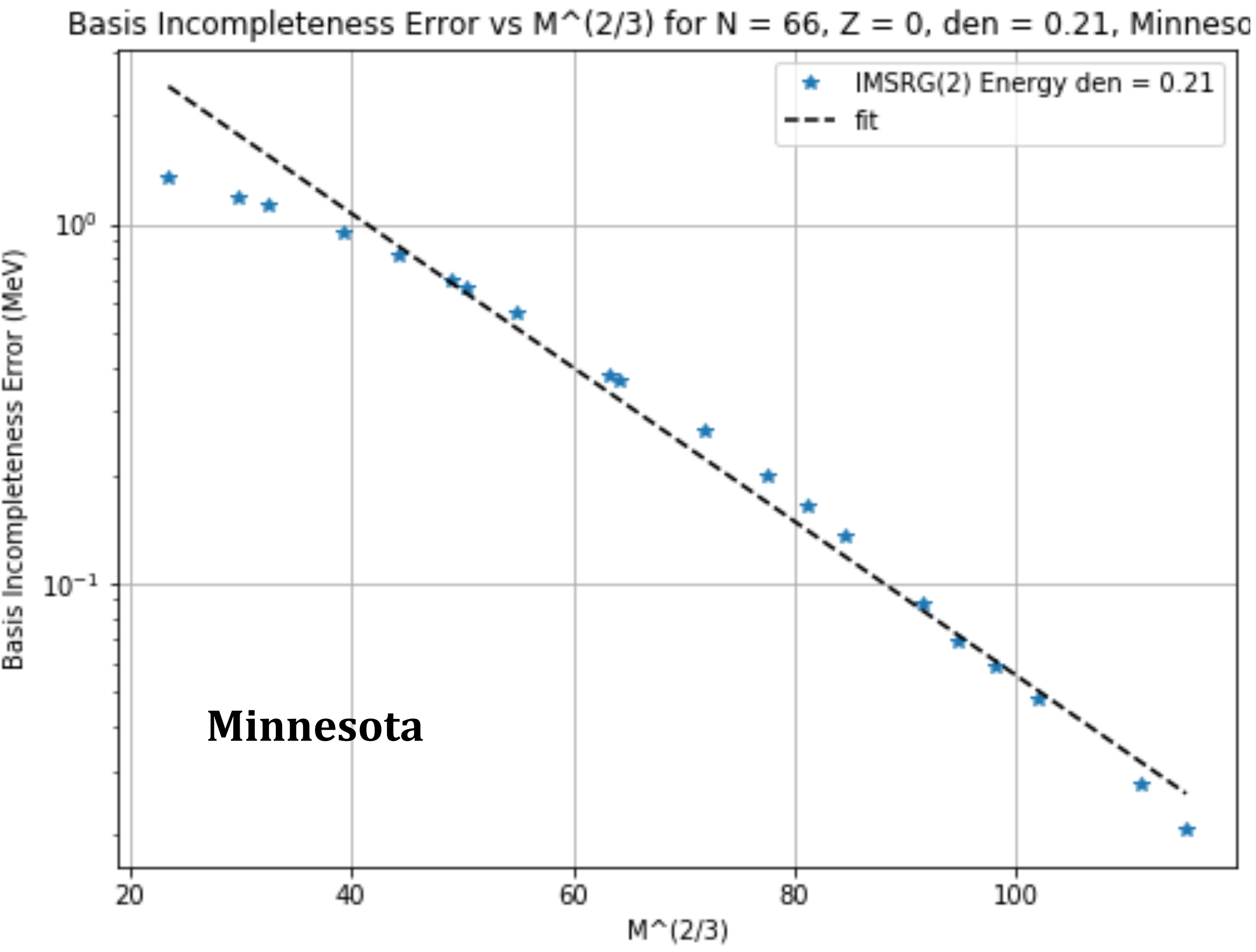
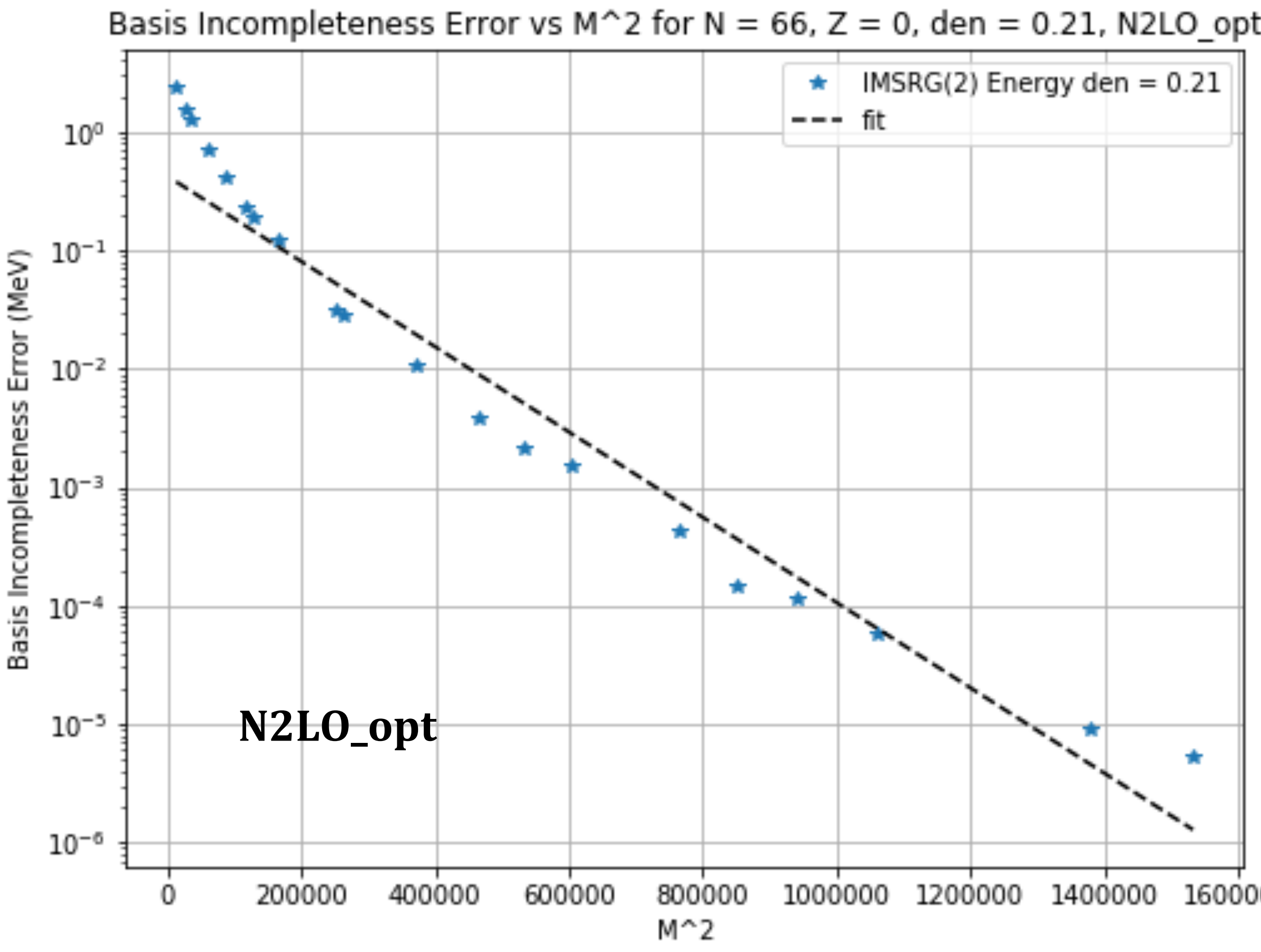




Results - Basis Extrapolation





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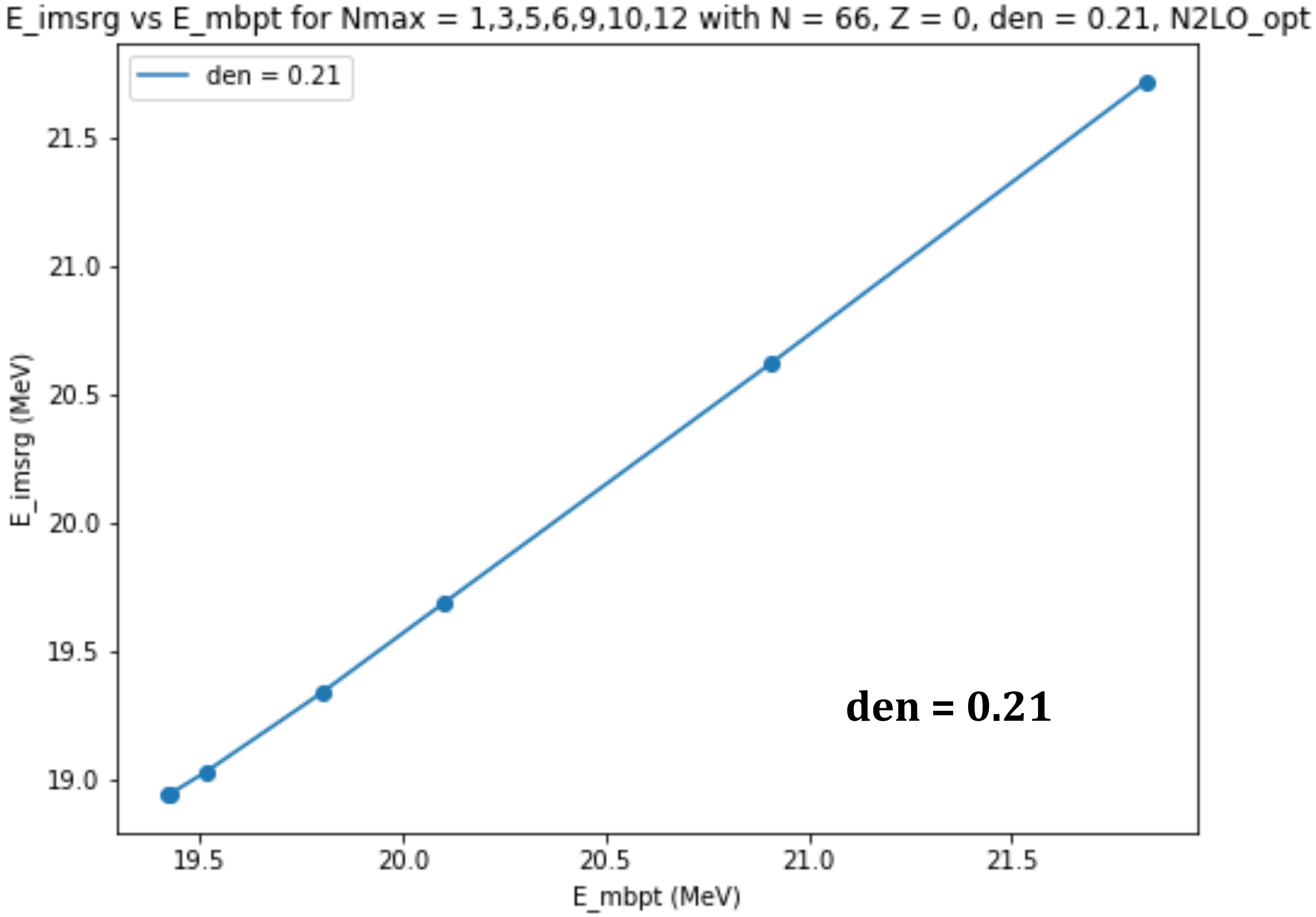
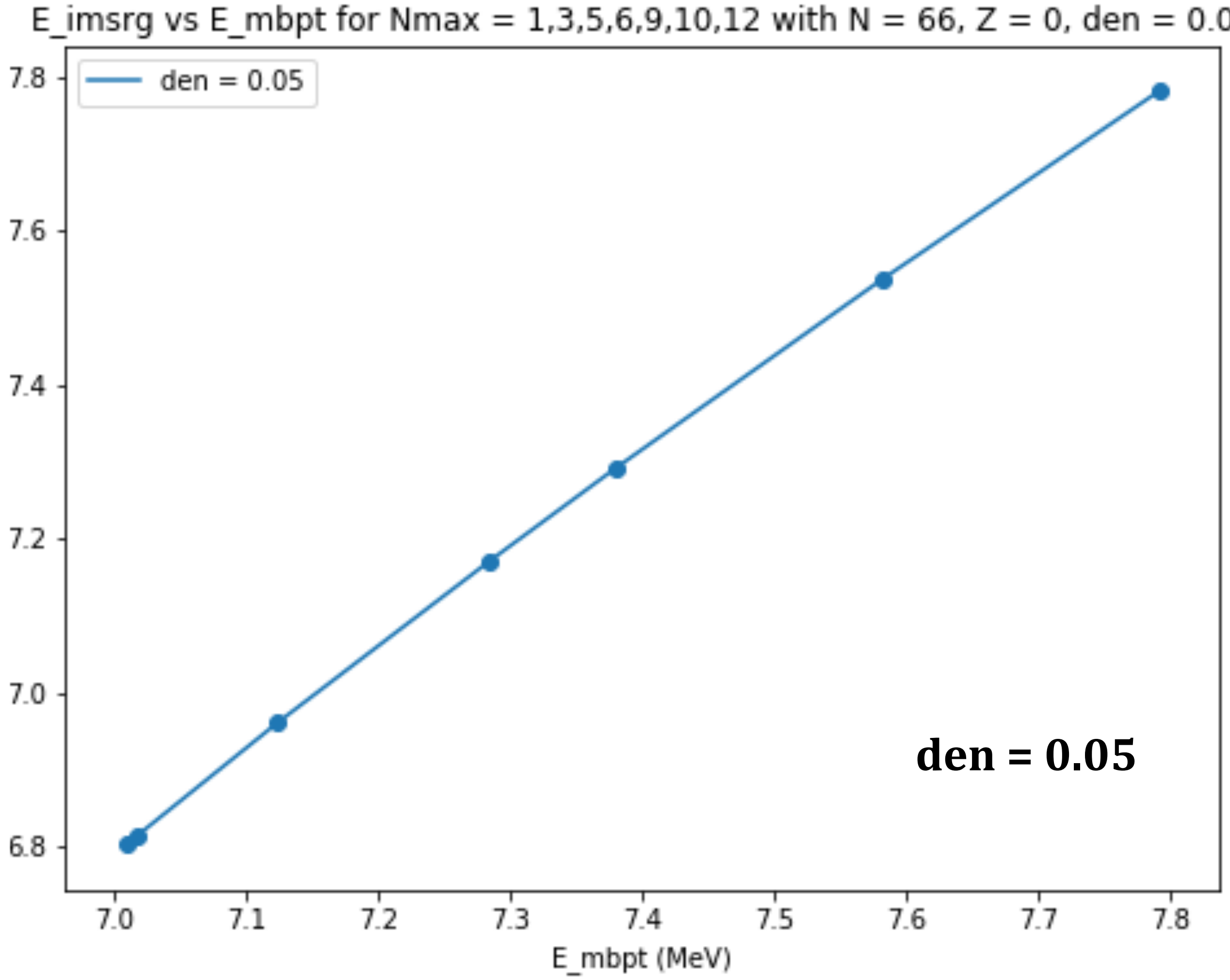
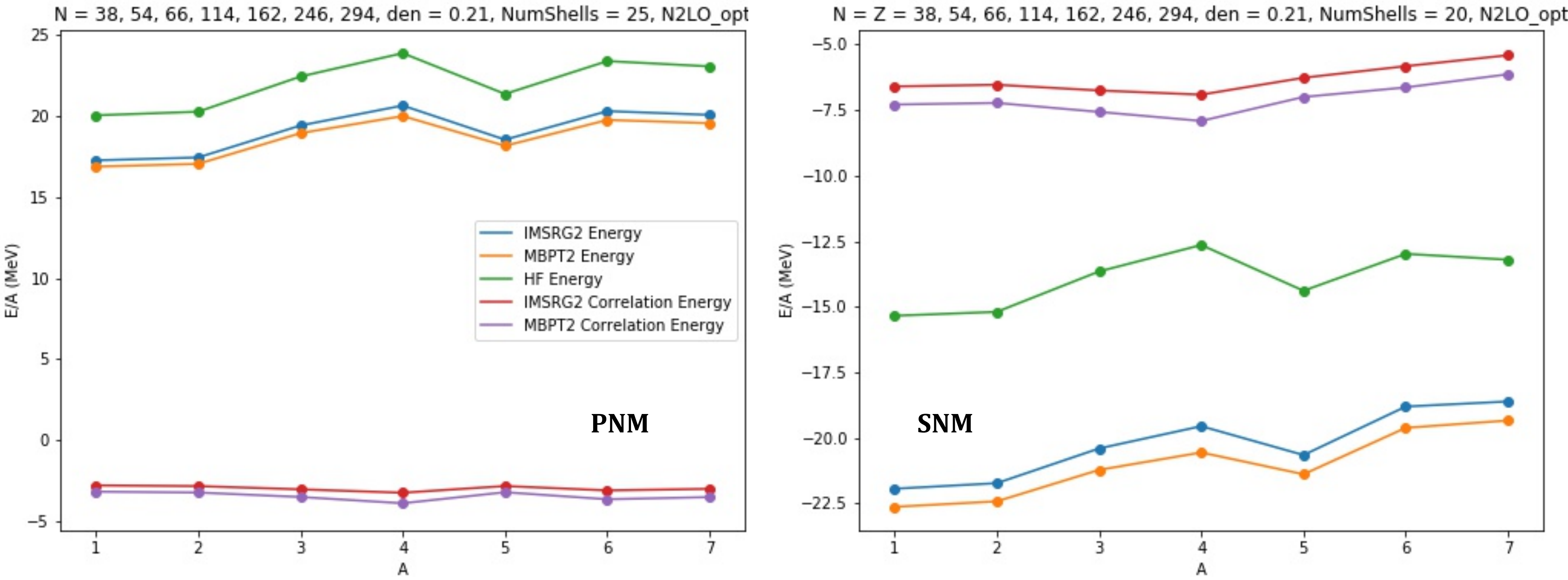




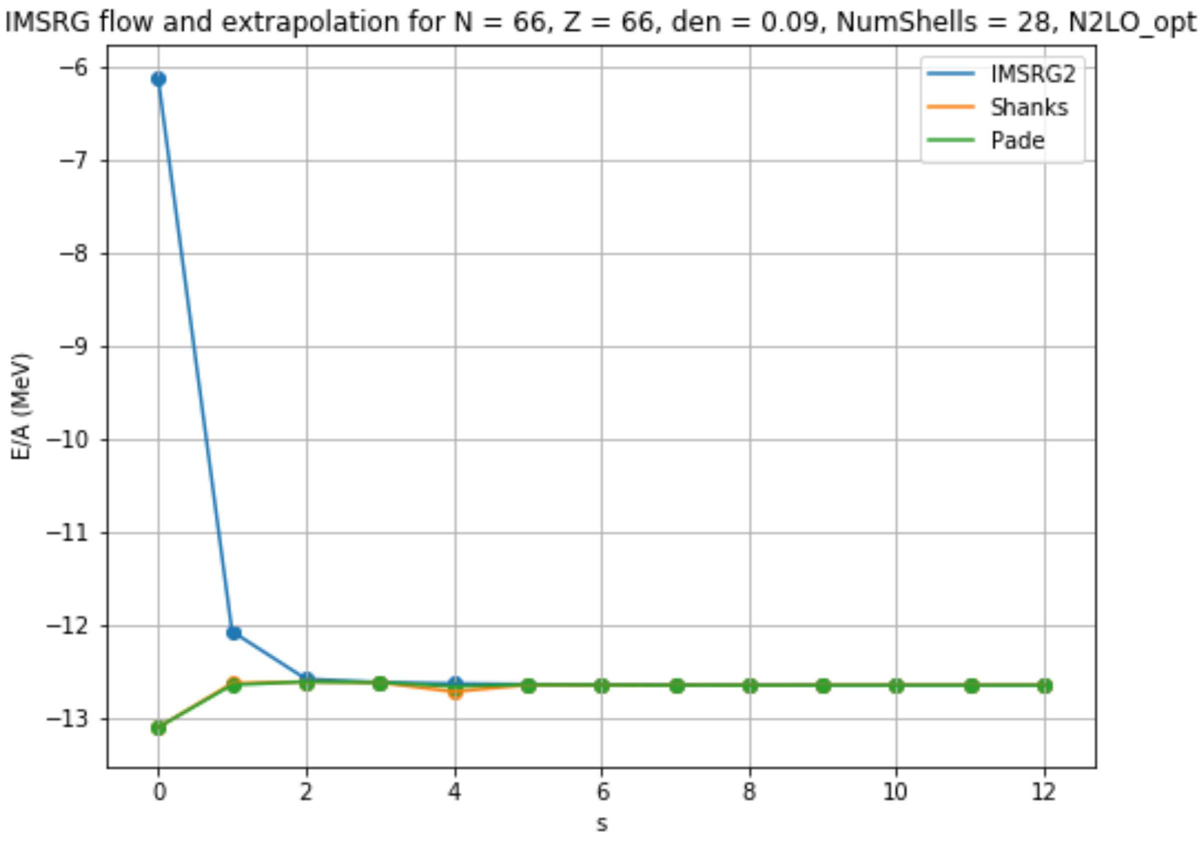
J. Butler et al, arXiv:2403.04645

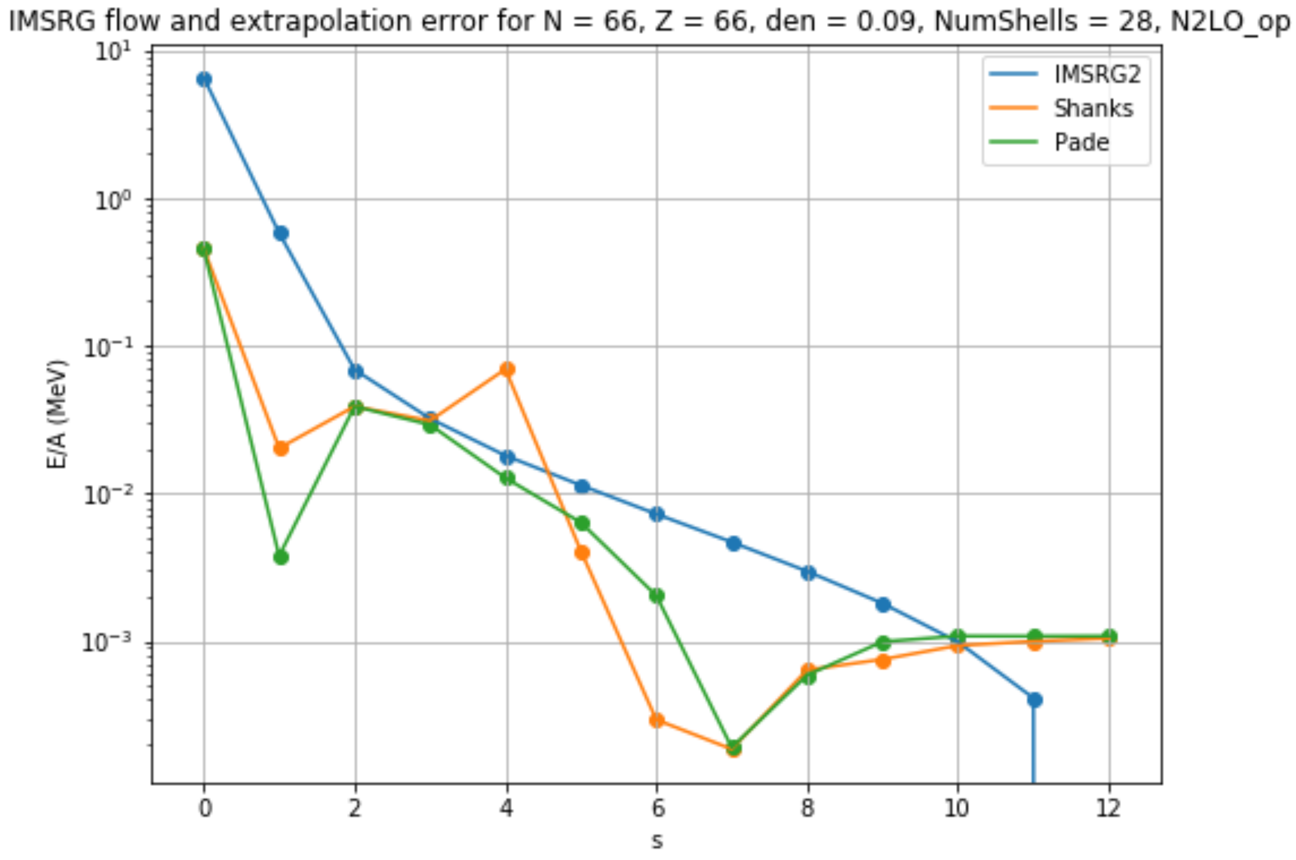
Results - Finite Size Effect



Numerical Techniques

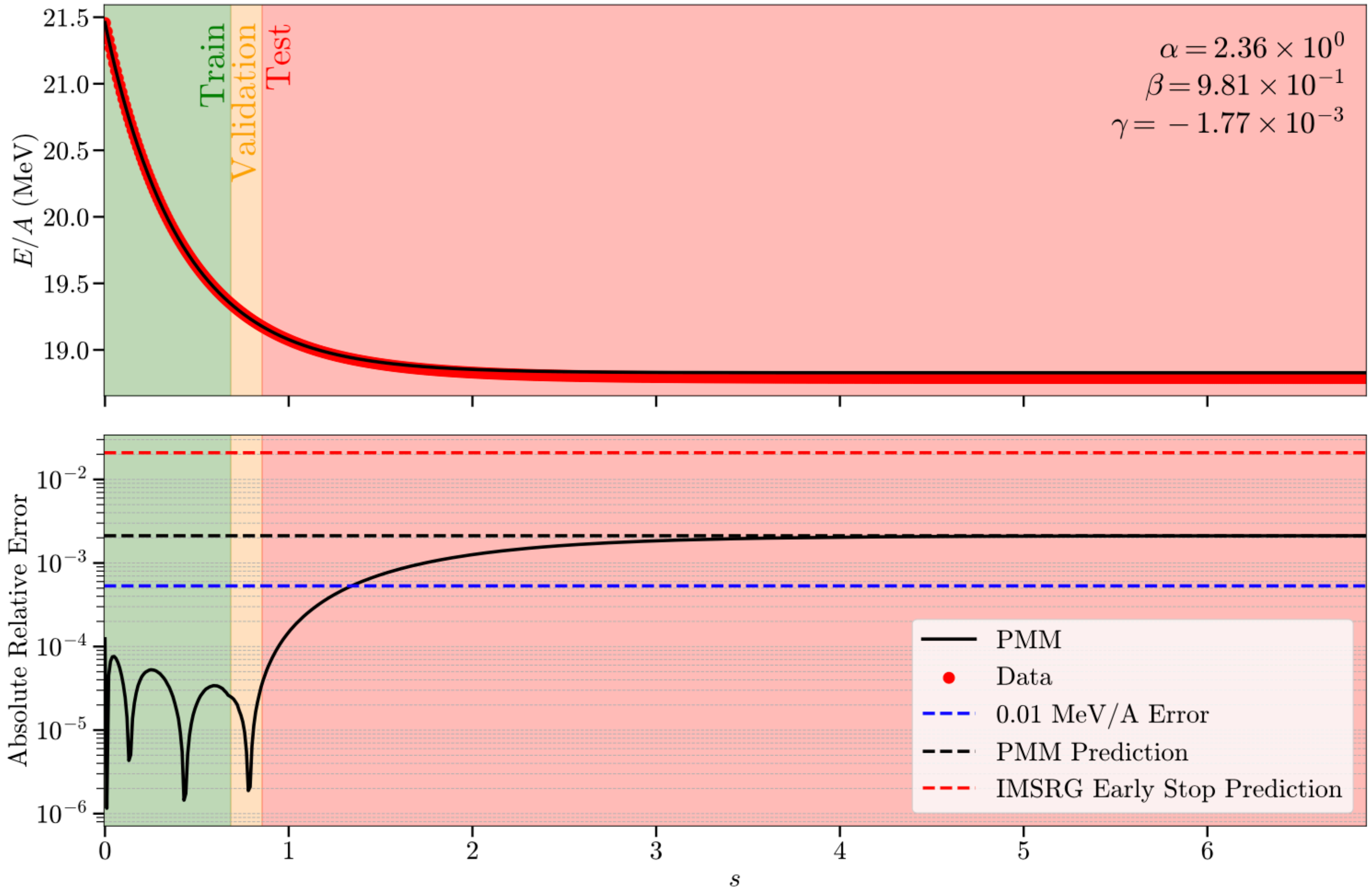
- Shanks Transformation & Padé Approximation





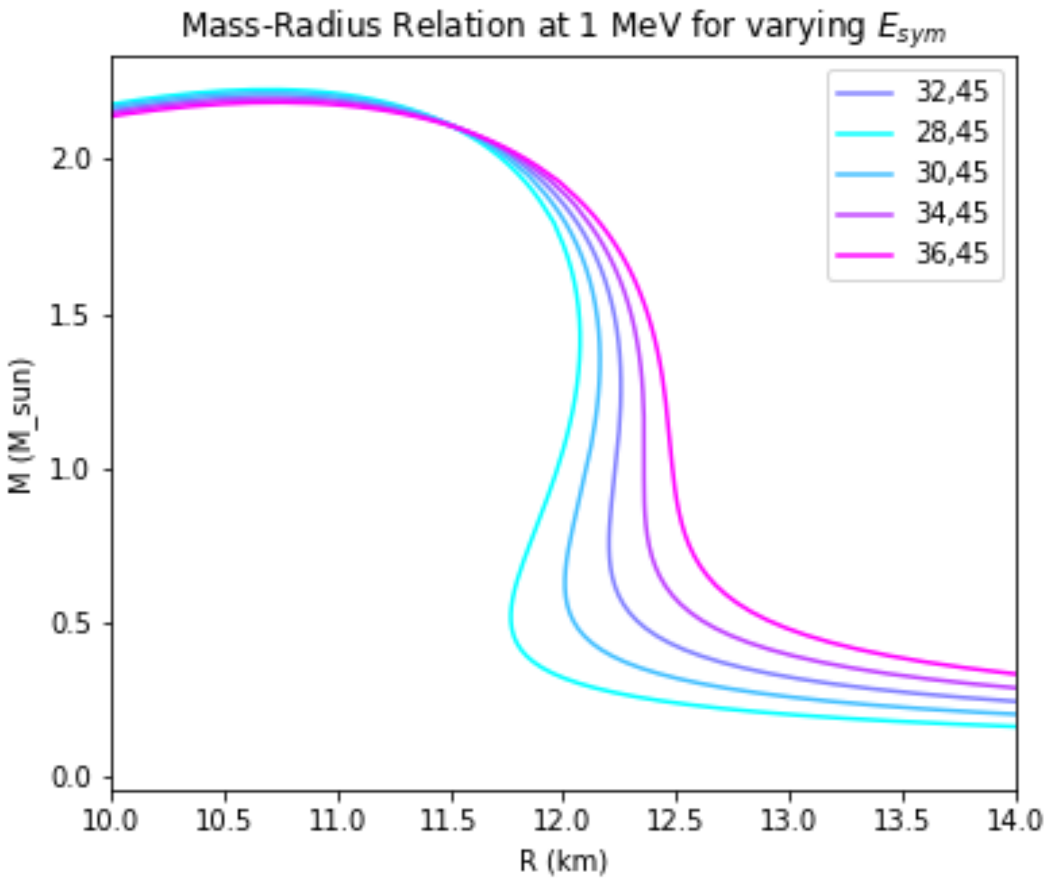
Numerical Techniques

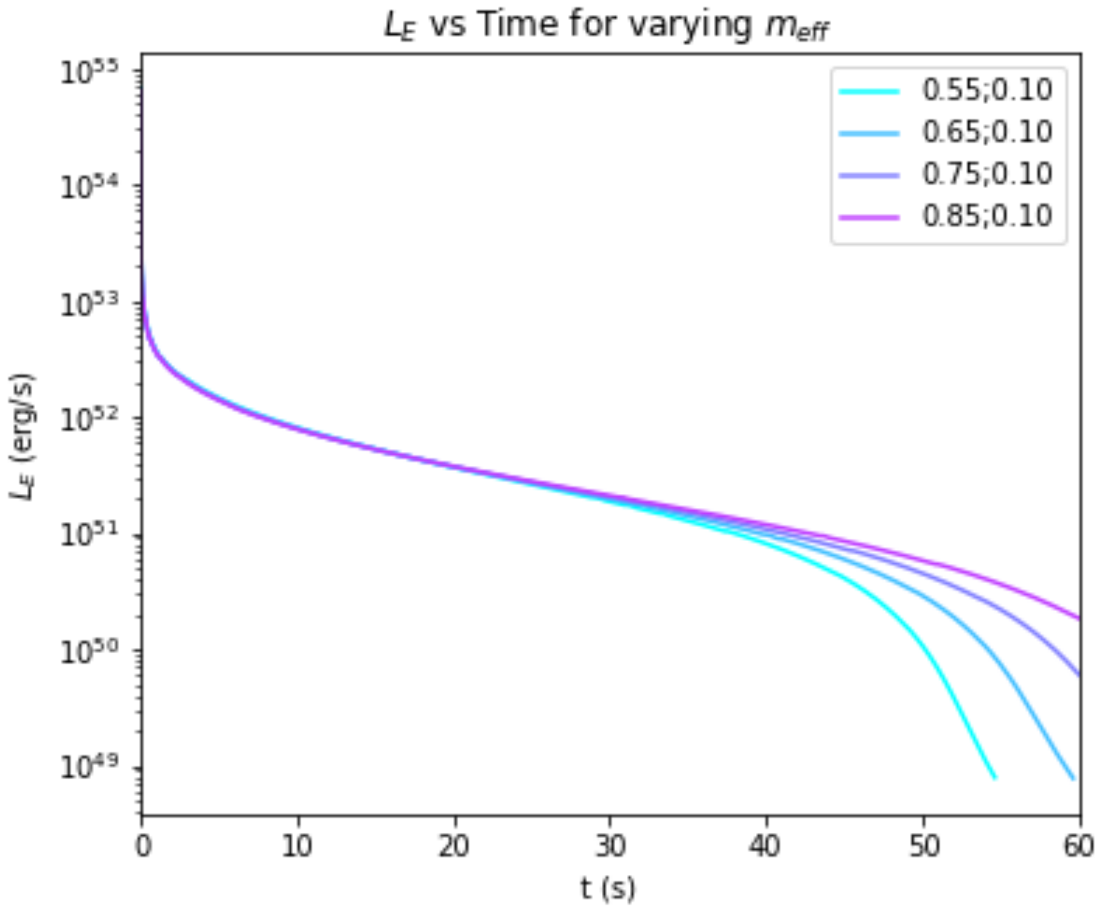
- Bayesian ML and Parametric Matrix Model (PMM)



Outlook - Astrophysical applications

- Outlook





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