Extracting GPDs from CFFs with NN

QCD AT THE FEMTOSCALE IN THE ERA OF BIG DATA

MARCO ZACCHEDDU – JEFFERSON LAB

Outline

Theory:

- What is a GPD?
- How do we construct a GPD?
- Where do we get them?
- Evolution equations
- How do we get the observable?

GPDs extraction with NN

- NN as pixel generator for Double Distributions
- Layers
- Results

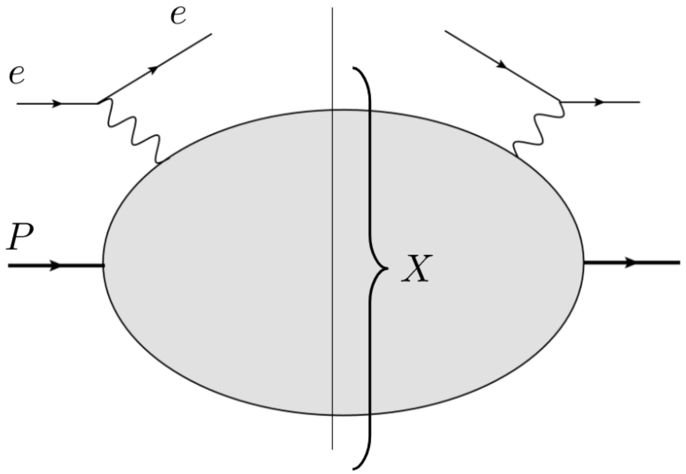
What is a GPD?

GPD : Generalized PDF

PDF : Parton Distribution Function

"Probability" of finding an unpolarized quark with momentum fraction x inside a nucleon

Deep Inelastic Scattering: $eP \to e\ X$



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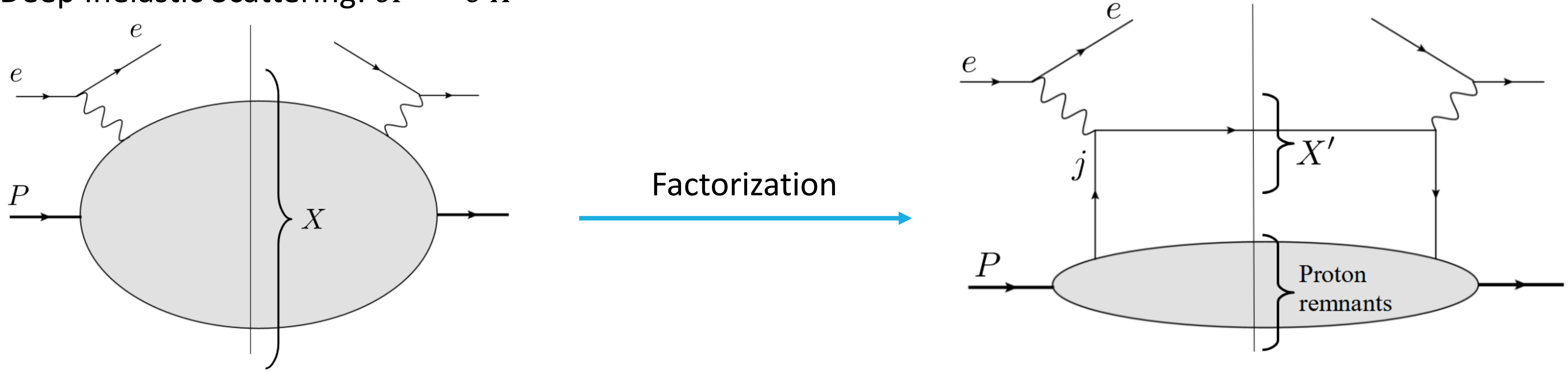

Factorization

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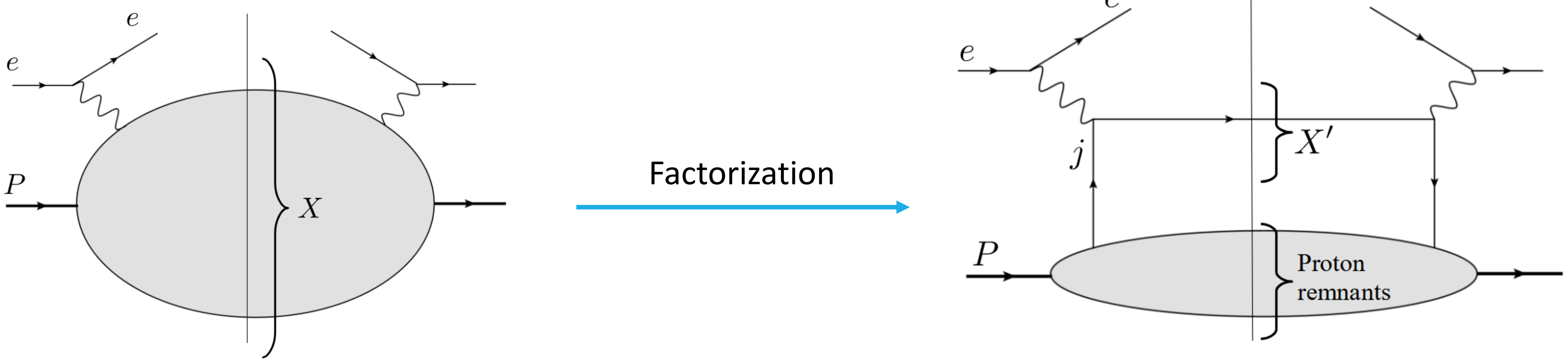

Cross section:

$$\sigma_{eP\to eX} \simeq \hat{\sigma}_{eq\to e} \otimes f_{q/P}(x)$$

PDF

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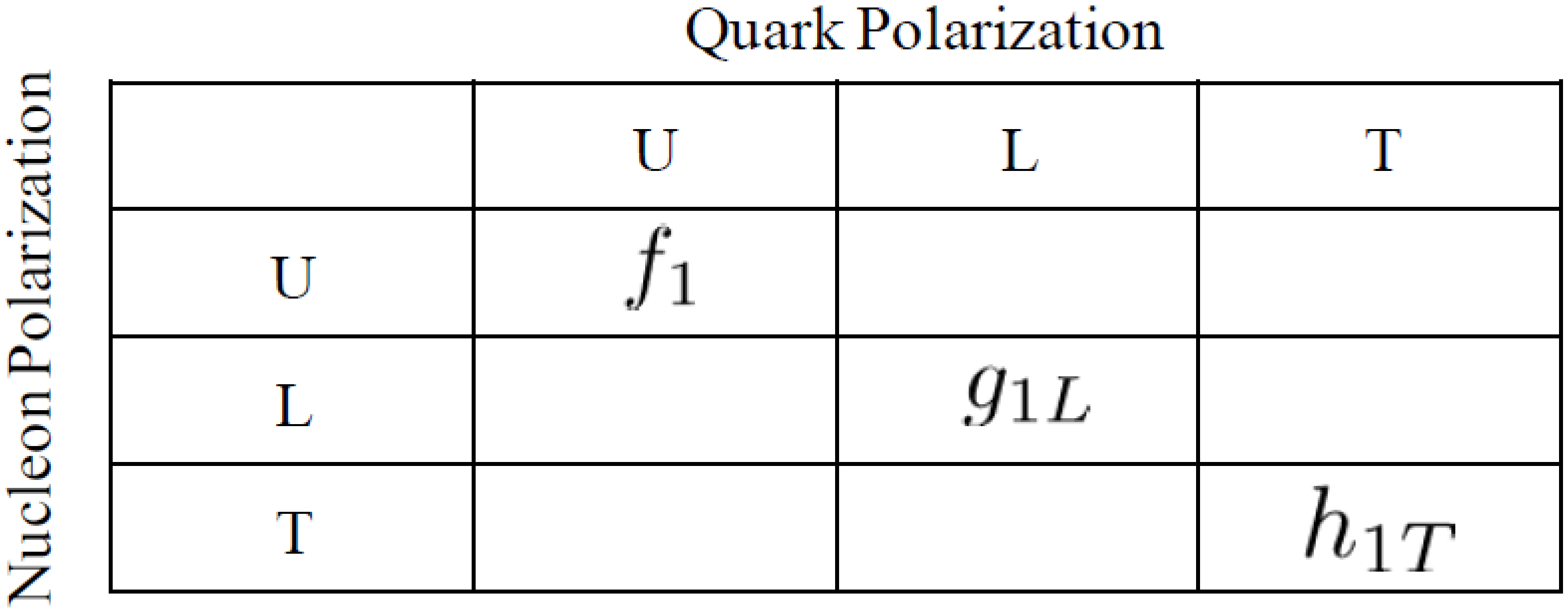

Quark Polarization

Nucleon Polarization

	U	L	T
U	f_1		
L		g_{1L}	
T			h_{1T}

Different PDFs for different parton/nucleon spin configuration

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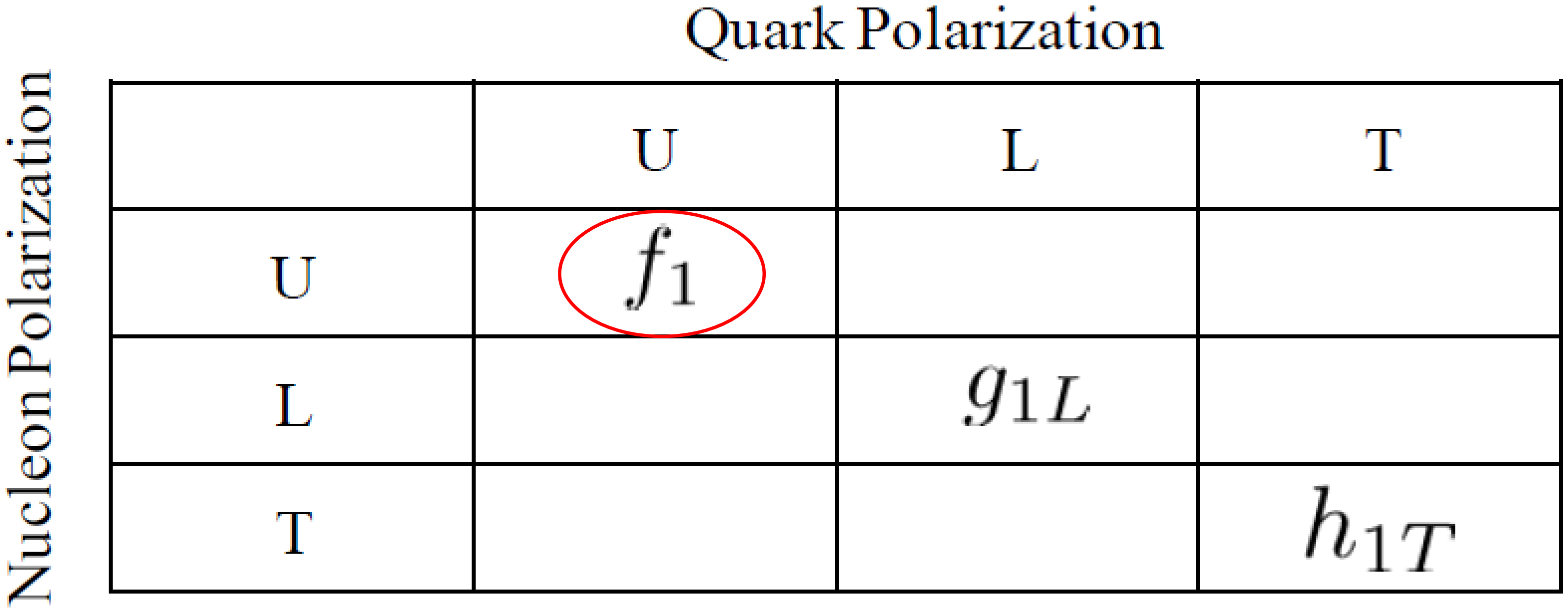

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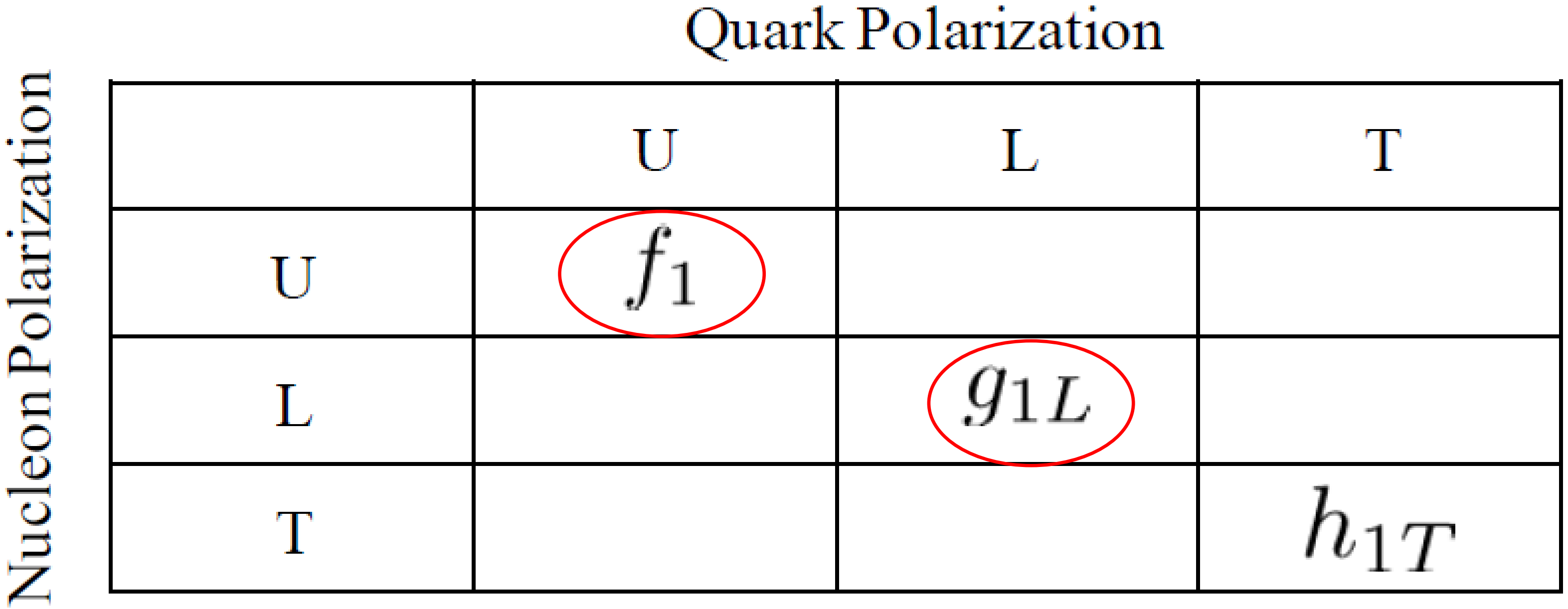

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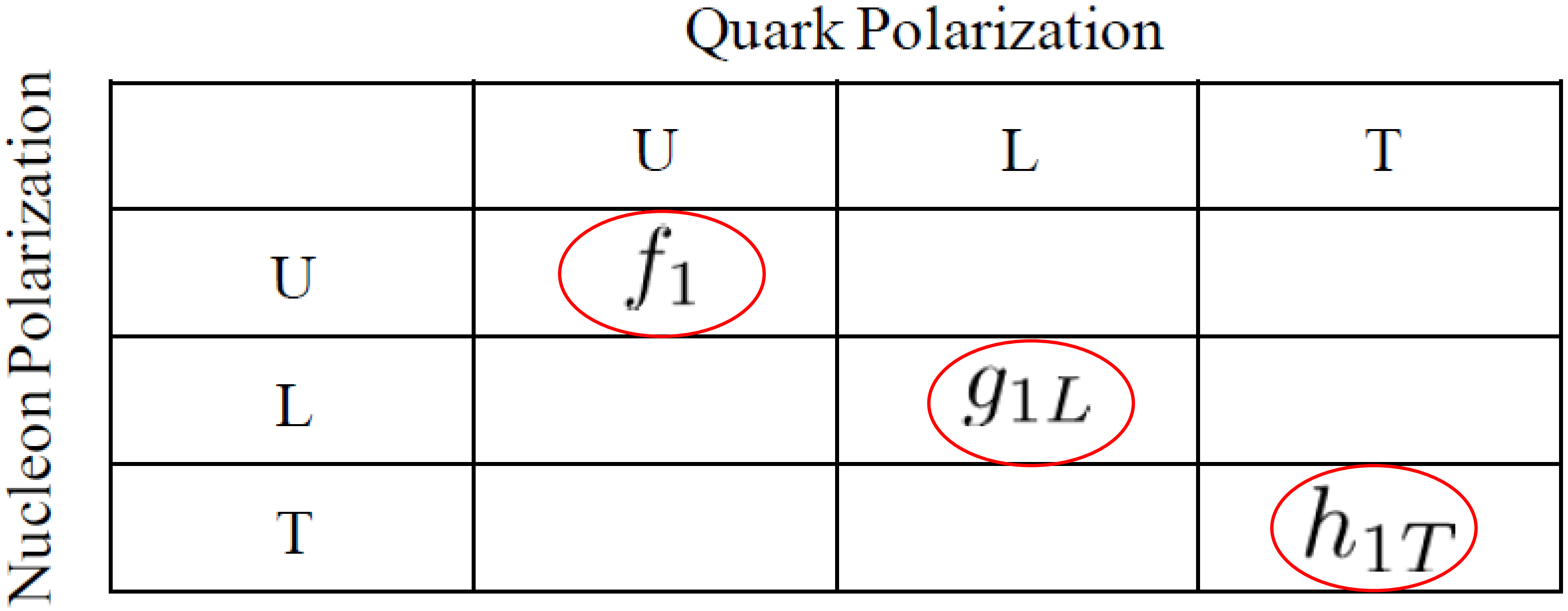

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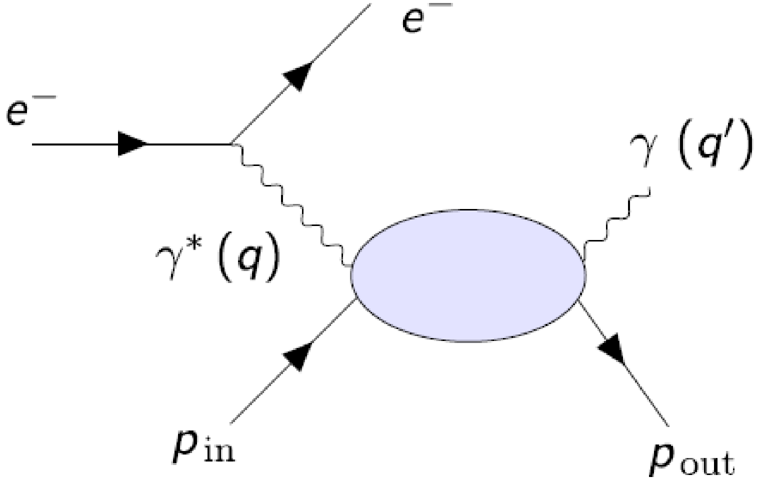
What is a GPD?

DVCS: Deeply Virtual Compton Scattering

$$e^- P_{in} \rightarrow e^- P_{out}\, \gamma$$



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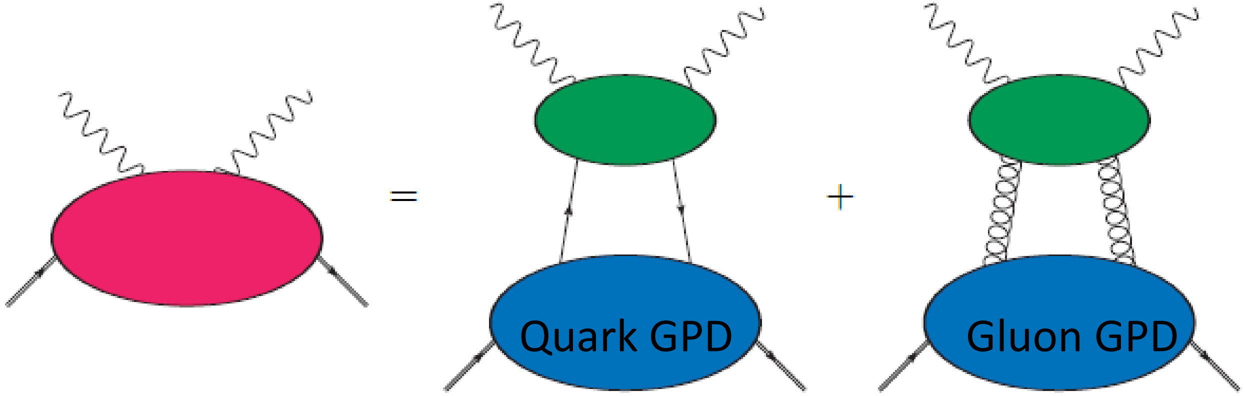

$$F(x,\xi,t)$$ GPDs depend on 3 variables

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$ Skewness

$$P = \frac{p+p'}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2$$

Q^2 dependence → Evolution equations

What is a GPD?

$$\begin{aligned} F^q &= \frac{1}{2}\int\frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^+=0,\,\mathbf{z}=0} \\ &= \frac{1}{2P^+}\left[\boxed{H^q(x,\xi,t)}\,\bar{u}(p')\gamma^+u(p)+\boxed{E^q(x,\xi,t)}\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right] \end{aligned}$$

Unpolarized GPDs for quarks

$$\begin{aligned} \tilde{F}^q &= \frac{1}{2}\int\frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\, q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^+=0,\,\mathbf{z}=0} \\ &= \frac{1}{2P^+}\left[\boxed{\tilde{H}^q(x,\xi,t)}\,\bar{u}(p')\gamma^+\gamma_5u(p)+\boxed{\tilde{E}^q(x,\xi,t)}\,\bar{u}(p')\frac{\gamma_5\Delta^+}{2m}u(p)\right] \end{aligned}$$

Longitudinally polarized GPDs for quarks

Same number of functions for gluons

Polynomiality: the nth-moment are polynomials in ξ of order n+1

$$\int_{-1}^{1} dx\, x^n H^q(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (2\xi)^i A^q_{n+1,i}(t) + \text{mod}(n,2)\,(2\xi)^{n+1} C^q_{n+1}(t).$$

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How do we construct a GPD?

We can obtain the GPDs integrating Double Distribution

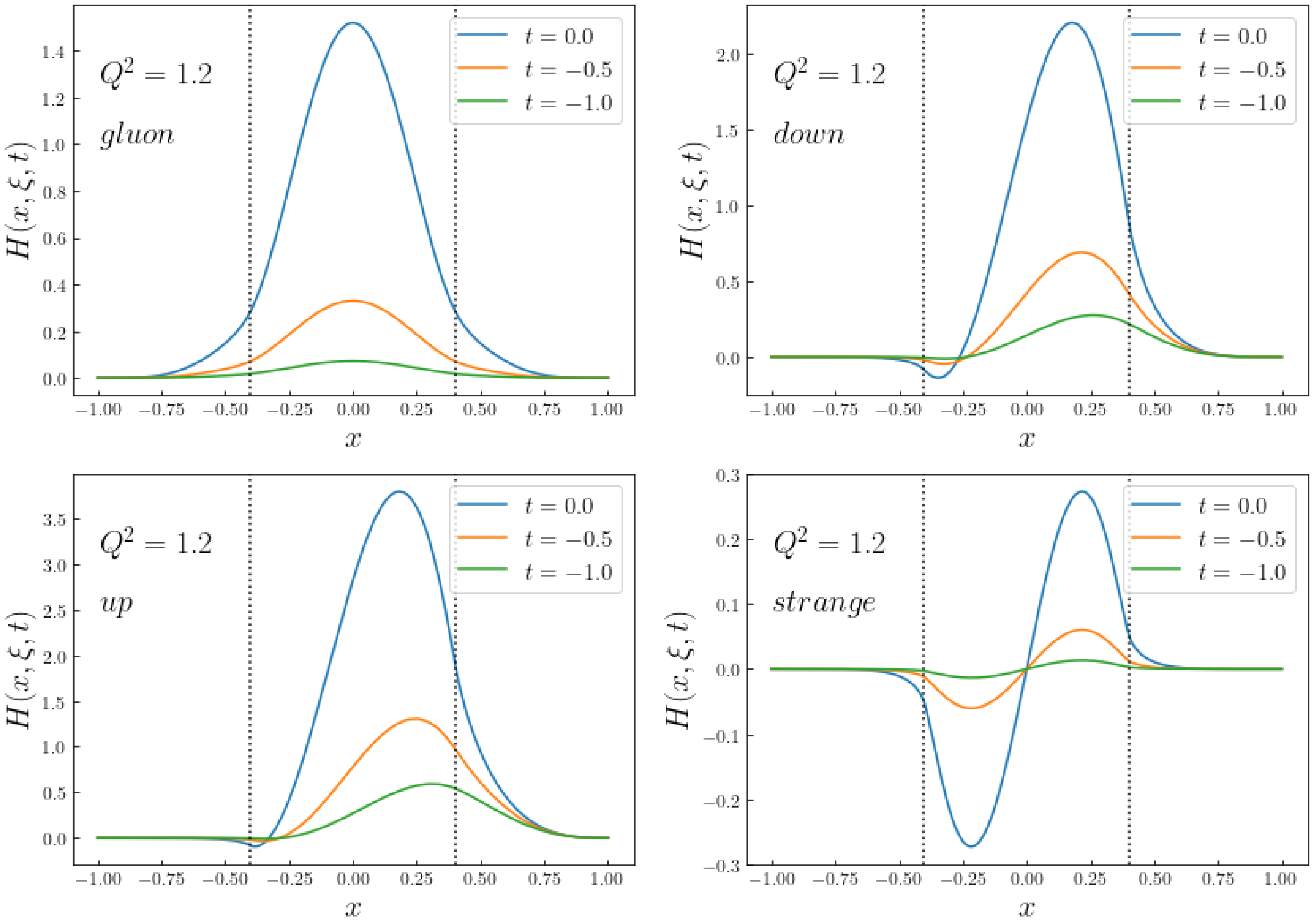
$$\begin{array}{rcl} H^q(x,\xi,t) & = & \int d\beta\, d\alpha\, \delta(x-\beta-\xi\alpha) \\ E^q(x,\xi,t) & = & \int d\beta\, d\alpha\, \delta(x-\beta-\xi\alpha) \end{array} \boxed{\begin{array}{l} f^q(\beta,\alpha,t) \\ k^q(\beta,\alpha,t) \end{array}}$$

They can be represented by 3D tensors

Properties:
DD generate GPDs which automatically satisfy the polynomiality

How do we construct a GPD?

Goloskokov – Kroll model: arxiv.org/pdf/1210.6975



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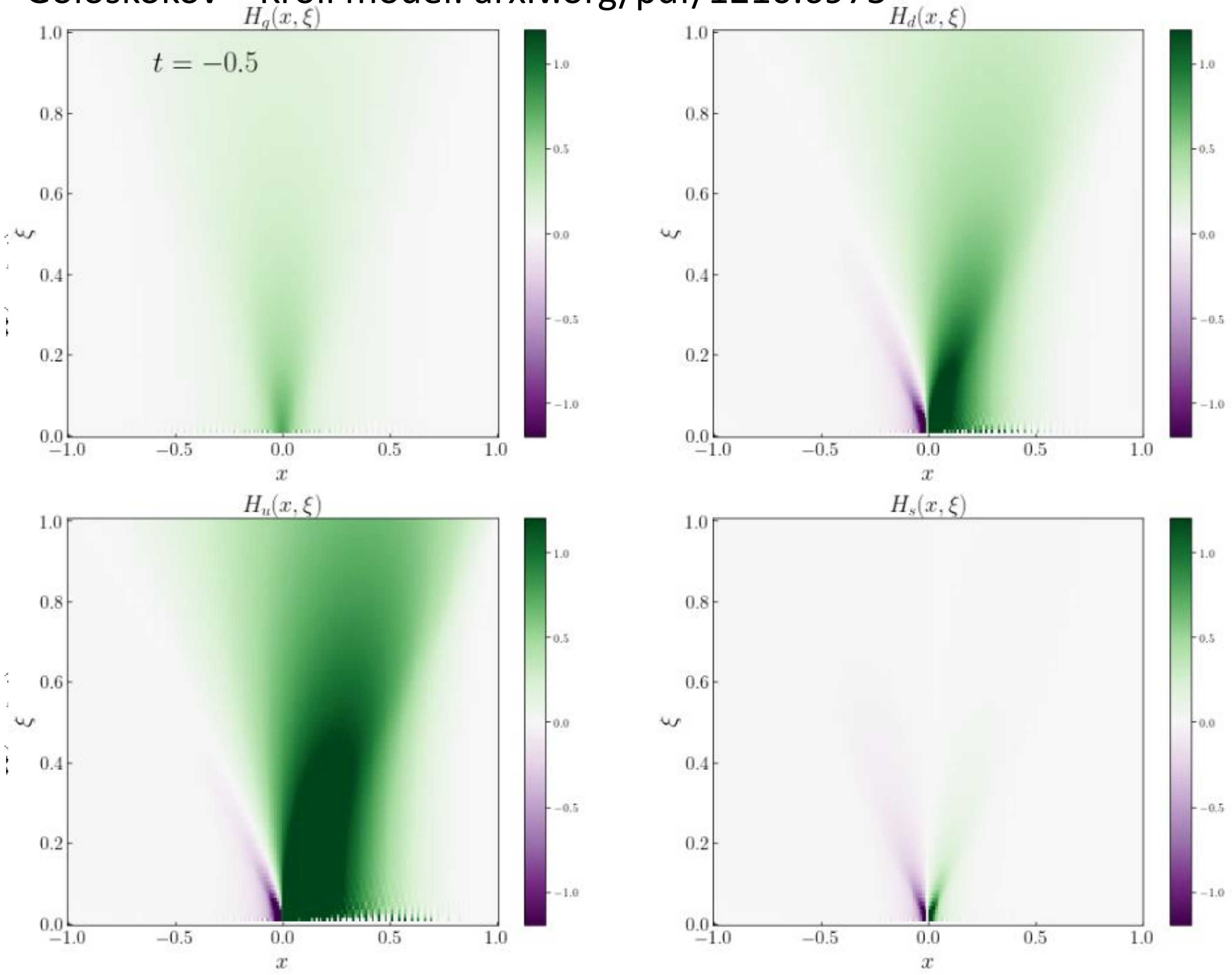

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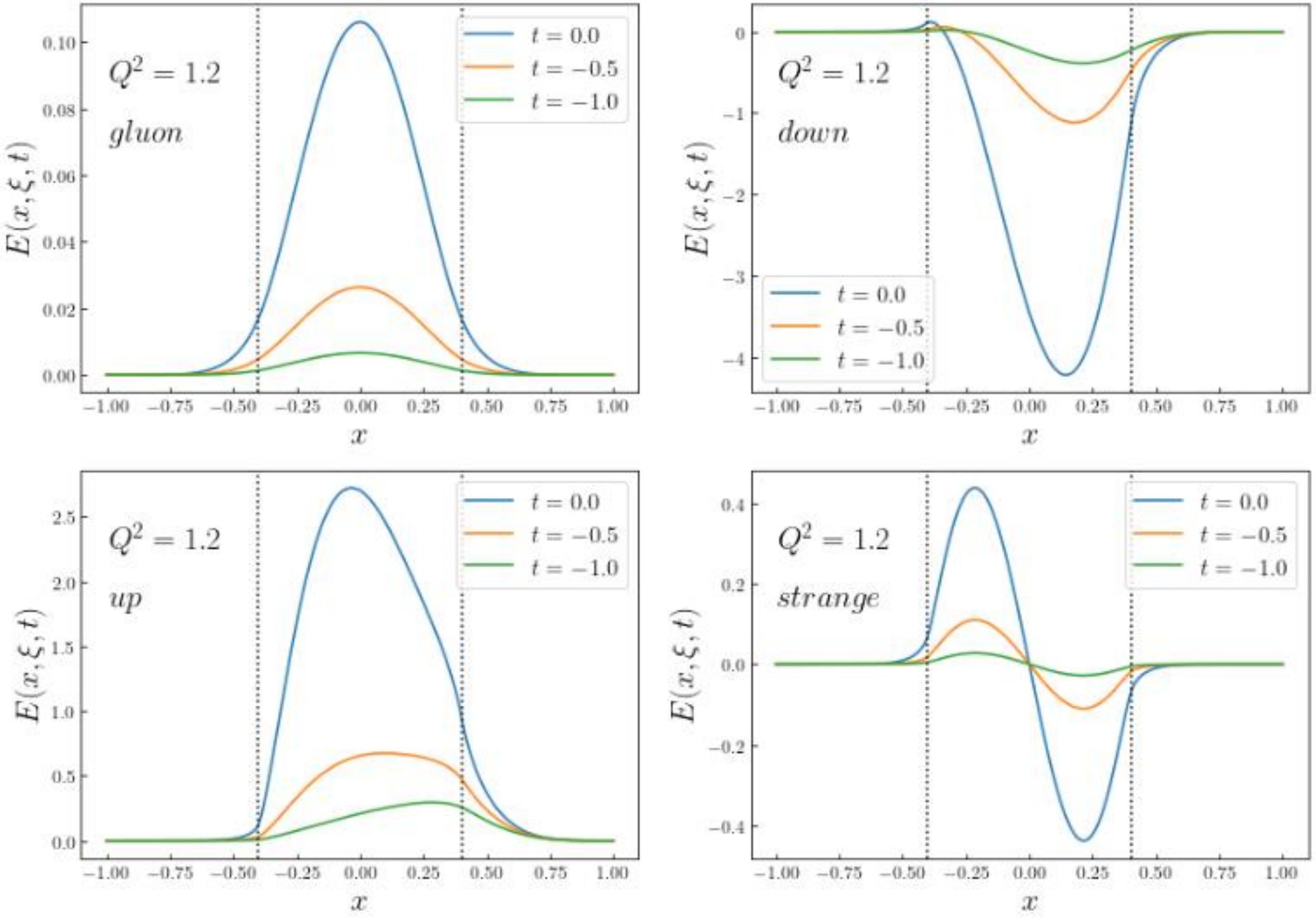

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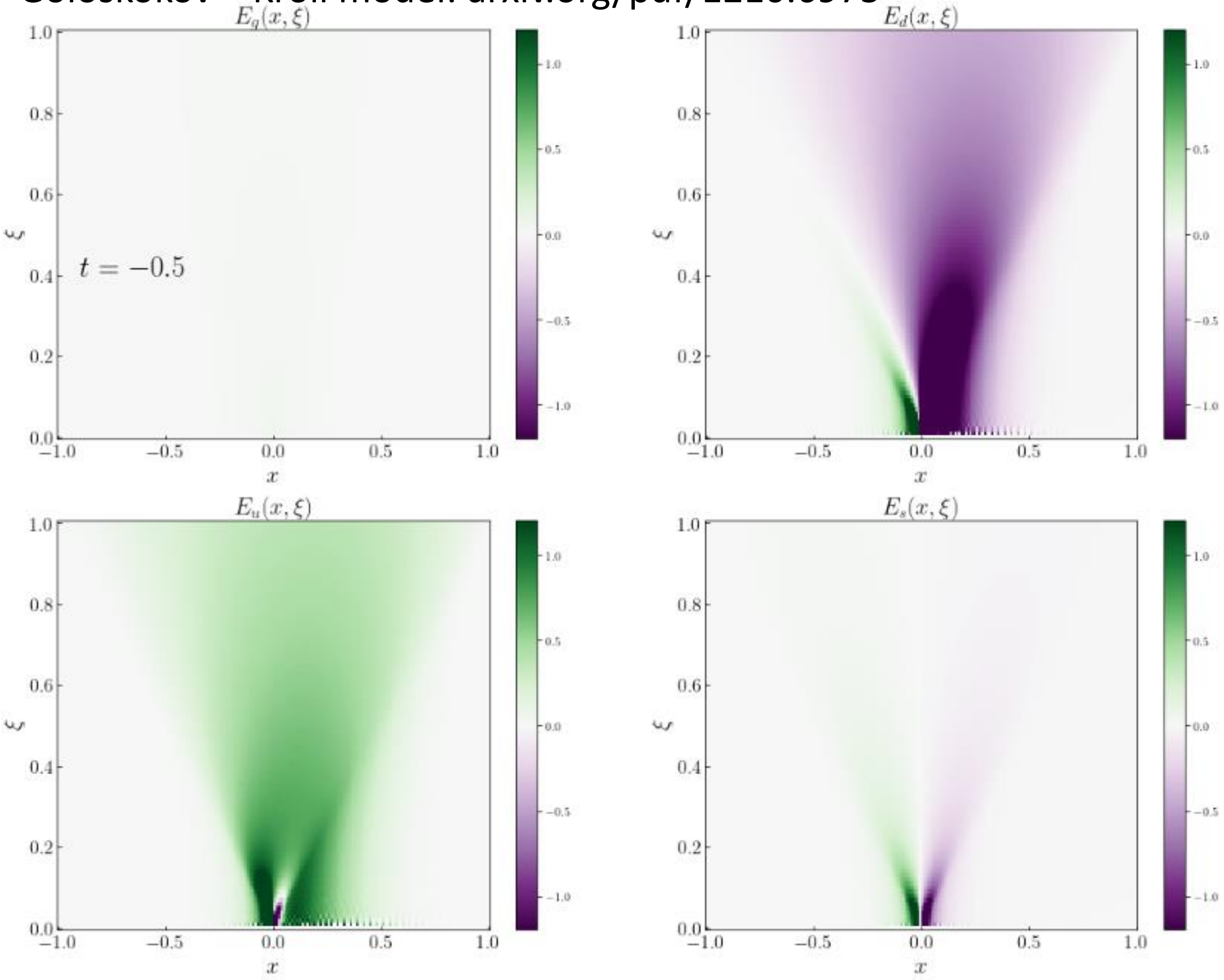

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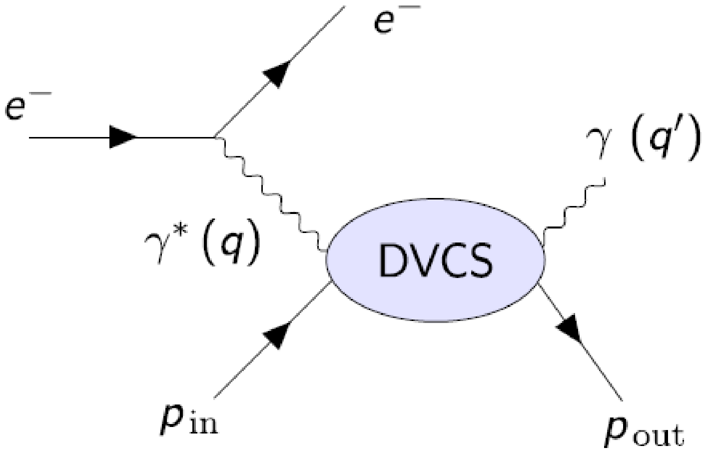
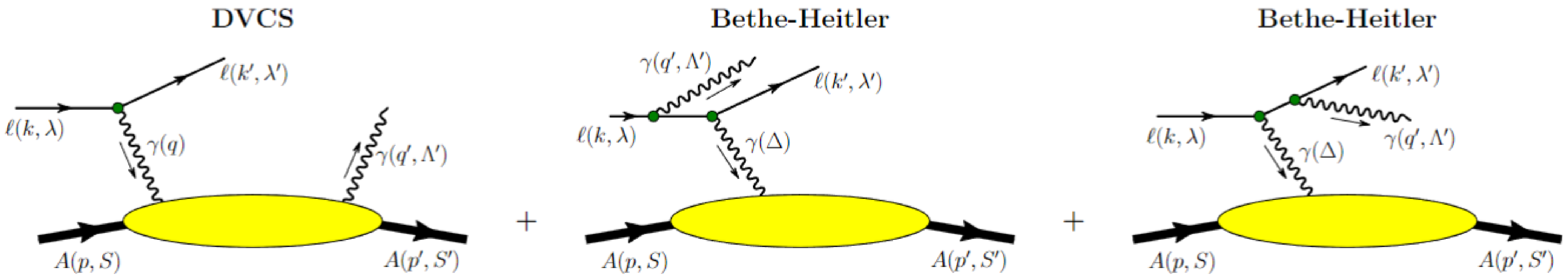
Where do we get them?

DVCS: Deeply Virtual Compton Scattering



Interference with



Where do we get them?

The cross section and the asymmetries are written in terms of the following amplitudes

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = |\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{UU}} + \lambda\, |\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{LU}} + \Lambda \cos\theta \left(|\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{UL}} + \lambda\, |\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{LL}} \right) + \sin\theta \left(|\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{UT}} + \lambda\, |\mathcal{T}_{\mathrm{DVCS}}|^2_{\mathrm{LT}} \right),$$

$$|\mathcal{T}_{\mathrm{BH}}|^2 = |\mathcal{T}_{\mathrm{BH}}|^2_{\mathrm{UU}} + \lambda\, |\mathcal{T}_{\mathrm{BH}}|^2_{\mathrm{LU}} + \Lambda \cos\theta \left(|\mathcal{T}_{\mathrm{BH}}|^2_{\mathrm{UL}} + \lambda\, |\mathcal{T}_{\mathrm{BH}}|^2_{\mathrm{LL}} \right) + \sin\theta \left(|\mathcal{T}_{\mathrm{BH}}|^2_{\mathrm{UT}} + \lambda\, |\mathcal{T}_{\mathrm{BH}}|^2_{\mathrm{LT}} \right)$$

$$\mathcal{I} = \mathcal{I}_{\mathrm{UU}} + \lambda I_{\mathrm{LU}} + \Lambda \cos\theta \left(I_{\mathrm{UL}} + \lambda I_{\mathrm{LL}} \right) + \sin\theta \left(I_{\mathrm{UT}} + \lambda I_{\mathrm{LT}} \right)$$

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Only terms depending on GPDs through CFFs

Where do we get them?

CFFs: Compton Form Factors

$$\{\mathcal{H},\mathcal{E}\}(\xi,t;Q^2) = \int_{-1}^{+1} dx\, C^{(-)}(x,\xi)\,\{H,E\}(x,\xi,t;Q^2)$$

$$\{\tilde{\mathcal{H}},\tilde{\mathcal{E}}\}(\xi,t;Q^2) = \int_{-1}^{+1} dx\, C^{(+)}(x,\xi)\,\{\tilde{H},\tilde{E}\}(x,\xi,t;Q^2)$$

$$C^{(\pm)}(x,\xi) = e_q^2\left(\frac{1}{\xi - x - i\epsilon} \pm \frac{1}{\xi + x - i\epsilon}\right)$$

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$$Re\,\{\mathcal{H},\mathcal{E}\}(\xi,t;Q^2) \;=\; e_q^2\Bigg\{\int_{-1}^{+1} dx\, \frac{\{H,E\}^+(x,\xi,t;Q^2) - \{H,E\}^+(\xi,\xi,t;Q^2)}{\xi - x} + \;\{H,E\}^+(\xi,\xi,t;Q^2)\log\left(\frac{1-\xi}{1+\xi}\right)\Bigg\}$$

$$Im\,\{\mathcal{H},\mathcal{E}\}(\xi,t;Q^2) = e_q^2\,\pi\{H,E\}^+(\xi,\xi,t;Q^2)$$

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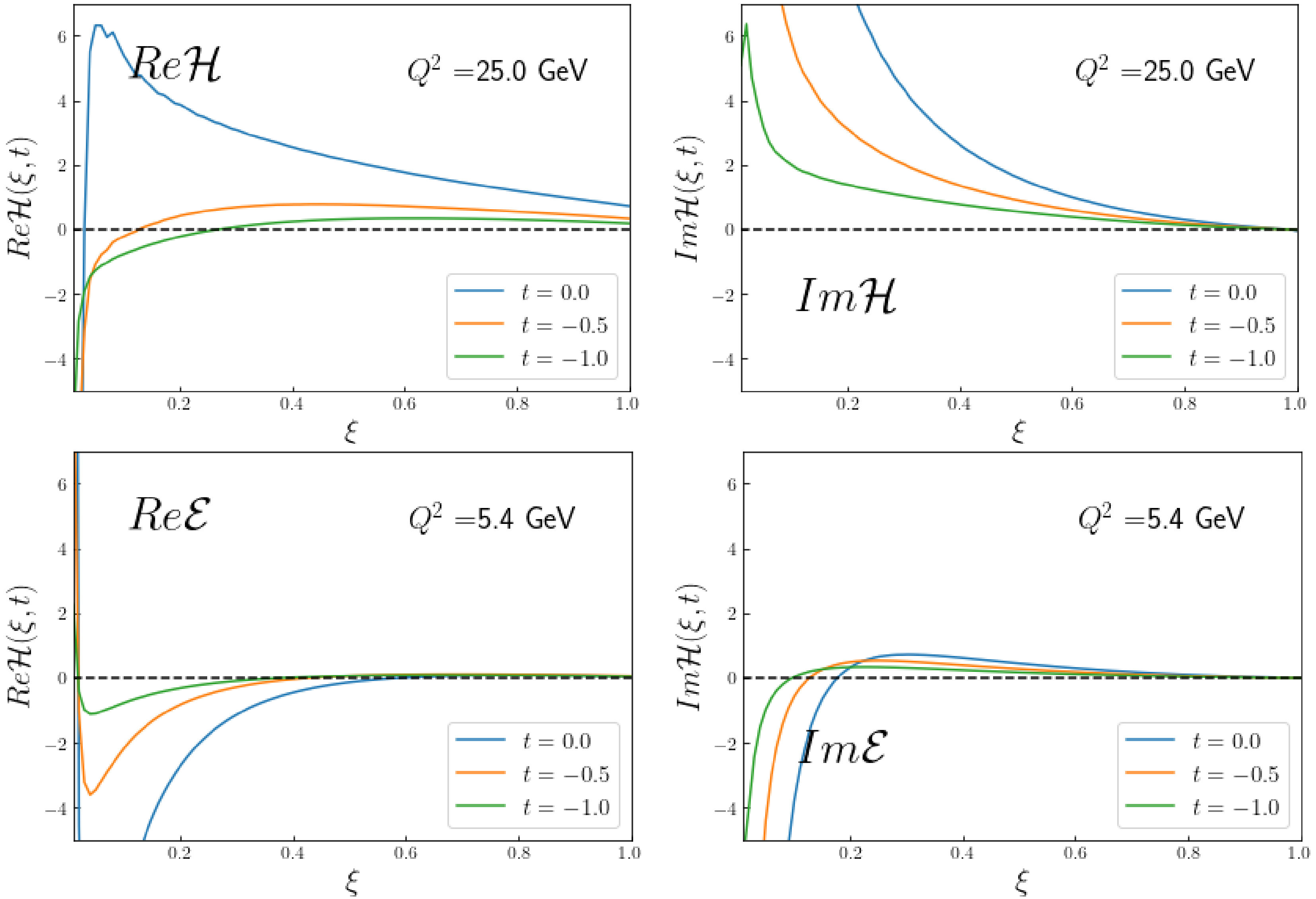
$$Im\,\{\mathcal{H},\mathcal{E}\}(\xi,t;Q^2) = e_q^2\,\pi\{H,E\}^+(\xi,\xi,t;Q^2)$$

$$\{\tilde{\mathcal{H}},\tilde{\mathcal{E}}\}(\xi,t;Q^2) = Re\{\tilde{\mathcal{H}},\tilde{\mathcal{E}}\}(\xi,t;Q^2) + iIm\{\tilde{\mathcal{H}},\tilde{\mathcal{E}}\}(\xi,t;Q^2)$$

Where do we get them?



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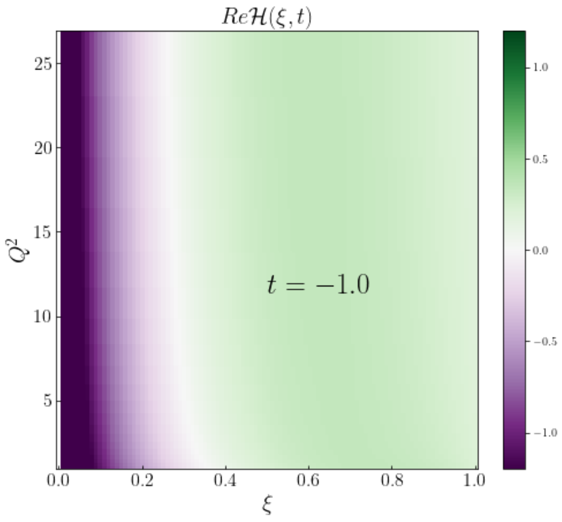
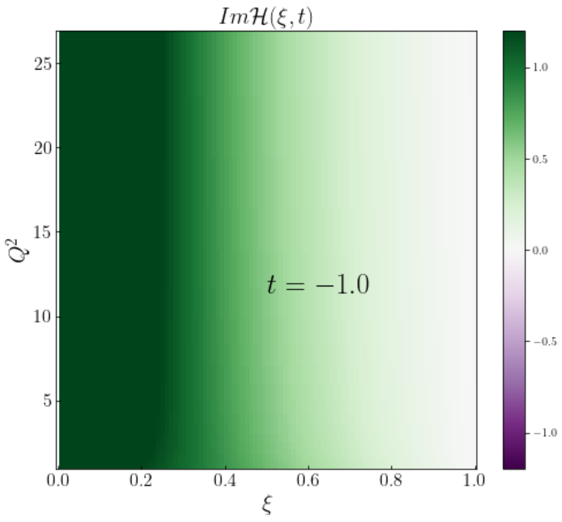

$Re\mathcal{H}(\xi,t)$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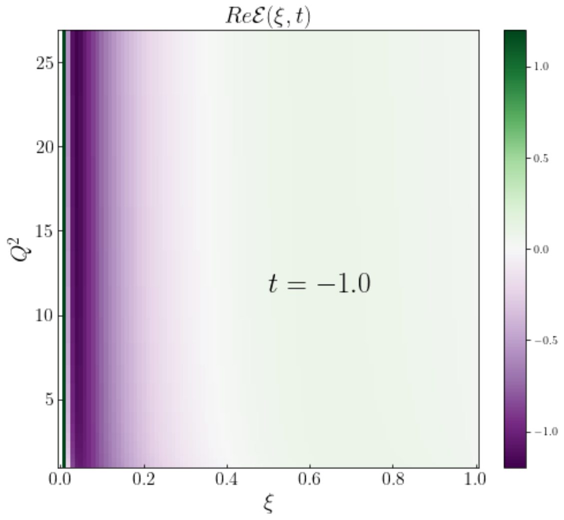
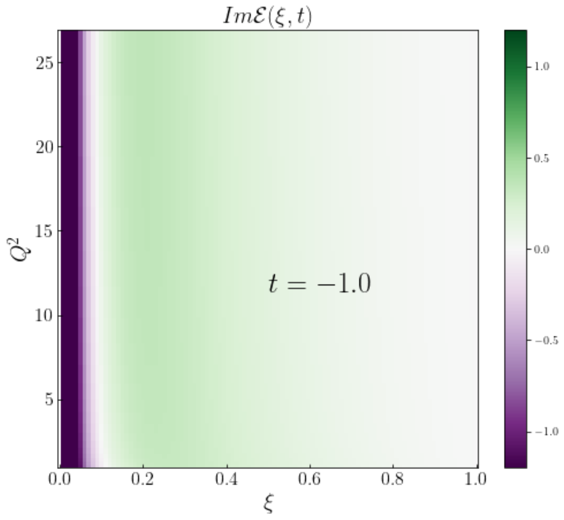

$Re\mathcal{E}(\xi,t)$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$$Im\,\{\mathcal{H},\mathcal{E}\}(\xi,t;Q^2) = e_q^2\,\pi\{H,E\}^+(\xi,\xi,t;Q^2)$$

Inversion Problem: CFFs depend only on ξ. We cannot recover the x dependence of the GPDs

Q^2 dependence → Evolution equations

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Evolution equations

Non-Singlet evolution equation:

$$\mu^2 \frac{d}{d\mu^2} H^{q(-)}(x,\xi,t) = \int_{-1}^{1} dx' \, \frac{1}{|\xi|} V_{\rm NS}\Big(\frac{x}{\xi}, \frac{x'}{\xi}\Big) H^{q(-)}(x',\xi,t)$$

Solve evolution equation with Runge-Kutta 4

Singlet evolution equation:

$$H^{q(-)} \to \left(\begin{array}{c} (2n_f)^{-1} \sum_{q}^{n_f} H^{q(+)} \\ H^g \end{array} \right)$$

$$V_{\rm NS}\Big(\frac{x}{\xi}, \frac{x'}{\xi}\Big) \to \left(\begin{array}{cc} V^{qq}\Big(\frac{x}{\xi}, \frac{x'}{\xi}\Big) & \frac{1}{\xi} V^{qg}\Big(\frac{x}{\xi}, \frac{x'}{\xi}\Big) \\ \xi V^{gq}\Big(\frac{x}{\xi}, \frac{x'}{\xi}\Big) & V^{gg}\Big(\frac{x}{\xi}, \frac{x'}{\xi}\Big) \end{array} \right)$$

Evolution equations

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$$\frac{d\,H(x,\xi,t)}{d\,ln\mu^2} = \int_{-1}^{1} dy\, K(x,y,\xi)\, H(y,\xi,t)$$

$$= \sum_g w_g K(x_j,\eta_g,\xi_n) H(\eta_g,\xi_n,t_t)$$

→ Gaussian quadrature

$$= \sum_g w_g K(x_j,\eta_g,\xi_n) L(\eta_g,x_l) H(x_l,\xi_n,t_l)$$

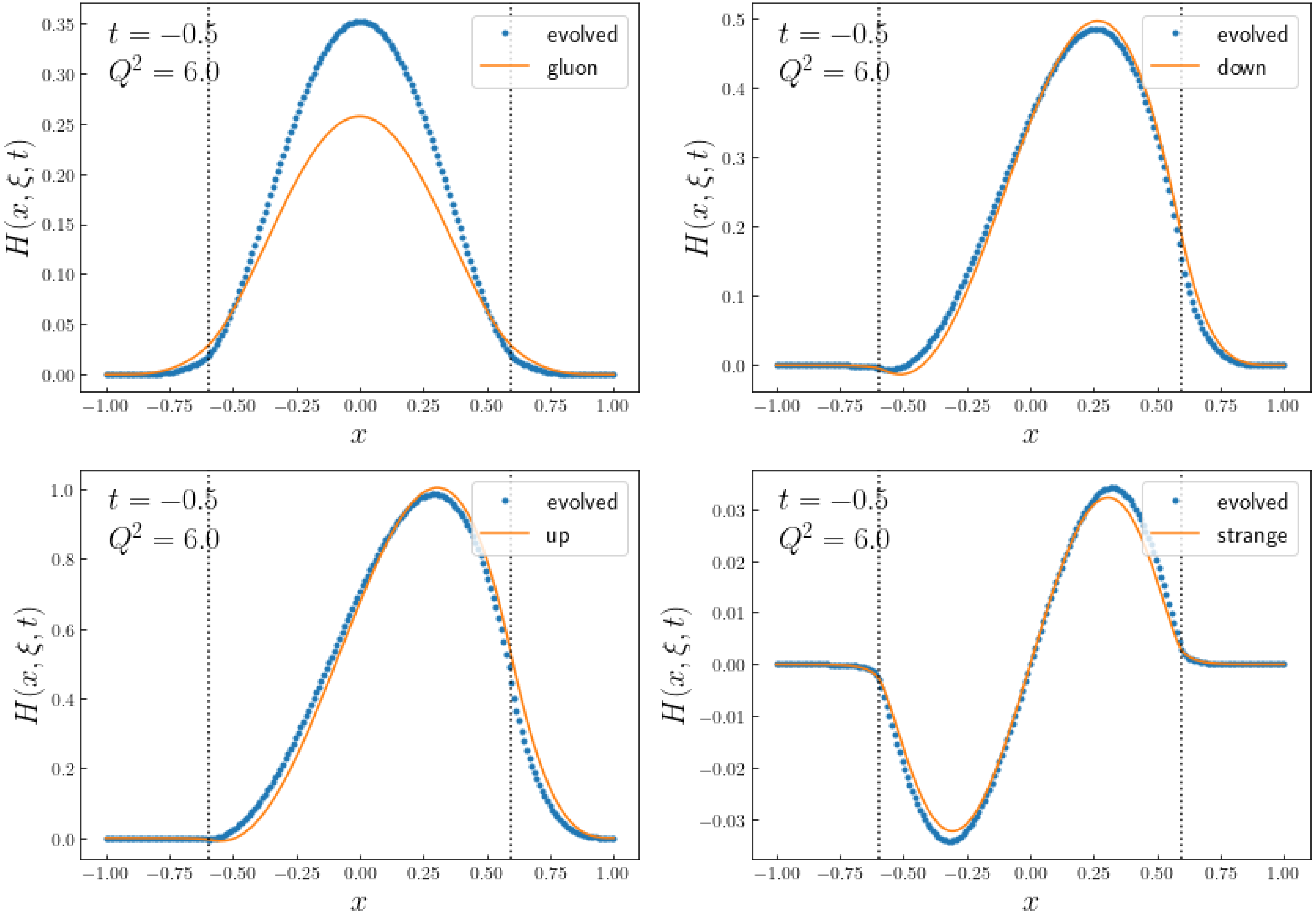
→ Interpolation

$$= \sum_g w_g K_{jgn} L_{jgl} H_{lnt}$$

$$= M_{jln} H_{lnt}$$

→ Matrix multiplication

Evolution equations



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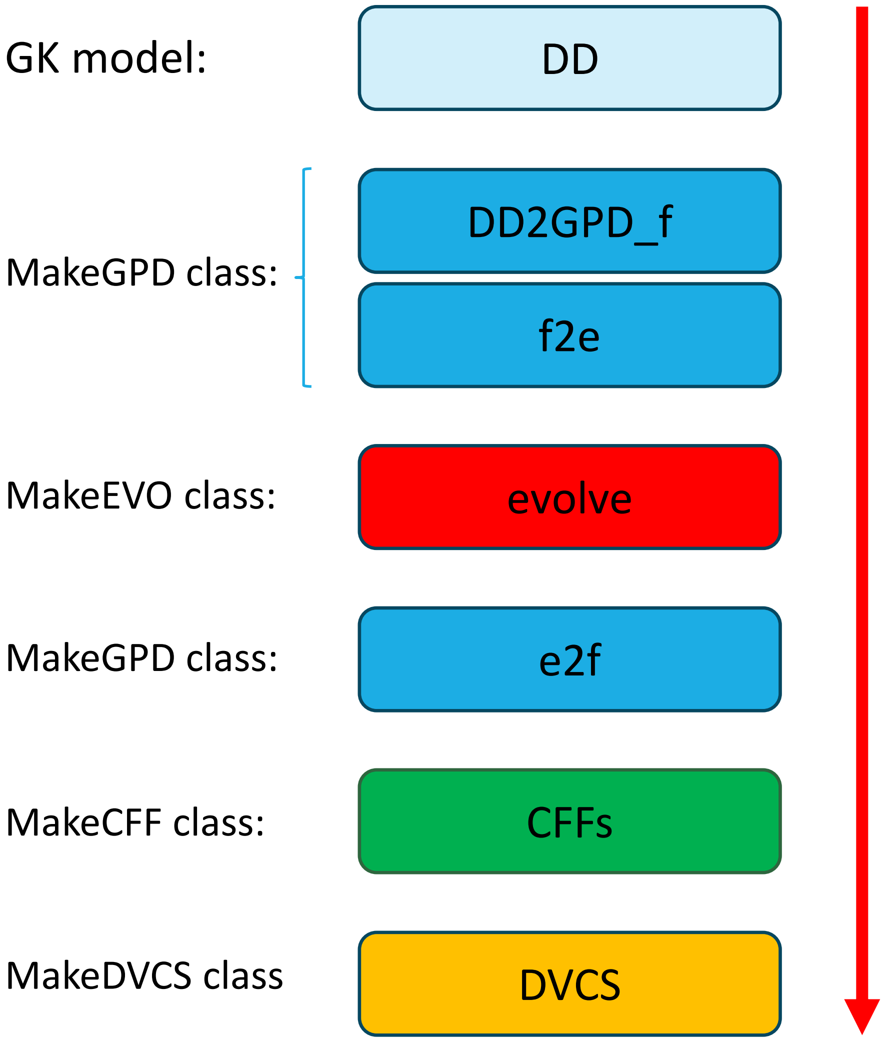

GK model: DD

MakeGPD class: DD2GPD_f, f2e

MakeEVO class: evolve

MakeGPD class: e2f

MakeCFF class: CFFs

MakeDVCS class DVCS

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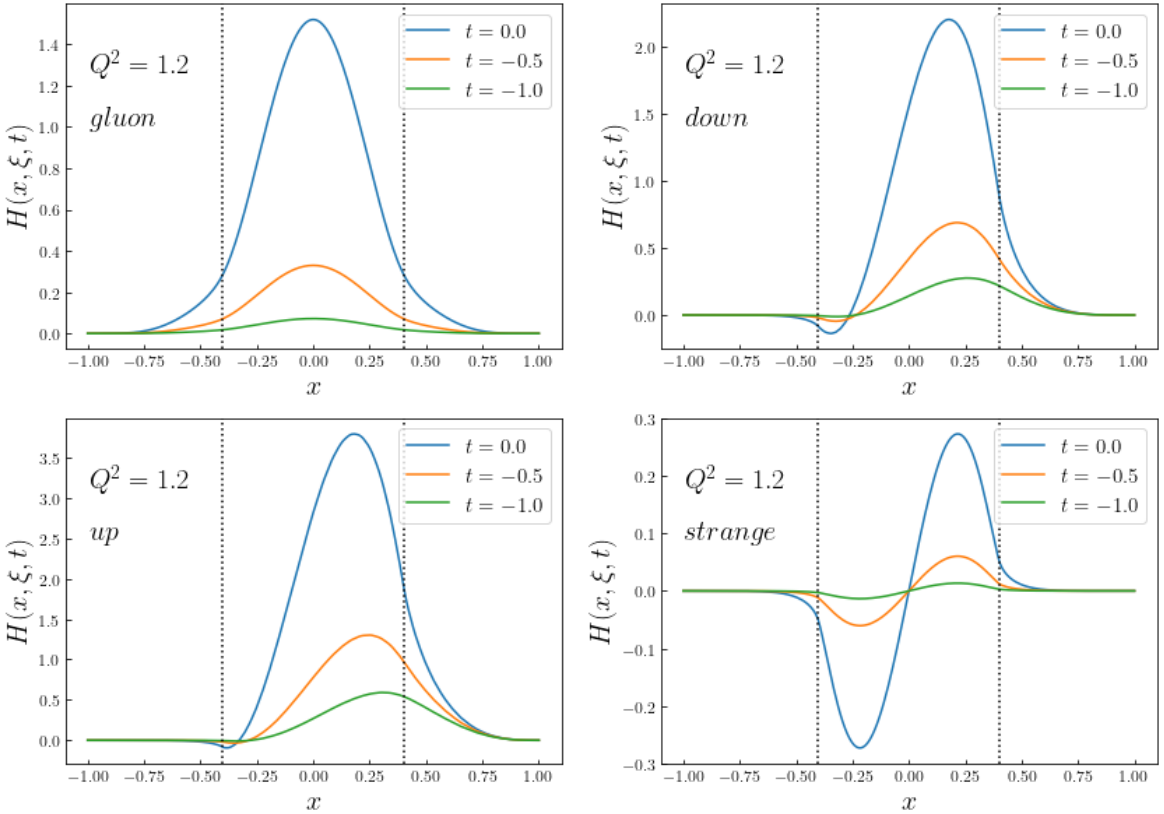

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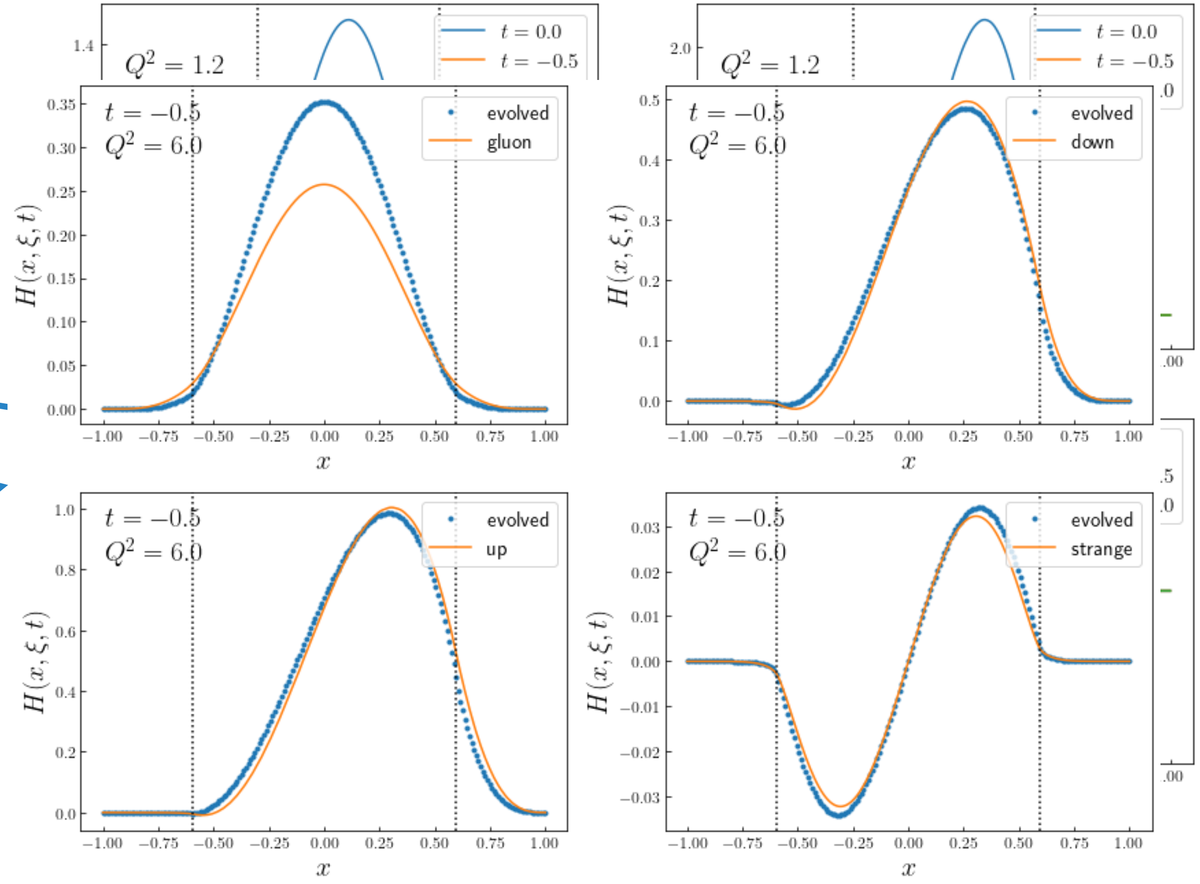

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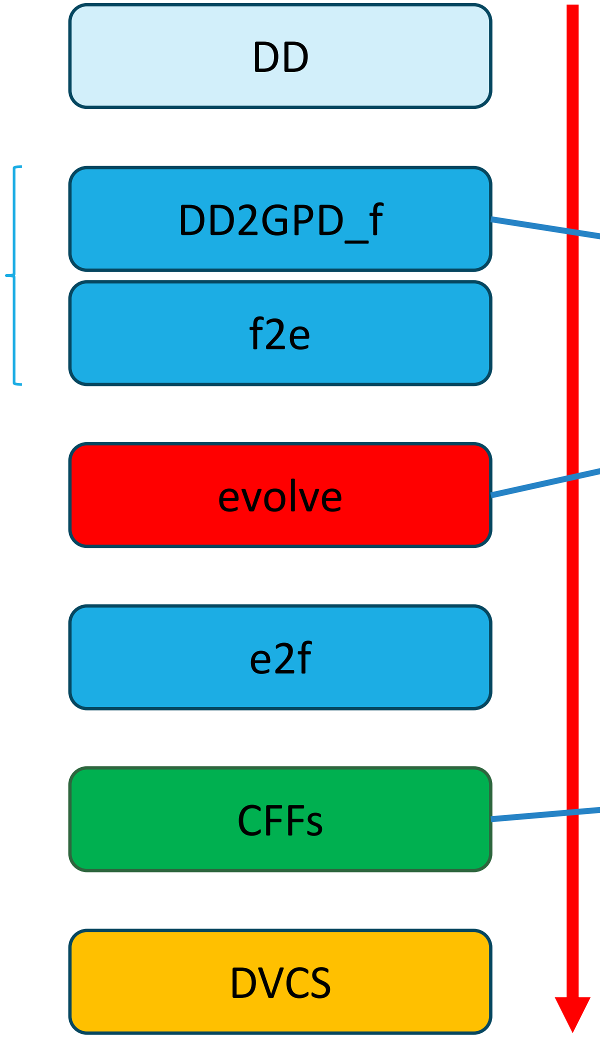

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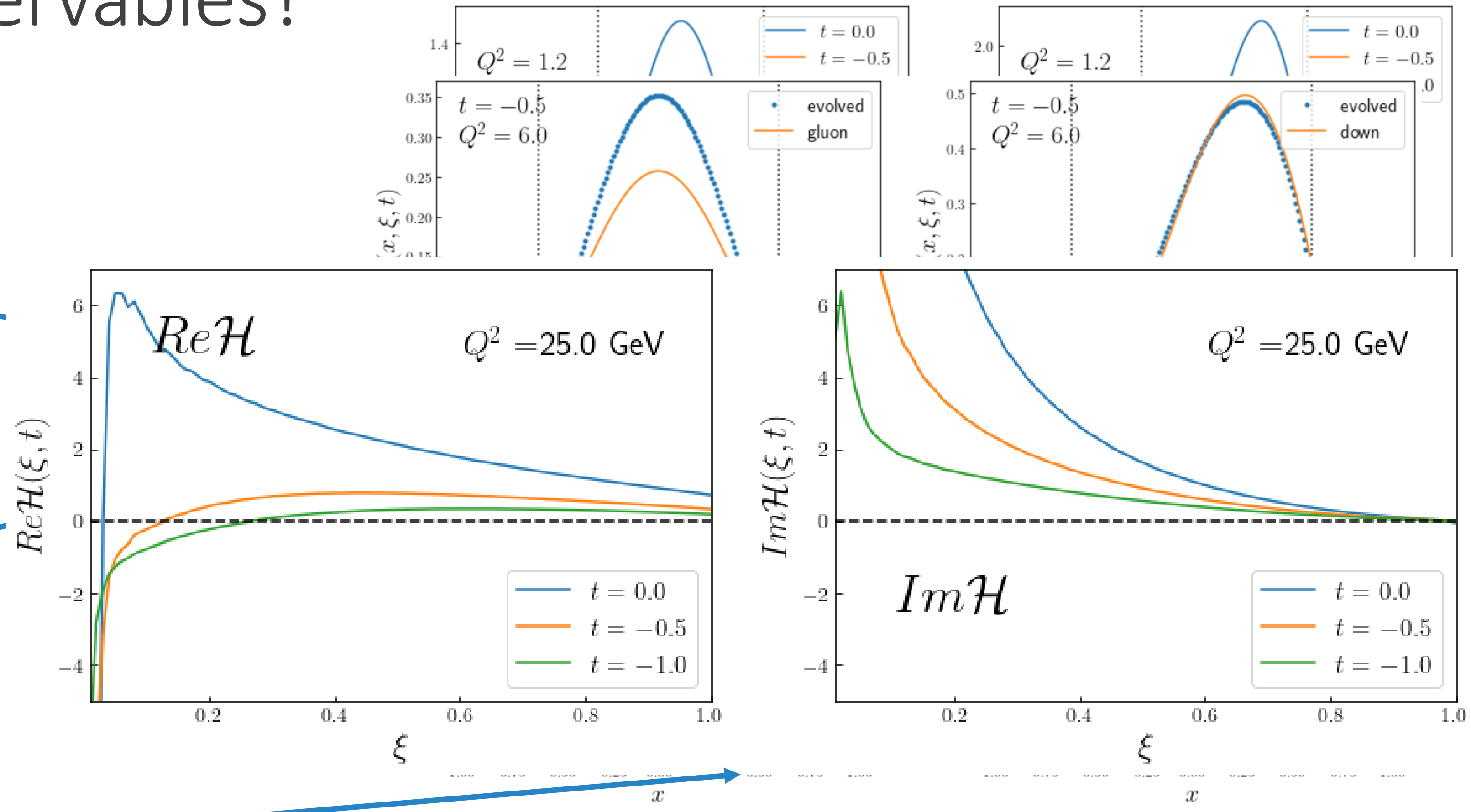

How do we get the observables?

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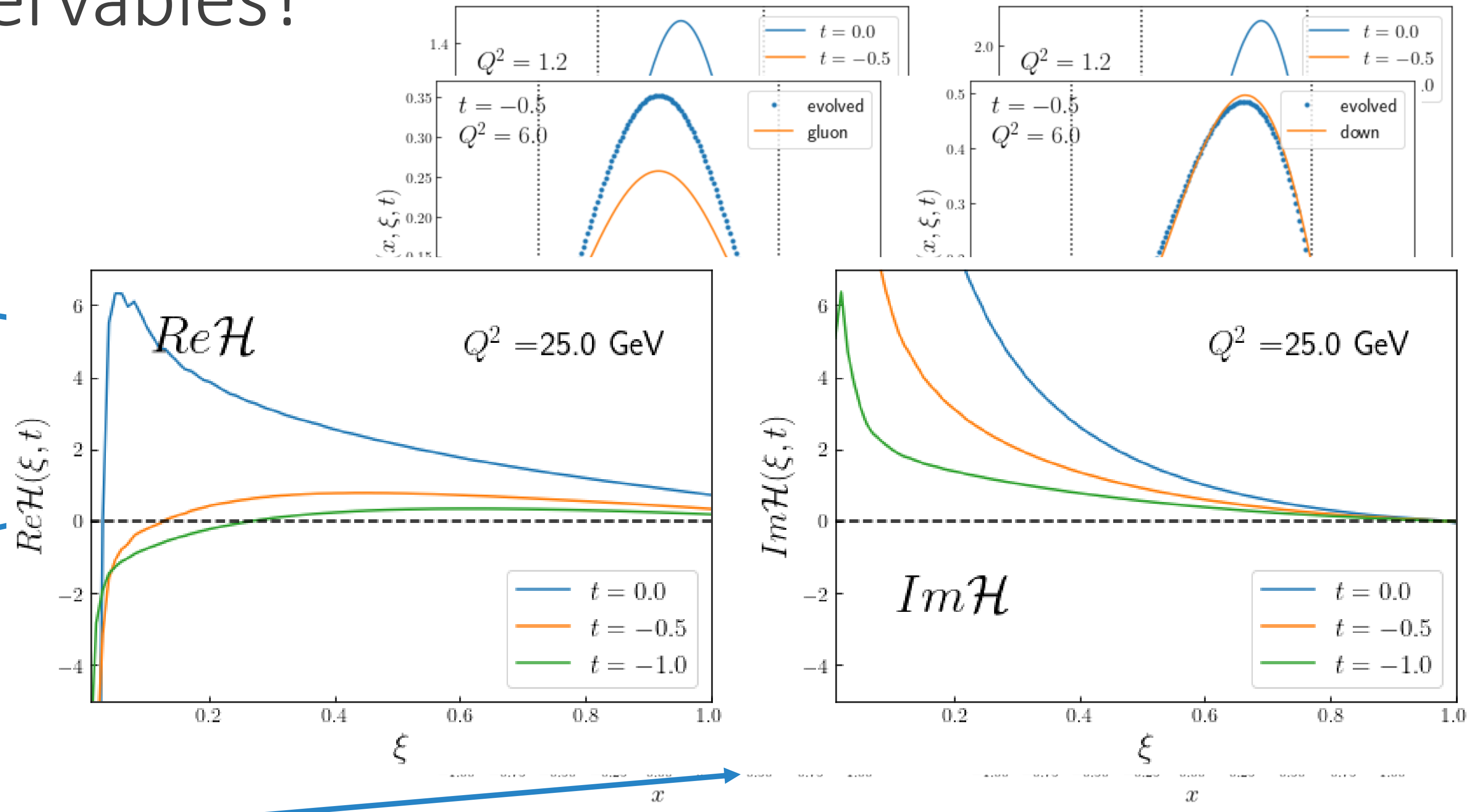
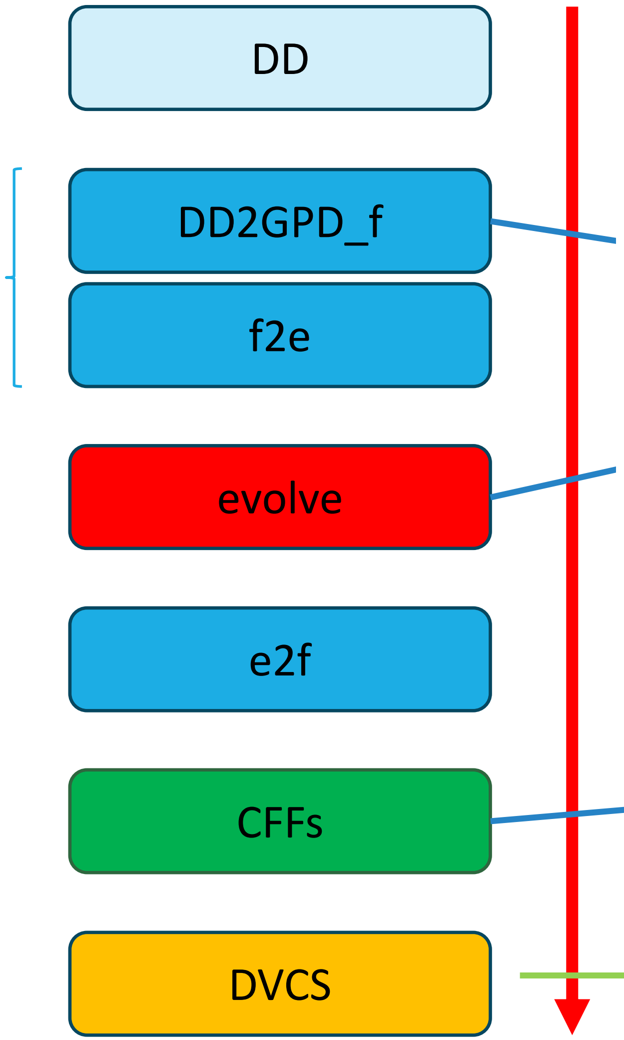


In progress

How do we get the observables?



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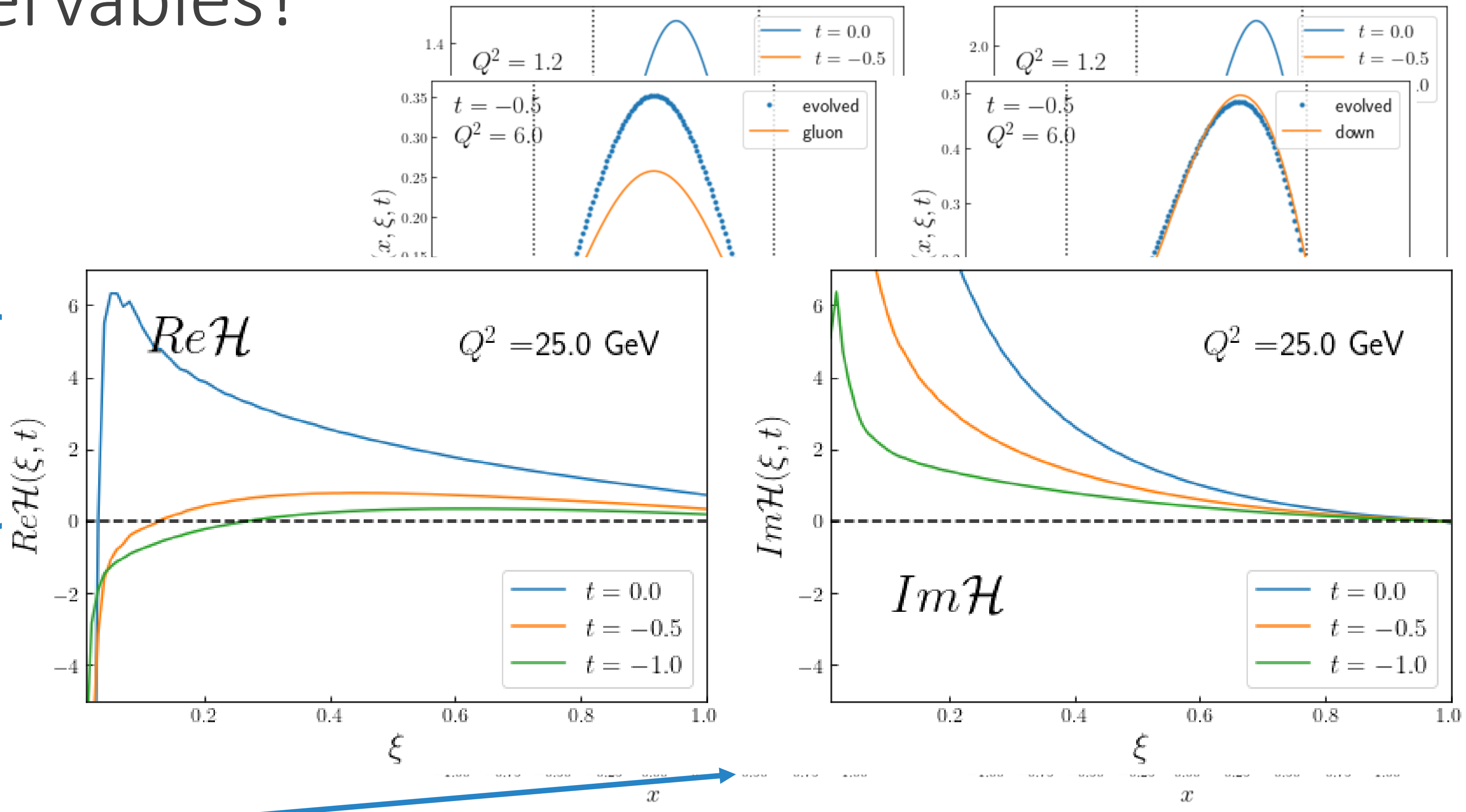

Closure test and extract the GPDs

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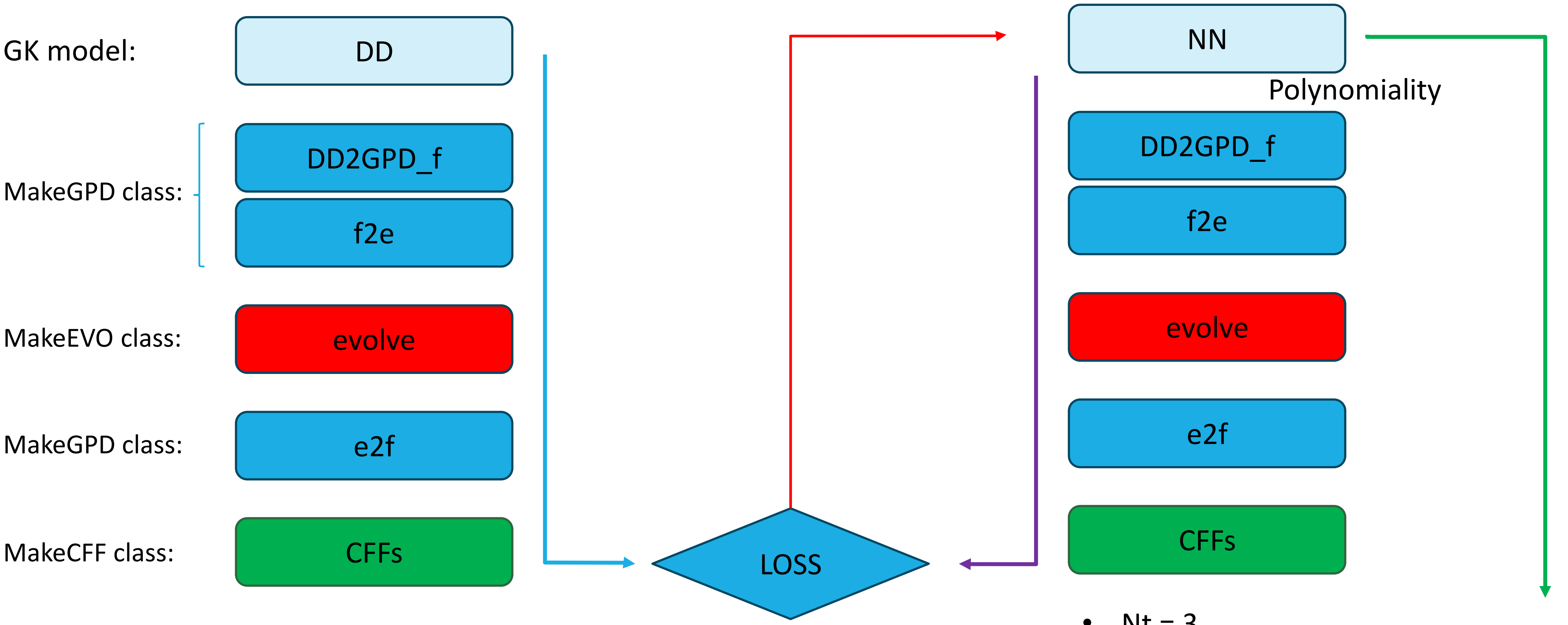
- How do we construct a GPD?
- Where do we get them?
- Evolution equations
- How do we get the observable?

GPDs extraction with NN
- NN as pixel generator for Double Distributions
- Layers
- Results

Extracting GPDs with NN

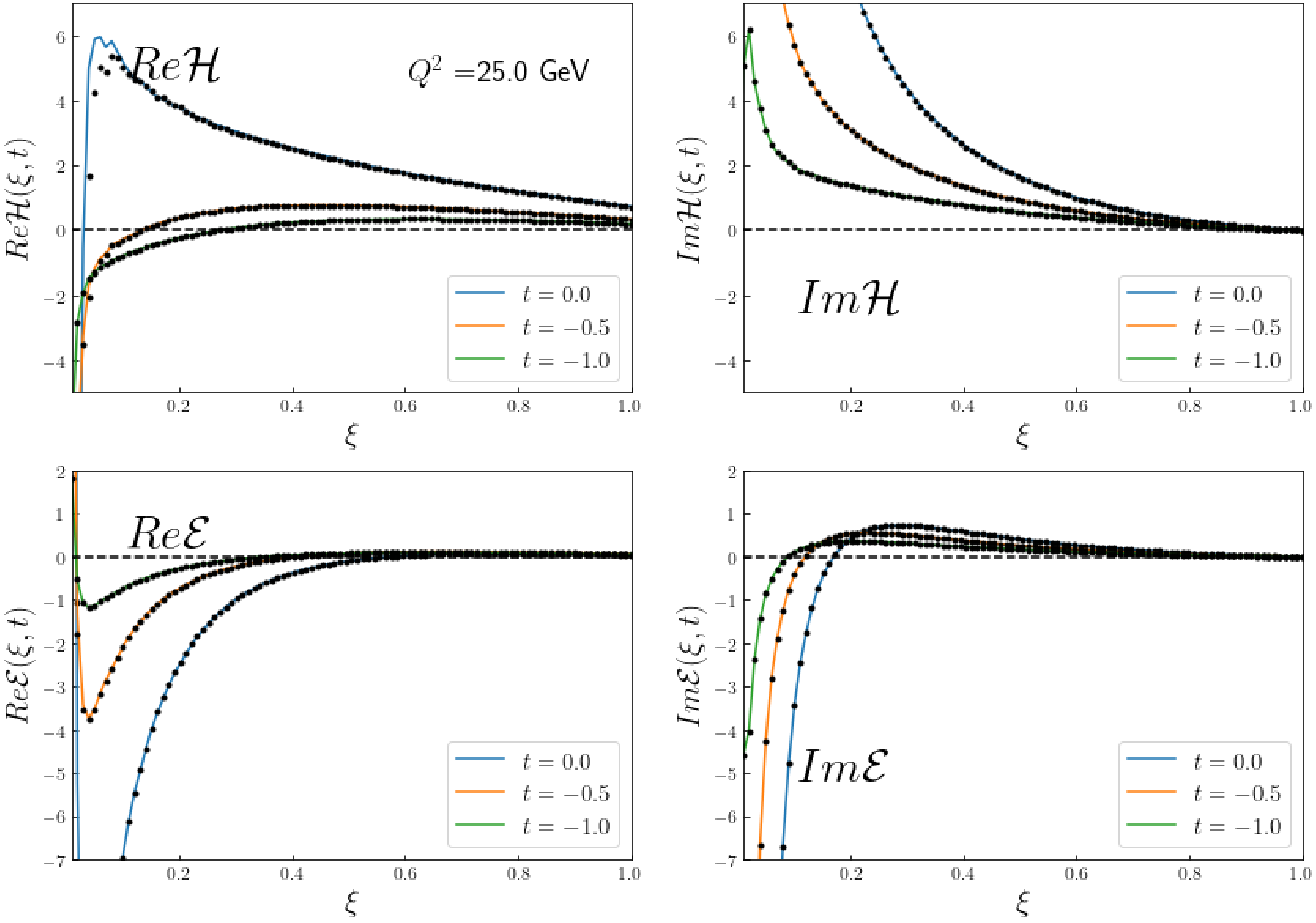


- Nt = 3
- Nx = 199
- Nxi = 100

Linear layer that generates 3.712.875 pixels

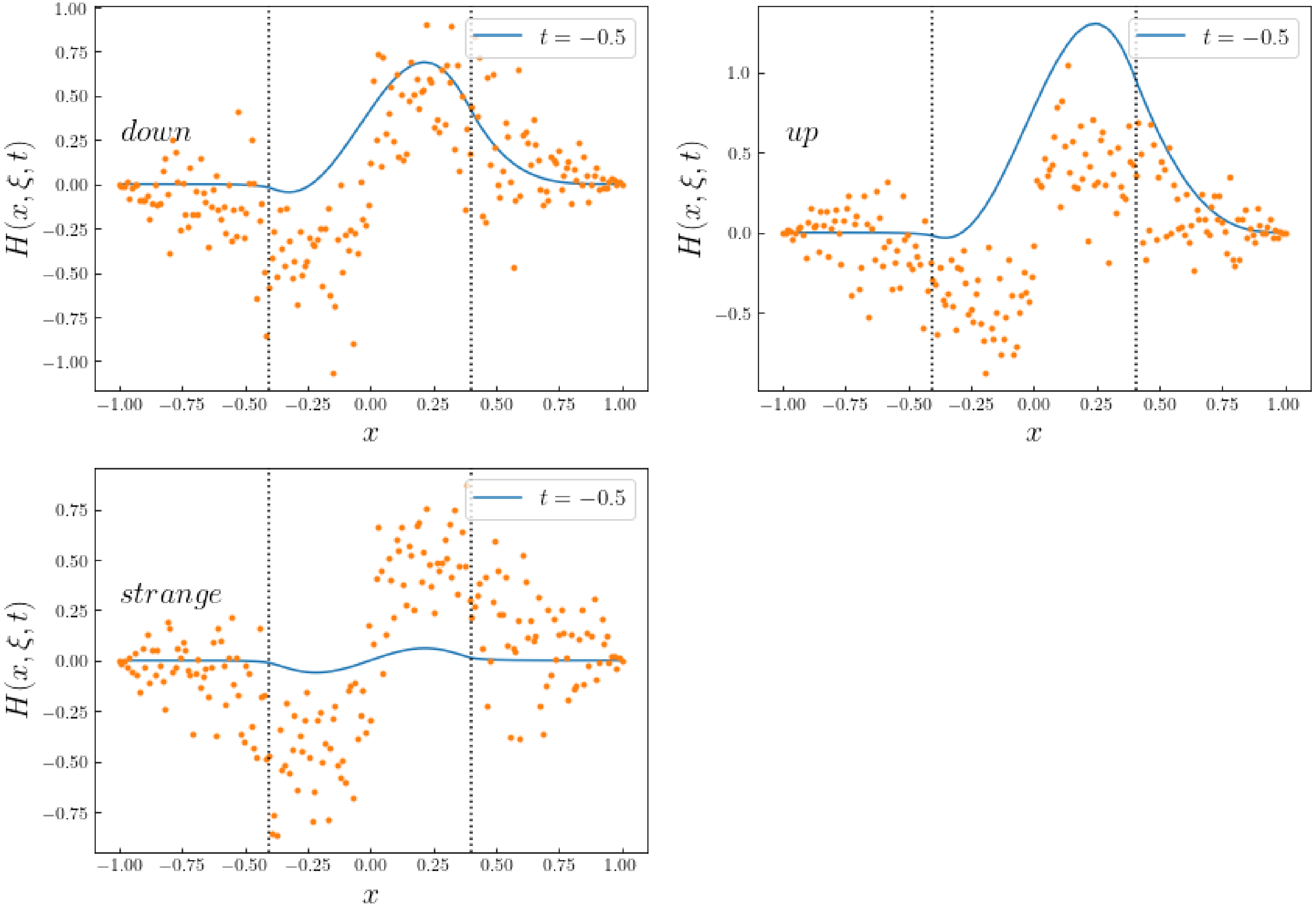
We train at the CFFs level, considering them as observables

Extracting GPDs with NN

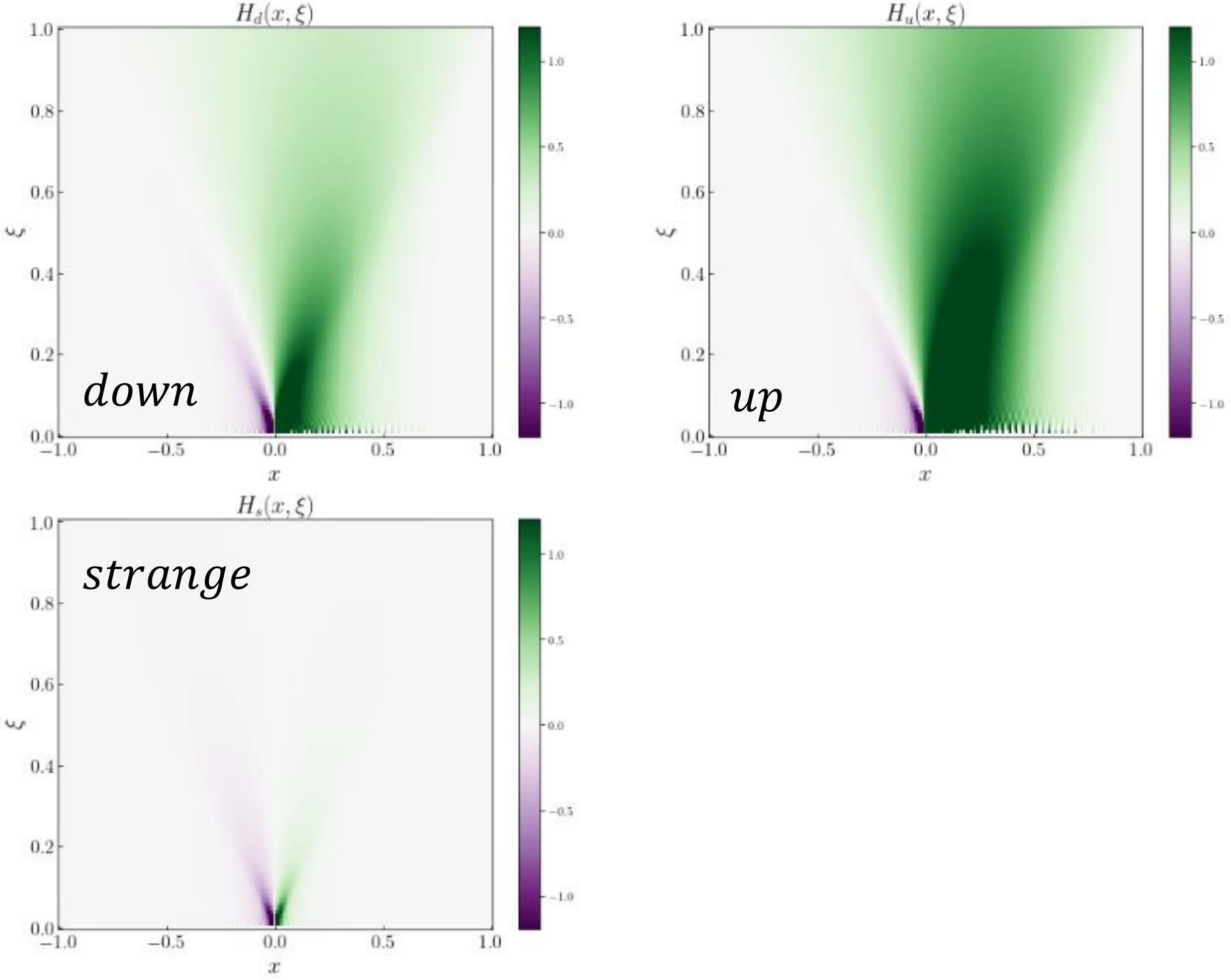


Solid lines: true
Black point: NN

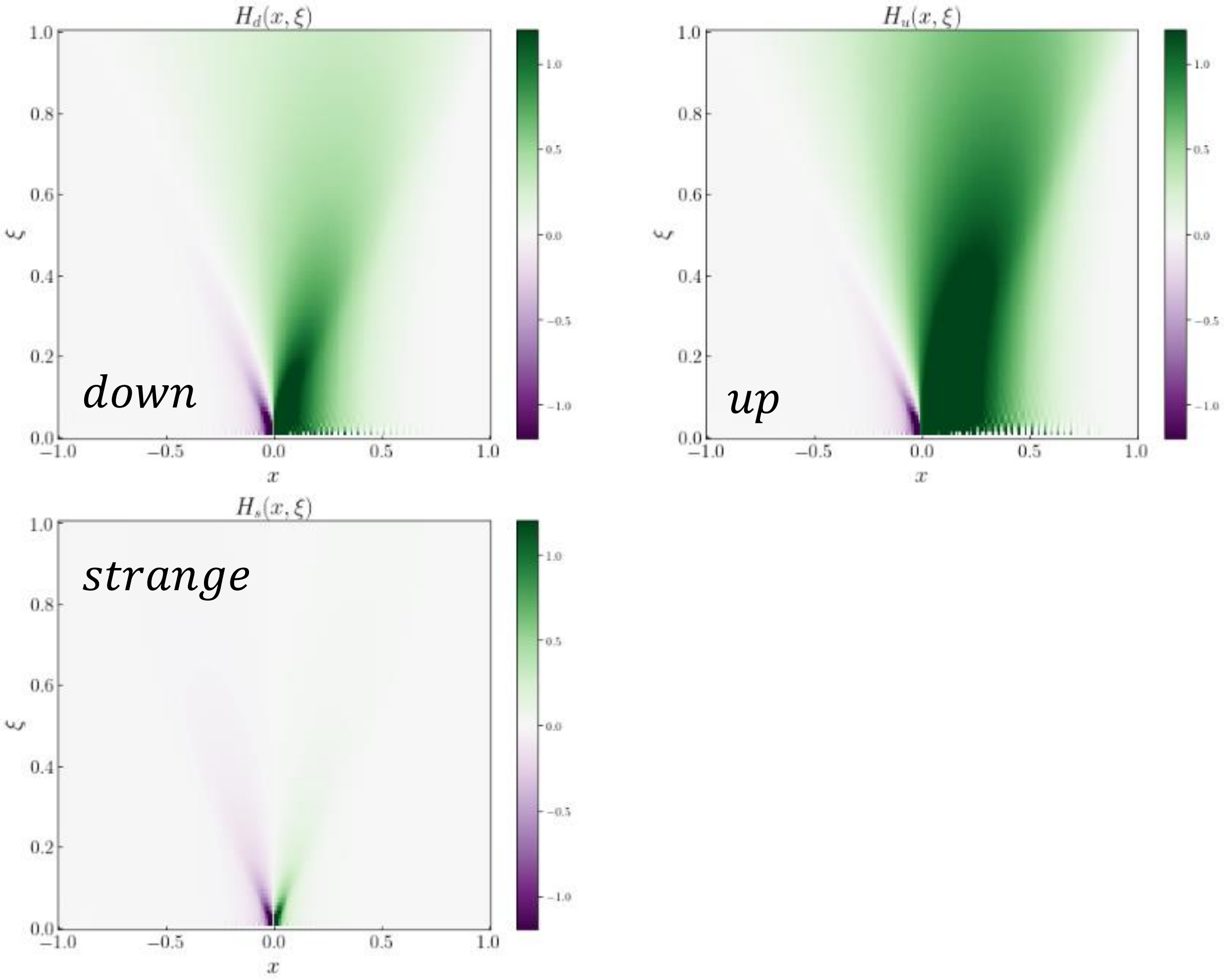
Extracting GPDs with NN

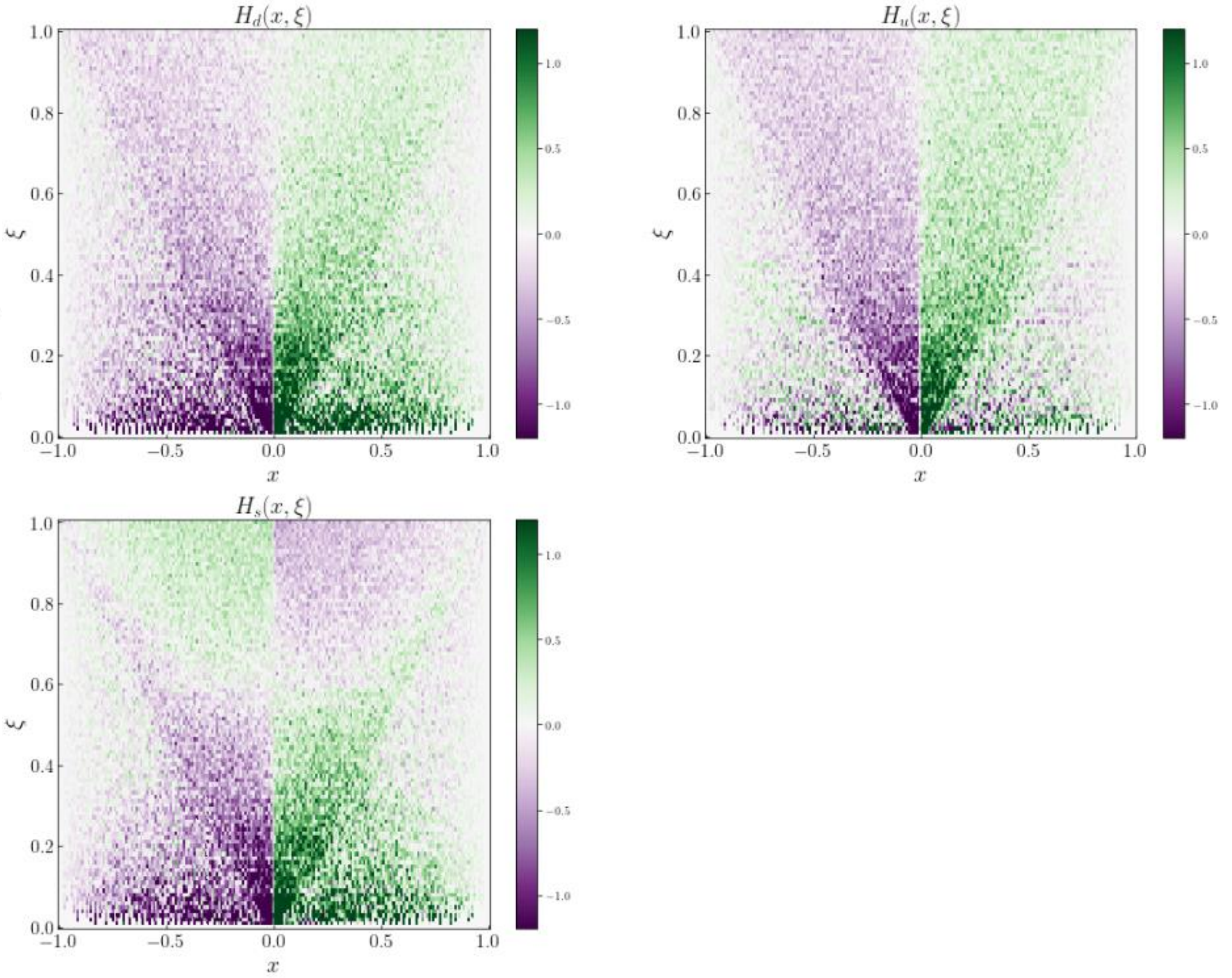


Extracting GPDs with NN

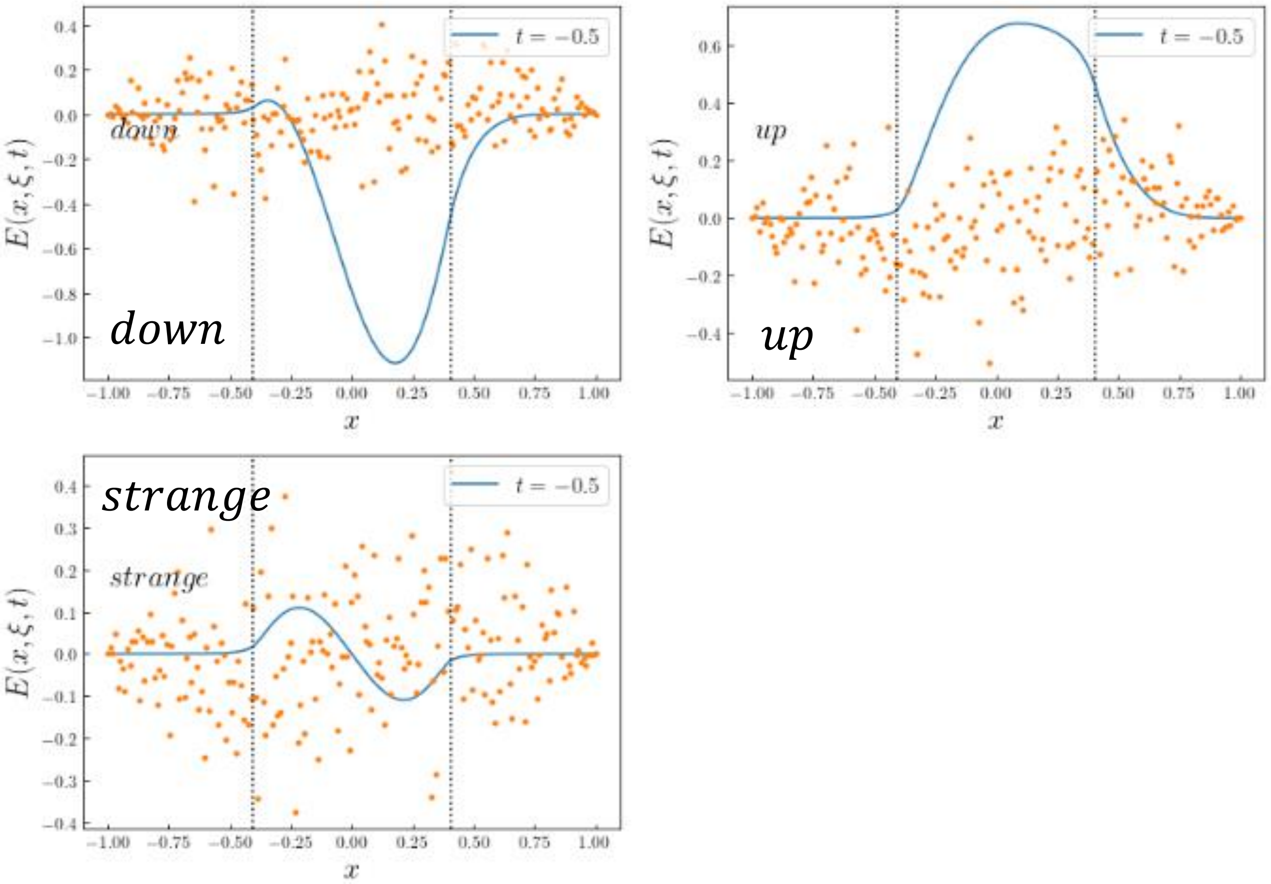


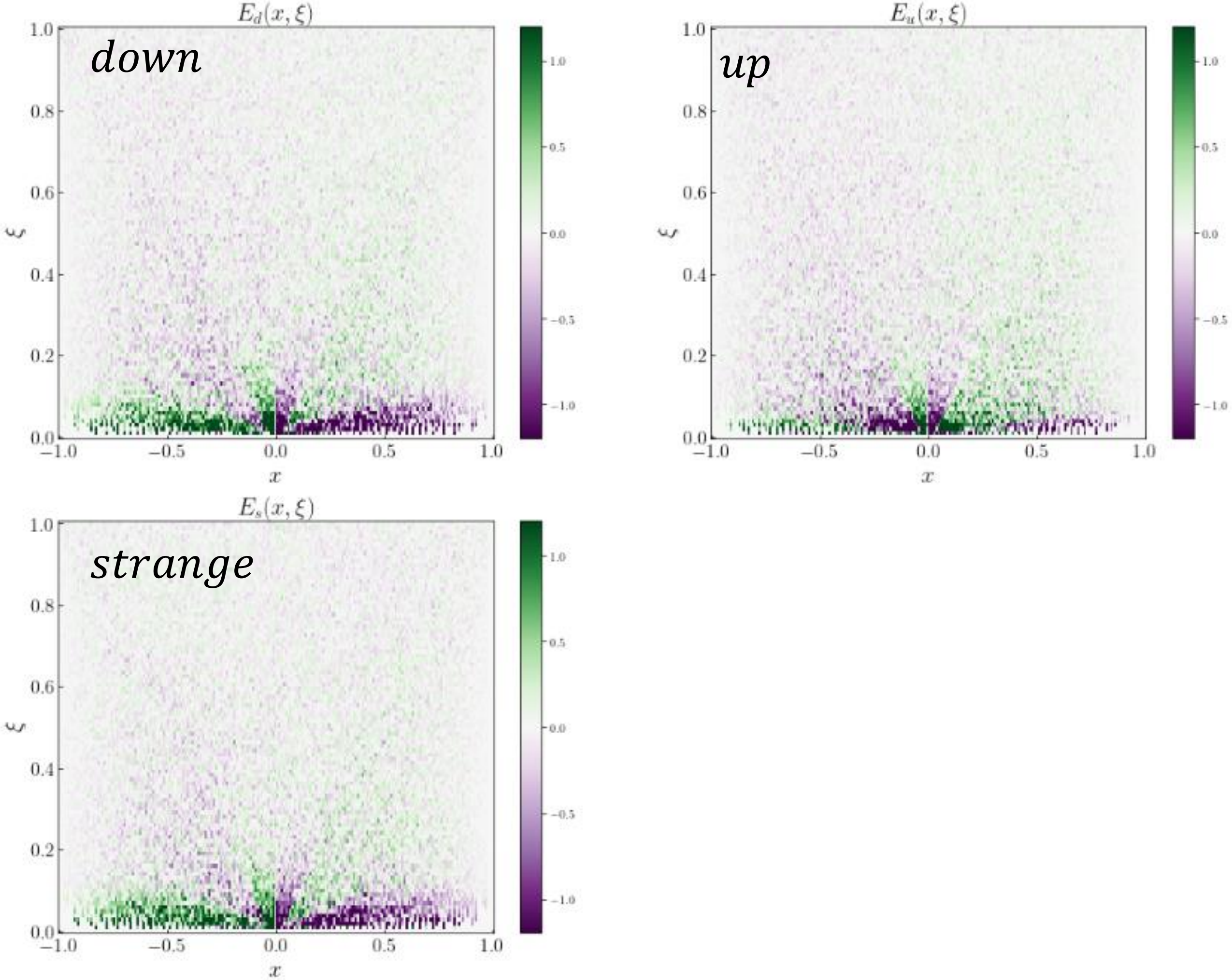
Extracting GPDs with NN





Extracting GPDs with NN





Conclusions

Theory:

- What is a GPD?
- How do we construct a GPD?
- Where do we get them?
- Evolution equations
- How do we get the observable?

To do:Evolution equations for Long. Pol. GPDs and at NLO

To do: DVCS cross section and asymmetries