Superfluid Fraction of the Inner Crust of Neutron Stars

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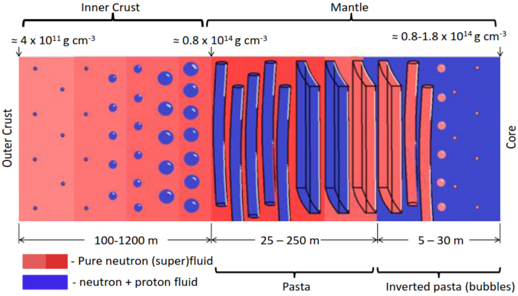
Outline

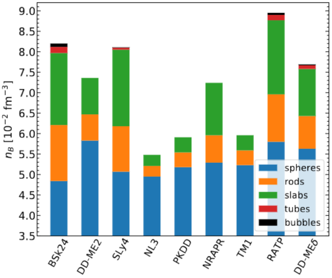
- Motivation
- Hartree-Fock-Bogoliubov in the slab and rod phases
- Conclusions

Inner crust of neutron stars: crystal and pasta phases

- Coulomb lattice of Clusters surrounded by a superfluid neutron gas
- in the deep layers of the inner crust, the clusters can take rod or slab shape (“pasta” phases)



[W.G. Newton]



[H. Dinh Thi et al., A&A 654, A114 (2021)]

Entrainment: band theory vs. hydrodynamics

Relative flow of neutrons vs. clusters:
Some neutrons are entrained by the clusters.
How many neutrons are superfluid?

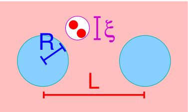
- Normal band theory
  [N. Chamel & P. Haensel, Liv. Rev. Relativity 11 (2008)]

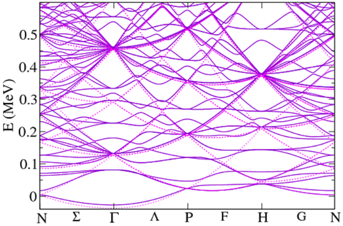
  valid for weak coupling ($\Delta \to 0$)

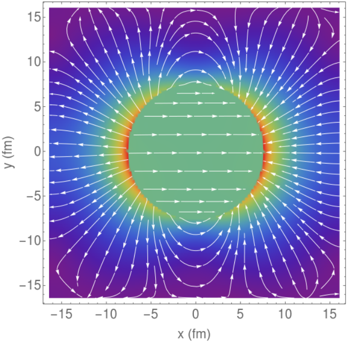
- Superfluid hydrodynamics
  [N. Martin & MU 2016, PRC 94 (2016)]

  valid for strong coupling

$$\xi \propto \frac{k_F}{\pi m \Delta} \ll L$$



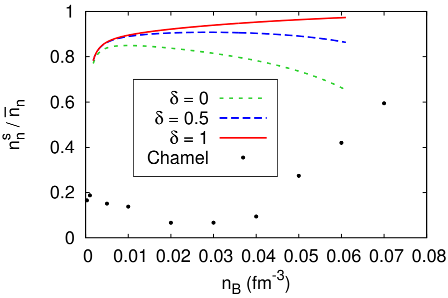




Superfluid fraction and Vela glitches

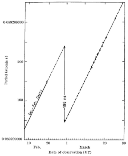
- Superfluid hydrodynamics predicts much weaker entrainment (= larger suberfluid fraction) than normal band theory

  $\xi \sim R \rightarrow$ parameter $\delta < 1$ to account for reduction of superfluidity inside clusters (nuclear moments of inertia suggest $\delta \sim 0.5$)

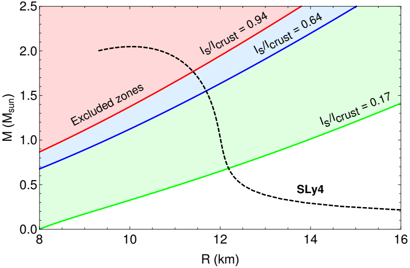


- Pulsar glitches

  [Radhakrishnan & Manchester Nature 222 (1969)]



- Superfluid part of the crust moment of inertia in band theory ~ 0.17 $\rightarrow$too small to explain Vela glitches



- In hydrodynamics, even with $\delta = 0$, there is enough superfluid density in the crust

Superfluid fraction and phonon velocities

larger superfluid fraction
(= weaker entrainment) implies:

- decreased effective cluster mass
- higher speeds v_i of phonons

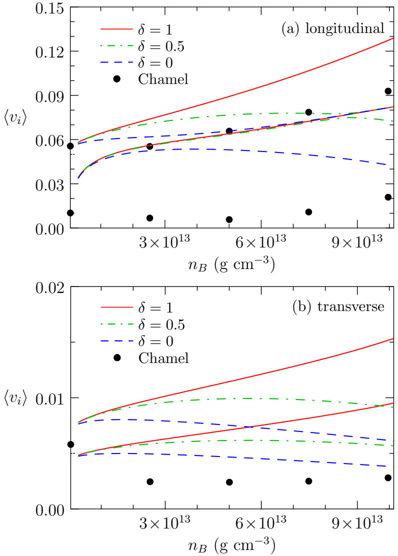
  coupling between lattice and superfluid phonons
  [C. Pethick, S. Reddy & N. Chamel, Prog. Theor. Phys. Suppl. 186 (2010); D. Durel & MU, PRC 97, 065805 (2018)]

- suppressed lattice specific heat:

$$c_v = \sum_{i=1}^{4} \frac{2\pi^2 T^3}{15\langle v_i\rangle^3}$$

angle average:

$$\langle v_i\rangle = \left(\int \frac{d\Omega}{4\pi}\,\frac{1}{v_i^3}\right)^{-1/3}$$



[D. Durel & MU, PRC 97, 065805 (2018)]

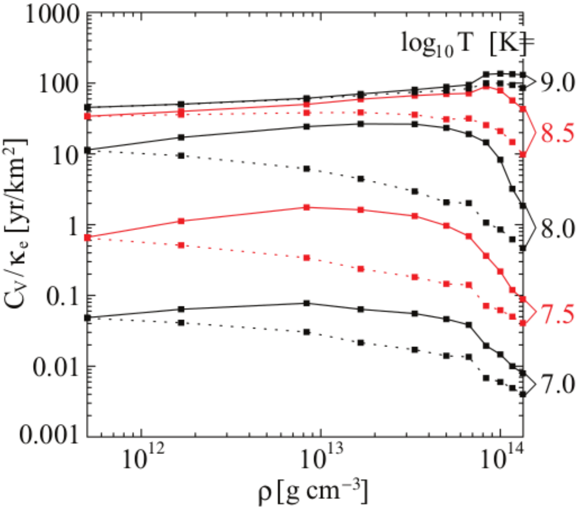
Relevance for cooling

Effect of entrainment on specific heat C_V and heat conductivity κ_e:

Figure taken from Chamel, Page & Reddy, PRC 87, 035803 (2013)

- solid lines: with strong entrainment (from normal band theory)
- dotted lines: without entrainment

(curves include phonon and electron specific heat, neutron specific heat suppressed by pairing)



Hartree-Fock-Bogoliubov (HFB) with periodicity

HFB can interpolate between normal band theory in weak coupling and superfluid hydrodynamics in strong coupling

$$\begin{pmatrix} h-\mu & -\Delta \\ -\Delta^\dagger & -\bar{h}+\mu \end{pmatrix} \begin{pmatrix} U_\alpha^\star \\ -V_\alpha \end{pmatrix} = E_\alpha \begin{pmatrix} U_\alpha^\star \\ -V_\alpha \end{pmatrix}$$

work in momentum space $\rightarrow$ matrices in discrete (band) indices, diagonal in the (continuous) Bloch and parallel (for rods, slabs) momenta

$$h_{kk'} = \left(\frac{1}{2m}\right)_{kk'} k\cdot k' + U_{kk'} - \hbar\, k\cdot v\, \delta_{kk'}$$

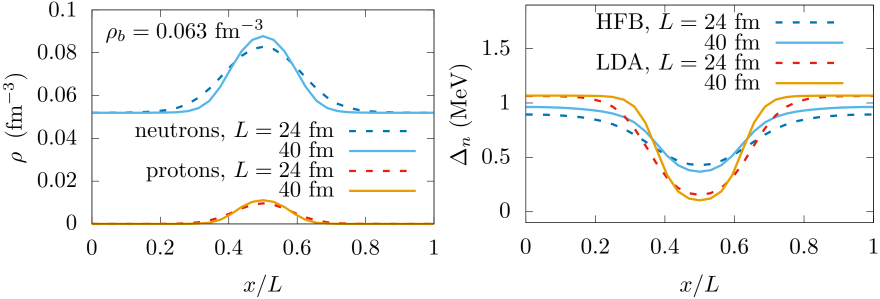
v = velocity of the slabs or rods in the rest frame of the superfluid

mean field: $U_{kk'} = -\sum_{pp'} V_{kpk'p'}\rho_{p'p}$ (Skyrme functional)

gap: $\Delta_{kk'} = -\sum_{pp'} V_{kk'p'p}\kappa_{p'p}$ (separable interaction $\sim V_{\text{low}-k}$)

"BCS approximation" (only diagonal elements of Δ are retained) not sufficient [Minami & Watanabe, Phys. Rev. Res. 4 (2022)]

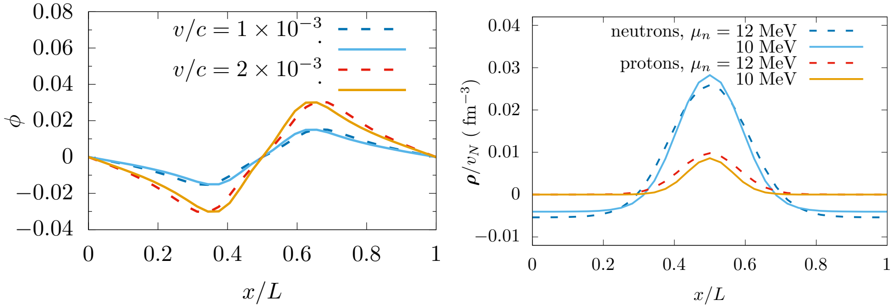
Slab phase (lasagna): density profile and gap



Almirante & Urban, Phys. Rev. C 109, 045805 (2024)

- gap inside the slab is smaller than in the neutron gas
- local-density approximation (LDA) overestimates this suppression

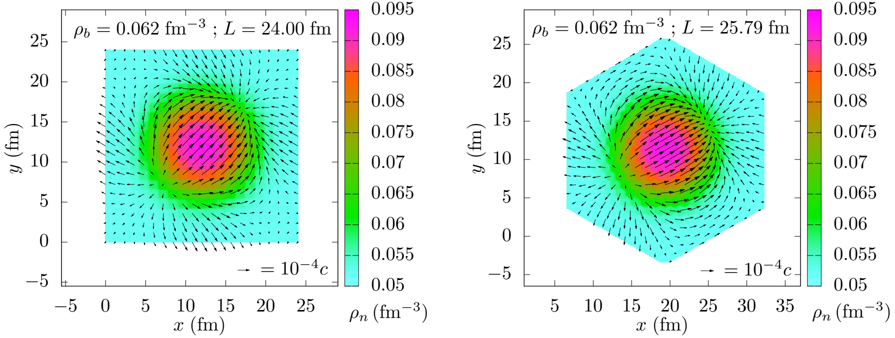
Slab phase (lasagna): phase of the gap and current



Almirante & Urban, Phys. Rev. C 109, 045805 (2024)

- phase $\phi \propto v \;\rightarrow\;$ linear regime
- proton current $= v\times$ proton density
- neutron current shifted down by a constant $\;\rightarrow\;$ superfluid part doesn't move

Rod phase (spaghetti): density and current in square and hexagonal lattices



Almirante & Urban, Phys. Rev. C 110, 065802 (2024)

Results for superfluid fraction

Spaghetti

μ_n (MeV)	L (fm)	ρ_b (fm^{-3})	$\rho_S/\bar{\rho}_n$ (HFB %)	$\rho_S/\bar{\rho}_n$ (Carter[a] %)
12	24	0.0619	94.5	75
	28	0.0617	95.7	
12.5	24	0.0670	95.4	82
	28	0.0668	96.7	

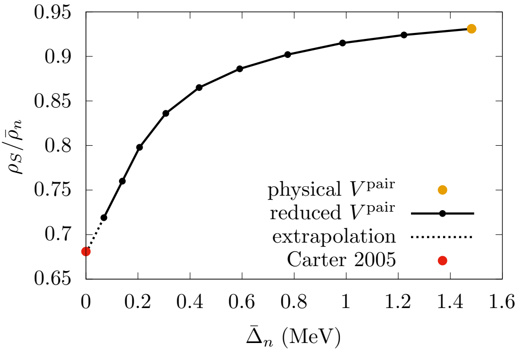
Lasagna

μ_n (MeV)	L (fm)	ρ_b (fm^{-3})	$\rho_S/\bar{\rho}_n$ (HFB %)	$\rho_S/\bar{\rho}_n$ (Carter[a] %)
13	20	0.0723	96.3	93
	24	0.0720	96.2	
13.5	20	0.0768	97.2	94
	24	0.0766	97.1	

[a] normal band theory: B. Carter, N. Chamel & P. Haensel, NPA 748 (2005).

Band structure effects vs. pairing gap

- Normal band theory should be valid in the weak-coupling limit ($\Delta \to 0$)
- Superfluid hydrodynamics only valid for $\xi \ll L \;\to\; \Delta \gg \dfrac{k_F}{\pi m L}$
- HFB should be valid all the way between these two limits!
- Varying artificially the strength of the pairing interaction:



Rod phase,
$\bar{\rho}_n = 0.059$ fm^{-3}
$L = 27.17$ fm,
for same conditions as in Carter, Chamel & Haensel, NPA 748 (2005)

Conclusions

- Superfluid fraction important for glitches and cooling
- Strong discrepancy between normal band theory (valid for weak coupling) and superfluid hydrodynamics (valid for strong coupling)
- HFB theory interpolates between these two limits
- HFB for slab and rod phases gives larger superfluid fraction (closer to hydrodynamic result) than normal band theory
- HFB for crystalline phase: work in progress